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# Aggregation of ranked votes considering different relative gaps between rank positions

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This paper considers ranked voting systems to determine the rank order of candidates who compete for a limited number of positions. We show that the preferential voting problems based on the data envelopment analysis (DEA) (Wang *et al*, 2007) can be solved using the extreme points of constraints on rank position importance incorporated in the formulation. This is basically due to the fact that the so-called inverse positive property of the constraints makes it possible to easily find their extreme points. Further, we emphasize that this finding is not restricted to Wang *et al*'s two linear models, but is also applicable to other DEA-based preferential voting problems, which include the constraints accounting for different relative gaps between rank positions.

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## 1. Introduction

Decision-making involves choosing the best alternative (course of action, project, candidate, option, or system) or constructing a total or partial order over a multitude of alternatives. In almost all decision-making problems, there are multiple bases (criteria, attributes, objectives, scenarios, or voters) on which to judge the alternatives. Ranking methods can be placed into two basic categories: cardinal and ordinal. Cardinal methods require a decision-maker to express his/her degree of preference for one alternative over another for each criterion. Ordinal methods, on the other hand, require that only the rank order of the alternatives be known for each criterion. Many ordinal ranking methods have been presented during the past two centuries, and they fall into one of several categories, including positional voting, mathematical programming, outranking techniques, and fuzzy ranking (Lansdowne, 1996).

The data envelopment analysis (DEA)-based preferential voting models pioneered by Cook and Kress (1990) have attracted much attention because of their innovative and practical approach. A key concern common to ordinal approaches, however, is how to discriminate between rank positions. With regard to this, we find that various forms of constraints on rank position importance are incorporated in DEA models for the purpose of obtaining a clear ranking of candidates (Hashimoto, 1997; Noguchi *et al*, 2002; Wang *et al*, 2007).

The purpose of this paper is to derive the extreme points of those constraints and then to solve the DEA-based linear programming (LP) problems proposed by Wang *et al* (2007), using the identified extreme points. This naturally extends to the DEA-based preferential voting problems, which include other types of constraints that account for the relative gaps between rank positions. Finally, we emphasize that the proposed method can be used to derive the extreme points of incomplete criteria weights frequently found in the multi-criteria decision-making (MCDM) field.

## 2. Ranking candidates with constraints on rank position importance

Cook and Kress (1990) developed a DEA-based model to aggregate votes into an overall index in a way that allows each candidate to be assessed in a fair manner. In their research framework, multiple voters select $m$ candidates from a set of $n$ ($n \geq m$) candidates by ranking them from first to the $m$th place. Briefly, the problem is to determine an ordering of all $n$ candidates by computing a total aggregated score $Z_i = \sum_{j=1}^{m} y_{ij} u_j$ for each candidate $i = 1, \ldots, n$ where $y_{ij}$ is the number of the $j$th place votes received by the $i$th candidate and $u_j$ the weights given to the $j$th place (i.e., rank position). The resulting DEA-based mathematical model appears as

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$$\begin{aligned}&\text{Maximize} \quad \sum_{j=1}^{m} y_{ij}u_j\\&\text{s.t.} \quad \sum_{j=1}^{m} y_{ij}u_j \le 1, \quad j=1,\ldots,n\end{aligned} \tag{1a}$$

$$\begin{aligned}&u_j - u_{j+1} \ge d(j,\epsilon), \quad j=1,\ldots,m-1\\&u_m \ge d(m,\epsilon)\end{aligned} \tag{1b}$$

where $d(j,\epsilon)$ is a positive function implying the minimum gap between successively ranked weights, the so-called discrimination intensity function.

