Reviewers' comments and authors' replies for the revised version of the paper:
A robust nonlinear position observer for synchronous motors with relaxed excitation conditions

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We would like to thank the Editor in Chief and the reviewers for their interest in our manuscript and also for providing many constructive comments and valuable suggestions. Their comments and suggestions have helped us to improve the quality of the paper, and have been included in the revised manuscript.

Reply to Reviewer 1

1. `The presentation of the theoretical results leads to the idea that the authors successfully remove the stringent PE condition and replace it by A5*. In contrast to what required, the authors do not explicitly emphasize in the paper that the new non-square integrability assumption leads to a different non exponential - type of convergence. This substantial drawback must be described everywhere in the paper (Abstract, Introduction,...).`

   We would like to emphasize that the new DREM-based observer, as the one with gradient (or least squares) estimators, ensures parameter convergence is *exponentially* fast if its regressor is PE. If $\phi(t)$ is only non-square integrable (but not PE) then the convergence of the DREM-based observer is still guaranteed, however, convergence is not exponential. For the gradient estimator, in its turn, there is *no analytical* proof of convergence if its regressor $m(t) \notin$ PE. In this sense, the DREM estimator is superior because, even in $\phi(t)$ is not PE convergence is ensured, provided $\phi(t)$ is non-square integrable. It should be underscored that the possibility of choosing suitable operators for the filtered signals in DREM gives an additional useful degree of freedom to satisfy the non-square integrability condition—degree of freedom that is absent in standard gradient estimators. We clarified this important points in the Abstract and Introduction.

2. `About the consequent lack of guaranteed robustness in the case in which` ω `is not PE (under which the price to be paid in terms of convergence can be reasonably sustained), the authors simply added a simulation whose results are reported in Fig. 6.`

   We agree with the reviewer that exponential stability ensures, via total stability arguments [†], some robustness properties to the system. However, it should be mentioned that this condition is not necessary and asymptotically stabilising systems do not necessarily "lack guaranteed robustness." This fact is depicted with the classical example

$$\dot{x} = -a^2 x^3,$$

   whose zero equilibrium is robustly (globally) asymptotically stable—where the qualifier *robustly*, stems from the fact that the property holds for all nonzero values of the parameter a. However, notice that the zero equilibrium, is *not exponentially* stable as clearly seen from its solutions

$$x(t) = \frac{\pm 1}{\sqrt{2a^2 t + k}}$$

   with k a constant depending on initial conditions.

   We would like also to highlight that even globally exponentially stable systems may exhibit undesirable behavior in the presence of disturbances. Recall, for example, [‡], where it is shown that the trajectories of a globally exponentially stable system can be driven to infinity by an exponentially decaying input.

† Anderson, B.D.O., Bitmead, R.R., Johnson, C.R., Kokotovic, P.V., Kosut, R.L., Mareels, I.M.Y., Praly, L., Riedle, B.D. (1986). *Stability of Adaptive Systems: Passivity and Averaging Analysis.* Cambridge, MA and London: MIT Press.

‡ A. R. Teel and J. Hespanha. (2004) Examples of GES Systems That can be Driven to Infinity by Arbitrarily Small Additive Decaying Exponentials, *IEEE Transactions on Automatic Control*, Vol. 49, No. 8, pp. 1407-1410.

3. `The simulation is to be changed according to the following questions:`

   `1) Why do the authors consider a zero load torque (which application are they describing)?`

   Zero load torque may correspond to an idle running of the motor. In the revised version we provided more simulations including non-zero load torque scenarios.

   `2) Which is the magnitude of the current sensor noises? Realistic noises must be inserted and their effect (along with the one related to a step load torque) is to be evaluated in a longer simulation (since the robustness properties are not uniform).`

   Thank you for this remark. In the revised version we specified that current sensor measurements contain band-limited white noises. The magnitude of noises is about $10-15\%$ of the useful signal. Simulation time is increased to 20 seconds.

   `3) Why do the authors use a (so small) sampling time (2` μ `s)?`

Sampling time depends on the type of the inverter the power system uses and influences the accuracy of model calculation. In our realistic simulations we verify the efficiency with three-phase IGBT switches which are based on a Universal Bridge Block of Matlab. 2 μs is the typical value for similar test schemes – please see, for instance, PMSM with speed regulation typing 'ac6_example' in Matlab command line.

`4) Why do the authors use the same gains for the observers I (the new one) and II? Since the authors vary their gains as the simulation changes and I and II are different observers (there is no sense in considering the same gains), did the authors choose the best gains for II?`

The difference between both observers is in the parameter estimator only, in Observer I it is a DREM while for Observer II it is a standard gradient. Notice that the DREM estimator is also a gradient one, but designed for the new scalar regressors. Therefore, to evaluate the effect of the new elements introduced by DREM, namely, the definition of the extended regressor and the data mixing (induced by the computation of the determinant), it is reasonable to select the gains of both estimators equal. In any case, additional simulations have shown that the performance of Observer II is not significantly modified selecting other gains.

Reply to Reviewer 2

1. `I am happy to see that the authors have included a Simulink simulation example for their algorithm so it is one step closer to applications.`

   The authors thank the reviewer for his(her) appreciation of our work.

2. `However, section 2 is really needed to be improved. The parameters lambda1, lambad2, lambda_m are not defined.`

   At the beginning of Sec. 2 we defined the stator flux $\lambda \in \mathbb{R}^2$. This is a common denotation which means that flux is a vector of two components $\lambda = \mathrm{col}(\lambda_1, \lambda_2)^\top$. The same applies to the currents and voltages. In the revised version we clarified that λ_m is a constant flux from permanent magnets – thank you.

3. `Also, it is not obvious to get equation (4) from equation (1). The problem can be easily fixed by introducing the PMSM model in alpha-beta reference frame (see equation (1.24) and (1.25)) in Wang et al. (2015)).`

   We consider classical two-phase model of PMSM presented exactly in the $\alpha\beta$ reference frame—see the very beginning of Section 2. To derive equation (4) first represent (1) as a system of two equations

$$\lambda_1 - Li_1 = \lambda_m \cos(n_p\theta), \tag{*}$$

$$\lambda_2 - Li_2 = \lambda_m \sin(n_p\theta). \tag{**}$$

Dividing (**) by (*) and using the well known identity $\tan x = \frac{\sin x}{\cos x}$ yields

$$\tan(n_p\theta) = \frac{\lambda_2 - Li_2}{\lambda_1 - Li_1}, \tag{***}$$

whence one can get the equation

$$\theta = \frac{1}{n_p} \arctan\left\{\frac{\lambda_2 - Li_2}{\lambda_1 - Li_1}\right\}. \tag{****}$$