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On Robust Stability and Set Invariance of Switched Linear Parameter Varying Systems

Maria M. Seron and José A. De Doná

Priority Research Centre for Complex Dynamic Systems and Control School of Electrical Engineering and Computer Science The University of Newcastle, Australia

Abstract

In this paper we address the problem of robust stability and set invariance of switched linear parameter varying (LPV) systems affected by bounded disturbances. A switched LPV system is a collection of single LPV subsystems with a switching rule that decides which subsystem is active at each time. This framework is useful for LPV systems with large parameter variation range for which it may be difficult to assess or guarantee stability over the whole parameter range. Our first contribution is to derive a result for a single LPV system that extends and generalises previous work for switched systems. Our second, and main, contribution is to apply this seed result in the derivation of dwell-time-type conditions on the switched-LPV switching rule, which ensure robust closed-loop stability and set invariance of the trajectories of the switched-LPV system across the whole parameter range. As an application of the results, we consider the problem of reference tracking for LPV systems using switched LPV state feedback and feedforward from an LPV reference system. Two examples are presented to illustrate the results: a two-mass-spring system with a varying spring characteristic, and the nonlinear model of a coupled-tank system which is embedded into a switched LPV system description.

1 Introduction

Linear parameter varying (LPV) systems are a class of linear systems whose state-space matrices are function of time-varying parameters that are not known in advance, but can be measured upon operation of the system. In recent years the LPV modelling approach has received major attention from the control research community as a useful technique to obtain tractable mathematical descriptions for nonlinear systems. Indeed, a nonlinear model can be embedded in an LPV description by redefining the nonlinearities in the model as varying parameters. This embedding technique is attractive since it allows the application of powerful linear-like design tools to a wide range of complex nonlinear models.

Numerous techniques have been developed to assess robust stability of LPV systems, especially for those systems that have a convex polytopic model description, that is, the system matrices can be expressed as a convex combination of a set of matrix vertices. Most of the available robust stability techniques lend themselves to the application of efficient computational methods such as those based on linear matrix inequalities, see, e.g., [6, 5]. In the present paper we employ an alternative robust stability technique, which is based on the work for switched linear systems developed in [9]. The key idea of this method is to search for a common transformation defining a time-invariant positive system that bounds the LPV system trajectories. Although not as general as the parameter dependent Lyapunov function approach [5], the 'common transformation' technique has the advantage of simultaneously providing an easily computable invariant set for the LPV system's trajectories. Invariant sets have numerous applications such as in model predictive control [4], fault diagnosis [17], fault tolerant control [14], etc.

For LPV systems with a large parameter variation range, however, it may be impossible, or very difficult, to assess or guarantee stability over the whole range by regarding it as a single parameter region. Moreover, the use of a single parameter region may be conservative if it is required to attain different performance goals in different parameter subregions, which may correspond to different operating conditions of the system. An approach proposed in [13] to avoid these problems is to divide the parameter set in several subregions and design different LPV controllers, each suitable for a specific parameter subregion, and switch among them to be able to ensure stability and achieve better performance. LPV systems of this type are called switched LPV systems. Two switching logics are considered in [13], hysteresis switching and switching with average dwell time. The control synthesis conditions for both switching logics are based on the multiple parameter-dependent Lyapunov function approach, and are formulated as generally non-convex matrix optimisation problems, which can be convexified under some simplifying assumptions. A simpler, yet more conservative, design based on a common parameter-dependent Lyapunov function is employed in [10], where the induced L_2 -norm performance of switched LPV systems is considered. In [15], the problem of trajectory tracking for an omnidirectional mobile robot application is approached by modelling the system as a switched LPV system. The use of this modelling framework in fault tolerant control has been reported in [16].

In this paper we address the problem of robust stability and set invariance of switched LPV systems affected by bounded disturbances. We first derive a result for a single LPV system that extends and generalises work originally presented in [9] for switched systems. We then use this seed result to derive dwell-time-type conditions on the switched-LPV switching sequences, which ensure robust closed-loop stability and set invariance of the trajectories of the switched-LPV system across the whole parameter range. As an application of the results, we consider the problem of reference tracking for LPV systems using switched LPV state feedback and feedforward from an LPV reference system. Two examples are presented to illustrate the results. The first example considers a two-mass-spring system where the varying parameter is the spring characteristic, assumed to be provided externally and independent of the system dynamics. In the second example, the nonlinear model of a coupled-tank system is embedded into an LPV system and overlapping subsets in a 2-dimensional parameter space are defined. This example also illustrates the applicability of the results in fault diagnosis.

2 Problem Formulation

Consider the switched LPV discrete-time model

$$x(t+1) = A_{\sigma}(\rho)x(t) + E_{\sigma}(\rho)w_{\sigma}(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state (at discrete-time $t \ge 0$). The "parameter" $\rho = \rho(t) \in \mathcal{P}$ (where $\mathcal{P} \subset \mathbb{R}^L$ is a bounded closed convex set) is an *a priori* unknown time-varying parameter whose measurement is available at each sample time. We consider the parameter set \mathcal{P} divided into a finite number of closed convex subsets $\{\mathcal{P}_i\}_{i\in \underline{M}}$, where $\underline{M} \triangleq \{1, 2, \ldots, M\}$ is the index set and $\mathcal{P} = \bigcup_{i=1}^M \mathcal{P}_i$. The subsets may have overlapping or disjoint interiors. We define the 'active' set $\mathcal{I}(t) \triangleq \{i \in \underline{M} : \rho(t) \in \mathcal{P}_i\}$ as the collection of indices of all subsets that contain the parameter $\rho(t)$ at each time $t \ge 0$. The *switching sequence* $\sigma = \sigma(t)$ then selects at each time an index within the active set, that is, it satisfies $\sigma(t) \in \mathcal{I}(t) \subset \underline{M}$. Which index in $\mathcal{I}(t)$, if there is more than one, is selected at each time is decided by the switching logic to be devised later.

For each fixed $i \in \underline{M}$, system (1) is an LPV system with matrices that can be expressed in the convex polytopic form

$$A_{i}(\rho) = \sum_{\ell=1}^{N_{i}} \xi_{i\ell}(\rho) A_{i\ell}, \quad E_{i}(\rho) = \sum_{\ell=1}^{N_{i}} \xi_{i\ell}(\rho) E_{i\ell},$$
(2)

for certain constant matrices $A_{i\ell} \in \mathbb{R}^{n \times n}$, $E_{i\ell} \in \mathbb{R}^{n \times r_i}$ and continuous functions $\xi_{i\ell} : \mathcal{P}_i \to \mathbb{R}$ that satisfy

$$\xi_{i\ell}(\rho) \ge 0 \quad \text{and} \quad \sum_{\ell=1}^{N_i} \xi_{i\ell}(\rho) = 1, \quad \text{for all } \rho \in \mathcal{P}_i.$$
 (3)

Also for each $i \in \underline{M}$, $w_i(t) \in \mathbb{R}^{r_i}$ is a bounded process disturbance satisfying the componentwise bound¹

$$|w_i(t)| \le \bar{w}_i,\tag{4}$$

where $\bar{w}_i \in \mathbb{R}^{r_i}$ is a nonnegative constant vector.