As can be seen in the model (1), the discrimination intensity function $d(j,\epsilon)$ plays an important role in determining the final ranking of candidates and, in due course, dissimilar rankings are induced by different forms of the function. Hashimoto (1997) introduced a DEA/AR exclusion model where the constraints (2a) and (2a) are incorporated for the purpose of restricting the weight space:

$$u_j - u_{j+1} \ge \epsilon, \quad j=1,\ldots,m-1, \;\; u_m \ge \epsilon \tag{2a}$$

$$u_j - u_{j+1} \ge u_{j+1} - u_{j+2}, \quad j=1,\ldots,m-2. \tag{2b}$$

Here we note that if $\epsilon = 0$ in (2a), only (2b) affects the ranking of candidates since (2a) is rendered redundant. When we denote $q_j = u_j - u_{j+1}$ in (2b), a set of constraints (2b) simply becomes $q_j \ge q_{j+1} \ge 0$ $j=1,\ldots,m-2$, implying $q_j \ge 0$ and $q_{j+1} \ge 0$ as in (2a). Noguchi *et al* (2002) employed a strong ordering that emphasizes the complete categorization of ranking by imposing the following strict ordinal relations:

$$u_1 \ge 2u_2 \ge \cdots \ge mu_m, \quad u_m \ge \epsilon = \frac{2}{Nm(m+1)}$$

where $N$ is the number of voters. Wang *et al* (2007) proposed three new models for preference voting and aggregation; two are linear models, partially based on Noguchi *et al*'s model, and one is a nonlinear model.

In this paper, we present an easy method for solving the two linear models proposed by Wang *et al* (2007) and extend these models by incorporating other constraints on rank position importance. First, we denote a strict ordering *with* the sum-to-unity constraint as $S_{\mathrm{W}}$ in (3a) and a strict ordering *without* the sum-to-unity constraint as $S_{\mathrm{WO}}$ in (3b), respectively:

$$S_{\mathrm{W}} = \left\{ u_1 \ge 2u_2 \ge \cdots \ge mu_m \ge 0, \sum_{j=1}^{m} u_j = 1 \right\} \tag{3a}$$

$$S_{\mathrm{WO}} = \{1 \ge u_1 \ge 2u_2 \ge \cdots \ge mu_m \ge 0\}. \tag{3b}$$

Then, we attempt to find a set of extreme points $\mathbf{H}$ of $S_{\mathrm{WO}}$ and $\mathbf{K}$ of $S_{\mathrm{W}}$ in sequence. Rewriting $S_{\mathrm{WO}}$ in terms of matrix notation yields

$$\mathbf{Au} \ge 0, \quad \mathbf{u} \ge 0$$

where

$$\mathbf{A} = \begin{pmatrix} 1 & -2 & 0 & \cdots & 0\\ 0 & 2 & -3 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & m-1 & -m\\ 0 & 0 & 0 & \cdots & m \end{pmatrix} \text{ and } \mathbf{u}^{\mathrm{T}} = (u_1,\ldots,u_m).$$

The lemma below provides a theoretical background for finding the extreme points of two types of constraints on rank position importance, $S_{\mathrm{WO}}$ and $S_{\mathrm{W}}$.

**Lemma** *The nonsingular matrix $\mathbf{A}_{m\times m}$ is an M-matrix, a class of inverse-positive matrices, of which the inverse matrix $\mathbf{A}^{-1}$ yields the extreme directions of the set $S_{\mathrm{WO}}$ and their normalized vectors result in the extreme points of the set $S_{\mathrm{W}}$.*

**Proof** The inverse matrix of $\mathbf{A}$, denoted below in (4) as $\mathbf{H} = \mathbf{A}^{-1}$, is surely inverse-positive where all elements are nonnegative. A closed convex cone $C$, defined by $C = \{\mathbf{u} \in R^m : \mathbf{Au} \ge 0, \mathbf{u} \ge 0\}$, is a simplicial cone with exactly $m$ extremal rays since $\mathbf{A}$ is a nonsingular matrix of order $m$. Then, it follows that $\left(\mathbf{A}R_{+}^{m}\right)^{*} = \left(\mathbf{A}^{-1}\right)^{\mathrm{T}} R_{+}^{m}$, based on the dual of $C$, defined by $C^{*} = \{\mathbf{y} \in R^m : \mathbf{s} \in C \rightarrow \mathbf{s}\cdot\mathbf{y} \ge 0\}$ where $\mathbf{s}\cdot\mathbf{y}$ denotes the inner product (Berman and Plemmons, 1994). Therefore, a set of extreme vectors of $C$ is composed of $\mathbf{h}_i$, $i=1,\ldots,m$, where $\mathbf{h}_i$ is the $i$th column vector of $\mathbf{A}^{-1}$ as shown below:

