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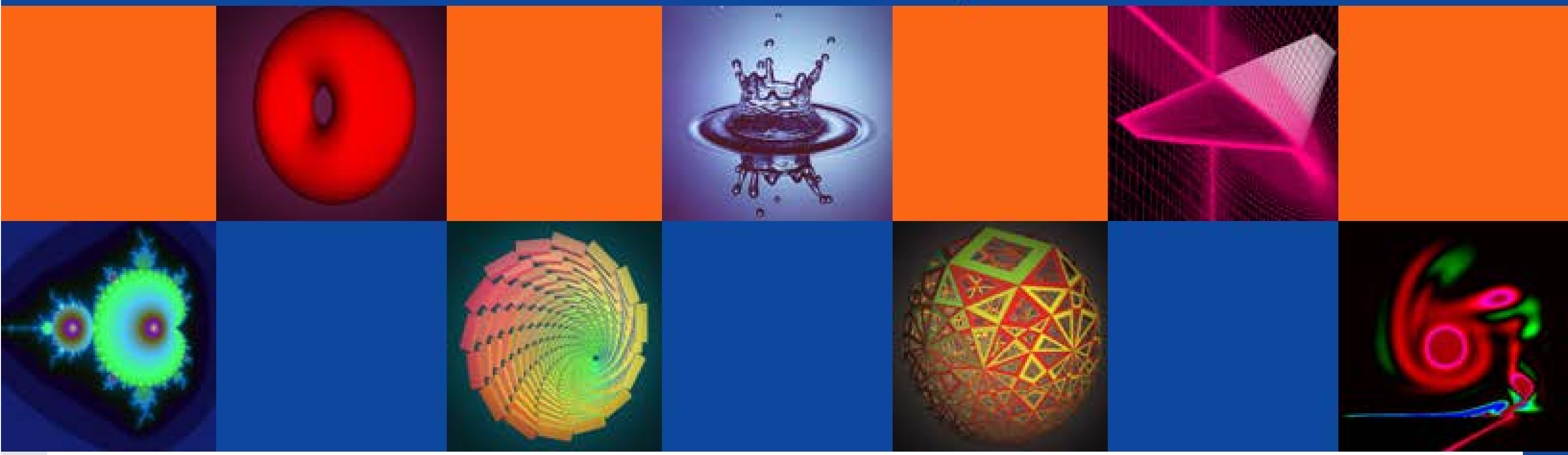

High-order mixed weighted compact and non-compact scheme for shock/discontinuity and small length scale interaction

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High-order mixed weighted compact and non-compact scheme for shock/discontinuity and small length scale interaction

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Abstract

It is critical for numerical scheme to obtain numerical results as accurate as possible with limited computational resources. Turbulent processes are very sensitive to numerical dissipation, which may dissipate the small length scales. On the other hand, dealing with shock waves, capturing and reproducing of the discontinuity may lead to non-physical oscillations for non-dissipative schemes. In the present work, a new high-order mixed weighted compact and non-compact (WCNC) difference scheme is proposed for accurate approximation of the derivatives in the governing Euler equations although the idea was given by Xie and Liu in 2007. The basic idea is to recover the non-dissipative high-order weighted compact scheme (WCS) where solution is smooth, while using a lower-ordered non-compact scheme for near-shock areas. Formulation and numerical tests are performed for the one dimensional case and results are compared with the well established weighted essentially non-oscillatory (WENO) scheme and WCS.

1. Introduction

It is desirable for a numerical scheme to attain high-order accuracy having limited computational resources. In the past two decades, many efforts have been made in development such high-order schemes, such as compact difference schemes (Lele, 1992; Visbal, et al., 2002; Yee, 1997), essentially non-oscillatory (ENO) schemes (Shu, et al., 1988; Shu, et al., 1989; Harten, et al., 1997) and their weighted variant (WENO) (Liu, et al., 1994; Jiang, et al., 1996; Shu, 2009), discontinuous Gelerkin (DG) methods (Cockburn, et al., 1989; Cockburn, et al., 1998; Cockburn, 2003; Bassi, et al., 1997), spectral element (SE) methods (Patera, 1984), spectral volume methods (SVM) (Wang, 2002; Liu, et al., 2006), spectral difference methods (SDM) (Liu, et al., 2006; Sun, et al., 2007), low dissipative high-order schemes (Yee, et al., 2000), group velocity control schemes (Ma, et al., 2001), and hybrid schemes (Adams, et al., 1996 ; Wang, et al., 2002).

Physical processes usually have various different length scales. In the case of flow transition and turbulence, for example, small length scales are of great interest and very sensitive to any artificial numerical dissipation. A high order central compact scheme (Lele, 1992; Visbal, et al., 2002) is non-dissipative with high-order and high-resolution, and thus appropriate for the solution of flow transition and turbulence cases. However, in many engineering applications such as shock-boundary layer

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interaction, shock-acoustic interaction, image process, flow in porous media, multiphase flow and detonation wave, there is a presence of both different length scales and shocks/discontinuities.

The shock can be considered as a discontinuity or a mathematical singularity (there is no classical unique solution and the derivatives are not bounded). In the near-shock region, continuity and differentiability of the governing Euler equations are lost and only the weak solution can be obtained. In fluid dynamics it is possible to have a shock solution when considering, for instance, the super-sonic regime of the Euler equations, which are hyperbolic. Hyperbolic systems can be solved taking advantage of the characteristic lines and Riemann invariants. The physics of the shock indicate that the derivative across the shock is not finite, and that the downstream region cannot influence the upstream one. In the framework of finite differences, it makes no sense to use, for instance, a high order compact scheme, which takes all grid point on both sides of a shock into account for the numerical approximation of the derivatives. Apparently, the upwind strategies are more suitable than compact schemes in dealing with shocks, and indeed history has shown a great success of upwind technologies applied to hyperbolic systems. Among upwind or bias upwind schemes that are capable to capture a shock sharply, there are Godunov (Godunov, 1959), Roe (Roe, 1981), MUSCL (van Leer, 1997), TVD (Harten, 1983), ENO (Harten, et al., 1997; Shu, et al., 1988; Shu, et al., 1989) and WENO (Liu, et al., 1994; Jiang, et al., 1996). All mentioned schemes above are based on upwind or bias upwind technology and are well suited for hyperbolic systems. On the other hand, upwinding strategies are not desirable for solving Navier-Stokes systems, which present a parabolic behavior, and are very sensitive to any numerical dissipation especially when tackling the problem of transitional and turbulent flow, where small length scales are important.