We consider the following problem. Suppose for each fixed $i \in \underline{M}$ subsystem (1) is robustly stable (in a sense to be defined below) and attractive invariant sets can be derived for its trajectories. Derive a switching logic that ensures robust stability and set invariance of the switched LPV system (1)–(4).

3 Robust Stability and Set Invariance for a Single LPV System

We assume that for each fixed $i \in \underline{M}$ system (1) is robustly stable according to the following theorem (based on the work in [9] for switched systems).

Theorem 3.1. Consider the convex polytopic uncertain system (1)–(4) for each fixed $i \in \underline{M}$, that is,

$$x(t+1) = A_i(\rho)x(t) + E_i(\rho)w_i(t).$$
(5)

Suppose an invertible transformation $V_i \in \mathbb{C}^{n \times n}$ exists such that the matrix

$$\Lambda_i \triangleq \max_{\ell \in \{1, \dots, N_i\}} |V_i^{-1} A_{i\ell} V_i| \quad is \ a \ Schur \ matrix,^2 \tag{6}$$

and define

$$b_i \triangleq (I - \Lambda_i)^{-1} \max_{\ell \in \{1, \dots, N_i\}} |V_i^{-1} E_{i\ell}| \overline{w}_i.$$
(7)

Then for any initial condition x(0) the trajectories of (5) are bounded and

(a) ultimately converge to the set

$$\mathcal{S}_i \triangleq \left\{ x \in \mathbb{R}^n : |V_i^{-1}x| \le b_i \right\},\tag{8}$$

which is an invariant set for the dynamics of (5).

(b) converge in finite time $t_i = t_i[x(0), S_i(\varepsilon_i)]$ defined as

$$t_i \triangleq \min\left\{\bar{t} \in \{0, 1, \dots\} : \Lambda_i^t | V_i^{-1} x(0) | \le \varepsilon_i \quad \forall t \ge \bar{t} \right\},\tag{9}$$

to the invariant set

$$\mathcal{S}_{i}(\varepsilon_{i}) \triangleq \left\{ x \in \mathbb{R}^{n} : |V_{i}^{-1}x| \leq b_{i} + \varepsilon_{i} \right\},$$
(10)

where $\varepsilon_i > 0$ is a vector with (arbitrarily small) positive elements satisfying $\Lambda_i \varepsilon_i \leq \varepsilon_i$.

¹Here, and in the remainder of the paper, the bars |.| denote elementwise magnitude (absolute value) and the inequalities and max operations are interpreted elementwise.

²A Schur matrix has all its eigenvalues with magnitude less than one.

Proof. Define

$$\zeta(t) \triangleq V_i^{-1} x(t). \tag{11}$$

Then, from (5) with initial condition x(0), we have

$$\zeta(t+1) = V_i^{-1} A_i(\rho) V_i \zeta(t) + V_i^{-1} E_i(\rho) w_i(t), \qquad \zeta(0) = V_i^{-1} x(0).$$
(12)

Taking magnitudes in (12) and using (2)-(4), (6) and (7), yields

$$\begin{aligned} |\zeta(t+1)| &\leq |V_{i}^{-1}A_{i}(\rho)V_{i}||\zeta(t)| + |V_{i}^{-1}E_{i}(\rho)||w_{i}(t)|. \\ &\leq \sum_{\ell=1}^{N_{i}} \xi_{i\ell}(\rho)|V_{i}^{-1}A_{i\ell}V_{i}||\zeta(t)| + \sum_{\ell=1}^{N_{i}} \xi_{i\ell}(\rho)|V_{i}^{-1}E_{i\ell}||w_{i}(t)| \\ &\leq \underbrace{\sum_{\ell=1}^{N_{i}} \xi_{i\ell}(\rho)}_{=1} \underbrace{\max_{\ell \in \{1,...,N_{i}\}} |V_{i}^{-1}A_{i\ell}V_{i}||\zeta(t)| + \underbrace{\sum_{\ell=1}^{N_{i}} \xi_{i\ell}(\rho)}_{=1} \underbrace{\max_{\ell \in \{1,...,N_{i}\}} |V_{i}^{-1}E_{i\ell}|\overline{w}_{i}}_{(I-\Lambda_{i})b_{i}} \\ &= \Lambda_{i} |\zeta(t)| + (I - \Lambda_{i})b_{i}. \end{aligned}$$
(13)

(a) Define a new variable y such that

$$y(t+1) = \Lambda_i y(t) + (I - \Lambda_i)b_i, \qquad y(0) \ge |\zeta(0)|.$$
 (14)

Since Λ_i is a Schur matrix by assumption then the trajectories of (14) asymptotically converge to b_i . Noticing, from (11) and (13)–(14), that

$$|V_i^{-1}x(t)| = |\zeta(t)| \le y(t), \quad \forall t \ge 0,$$
(15)

it then follows that the trajectories of (5) ultimately converge to the set S_i defined in (8). To see that this set is invariant, let $x(t) \in S_i$ for some $t \ge 0$. Hence, $|V_i^{-1}x(t)| = |\zeta(t)| \le b_i$ and using (13) yields

$$|V_i^{-1}x(t+1)| = |\zeta(t+1)| \le \Lambda_i |\zeta(t)| + (I - \Lambda_i)b_i \le \Lambda_i b_i + (I - \Lambda_i)b_i = b_i.$$

(b) Define the 'forced' and 'initial condition' responses of ζ , denoted by $\overline{\zeta}$ and $\overline{\zeta}$ respectively, satisfying

$$\bar{\zeta}(t+1) = V_i^{-1} A_i(\rho) V_i \,\bar{\zeta}(t) + V_i^{-1} E_i(\rho) w_i(t), \qquad \bar{\zeta}(0) = 0, \tag{16}$$

and

$$\tilde{\zeta}(t+1) = V_i^{-1} A_i(\rho) V_i \tilde{\zeta}(t), \qquad \tilde{\zeta}(0) = \zeta(0) = V_i^{-1} x(0).$$
 (17)

Clearly, $\zeta(t)$ satisfying (12) is such that

$$\zeta(t) = \bar{\zeta}(t) + \tilde{\zeta}(t), \qquad \forall t \ge 0.$$
(18)