$$\mathbf{H} = (\mathbf{h}_1,\mathbf{h}_2,\ldots,\mathbf{h}_m) = \begin{pmatrix} 1 & 1 & 1 & \cdots & 1\\ 0 & \frac{1}{2} & \frac{1}{2} & \cdots & \frac{1}{2}\\ 0 & 0 & \frac{1}{3} & \cdots & \frac{1}{3}\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & \cdots & \frac{1}{m} \end{pmatrix} \tag{4}$$

A set of extreme points $\mathbf{K}$ of $S_{\mathrm{W}}$ can be determined by dividing each column vector $\mathbf{h}_i$ by its column sum $\mathbf{h}_i^{\mathrm{T}}\cdot\mathbf{1}$ to satisfy the sum-to-unity constraint as shown below:

$$\begin{aligned}\mathbf{K} &= (\mathbf{k}_1,\mathbf{k}_2,\ldots,\mathbf{k}_m)\\ &= \begin{pmatrix} 1 & \frac{2}{3} & \frac{6}{11} & \cdots & 1\Big/\sum_{j=1}^{m}\frac{1}{j}\\ 0 & \frac{1}{3} & \frac{3}{11} & \cdots & 1\Big/\sum_{j=1}^{m}\frac{2}{j}\\ 0 & 0 & \frac{2}{11} & \cdots & 1\Big/\sum_{j=1}^{m}\frac{3}{j}\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & \cdots & 1\Big/\sum_{j=1}^{m}\frac{m}{j} \end{pmatrix}\end{aligned} \tag{5}$$

Other approaches to find the extreme points of $S_W$ have also been presented (Carrizosa *et al*, 1995; Mármol *et al*, 1998; Puerto *et al*, 2000; Mármol *et al*, 2002; Ahn, 2015).

In what follows, we present how to solve the two DEA-based LP models, so-called LP-1 and LP-2 to aggregate preferential votes and thus to rank candidates (Wang *et al*, 2007).

$$\begin{array}{lll} \text{LP-1:} & & \\ \text{Maximize} & \alpha & \\ \text{s.t.} & Z_i = \sum_{j=1}^{m} y_{ij}u_j \geq \alpha, & i = 1,\ldots,n \\ & u_1 \geq 2u_2 \geq \cdots \geq mu_m \geq 0 & \\ & \sum_{j=1}^{m} u_j = 1 & \end{array}$$

**Theorem 1** *The optimal solution to LP-1 is obtained by* $\alpha^* = \max_{1\leq j\leq m}[\underline{\alpha}_j]$, $\underline{\alpha}_j = \min_{1\leq i\leq n}[\boldsymbol{y}_i^{\mathrm{T}}\cdot\boldsymbol{k}_j]$, $\boldsymbol{y}_i^{\mathrm{T}} = (y_{i1},\ldots,y_{im})$, $\boldsymbol{k}_j \in \boldsymbol{K}$ *and* $\boldsymbol{u}^* = \boldsymbol{k}_j$ *for* $\{j|\alpha^* = \max_{1\leq j\leq m}[\underline{\alpha}_j]\}$.