Efforts have been made in developing high-order numerical schemes with high resolution for small length scales, but at the same time capable of sharply capture the shock/discontinuity without generating visible numerical oscillations. A combination of WENO and standard central scheme is proposed by (Kim, et al., 2005; Costa, et al., 2007), and a combination of WENO and upwinding compact scheme (UCS) is proposed by (Ren, et al., 2003), but the mixing function is still complex and has a number of case related adjustable coefficients, which is not convenient to use.

A weighted compact scheme (WCS) is developed by (Jiang, et al., 2001). WCS is based on using WENO (Liu, et al., 1994) weighting method for evaluating candidates which use the standard compact scheme. The building block for each candidate is a Lagrange polynomial in WENO, but is Hermite in WCS, obtaining for the latter a higher order of accuracy with the same stencil width. In shock regions, the WCS controls the contributions of different candidate stencils to minimize the influence of candidates containing a shock/discontinuity. On the other hand, in regions with smooth solution, WCS recovers the standard compact scheme (Lele, 1992) to achieve high accuracy and resolution. Numerical tests reveal that the original WCS works well in some cases such as Burgers' equation, but is not suitable for solving the Euler equations. As mentioned, the usage of derivatives by compact schemes results in global dependency. WCS minimizes the influence of a shock-containing candidate stencil by assigning a smaller weight, but still uses all of the candidates, resulting in global dependency.

In order to overcome the drawback of WCS, local dependency has to be achieved in shock areas, while recovering global dependency in smooth regions. This fundamental idea leads naturally to the combination of compact and non-compact schemes, that is, to the mixed high order weighted compact and non-compact scheme (WCNC, (Xie, et al., 2007)). However, the mixing coefficient was not defined well and the scheme was still very dissipative.

The aim of this work is to develop a high order scheme for those cases where both discontinuities (e.g. shocks) and small length scales (e.g. sound wave, turbulence) are important. The proposed scheme captures the discontinuity (shocks) very sharply by upwinding dominant weights, and recovers the high order compact scheme for high accuracy and high resolution in the smooth area. A black-box type subroutine is developed, which can be used for any discrete data set to achieve high order accuracy for derivatives.

The present work is organized as follows. In section 2 the WCNC formulation is described, section 3 reports numerical results of test cases for the solution of the Euler equation in the one-dimensional case, and in section 4 conclusions are drawn along with the future work.

2. The numerical scheme

As mentioned, the WCS (Jiang, et al., 2001) uses the ideas of WENO (Liu, et al., 1994) for controlling the contributions of different candidate stencils with the aim of minimizing the influence of the candidate containing a discontinuity. In smooth solution regions, WCS recovers the standard compact scheme of high order and high resolution. For a function f, a one-parameter family compact scheme (Lele, 1992) may be written as

$$\alpha f'_{i-1} + f'_i + \alpha f'_{i+1} = \frac{1}{h}\left[-\frac{1}{12}(4\alpha-1)f_{i-2} - \frac{1}{3}(\alpha+2)f_{i-1} + \frac{1}{3}(\alpha+2)f_{i+1} + \frac{1}{12}(4\alpha-1)f_{i+2}\right] \quad \textbf{(2.1)}$$

where f_i and f_i' denote the function's known value and its unknown derivative, respectively, at the i-th point, and α is the parameter to be determined for highest order. If $\alpha = 1/3$, the scheme **(2.1)** recovers the standard sixth order compact scheme, corresponding to a symmetric and tri-diagonal system. If $\alpha = 0$ only the fourth order non-compact central scheme is recovered.

WCS has shown to solve well certain test cases, such as the convection equation and Burgers' equation, but unfortunately numerical solutions of the Euler equations are severely affected by numerical oscillations due to the use of derivatives by the compact scheme, which results in global dependency. WCNC is based on WCS (Jiang, et al., 2001) and is aimed to achieve local dependency in areas where discontinuities are present.

2.1. Derivation of the WCNC scheme

For a given point i, three candidate stencils containing the point are defined as $S_0=(x_{i-2},x_{i-1},x_i)$, $S_1=(x_{i-1},x_i,x_{i+1})$ and $S_2=(x_i,x_{i+1},x_{i+2})$, as shown in Figure 1.

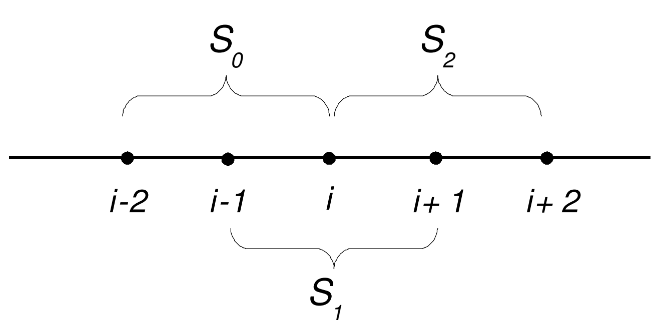


Figure 1: Candidate stencils for an interior point *i*.