For $\bar{\zeta}$ in (16), noticing that $V_i \bar{\zeta}(0) = 0 \in S_i$, and that S_i defined in (8) is invariant, we have

$$|\bar{\zeta}(t)| = |V_i^{-1}[V_i\bar{\zeta}(t)]| \le b_i, \qquad \forall t \ge 0.$$
(19)

For $\tilde{\zeta}$ in (17), proceeding as in (13) we obtain the bound

$$|\tilde{\zeta}(t+1)| \le \Lambda_i |\tilde{\zeta}(t)|,$$

and defining \tilde{y} such that $\tilde{y}(t+1) = \Lambda_i \tilde{y}(t), \tilde{y}(0) = |\tilde{\zeta}(0)| = |V_i^{-1}x(0)|$, we have

$$|\tilde{\zeta}(t)| \le \tilde{y}(t) = \Lambda_i^t \tilde{y}(0) = \Lambda_i^t |\tilde{\zeta}(0)| = \Lambda_i^t |V_i^{-1} x(0)|, \quad \forall t \ge 0.$$
(20)

Combining (18)-(20) and using (11) yields

$$|V_i^{-1}x(t)| = |\zeta(t)| \le b_i + \Lambda_i^t |V_i^{-1}x(0)|, \qquad \forall t \ge 0.$$
(21)

Since Λ_i is a Schur matrix, given $\varepsilon_i > 0$ and for each initial condition x(0) there exists a time $t_i < \infty$, defined as in (9), such that $\Lambda_i^t |V_i^{-1}x(0)| \le \varepsilon_i$ for all $t \ge t_i$. This shows convergence in finite time of the trajectories of (5) to $S_i(\varepsilon_i)$ defined in (10). To see that this set is invariant, let $x(t) \in S_i(\varepsilon_i)$ for some $t \ge 0$. Hence, $|V_i^{-1}x(t)| = |\zeta(t)| \le b_i + \varepsilon_i$ and using (13) and the property $\Lambda_i \varepsilon_i \le \varepsilon_i$ yields

$$|V_i^{-1}x(t+1)| = |\zeta(t+1)| \le \Lambda_i |\zeta(t)| + (I - \Lambda_i)b_i \le \Lambda_i (b_i + \varepsilon_i) + (I - \Lambda_i)b_i \le b_i + \varepsilon_i.$$

Theorem 3.1 generalises to LPV systems some of the results of [9] for switched systems (specifically, those that apply to switched systems with disturbances having constant bounds). Moreover, it extends these results by deriving (in part (b)) invariant sets that attract the LPV system trajectories in finite time and by providing an expression for the convergence time to these finite-time-attractive invariant sets.

The invariant sets derived in Theorem 3.1 are over-approximations of the minimal robust invariant set for system (5) (see, e.g., [7]). Despite the possible conservatism, in this paper we use the invariant sets of the form (10) due to its simple characterisation and the direct way to compute the convergence time to the set via the formula (9). These tools will be useful to obtain conditions guaranteeing robust stability of the switched LPV system (1), which we present in Section 4.

Remark 3.2. We show here that $\varepsilon_i > 0$ satisfying $\Lambda_i \varepsilon_i \leq \varepsilon_i$, as required in part (b) of Theorem 3.1, can always be found for a Schur nonnegative matrix Λ_i . Let $\Lambda_+ > 0$ be a slight perturbation of Λ_i such that $\Lambda_+ \geq \Lambda_i$ and Λ_+ is Schur. Then by the Perron-Frobenius Theorem (see e.g., [11]) there exists a positive eigenvalue r (the Perron-Frobenius eigenvalue) and an eigenvector ε_i with positive elements such that $\Lambda_+\varepsilon_i = r\varepsilon_i$. Since Λ_+ is Schur then r < 1 and we have

$$\Lambda_i \varepsilon_i \le \Lambda_+ \varepsilon_i = r \varepsilon_i < \varepsilon_i.$$

0

Remark 3.3. As mentioned in [9], to find the transformation V_i required in Theorem 3.1 a numerical search routine can be readily implemented, for example,

Minimise the spectral radius of
$$\Lambda_i$$
 in (6) over $V_i \in \mathbb{C}^{n \times n}$ invertible. (22)

In fact, any feasible solution V_i of the above optimisation problem such that the spectral radius of Λ_i is less than one can be used. \circ

4 Robust Stability of the Switched LPV System

In this section we provide a sufficient condition for robust stability and boundedness of the trajectories of the switched LPV system by ensuring that the switching sequences keep selecting subsystems for sufficiently long 'dwell times', which can be computed using the techniques described above.

Theorem 4.1. Consider the convex polytopic uncertain system (1)–(4) and suppose for each $i \in \underline{M}$ the conditions of Theorem 3.1 hold and an invariant set of the form (10) can be obtained. Suppose $x(0) \in S_i(\varepsilon_i)$ for some $i \in \underline{M}$. Let

$$T_{ij} \triangleq \max_{x \in \mathcal{S}_i(\varepsilon_i)} t_j[x, \mathcal{S}_j(\varepsilon_j)],$$
(23)

with t_j as defined in part (b) of Theorem 3.1 (see eq. (9)). Then the trajectories of system (1)–(4) remain bounded for all switching sequences $\sigma(t)$ that are such that whenever $\sigma(t^* - 1) = i$ and $\sigma(t^*) = j$, then $x(t^* - 1) \in S_i(\varepsilon_i)$ and $\sigma(t) = j$ for $t \in [t^*, t^\circ]$ with $t^\circ \ge t^* + T_{ij}$. (Note that this implicitly requires jto remain active, that is $j \in \mathcal{I}(t)$ for $t \in [t^*, t^\circ]$.)

Proof. Suppose, without loss of generality, that $x(t^* - 1) \in S_i(\varepsilon_i)$, $i \in \underline{M}$ and $\sigma(t^* - 1) = i$. Then if a switching sequence as described in the statement of the theorem is such that $\sigma(t^*) = j$, it will remain selecting subsystem j for at least T_{ij} time steps. By its definition in (23) (cf. (9)) T_{ij} gives enough time for all possible trajectories starting in $S_i(\varepsilon_i)$ to converge to $S_j(\varepsilon_j)$. Hence any possible switch to another subsystem k occurring from $t^* + T_{ij}$ onwards has a well defined originating set of states, $S_j(\varepsilon_j)$, from which to evaluate the next dwell time T_{jk} that needs to elapse before the switch from subsystem k to another subsystem is allowed to occur. Since all the invariant sets are bounded and the times during which the trajectories transit between sets are finite, it follows that the trajectories are bounded at all times, proving the result.