**Proof** Let us denote the preference voting data by $\mathbf{Y} = [y_{ij}], i = 1,\ldots,n,\ j = 1,\ldots,m$. Then, LP-1 can be equivalently written by

$$\begin{array}{ll} \text{Maximize} & \alpha \\ \text{s.t.} & \mathbf{Y}\cdot\mathbf{u} \geq \boldsymbol{\alpha} \\ & \mathbf{u}\in\mathbf{K}, \quad \boldsymbol{\alpha}^{\mathrm{T}} = (\alpha,\ldots,\alpha) \end{array}$$

Note that $Z_i = \boldsymbol{y}_i^{\mathrm{T}}\cdot\mathbf{u}$ represents the $i$th candidate's aggregated score evaluated by $\mathbf{u}$. For a given extreme weighting vector $\mathbf{u} = \mathbf{k}_j \in \mathbf{K}$, we can always find a feasible $\underline{\alpha}_j > 0$ such that $\mathbf{Y}\cdot\mathbf{k}_j \geq \underline{\alpha}_j$, $\underline{\alpha}_j = \min_{1\leq i\leq n}[\alpha_{ij}]$, $\alpha_{ij} = \boldsymbol{y}_i^{\mathrm{T}}\cdot\mathbf{k}_j$. Therefore, the optimal objective value is achieved at $\max_{1\leq j\leq m}[\underline{\alpha}_j]$ when evaluated by every extreme weighting vector, thus yielding $\alpha^* = \max_{1\leq j\leq m}[\underline{\alpha}_j]$. Accordingly, the optimal weighting vector is given by $\mathbf{u}^* = \mathbf{k}_j$ for $\{j|\alpha^* = \max_j[\underline{\alpha}_j]\}$.

According to Theorem 1, the final rank order of candidates can be obtained by arranging the elements of $\mathbf{Y}\cdot\mathbf{u}^*$ in descending order. In addition to LP-1, Wang *et al* (2007) introduced a second DEA-based LP, so-called LP-2 as follows[1]:

$$\begin{array}{lll} \text{LP-2:} & & \\ \text{Maximize} & \alpha & \\ \text{s.t.} & \alpha \leq Z_i = \sum_{j=1}^{m} y_{ij}u_j \leq 1, & i = 1,\ldots,n \\ & 1 \geq u_1 \geq 2u_2 \geq \cdots \geq mu_m \geq 0 & \end{array}$$

**Theorem 2** *The optimal solution to LP-2 is obtained by* $\alpha^* = \min\left[\left(\frac{1}{\beta}\right)\boldsymbol{Y}\cdot\boldsymbol{h}_m\right]$ *and the optimal weighting vector* $\boldsymbol{u}^* = \left(\frac{1}{\beta}\right)\boldsymbol{h}_m$ *where* $\beta = \max[\boldsymbol{Y}\cdot\boldsymbol{h}_m]$.

[1]Even though $\{u_1 \leq 1\}$ was not designated in the original LP-2 model, it is legitimate to consider it such a way because of $Z_i = \sum_{j=1}^{m} y_{ij}u_j \leq 1$.

**Proof** It is obvious that $\mathbf{y}_i^{\mathrm{T}}\cdot\mathbf{h}_m > \mathbf{y}_i^{\mathrm{T}}\cdot\mathbf{h}_j$, $i = 1,\ldots,n$, $j = 1,\ldots,m-1$ when evaluating each candidate $Z_i = \mathbf{y}_i^{\mathrm{T}}\cdot\mathbf{u}$ in terms of the extreme point of $S_{WO}$ (thus we only have to focus on the extreme weighting vector $\mathbf{h}_m$). Furthermore, we obtain the following feasible set of constraints by dividing each $\mathbf{y}_i^{\mathrm{T}}\cdot\mathbf{h}_m$ by $\beta = \max[\mathbf{Y}\cdot\mathbf{h}_m] > 1$:

$$\begin{cases} \left(\frac{1}{\beta}\right)\mathbf{y}_k^{\mathrm{T}}\cdot\mathbf{h}_m = 1 & \text{for some } \{k|\max_k[\mathbf{y}_k^{\mathrm{T}}\cdot\mathbf{h}_m]\} \\ \left(\frac{1}{\beta}\right)\mathbf{y}_j^{\mathrm{T}}\cdot\mathbf{h}_m < 1, & j \neq k \end{cases}$$