The one-parameter α-family of the compact scheme (Lele, 1992) is used. A compact scheme, namely F_0, F_1 and F_2, is derived on each of the stencils S_0, S_1 and S_2, respectively, as

$$
\begin{aligned}
&S_0: F_0 \qquad && \alpha_0^- f'_{i-1} + f'_i = \frac{1}{h}[b_0^- f_{i-2} + a_0^- f_{i-1} + c_0 f_i] \\
&S_1: F_1 \qquad && \alpha_1^- f'_{i-1} + f'_i + \alpha_1^+ f'_{i+1} = \frac{1}{h}[a_1^- f_{i-1} + c_1 f_i + a_1^+ f_{i+1}] \\
&S_2: F_2 \qquad && f'_i + \alpha_2^+ f'_{i+1} = \frac{1}{h}[c_2 f_i + a_2^+ f_{i+1} + b_2^+ f_{i+2}]
\end{aligned}
\qquad \textbf{(2.2)}
$$

As for WCS, each compact scheme F_0, F_1 and F_2, will be multiplied with an associated linear weight C_0,C_1 and C_2, respectively, and the summation of the three contributions will lead to the final scheme F as

$$F = C_0 F_0 + C_1 F_1 + C_2 F_2 \qquad \textbf{(2.3)}$$

where the condition on the linear weight has to be satisfied

$$C_0 + C_1 + C_2 = 1 \qquad \textbf{(2.4)}$$

At this stage, the total number of 16 free parameters has to be determined:

$$
\begin{array}{llllll}
C_0, & \alpha_0^-, & b_0^-, & a_0^-, & c_0, & \\
C_1, & \alpha_1^-, & \alpha_1^+, & a_1^-, & c_1, & a_1^+, \\
C_2, & \alpha_2^+, & a_2^+, & b_2^+, & c_2. &
\end{array}
$$

By matching the Taylor series coefficients for each lower order compact scheme F_0, F_1 and F_2, and solving for the right-hand side free parameters in **(2.2)**, the following conditions are obtained

$$
\begin{array}{lll}
b_0^- = \dfrac{1-\alpha_0^-}{2}, & a_0^- = -2, & c_0 = \dfrac{3+\alpha_0^-}{2}, \\
a_1^- = \dfrac{\alpha_1^+ - 3\alpha_1^- - 1}{2}, & c_1 = 2(\alpha_1^- - \alpha_1^+), & a_1^+ = -\dfrac{\alpha_1^- - 3\alpha_1^+ - 1}{2}, \\
a_2^+ = 2, & b_2^+ = -\dfrac{1-\alpha_2^+}{2}, & c_2 = -\dfrac{3+\alpha_2^+}{2}.
\end{array}
\qquad \textbf{(2.5)}
$$

In order to reassemble the standard sixth order compact scheme **(2.1)**, the following conditions have to be satisfied

$$C_0\alpha_0^- + C_1\alpha_1^- = \alpha, \qquad C_1\alpha_1^+ + C_2\alpha_2^+ = \alpha,$$

$$C_0 a_0^- + C_1 a_1^- = -\frac{1}{3}(\alpha+2), \qquad C_0 c_0 + C_1 c_1 + C_2 c_2 = 0, \qquad C_1 a_1^+ + C_2 a_2^+ = \frac{1}{3}(\alpha+2), \qquad \textbf{(2.6)}$$

$$C_0 b_0^- = -\frac{1}{12}(4\alpha-1), \qquad C_2 b_2^+ = \frac{1}{12}(4\alpha-1)$$

where α is treated as a parameter. The above system of equations, together with condition **(2.4)** for the linear weights, and the condition of symmetric scheme $\alpha_1^- = \alpha_1^+$, is non-linear, and is not closed for solving the 16 unknowns listed above. An artificial condition may be added in order to close the system. In this preliminary study, it is chosen

$$\alpha_1^+ = \frac{3\alpha}{4} \qquad \textbf{(2.7)}$$

but future study may lead to a different choice for closing the system of equations.
Solutions for the 16 unknowns of WCNC scheme are reported in Table 1.

Table 1: Coefficients for WCNC for condition (2.7) on each of the three stencils $S_k, k = 1, 2, 3$.

	C_k	α_k^-	α_k^+	b_k^-
S_0	$\frac{5\alpha-2}{6(3\alpha-2)}$	$\frac{6\alpha(2\alpha-1)}{5\alpha-2}$		$\frac{1}{2}-\frac{3\alpha(2\alpha-1)}{5\alpha-2}$
S_1	$\frac{4(\alpha-1)}{3(3\alpha-2)}$	$\frac{3\alpha}{4}$	$\frac{3\alpha}{4}$	
S_2	$\frac{5\alpha-2}{6(3\alpha-2)}$		$\frac{6\alpha(2\alpha-1)}{5\alpha-2}$	

	a_k^-	c_k	a_k^+	b_k^+
S_0	-2	$\frac{3}{2}+\frac{3\alpha(2\alpha-1)}{5\alpha-2}$		
S_1	$-\frac{3\alpha}{4}-\frac{1}{2}$	0	$\frac{3\alpha}{4}+\frac{1}{2}$	
S_2		$-\frac{3}{2}-\frac{3\alpha(2\alpha-1)}{5\alpha-2}$	2	$-\frac{1}{2}+\frac{3\alpha(2\alpha-1)}{5\alpha-2}$