Note that T_{ij} in (23) is a sufficiently long 'dwell time' for the switching sequence to remain on subsystem j when coming from subsystem i. A (more conservative) dwell time for each subsystem independent from the previously active subsystem can be considered by letting $S \triangleq \bigcup_{i \in \underline{M}} S_i(\varepsilon_i)$ and defining

$$T_j \triangleq \max_{x \in \mathcal{S}} t_j[x, \mathcal{S}_j(\varepsilon_j)],$$
(24)

with t_j as defined in (9). Similar computations of dwell times have been reported in the literature for systems that switch between linear time invariant (non LPV) subsystems, see, for example, [3, 8].

5 Application to Reference Tracking for a Single LPV System Using Switched State Feedback

Consider the single LPV system

$$x(t+1) = A^{o}(\rho)x(t) + Bu(t) + Ew(t),$$
(25)

where u is a control input and the parameter $\rho = \rho(t)$ belongs to a bounded closed convex set \mathcal{P} . The matrix $A^{o}(\rho)$ is assumed to depend affinely on the parameter ρ and the matrices B and E (as well as the bounded disturbance signal w) are assumed parameter independent for simplicity (this assumption can be easily relaxed at the expense of more intricate notation).

We want to design a parameter dependent, static state feedback controller such that the states of (25) are ultimately bounded in a set centred at a desired bounded state reference signal $x_{ref}(t)$ satisfying

$$x_{ref}(t+1) = A^{o}(\rho)x_{ref}(t) + Bu_{ref}(t),$$
(26)

where $u_{ref}(t)$ is a bounded input reference signal. Different methodologies can be used to design the above reference system, see, for example, [1] and [15].

When the parameter variations are large, it is usually difficult to design a single LPV controller that achieves the control objective over the whole parameter range. A solution may then be found by dividing

the parameter set \mathcal{P} in closed convex subsets \mathcal{P}_i , $i \in \underline{M} \triangleq \{1, 2, \dots, M\}$, as explained in Section 2, designing an LPV controller for each region and then using an appropriate strategy to switch among the controllers so that the conditions of Theorem 4.1 hold. To this end, for each convex subset \mathcal{P}_i with vertices $\{\nu_{i1}, \nu_{i2}, \dots, \nu_{iN_i}\}$, consider the convex decomposition (see, for example, [2])

$$\rho = \sum_{\ell=1}^{N_i} \xi_{i\ell}(\rho) \,\nu_{i\ell},\tag{27}$$

where the functions $\xi_{i\ell}$ satisfy (3). Since $A^o(\rho)$ is affine in ρ then the decomposition (27) can be directly used for this matrix whenever $\rho = \rho(t) \in \mathcal{P}_i$, that is,

$$A^{o}(\rho) = A^{o}_{i}(\rho) = \sum_{\ell=1}^{N_{i}} \xi_{i\ell}(\rho) A^{o}_{i\ell}, \quad A^{o}_{i\ell} \triangleq A^{o}(\nu_{i\ell}), \quad \text{for } \rho \in \mathcal{P}_{i}.$$

$$(28)$$

Considering the same parameterisation for the controller gains, define the switching state feedback control

$$u = u_{ref} + K_{\sigma}(\rho)(x - x_{ref}), \qquad (29)$$

where $\sigma = \sigma(t) \in \underline{M}$ is the switching sequence to be designed and, for $i \in \underline{M}$,

$$K_i(\rho) = \sum_{\ell=1}^{N_i} \xi_{i\ell}(\rho) K_{i\ell}, \quad K_{i\ell} \triangleq K_i(\nu_{i\ell}), \quad \text{for } \rho \in \mathcal{P}_i.$$
(30)

When $\rho(t)$ belongs to the intersection of two or more parameter subsets then any of the parameterisations corresponding to those subsets can be used. The switching sequence $\sigma = \sigma(t)$ can then be designed to appropriately select the active subset with index in the active set of indices $\mathcal{I}(t) \triangleq \{i \in \underline{M} : \rho(t) \in \mathcal{P}_i\}$, as we next discuss.

Define the tracking error $z \triangleq x - x_{ref}$, which satisfies, using (25), (26) and (29),

$$z(t+1) = [A^{o}(\rho) + BK_{\sigma}(\rho)]z(t) + Ew(t).$$
(31)

Using the parameterisations (28) and (30) we can define

$$A_{i}(\rho) \triangleq A_{i}^{o}(\rho) + BK_{i}(\rho) = \sum_{\ell=1}^{N_{i}} \xi_{i\ell}(\rho) \underbrace{(A_{i\ell}^{o} + BK_{i\ell})}_{\triangleq A_{i\ell}},$$
(32)

and hence system (31)-(32) fits the formulation of Section 2 (cf. (1) and (5)), and thus invariant sets (10) can be computed for each subsystem and the switching sequence $\sigma = \sigma(t)$ can be devised to comply with the requirements of Theorem 4.1.

Although we have presented the above derivations for single LPV systems (i.e., where the only switched element is the state feedback controller), the technique can be applied with minor changes to the case where both the plant and the reference systems are also switched LPV systems, in which case the parameter space partition is usually determined by the plant.

6 Example: Two-Mass-Spring System

We consider the two-mass-spring system as described in [4] (see also [12]), consisting of two carts connected through a spring that has a time-varying characteristic. The control input is a force applied to the



Figure 1: Varying parameter $\rho(t)$ and associated subintervals.

first cart and the objective is to make the position of the second cart follow a desired reference. Using a first-order Euler approximation for the derivative and a sampling time T_s , the discrete-time state space equations are

$$x(t+1) = \underbrace{\begin{bmatrix} 1 & 0 & T_s & 0 \\ 0 & 1 & 0 & T_s \\ -T_s \rho/m_1 & T_s \rho/m_1 & 1 & 0 \\ T_s \rho/m_2 & -T_s \rho/m_2 & 0 & 1 \end{bmatrix}}_{A^o(\rho)} x(t) + \underbrace{\begin{bmatrix} 0 \\ 0 \\ T_s/m_1 \\ 0 \end{bmatrix}}_{B=E} [u(t) + w(t)], \quad (33)$$

where the first two states are the positions of the two carts, the last two states are their respective velocities, m_1 and m_2 are the masses of the carts, u is the control input and w is an input disturbance. We let $m_1 = m_2 = 1$, $T_s = 0.1$ s, and $|w(t)| \le \bar{w} = 0.001$, for $t \ge 0$. The spring parameter $\rho = \rho(t)$ has the following form:

$$\rho(t) = \alpha_t \rho + (1 - \alpha_t)\overline{\rho}, \quad \alpha_t = |\sin(0.01\pi t + \phi)|, \tag{34}$$

where $\rho = 0.25$, $\overline{\rho} = 1$, and ϕ is a phase shift. The parameter evolution over 100 samples (10s) is plotted with a thick solid line in Figure 1.