Then, the optimal objective value is given by $\alpha^* = \min\left[\left(\frac{1}{\beta}\right)\mathbf{Y}\cdot\mathbf{h}_m\right] > \min\left[\left(\frac{1}{\beta}\right)\mathbf{Y}\cdot\mathbf{h}_j\right]$ for $j \neq m$, which is the maximum that $\alpha$ can attain and accordingly, the optimal weighting vector is $\mathbf{u}^* = \left(\frac{1}{\beta}\right)\mathbf{h}_m$.

### Example

We illustrate the proposed method with an example adopted from Cook and Kress (1990) where 20 voters are involved in selecting four among six candidates via ranking. The detailed preference votes are recorded in Table 1.

First, we attempt to determine the rank order of candidates by solving LP-1 with the preferential voting data in Table 1. The product of $\mathbf{Y}\cdot\mathbf{K}$ results in the aggregated preference scores evaluated by the extreme points of $S_W$ where $\mathbf{K}$ is given by

$$\mathbf{K} = \begin{pmatrix} 1 & \frac{2}{3} & \frac{6}{11} & \frac{12}{25} \\ 0 & \frac{1}{3} & \frac{3}{11} & \frac{6}{25} \\ 0 & 0 & \frac{2}{11} & \frac{4}{25} \\ 0 & 0 & 0 & \frac{3}{25} \end{pmatrix}.$$

The optimal objective value is determined by $\alpha^* = \max\left[0, \frac{4}{3}, \frac{18}{11}, \frac{48}{25}\right] = \frac{48}{25}$ and $\mathbf{u}^{*\mathrm{T}} = \mathbf{k}_4^{\mathrm{T}} = \left(\frac{12}{25}, \frac{6}{25}, \frac{4}{25}, \frac{3}{25}\right)$ according to Theorem 1. Thus, the resulting rank order of candidates is

$$D(4.4) \succ B(4.16) \succ C(4.08) \succ A(3.16) \succ F(2.28) \succ E(1.92).$$

**Table 1** Preference votes received by six candidates

| *Candidate* | *First place* | *Second place* | *Third place* | *Fourth place* |
|---|---|---|---|---|
| A | 3 | 3 | 4 | 3 |
| B | 4 | 5 | 5 | 2 |
| C | 6 | 2 | 3 | 2 |
| D | 6 | 2 | 2 | 6 |
| E | 0 | 4 | 3 | 4 |
| F | 1 | 4 | 3 | 3 |

To solve LP-2, on the other hand, we use a set of extreme points $\mathbf{H}$ as shown below instead of $\mathbf{K}$ in LP-1:

$$\mathbf{H} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{4} \end{pmatrix}.$$

Based on Theorem 2, we obtain $\alpha^* = \min\left[\left(\frac{1}{\beta}\right)\mathbf{Y}\cdot\mathbf{h}_m\right] = \frac{48}{110}$ where $\beta = \max[\mathbf{Y}\cdot\mathbf{h}_4] = \max\left[\frac{79}{12},\frac{104}{12},\frac{102}{12},\frac{110}{12},\frac{48}{12},\frac{57}{12}\right] = \frac{110}{12}$. Therefore, the optimal weighting vector becomes $\mathbf{u}^{*\mathrm{T}} = \left(\frac{1}{\beta}\right)\mathbf{h}_4 = \frac{12}{110}\left(1,\frac{1}{2},\frac{1}{3},\frac{1}{4}\right) = \left(\frac{12}{110},\frac{6}{110},\frac{4}{110},\frac{3}{110}\right)$ and the resulting rank order of candidates is

$$D(1) \succ B(0.9455) \succ C(0.9273) \succ A(0.7182) \succ F(0.5182) \succ E(0.4364).$$

Constraints other than (3a) and (3b) can be considered to account for different relative gaps between rank positions. Specifically, the constraints in (2a) and (2b) are good candidates for that purpose and the proposed method can be directly applied to solve the DEA-based preferential voting problems. Aguayo *et al* (2014) presents a different approach for finding these extreme points.