All the coefficients in Table 1 vary smoothly and monotonically when the parameter α varies from 0 to 1/3. Substituting the found coefficients, **(2.2)** become

$$S_0\colon F_0 \quad \frac{6\alpha(2\alpha-1)}{5\alpha-2}f'_{i-1}+f'_i=\frac{1}{h}\left[\left(\frac{1}{2}-\frac{3\alpha(2\alpha-1)}{5\alpha-2}\right)f_{i-2}-2f_{i-1}+\left(\frac{3}{2}+\frac{3\alpha(2\alpha-1)}{5\alpha-2}\right)f_i\right]$$

$$S_1\colon F_1 \quad \frac{3\alpha}{4}f'_{i-1}+f'_i+\frac{3\alpha}{4}f'_{i+1}=\frac{1}{h}\left[-\left(\frac{3\alpha}{4}+\frac{1}{2}\right)f_{i-1}+\left(\frac{3\alpha}{4}+\frac{1}{2}\right)f_{i+1}\right] \qquad (\mathbf{2.8})$$

$$S_2\colon F_2 \quad f'_i+\frac{6\alpha(2\alpha-1)}{5\alpha-2}f_{i+1}=\frac{1}{h}\left[-\left(-\frac{3}{2}-\frac{3\alpha(2\alpha-1)}{5\alpha-2}\right)f_i+2f_{i+1}-\left(-\frac{1}{2}+\frac{3\alpha(2\alpha-1)}{5\alpha-2}\right)f_{i+2}\right]$$

For the compact schemes F_0 and F_2, corresponding to stencils S_0 and S_2 in **(2.8)**, the calculation of the derivative at the *i*-th grid point, namely f'_i, involves the function value at three grid points and the value of the function's derivative at one single grid point. Thus the schemes are one-sided, and at least second-order accurate (third-order accurate when $\alpha = 1/3$). For the compact scheme F_1, corresponding to the stencil S_1in **(2.8)**, the calculation of the punctual derivative f'_i involves the function value at two grid points and the function derivative at two grid points. Thus the scheme is centered, and at least second-order accurate (fourth-order when $\alpha = 1/3$). As mentioned, the new scheme is obtained by multiplying each equation F_0, F_1 and F_2 in **(2.8)** with the specific weight assigned, C_0, C_1 and C_2 in Table 1, respectively, as specified in **(2.3)**. The proposed scheme reproduces the standard sixth-order compact scheme **(2.1)** when $\alpha = 1/3$, and is fourth-order accurate for $0 \le \alpha < 1/3$.

As mentioned above, WCNC uses the WENO (Jiang, et al., 1996) idea for calculating the weights for the final scheme. Instead of using directly C_0, C_1 and C_2 of Table 1, the WENO weights ω_0, ω_1 and ω_2 are used, so **(2.3)** becomes

$$F = \omega_0 F_0 + \omega_1 F_1 + \omega_2 F_2 \qquad \mathbf{(2.9)}$$

where

$$\omega_k = \frac{\gamma_k}{\Sigma_{j=0}^{2}\gamma_j}, \qquad \gamma_k = \frac{C_k}{(\varepsilon + IS_k)^p} \qquad \mathbf{(2.10)}$$

where ε is a small parameter in order to avoid divisions per zero, C_k are the weights defined in Table 1, *p* is an important parameter for controlling the WENO weights, and IS_k are the smoothness measurements (Jiang, et al., 1996).

The final scheme obtained using **(2.9)** is

$$\left[\omega_0\frac{6\alpha(2\alpha-1)}{5\alpha-2}+\omega_1\frac{3\alpha}{4}\right]f'_{i-1}+f'_i+\left[\omega_1\frac{3\alpha}{4}+\omega_2\frac{6\alpha(2\alpha-1)}{5\alpha-2}\right]f'_{i+1}=$$

$$\frac{1}{h}\left\{\omega_0\left[\frac{1}{2}-\frac{3\alpha(2\alpha-1)}{5\alpha-2}\right]f_{i-2}-\left[2\omega_0+\omega_1\left(\frac{3\alpha}{4}+\frac{1}{2}\right)\right]f_{i-1}+(\omega_0-\omega_2)\left[\frac{3}{2}+\frac{3\alpha(2\alpha-1)}{5\alpha-2}\right]f_i+\right.$$

$$\left.\left[2\omega_2+\omega_1\left(\frac{3\alpha}{4}+\frac{1}{2}\right)\right]f_{i+1}+\omega_2\left[-\frac{1}{2}+\frac{3\alpha(2\alpha-1)}{5\alpha-2}\right]f_{i+2}\right\} \qquad (\mathbf{2.11})$$

2.2 Determination of the parameter α

The only parameter left undetermined so far in the final expression of WCNC scheme **(2.11)** is α, which is crucial for the schemes' performance. The parameter α is determined in relation to the smoothness of the function. As mentioned, the basic idea is to recover the standard compact scheme **(2.1)** in smooth regions, taking advantage of global dependency of the compact scheme for achieving high order, and to reassemble a lower order scheme, non-compact, in regions where discontinuities are present. Thus the role of the parameter α is to assume the value 1/3 in smooth regions, and to range from zero to 1/3 according to the smoothness of the function.

In this preliminary study, the formulation adopted is the following

$$\alpha = (1-\mu)\,\bar{\alpha} + \mu\frac{1}{3} \qquad \textbf{(2.12)}$$

where μ is a fixed parameter ($\mu = 0.95$ in the present study), and

$$\bar{\alpha} = \frac{1}{3} - \frac{1}{2}\left[\left(\overline{IS_0} - \frac{1}{3}\right)^2 + \left(\overline{IS_1} - \frac{1}{3}\right)^2 + \left(\overline{IS_2} - \frac{1}{3}\right)^2\right]^{\frac{1}{2}} \qquad \textbf{(2.13)}$$

The normalized smoothness parameters $\overline{IS_k}$ are

$$\overline{IS_k} = \frac{IS_k + \varepsilon}{\Sigma_{j=0}^{2}(IS_j + \varepsilon)} \qquad \textbf{(2.14)}$$

In smooth regions, the three normalized smoothness parameters $\overline{IS_k}$ are nearly equal, namely $\overline{IS_0} \cong \overline{IS_1} \cong \overline{IS_2} \cong 1/3$, thus $\bar{\alpha} \cong 1/3$ and also $\alpha \cong 1/3$, and the sixt-order standard compact scheme is recovered achieving global dependency and the best resolution. In shock/discontinuity regions, the worst case leads to dramatically different weights for the stencils. For instance, if $\overline{IS_0} = 1$ and $\overline{IS_1} = \overline{IS_2} = 0$, the parameter $\bar{\alpha} = 0$, so that the local dependency is introduced in the scheme and non-oscillatory property is achieved.