To design a switched LPV controller, we divide the parameter interval $\mathcal{P} = [0.25, 1]$ into 4 subintervals

$$\mathcal{P}_i = [\underline{\rho}_i, \overline{\rho}_i] = \{\underline{\rho}\} \oplus \Delta[i-1, i], \quad i = 1, 2, 3, 4,$$
(35)

of equal length $\Delta \triangleq (\overline{\rho} - \underline{\rho})/4$ (the symbol \oplus denotes Minkowski sum of sets). These subintervals are shown in Figure 1 as the vertical edges of the shaded areas. Also shown as the horizontal edges of the shaded areas are the times the parameter spends in each subinterval (except, possibly, at the start of operation, depending on the phase shift ϕ). Note that some times are not explicitly shown since they can be obtained by symmetry. By inspection we obtain (as multiples of $T_s = 0.1$ s)

$$T_1 = 46 T_s, \quad T_2 = 11 T_s, \quad T_3 = 9 T_s, \quad T_4 = 16 T_s.$$
 (36)

The tracking objective is for the position of the second cart to track a 'smoothened' (low-pass filtered) version of a square-wave signal r(t) that varies between 0.5 and 1. To achieve this objective, we use the procedure of Section 5 and first design an LPV reference system of the form (26), where $u_{ref}(t) = K_0[x_{ref}(t) - [1 \, 1 \, 0 \, 0]^T r_f(t)]$, with the constant feedback gain $K_0 = -[75.75 \quad 164.25 \quad 14.50 \quad 269.50]$ designed to place the poles of the 'central closed-loop system' $A^o(\rho_0) + BK_0$, with $\rho_0 = (\rho + \overline{\rho})/2$, at (0.5, 0.6, 0.7, 0.75), and where $r_f(t)$ is a low-pass filtered version of the square wave r(t) through a first order discrete transfer function with unity gain and a pole at 0.8. We observe that the above heuristic design of the reference system has no a priori LPV stability guarantees; this, however, is not a problem since the reference system is part of the controller and can be simulated off-line to test its performance prior to implementation on the plant.

Next, to design a scheduled feedback controller of the form (29)–(30) using the subintervals (35), a parameterisation of $A^o(\rho)$ in (33) of the form (28) is obtained by defining $A_{i1}^o = A^o(\underline{\rho}_i)$, $A_{i2}^o = A^o(\overline{\rho}_i)$, and the functions $\xi_{i1}(\rho) = (\overline{\rho}_i - \rho)/(\overline{\rho}_i - \underline{\rho}_i) = 4(\alpha_t - 1) + i$, $\xi_{i2}(\rho) = (\rho - \underline{\rho}_i)/(\overline{\rho}_i - \underline{\rho}_i) = 4(1 - \alpha_t) + 1 - i$. A parameterisation of the controller gains in each subinterval \mathcal{P}_i as in (30) is obtained by computing K_{i1} and K_{i2} via LQR design using the system matrices (A_{i1}^o, B) and (A_{i2}^o, B) , and weights $Q_1 = Q_2 = \text{diag}\{100, 100, 0.01, 0.01\}$ and $R_1 = 0.009$, $R_2 = 0.0085$, respectively.

Having the closed-loop matrices $A_{i\ell} = A_{i\ell}^o + BK_{i\ell}$, for i = 1, 2, 3, 4 and $\ell = 1, 2$, transformations V_i , satisfying (6) are sought via the numerical search (22).³ We then compute the corresponding vectors b_i defined in (7) and the sets (10) using Remark 3.2 to find the required $\varepsilon_i > 0$ satisfying $\Lambda_i \varepsilon_i \le \varepsilon_i$.

To compute upper bounds on the convergence times T_{ij} defined in (23), (9), we proceed as follows:

$$T_{ij} = \max_{x \in \mathcal{S}_i(\varepsilon_i)} \min\left\{ \bar{t} \in \{0, 1, \dots\} : \Lambda_j^t | V_j^{-1} x | \le \varepsilon_j \quad \forall t \ge \bar{t} \right\}$$
$$\leq \min\left\{ \bar{t} \in \{0, 1, \dots\} : \Lambda_j^t \Big[\max_{x \in \mathcal{S}_i(\varepsilon_i)} | V_j^{-1} x | \Big] \le \varepsilon_j \quad \forall t \ge \bar{t} \right\}.$$

We first solve the maximisation inside the square brackets and then address the minimisation by numerical iteration of the mapping Λ_j^t for sufficiently long t to find the solution by inspection of the resulting trajectories. Note that it is only relevant to compute the convergence times between adjacent subintervals as the parameter variation has no jumps. This yields

$$T_{12} \le 11 T_s, \quad T_{21} \le 22 T_s, \quad T_{23} \le 3 T_s, \quad T_{32} \le 11 T_s, \quad T_{34} \le 5 T_s, \quad T_{43} \le 9 T_s.$$
 (37)

For the above times to be feasible, the maximum time the system needs to 'dwell' in $S_j(\varepsilon_j)$ has to be smaller that T_j , the time the parameter spends in \mathcal{P}_j , given in (36). That is, $\max\{T_{ij}, \forall i \text{ adjacent to } j\} \leq T_j$. This indeed holds, for example, $\max\{T_{12}, T_{32}\} \leq T_2$, etc.

We simulated the closed-loop system over 700 samples (70s). The results are shown in Figure 2. The first plot shows the position of the second cart, $x_2(t)$, in solid black line and the filtered reference signal $r_f(t)$ in dashed blue line. We observe a good tracking performance and a slightly different shape of the response at different times due to the effect of the varying parameter. (Note that the second state of the state reference signal, $x_{ref,2}(t)$, is not shown since the maximum difference $|z_2(t)| = |x_2(t) - x_{ref,2}(t)|$ is less than 0.01 and thus not discernible in the plot's scale.) The second plot shows the variation of the scheduling parameter $\rho(t)$. The third plot shows the active index set $\mathcal{I}(t)$, which corresponds to the index of the subinterval that contains $\rho(t)$ at each time. The last plot shows a test signal that checks whether the tracking error z(t) belongs to the corresponding set $\mathcal{S}_i(\varepsilon_i)$ when the scheduling parameter is in \mathcal{P}_i . The test signal is computed as

$$\mu(t) \triangleq \max\left[|V_{\mathcal{I}(t)}^{-1}z(t)| - (b_{\mathcal{I}(t)} + \varepsilon_{\mathcal{I}(t)})\right],$$

³We observe that considering the two-mass-spring system as a single LPV system over the whole parameter range $\mathcal{P} = [0.25, 1]$, and adopting an analogous LQR design at the vertex systems, the use of the same numerical search algorithm did not produce a single transformation V and associated matrix Λ satisfying condition (6). This justifies the partitioning of the parameter space within the proposed framework.