We illustrate the following sets of weights constraints, which are the hybrids of (2b) and (3a), and (2b) and (3b):

$$Q_{\mathrm{W}} = \left\{ 1 \geq u_1 - u_2 \geq 2(u_2 - u_3) \geq \cdots \geq (m-1)(u_{m-1} - u_m) \geq m u_m \geq 0, \sum_{j=1}^{m} u_j = 1 \right\}$$

$$Q_{\mathrm{WO}} = \{1 \geq u_1 - u_2 \geq 2(u_2 - u_3) \geq \cdots \geq (m-1)(u_{m-1} - u_m) \geq m u_m \geq 0\}.$$

To solve LP-1 (or LP-2) constrained by $Q_{\mathrm{W}}$ (or $Q_{\mathrm{WO}}$), we attempt to find the extreme points of the constraints. Consider $Q_{\mathrm{WO}}$ and denote $q_i = u_i - u_{i+1},\ i = 1,\ldots,m-1$ to give $Q'_{\mathrm{WO}} = \{q_1 \geq 2q_2 \geq \cdots \geq (m-1)q_{m-1} \geq m q_m \geq 0\}$. A set of extreme points of $Q'_{\mathrm{WO}}$ is given by $\mathbf{H}$ in (4) in terms of $q_i$. Thus, to derive the extreme points of $Q_{\mathrm{WO}}$, we only have to transform the extreme points in terms of $q_i$ into those in terms of $u_i$ by solving the following set of equations:

$$q_i = u_i - u_{i+1}, \quad i = 1,\ldots,m-1.$$

For example, for $\mathbf{q}_1^{\mathrm{T}} = (1,0,0,0)$, we solve a system of equations $u_1 - u_2 = 1,\ u_2 - u_3 = 0,\ u_3 - u_4 = 0,\ u_4 = 0$ to derive $\mathbf{u}_1^{\mathrm{T}} = (1,0,0,0)$. Similarly, $\mathbf{u}_2^{\mathrm{T}} = \left(1,\frac{1}{3},0,0\right)$ is obtained for $\mathbf{q}_2^{\mathrm{T}} = \left(1,\frac{1}{2},0,0\right)$ by solving $u_1 - u_2 = 1,\ u_2 - u_3 = \frac{1}{2},\ u_3 - u_4 = 0,\ u_4 = 0$. Finally, $\mathbf{u}_3^{\mathrm{T}} = \left(1,\frac{5}{11},\frac{2}{11},0\right)$ and $\mathbf{u}_4^{\mathrm{T}} = \left(1,\frac{13}{25},\frac{7}{25},\frac{3}{25}\right)$ correspond to $\mathbf{q}_3^{\mathrm{T}} = \left(1,\frac{1}{2},\frac{1}{3},0\right)$ and $\mathbf{q}_4^{\mathrm{T}} = \left(1,\frac{1}{2},\frac{1}{3},\frac{1}{4}\right)$, respectively. The extreme points of $Q_{\mathrm{W}}$ are obtained by normalizing each $\mathbf{u}_i$, $i = 1,\ldots,4$.

## 3. Concluding remarks

In the paper, we have shown that the two DEA-based LP models proposed by Wang *et al* (2007) can be solved by simple matrix computations when the extreme points of the constraints used in the models are determined. The constraints that account for the relative gaps between rank positions are revealed to be inverse-positive, which consequently makes it easier to find their extreme points. These findings can be effectively used to identify the extreme points of other types of incomplete criteria weights frequently found in the MCDM field.

Furthermore, other types of constraints could represent the decision-maker's quantifying metrics for rank position. To this end, we illustrated two sets of constraints and showed that the proposed method can also be applied to rank candidates when they are incorporated into the DEA-based preferential voting problems.

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