Apparently, the WCNC scheme relies heavily on the function which controls the mixing between the non dissipative standard compact scheme for smooth regions, and the dissipative non-compact scheme for shock areas. The approach for finding the optimum α in relation to the smoothness of the function will be subject of future study.

3.1 Numerical results

Numerical simulations of test cases are carried out using the proposed WCNC scheme, and compared with WCS (Jiang, et al., 2001) and the well established WENO (Jiang, et al., 1996). The proposed WCNC scheme is used in solving the Euler 1-D equations

$$\begin{gathered}\frac{\partial \boldsymbol{U}}{\partial t} + \frac{\partial \boldsymbol{F}}{\partial x} = 0 \\ \boldsymbol{U} = (\rho, \rho u, E)^T \\ \boldsymbol{F} = \big(\rho, \rho u + p, u(E+p)\big)^T\end{gathered} \qquad \textbf{(3.1.1)}$$

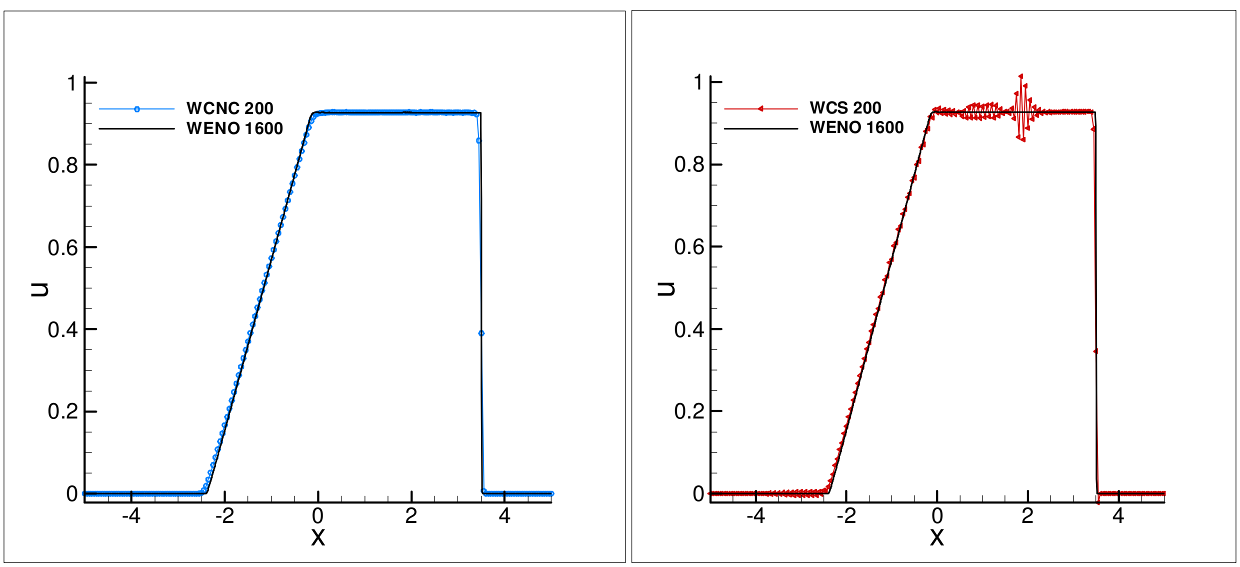
where ρ is the density, u is the velocity, E denotes energy, p is the pressure, $x \in \Omega = [-5,5]$ is the spatial coordinate and t is time. Steger-Warming (Steger, et al., 1981) flux vector splitting is used.

3.1.1 One-dimensional shock tube

In order to test the capability of shock capturing, Euler equations **(3.1.1.)** are solved with the initial conditions given as

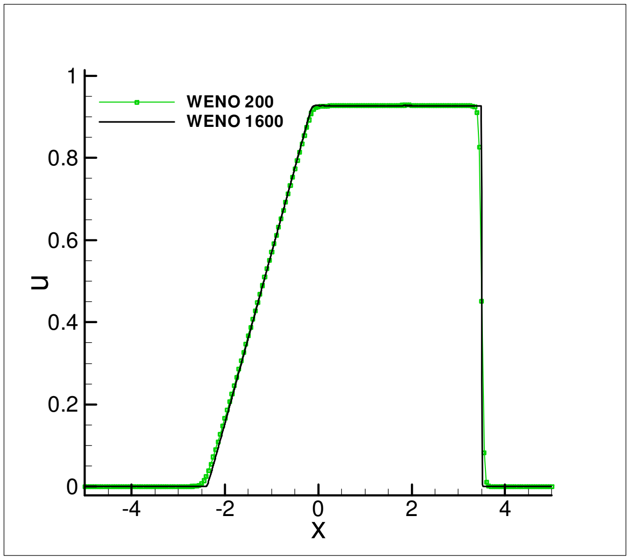
$$(\rho, u, p) = \begin{cases} (1, 0, 1) & t = 0, x < 0 \\ (0.125, 0, 0.1) & t = 0, x \geq 0 \end{cases} \qquad \textbf{(3.1.2)}$$

Figures 2 shows the plot of the solved velocity u, at time $t = 2$. Figures 2(a), (b) and (c) report the solution on the whole domain for WCNC, WCS and WENO schemes, respectively. The exact solution is regarded as the one obtained by the fifth-order WENO scheme using a mesh of 1600 points, labeled as WENO 1600. All the other calculations are made on a coarser mesh of 200 points. The solutions using WCNC (labeled as WCNC 200) and WENO scheme (labeled as WENO 200) are free from visible oscillations, which on the contrary, are present for WCS (labeled as WCS 200). Figure 2(d) report an enlargement of the downstream shock area, comparing the three different schemes. Using WCNC scheme, the discontinuity is captured more sharply and is less smeared compared to the fifth-order WENO, and the solution does not present unphysical oscillations, which affect the WCS.