Figure 2: Time domain simulation results.

which can be seen to be negative at all times, confirming that the tracking error always belongs to the required sets.

7 Example: Coupled-Tank System

In this example we "stretch" the theory presented in this paper, as we consider the varying parameters to depend on the system states through functions that are not globally bounded (i.e., \sqrt{x}/x); hence boundedness of the variation of the parameter in a compact set \mathcal{P} (as required in the developed theory) cannot be guaranteed a priori and has to be verified a posteriori. Also, the reference system (26) requires measurements of the parameter—a function of the state—and thus cannot be independently operated. Hence, this example is strictly speaking not covered by the theory presented. However, it serves to illustrate that the methodology is capable of dealing with more challenging applications.⁴ We also explain, by defining a suitable "residual signal", how the results can be used in fault diagnosis.

We consider the coupled-tank process described in [1], consisting of two equal cylindrical tanks positioned one directly above the other. Water flows from the upper tank through the lower tank to a water reservoir, and a pump is used to thrust water from the reservoir back up to the upper tank. The tank water levels are measured through pressure sensors located at the bottom of each tank. The performance objective is to control the pump so that the water level in the lower tank tracks the output of a reference model.

The continuous-time dynamics of the water levels $h_1(\tau)$ (upper tank) and $h_2(\tau)$ (lower tank) can be

⁴These issues are outside the scope of the present paper, which is based on the standard LPV approach that the parameter is a priori bounded, and will motivate theoretical supporting research to be reported in future work. Preliminary ideas to address these issues include hypothesising a bound on the parameter variation that can be later certified by invariant sets that guarantee the corresponding constraints on the system state.

modelled as

$$\dot{h}_{1}(\tau) = -(s_{1}/S_{1})\sqrt{2g}\sqrt{h_{1}(\tau)} + (k_{p}/S_{1})u(\tau) + w_{1}(\tau),$$

$$\dot{h}_{2}(\tau) = (s_{1}/S_{2})\sqrt{2g}\sqrt{h_{1}(\tau)} - (s_{2}/S_{2})\sqrt{2g}\sqrt{h_{2}(\tau)} + w_{2}(\tau),$$
(38)

where $u(\tau)$ is the voltage applied to the pump, $w_1(\tau)$, $w_2(\tau)$ are bounded state perturbations and the parameters are as follows: $S_1 = S_2 = S = 15.5179 \text{cm}^2$ is the cross-section area of the tanks; $s_1 = s_2 = s = 0.1781 \text{cm}^2$ the cross-section area of the tanks outflow orifice; $k_p = 3.3 \text{cm}^3/\text{Vs}$ the gain of the pump; and $g = 980 \text{cm/s}^2$ the gravitational constant.

An LPV model is derived by defining the parameters $\rho_i \triangleq \sqrt{h_i}/h_i = 1/\sqrt{h_i}$ for $h_i > 0$ cm, i = 1, 2. The use of this parameterisation leads to the continuous-time LPV model

$$\dot{x}(\tau) = \frac{s\sqrt{2g}}{S} \begin{bmatrix} -\rho_1 & 0\\ \rho_1 & -\rho_2 \end{bmatrix} x(\tau) + \frac{k_p}{S} \begin{bmatrix} 1\\ 0 \end{bmatrix} u(\tau) + \begin{bmatrix} w_1(\tau)\\ w_2(\tau) \end{bmatrix}.$$

We discretise the above model using an Euler approximation with sampling period $T_s = 1$ s, to obtain

$$x(t+1) = \underbrace{\begin{bmatrix} 1 - a \rho_1 & 0\\ a \rho_1 & 1 - a \rho_2 \end{bmatrix}}_{A^o(\rho_1, \rho_2)} x(t) + \underbrace{\begin{bmatrix} b\\ 0 \end{bmatrix}}_B u(t) + \underbrace{T_s \begin{bmatrix} w_1(t)\\ w_2(t) \end{bmatrix}}_{Ew(t)},$$
(39)

where $a \triangleq \frac{s\sqrt{2g}T_s}{S}$, $b \triangleq \frac{k_pT_s}{S}$, and where x(t+1) stands for $x((t+1)T_s)$, x(t) for $x(tT_s)$ and similarly for the remaining variables. Letting $\rho \triangleq (\rho_1, \rho_2)$ we see that (39) has the form (25).

From the parameter definition, for $0.5 \text{cm} \le h_1, h_2 \le 25 \text{cm}$ (the range of water level variations that we will design the tracking controller to operate on), ρ is bounded in the interval $\mathcal{P} = [1/\sqrt{25}, 1/\sqrt{0.5}] \times [1/\sqrt{25}, 1/\sqrt{0.5}]$. To approach the controller design as described in Section 5, we will consider two overlapping subsets $\mathcal{P}_1 = [1/\sqrt{25}, 1/\sqrt{3}] \times [1/\sqrt{25}, 1/\sqrt{3}]$ and $\mathcal{P}_2 = [1/\sqrt{8}, 1/\sqrt{0.5}] \times [1/\sqrt{8}, 1/\sqrt{0.5}]$, as shown in Figure 3. These subsets do not cover the whole parameter set \mathcal{P} but, as we will see later, for the considered trajectory (shown in red in the figure) this division is sufficient to illustrate the proposed switched LPV control technique.

Noting that each subset \mathcal{P}_i has the form

$$\mathcal{P}_i = [\underline{\rho}_i, \overline{\rho}_i] \times [\underline{\rho}_i, \overline{\rho}_i],$$

a parameterisation of $A^o(\rho_1, \rho_2)$ in (39) of the form (28) is obtained by defining

$$A^o_{i1} = A^o(\underline{\rho}_i, \underline{\rho}_i), \ A^o_{i2} = A^o(\underline{\rho}_i, \overline{\rho}_i), \ A^o_{i3} = A^o(\overline{\rho}_i, \underline{\rho}_i), \ A^o_{i4} = A^o(\overline{\rho}_i, \overline{\rho}_i)$$

and the functions

$$\xi_{i1}(\rho) = \alpha_i(\rho_1)[1 - \beta_i(\rho_2)], \quad \xi_{i2}(\rho) = \alpha_i(\rho_1)\beta_i(\rho_2), \xi_{i3}(\rho) = [1 - \alpha_i(\rho_1)][1 - \beta_i(\rho_2)], \quad \xi_{i4}(\rho) = [1 - \alpha_i(\rho_1)]\beta_i(\rho_2),$$
(40)

where

$$\alpha_i(\rho_1) = \frac{\overline{\rho}_i - \rho_1}{\overline{\rho}_i - \underline{\rho}_i}, \quad \beta_i(\rho_2) = \frac{\rho_2 - \underline{\rho}_i}{\overline{\rho}_i - \underline{\rho}_i}.$$
(41)

Accordingly, a parameterisation of the controller gains in each subset \mathcal{P}_i as in (30) is obtained by letting

$$K_{i1} = K_i(\underline{\rho}_i, \underline{\rho}_i), \ K_{i2} = K_i(\underline{\rho}_i, \overline{\rho}_i), \ K_{i3} = K_i(\overline{\rho}_i, \underline{\rho}_i), \ K_{i4} = K_i(\overline{\rho}_i, \overline{\rho}_i)$$



Figure 3: Parameter set and subsets.