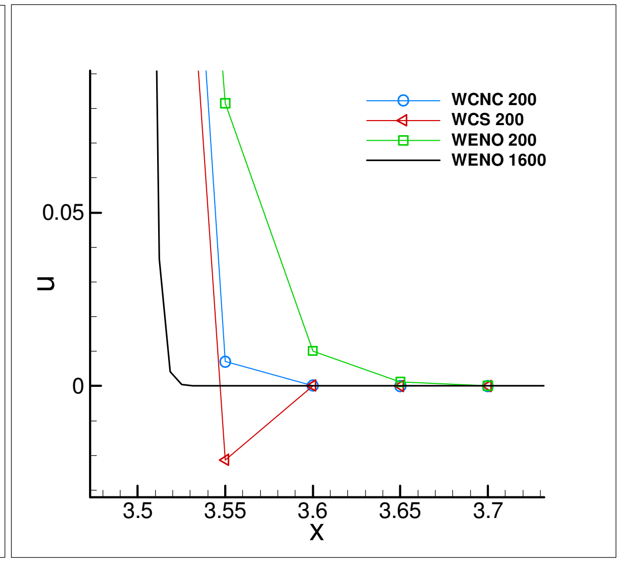


(a) **(b)**

Figure 2: Velocity *u* for the shock tube problem (3.1.1, 3.1.2) using (a) WCNC scheme, (b) WCS and (c) WENO scheme. (d) the detail enlargement of the downstream shock area is reported in order to compare the three different schemes.



(c)



(d)

3.1.2 One-dimensional shock/entropy wave interaction

In order to test both shock capturing capability and resolution, the proposed WCNC scheme is used to solve the shock-entropy interaction problem using 1-D Euler equations **(3.1.1.)**, with initial conditions given as

$$(\rho, u, p) = \begin{cases} (3.857143, 2.629369, 10.33333) & t = 0, x < -4 \\ (1 + 0.2\sin(5x)\,, 0, 1) & t = 0, x \geq -4 \end{cases} \qquad \textbf{(3.1.3)}$$

Figure 3 shows the result for the solved density distribution ρ at time $t = 1.8$. Figures 3(a), (b) and (c) report the solution on the whole domain for WCNC, WCS and WENO schemes, respectively. The exact solution is regarded as the one obtained by the fifth-order WENO scheme using a mesh of 1600 points, labeled as WENO 1600. All the other calculations are made on a coarser mesh of 200 points. The WCNC scheme (labeled WCNC 200) shows higher resolution and sharper shock capturing compared to WENO (labeled WENO 200). WCS (labeled WCS 200) is capable of capturing the high-frequencies waves generated in the upstream area of the shock, due to the intrinsic non-dissipative nature of the scheme. Figure 3(d), (e) and (f) report detail enlargements of discontinuity areas in the upstream shock region, comparing the three different schemes. It can be observed that WCNC solution, compared to the fifth-order WENO solution, can capture the shock more sharply, and is free from numerical oscillations. In certain areas (see, for instance, Figure 2(d)), the WCS appears to be very close to the reference solution, but is affected by numerical oscillations (see, for instance, Figure 3(e) and (f)).

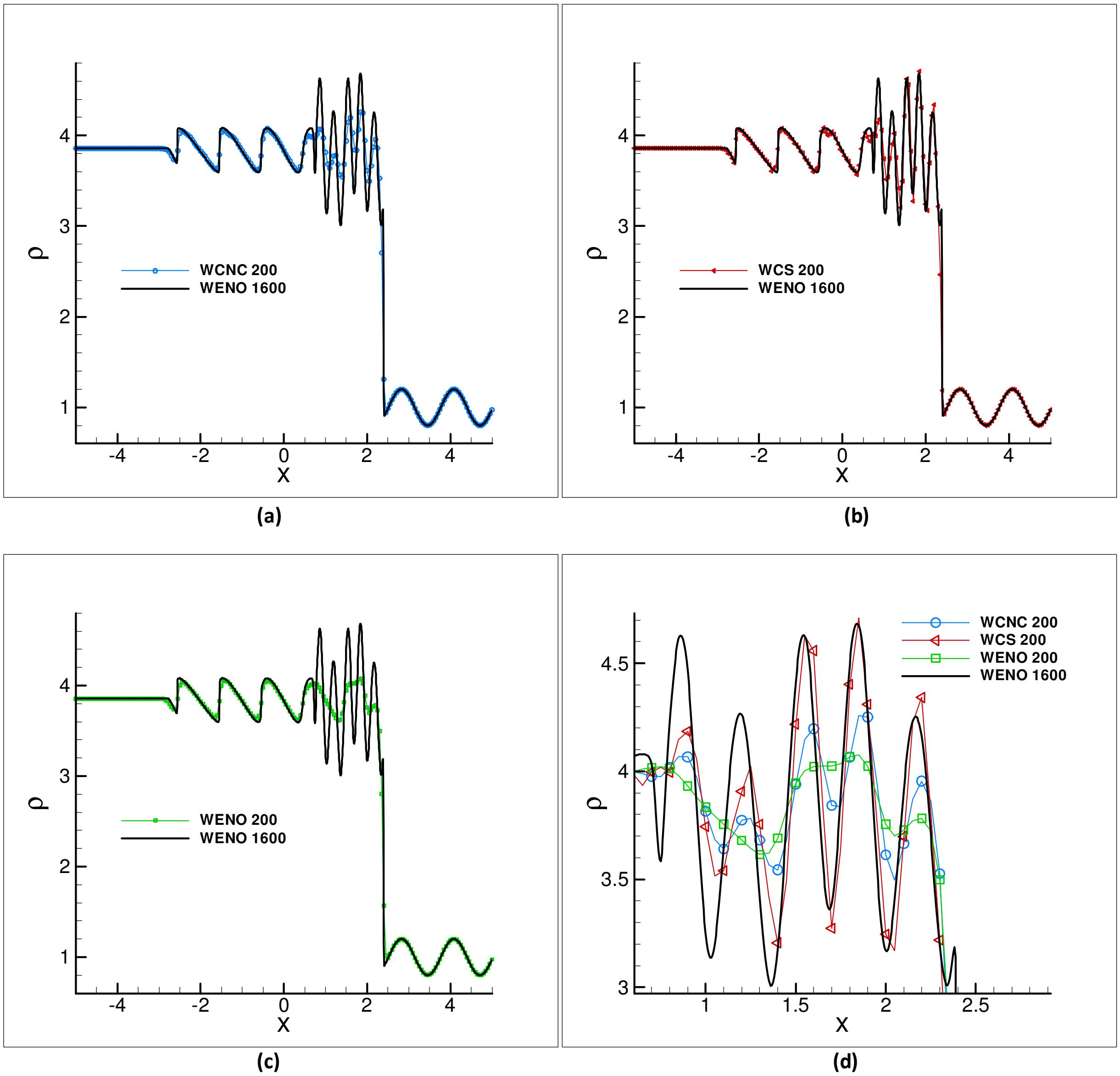
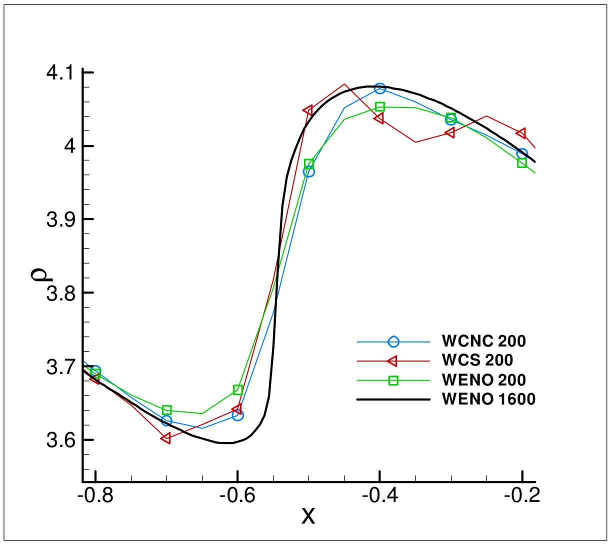
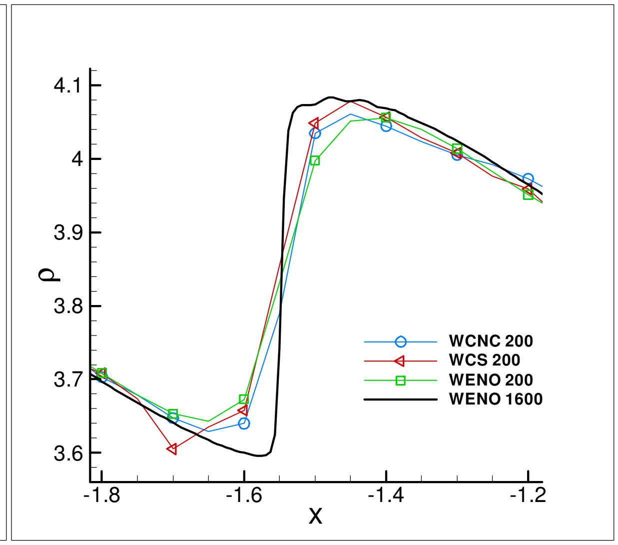


Figure 3: Density ρ for the shock tube problem (3.1.1, 3.1.3) using (a) WCNC scheme, (b) WCS and (c) WENO scheme. (d), (e) and (f): the detail enlargement of upstream shock areas are reported in order to compare the three different schemes.



(e)



(f)

4.1 Conclusions

The basic formulation for the high-order weighted compact and non-compact (WCNC) scheme is proposed for the calculation of derivatives with high-order accuracy of any function. The fundamental idea is to take advantage of the globally-dependent high-order compact scheme for smooth areas, and, through a weighting procedure, achieve local dependency of the non-compact scheme of lower order in shock regions. Numerical tests are carried out and compared to the well established WENO scheme and WCS. The WCNC scheme is shown to be capable of capturing the shock/discontinuity more sharply than WENO scheme, without generating the non-physical oscillations which were observed for WCS.

Future work will be dedicated to the development of an "optimum" mixing function for the control of the high-order WCNC, which apparently is of crucial importance for the proposed scheme. Simulations of multidimensional cases will be performed and reported in the AIAA Annual Scientific Conference.

References

Adams N. A. and Shariff K. A high-resolution hybrid compact-ENO scheme for shock-turbulence interaction problems [Journal] // Journal of Computational Physics. - 1996 . - Vol. 127. - pp. 27-51.

Bassi F. and Rebay S. A high-order accurate discontinuous finite element menthod for the numerical solution of the compressible Navier-Stokes equations [Journal] // Journal of Computational Physics. - 1997. - Vol. 131. - pp. 267-279.

Cockburn B. and Shu C. W. The local discontinuous Galerkin method for time-dependent convection-diffusion systems [Journal] // SIAM Journal on Numerical Analysis. - 1998. - Vol. 35. - pp. 2440-2463.