Here we propose to compute each gain $K_{i\ell}$ by Linear Quadratic Regulator (LQR) design using the system matrices $(A_{i\ell}^o, B)$ and weights $Q = \text{diag}\{1, 10\}$ and R = 0.1. Then, transformations V_i , for i = 1, 2, satisfying (6) are sought via the numerical search (22).⁵ This procedure yields

$$V_1 = \begin{bmatrix} 0.5425 + 1.8931j & -0.8245 - 0.5258j \\ 0.2942 - 1.0508j & 0.5418 + 0.0085j \end{bmatrix}, \quad V_2 = \begin{bmatrix} -1.3300 - 0.1181j & -1.0268 - 0.1426j \\ 1.0672 - 1.0493j & 0.6244 + 0.9799j \end{bmatrix},$$

where $j = \sqrt{-1}$ is the imaginary unit, and

$$\Lambda_1 = \begin{bmatrix} 0.6213 & 0.0699\\ 0.2834 & 0.6213 \end{bmatrix}, \quad \Lambda_2 = \begin{bmatrix} 0.6052 & 0.2234\\ 0.3707 & 0.6052 \end{bmatrix}.$$

Using the above matrices, E = I, and the bounds $|w(t)| \le \bar{w} = \begin{bmatrix} 1 & 1 \end{bmatrix}^T \times 10^{-3}$, for $t \ge 0$, we compute from (7)

$$b_1 = [0.0057 \ 0.0115]^T, \quad b_2 = [0.0087 \ 0.0112]^T$$

We next compute the sets (10) using Remark 3.2 to find the required $\varepsilon_i > 0$ satisfying $\Lambda_i \varepsilon_i \le \varepsilon_i$. Indeed, since $\Lambda_i > 0$ for i = 1, 2, we compute $\varepsilon_i > 0$ as (a scaled version of) the eigenvector associated with the Perron-Frobenius eigenvalue of Λ_i , obtaining

$$\varepsilon_1 = [0.0148 \ 0.0299]^T, \quad \varepsilon_2 = [0.0204 \ 0.0263]^T.$$

The resulting sets of the form (10) are the ellipses $S_1(\varepsilon_1)$ and $S_2(\varepsilon_2)$ shown in Figure 4. (The figure also shows the tracking error trajectory, this will be explained later.)

To compute upper bounds on the convergence times T_{ij} defined in (23), (9), we proceed as follows:

$$T_{ij} = \max_{x \in \mathcal{S}_i(\varepsilon_i)} \min\left\{ \bar{t} \in \{0, 1, \dots\} : \Lambda_j^t | V_j^{-1} x | \le \varepsilon_j \quad \forall t \ge \bar{t} \right\}$$
$$\leq \min\left\{ \bar{t} \in \{0, 1, \dots\} : \Lambda_j^t \Big[\max_{x \in \mathcal{S}_i(\varepsilon_i)} | V_j^{-1} x | \Big] \le \varepsilon_j \quad \forall t \ge \bar{t} \right\}.$$

⁵As in the previous example, when considering the coupled-tank system as a single LPV system over the whole parameter range \mathcal{P} in Figure 3, and adopting an analogous LQR design at the vertex systems, the use of the same numerical search algorithm did not produce a single transformation V and associated matrix Λ satisfying condition (6). This justifies the partitioning of the parameter space within the proposed framework.



Figure 4: Invariant sets for the tracking error z(t) and its trajectory.

As for the previous example, we first solve the maximisation inside the square brackets and then address the minimisation by numerical iteration of the mapping Λ_j^t for sufficiently long t to find the solution by inspection of the resulting trajectories. This yields $T_{12} = 5s$ and $T_{21} = 7s$.

Since in this example the parameter ρ is a function of the system states, the time it spends in each subset \mathcal{P}_1 and \mathcal{P}_2 depends on the control strategy and cannot be guaranteed independently. However, since the above computed times T_{12} and T_{21} are small, it is reasonable to expect that for slowly varying references the parameter will change subsets slowly enough so that the trajectories of the tracking error after each change will converge to the corresponding set. For example, when tracking a piecewise constant reference, the parameter can change sets during the transients but during the constant part of the reference signal it is likely to remain in the same set. Also, due to the subset overlap, it is possible to design switching sequences with different degrees of hysteresis; for example the switching between controller gains can be delayed as much as possible by considering the following switching law:

If
$$\sigma(t) = i \in \mathcal{I}(t)$$
, then $\sigma(t+1) = i$ if $i \in \mathcal{I}(t+1)$;
otherwise switch to (any, if more than 2 subsets) $i \in \mathcal{I}(t+1)$. (42)

The tracking objective is for the tank levels to approximately track a square-wave signal r(t) that varies between 4 and 20. Towards this goal, we use the procedure of Section 5 and first design a reference system of the form (26), where $A^o(\rho) = A^o(\rho_1, \rho_2)$ is as in (39) (i.e., ρ_1 and ρ_2 are computed based on the plant states and used in the reference system) and where $u_{ref}(t) = [k_0 x_{ref,1}(t) + (1 - k_0)r(t) - (1 - a\rho_1)x_{ref,1}(t)]/b$, with $k_0 = 0.5$. Note that this control 'linearises' the $x_{ref,1}$ -equation so that $x_{ref,1}$ evolves as an LTI first order system; the response of $x_{ref,2}$, on the other hand, cannot be adjusted separately but it can be simulated (with ρ_1 and ρ_2 computed based on the reference states) to have a reasonable indication of its performance.

We simulated a realistic implementation using the continuous-time nonlinear model (38) as the "plant". The proposed discrete-time switched LPV controller is interfaced with the plant via a zero-order hold. The results of the time simulation of the closed-loop system are shown in Figure 5. The top plot shows the tank levels from the initial condition $x(0) = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}^T$, with $x_1(t) = h_1(t)$ in solid black line and



Figure 5: Time domain simulation results.