Cockburn B. Discontinuous Galerkin methods [Journal] // Journal of Applied Mathematics and Mechanics. - 2003. - Vol. 11. - pp. 731-754.

Cockburn B., Lin S. Y. and Shu C. W. TVB Runge-Kutta local projection discontinuous Galerkin finite element method for conservation laws III: one-dimensional systems [Journal] // Journal of Computational Physics. - 1989. - Vol. 84. - pp. 90-113.

Costa B. and Don W. S. High order hybrid central—WENO finite difference scheme for conservation laws [Journal] // Journal of Computational and Applied Mathematics. - 2007. - Vol. 204. - pp. 209-218.

Godunov S. K. A difference method for numerical calculation of discontinuous solutions of the equations of hydrodynamics [Journal] // Mat. Sb. (N.S.). - 1959. - 89 : Vol. 47. - pp. 271-306.

Harten A. [et al.] Uniformly high order accurate essentially non-oscillatory schemes, III [Journal] // Journal of Computational Physics. - 1997. - Vol. 131. - pp. 3-47 .

Harten A. High resolution schemes for hyperbolic conservation laws [Journal] // Journal of Computational Physics. - 1983. - Vol. 49. - pp. 357-393.

Jiang G. S. and Shu C. W. Efficient implementation of weighted ENO scheme [Journal] // Journal of Computational Physics. - 1996. - Vol. 126. - pp. 202-228.

Jiang L., Shan H. and Liu C. Weighted compact scheme for shock capturing [Journal] // International Journal of Computational Fluid Dynamics. - 2001. - Vol. 15. - pp. 147-155 .

Kim D. and Kwon J. A high-order accurate hybrid scheme using a central flux scheme and a WENO scheme for compressible flowfield analysis [Journal] // Journal of Computational Physics. - 2005. - Vol. 210. - pp. 554-583.

Lele S.K. Compact finite difference schemes with spectral-like resolution [Journal] // Journal of Computational Physics. - 1992. - Vol. 103. - pp. 16-42.

Liu D., Osher S. and Chan T. Weighted essentially non-oscillatory schemes [Journal] // Journal of Computational Physics. - 1994. - Vol. 115. - pp. 200-212.

Liu Y., Vinokur M. and Wang Z. J. Spectral (finite) volume method for conservation laws on unstructured grids. V: extension to three-dimensional systems [Journal] // Journal of Computational Physics. - 2006. - Vol. 212. - pp. 454-472.

Liu Y., Vinokur M. and Wang Z. J. Spectral difference method for unstructured grids I: basic formulation [Journal] // Journal of Computational Physics. - 2006. - Vol. 216. - pp. 780-801.

Ma Y. and Fu D. Forth order accurate compact scheme with group velocity control (GVC) [Journal] // Science in China. - 2001. - Vol. 44. - pp. 1197-1204.

Patera A. A spectral element method for fluid dynamics: Laminar flow in a channel expansion [Journal] // Journal of Computational Physics. - 1984. - Vol. 54. - pp. 468-488 .

Ren Y., Liu M. and Zhang H. A characteristic-wise hybrid compact-WENO scheme for solving hyperbolic conservation laws [Journal] // Journal of Computational Physics. - 2003. - Vol. 192. - pp. 365-386.

Roe P. L. Approximate Riemann solvers, parameter vectors, and difference schemes [Journal] // Royal Aircraft Establishment, Bedford, United Kingdom. - 1981. - pp. 357-372 .

Shu C. W. and Osher S. Efficient implementation of essentially non-oscillatory shock-capturing scheme [Journal] // Journal of Computational Physics. - 1988. - Vol. 77. - pp. 439-471.

Shu C. W. and Osher S. Efficient implementation of essentially non-oscillatory shock-capturing schemes II [Journal] // Journal of Computational Physics. - 1989. - Vol. 83. - pp. 32-78.

Shu C. W. High oder weighted essentially non-oscillatory schemes for convection dominated problems [Journal] // SIAM Review. - 2009. - Vol. 51. - pp. 82-126.

Steger J. and Warming R. Flux vector splitting of the inviscid gasdynamic equations with applications to finite-difference methods [Journal] // Journal of Computational Physics. - 1981. - Vol. 40. - pp. 263-293.

Sun Y., Wang Z. J. and Liu Y. High-order multi-domain spectral difference method for the Navier-Stokes equations on unstructured hexahedral grids [Journal] // Communications in Computational Physics. - 2007. - Vol. 2. - pp. 310-333.

van Leer B. Towards the ultimate conservative difference scheme V. A second-order sequel to Godunov's method [Journal] // Journal of Computational Physics. - 1997. - Vol. 135. - pp. 229 - 248.

Visbal M. and Gaitonde D. On the use of higher-order finite-difference schemes on curvilinear and deforming meshes [Journal] // Journal of Computational Physics. - 2002. - Vol. 181. - pp. 155-158.

Wang Z. and Huang G. P. An essentially non-oscillatory high-order Padé-type (ENO-Padé) scheme [Journal] // Journal of Computational Physics. - 2002. - Vol. 177. - pp. 37-58.

Wang Z. J. Spectral (finite) volume method for conservation laws on unstructured grids: basic formulation [Journal] // Journal of Computational Physics. - 2002. - Vol. 178. - pp. 210-251.

Xie P. and Liu C. Weighted compact and non-compact scheme for shock tube and shock-entropy interaction [Conference] // AIAA Paper AIAA-2007-509. - 2007.

Yee H. C. [et al.] Progress in the development of a class of efficient low dissipative high order shock-capturing methods [Report] / Proceeding of the Symposium in Computational Fluid Dynamics for the 21st Century. - Kyoto : [s.n.], 2000.

Yee H. C. Explicit and implicit multidimensional compact high-resolution shock-capturing methods: formulation [Journal] // Journal of Computational Physics. - 1997. - Vol. 131. - pp. 216-232.