 $x_2(t) = h_2(t)$ in dashed blue line, both states displaying a good tracking performance of the square-wave signal. At 300s we simulated a sensor fault by introducing a negative pulse f(t) of amplitude 1 and duration 1s in the measurement $\hat{x}_2(t) = x_2(t) + f(t)$ of the second tank level. Small perturbations due to the faulty measurements can be seen in the responses, especially for the first state (see the zoomed area). The second plot shows the variation of the measured scheduling parameter $\rho(t)$, with $\rho_1 = 1/\sqrt{x_1}$ in black solid line and $\rho_2 = 1/\sqrt{\hat{x}_2}$ (note the use of the measured state, affected by the fault) in dashed blue line. The zoomed area shows the small perturbations due to the fault. The corresponding plot of ρ_2 versus ρ_1 in the parameter space is the red trajectory shown in Figure 3. As seen in this figure, the parameter starts in \mathcal{P}_2 (near the top right corner, corresponding to the plant initial condition) and then enters and remains in \mathcal{P}_1 during the whole simulation, with parts of the trajectory lying in the intersection between the two subsets. The switching signal is chosen as $\sigma(t) = 2$ if $\rho(t) \in \mathcal{P}_2$ and $\sigma(t) = 1$ otherwise, i.e., $\sigma(t) = 2$ is used both in \mathcal{P}_2 and in $\mathcal{P}_1 \cap \mathcal{P}_2$. This is indicated in the bottom plot of Figure 5 through different shaded sections: the first and third shaded sections (in green) correspond to $\sigma(t) = 2$ and the second and fourth shaded sections (in magenta) correspond to $\sigma(t) = 1$. This plot also shows the signal

$$\mu(t) \triangleq \max\left[|V_{\sigma(t)}^{-1} \hat{z}(t)| - (b_{\sigma(t)} + \varepsilon_{\sigma(t)}) \right],$$

which checks whether the measured tracking error $\hat{z}(t)$ belongs to the corresponding set $S_i(\varepsilon_i)$ when $\sigma(t) = i$. Note that, except for a short lapse around 300s, $\mu(t)$ is negative, confirming that before and a short while after the fault the measured tracking error always belongs to the 'active' invariant set. The signal $\mu(t)$ can then serve as a 'residual' signal to indicate the presence of faults in an FDI scheme. The corresponding tracking error trajectory is shown in blue dashed-starred line in Figure 4, where it can be seen that z(t) experiences a 'jump' due to the fault and after a short transient returns to the intersection of the attractive invariant sets.

8 Conclusions

We have considered the problem of robust stability and set invariance of switched LPV systems affected by bounded disturbances. We have first derived a result for a single LPV system, which is a generalisation of previous work for switched systems. We have then applied this seed result in the derivation of dwelltime-type conditions on the switched-LPV switching rule to ensure robust closed-loop stability and set invariance of the trajectories of the switched-LPV system across the whole parameter range. As an application of the results, we considered the problem of reference tracking for LPV systems using switched LPV state feedback and feedforward from an LPV reference system. The results were illustrated via an example of a two-mass-spring system with a varying spring characteristic, and via the nonlinear model of a coupled-tank system embedded into a switched LPV system description.

References

- A. Abdullah and M. Zribi. Model reference control of LPV systems. *Journal of the Franklin Institute*, 346:854– 871, 2009.
- [2] P. Apkarian, P. Gahinet, and G. Becker. Self-scheduled H_{∞} Control of Linear Parameter-varying Systems: a Design Example. *Automatica*, 31(9):1251–1261, 1995.
- [3] F. Blanchini, D. Casagrande, and S. Miani. Modal and transition dwell time computation in switching systems: A set-theoretic approach. *Automatica*, 46(9):1477–1482, 2010.
- [4] Y.-Y. Cao and Z. Lin. Minmax MPC algorithm for LPV systems subject to input saturation. *IEE Proceedings, Control Theory and Applications*, 152(3):266–272, 2005.
- [5] J. Daafouz and J. Bernussou. Parameter dependent Lyapunov functions for discrete time systems with time varying parametric uncertainties. *Systems and Control Letters*, 43:355–359, 2001.
- [6] M.C. de Oliveira, J. Bernussou, and J.C. Geromel. A new discrete-time robust stability condition. Systems and Control Letters, 37:261–265, 1999.
- [7] M. Dehghan and C.-J. Ong. Characterization and computation of disturbance invariant sets for constrained switched linear systems with dwell time restriction. *Automatica*, 48(9):2175–2181, 2012.
- [8] M. Dehghan and C.-J. Ong. Computations of mode-dependent dwell times for discrete-time switching system. *Automatica*, 49(6):1804–1808, 2013.
- [9] H. Haimovich and M.M. Seron. Bounds and invariant sets for a class of discrete-time switching systems with perturbations. *International Journal of Control*, (2):371–383, 2014.
- [10] X. He, G.M. Dimirovski, and J. Zhao. Control of Switched LPV Systems Using Common Lyapunov Function Method and an F-16 Aircraft Application. In *IEEE International Conference on Systems Man and Cybernetics* (SMC), pages 386–392, Istanbul, October 2010.
- [11] R.A. Horn and C.R. Johnson. Matrix Analysis. Cambridge University Press, 1985.
- [12] M.V. Kothare, V. Balakrishnan, and M. Morari. Robust constrained model predictive control using linear matrix inequalities. *Automatica*, 32:1361–1379, 1996.
- [13] B. Lu and F. Wu. Switching LPV control designs using multiple parameter-dependent Lyapunov functions. *Automatica*, 40(11):1973–1980, 2004.
- [14] S. Olaru, J.A. De Doná, M.M. Seron, and F. Stoican. Positive invariant sets for fault tolerant multisensor control schemes. *International Journal of Control*, 83(12):2622–2640, December 2010.
- [15] D. Rotondo, F. Nejjari, and V. Puig. Model Reference Switching Quasi-LPV Control of a Four Wheeled Omnidirectional Robot. In *Preprints of the 19th IFAC World Congress*, Cape Town, South Africa, August 2014.
- [16] D. Rotondo, V. Puig, and F. Nejjari. A Virtual Actuator Approach for Fault Tolerant Control of Switching LPV Systems. In *Preprints of the 19th IFAC World Congress*, Cape Town, South Africa, August 2014.
- [17] M.M. Seron, J.A. De Doná, and J. Richter. Invariant-set-based Fault Diagnosis in Lure Systems. *International Journal of Robust and Nonlinear Control*, 24(16):2405–2422, 2014.