Online Supplements

"Peer-regarding fairness in supply chain"

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APPENDIX A. Variable Notation

Table A.1: Variable Definitions

Notation	*Definition*
c	Unit production cost
d_i	Demand of market i, which is a decreasing function of p_i, i.e., $d_i = a - p_i$
p_i, w_i	Retailer i's retail price, the supplier's wholesale price to Retailer i
π_i, $\pi_{s,i}$	Profit functions for Retailer i and the supplier in Supply Chain i
δ	The peer-regarding fairness parameter (also the distributional fairness parameter in Benchmark Model 2), $\delta > 0$
λ	The sympathy parameter, $0 < \lambda < 1$
η	The schadenfreude parameter, $\eta > 0$
t	The unfairness Retailer 1 suffers, i.e., $(\pi_{s,1} - \pi_1)^+$
π_i^p	The profit of Supply Chain i
π^*	The profit of the distribution channel
π_s^*	The supplier's total profit in the distribution channel

APPENDIX B. Peer-regarding Fairness Model For Sympathy

APPENDIX B.1. Retailer 2's Best Response Function

When Retailer 2 experiences PF for sympathy, that is, $(\pi_{s,2}-\pi_2)+\lambda(\pi_{s,1}-\pi_1)^+ > 0$, Retailer 2's utility-maximization problem is given by

$$\max_{p_2} \quad \pi_2 - \delta(\lambda(\pi_{s,1} - \pi_1)^+ + (\pi_{s,2} - \pi_2)) \tag{B.1}$$

$$s.t. \quad \pi_{s,2} - \pi_2 > -\lambda(\pi_{s,1} - \pi_1)^+ \tag{B.2}$$

For convenience, we set $t = (\pi_{s,1} - \pi_1)^+$. Because of $p_1 = \frac{a+w_1}{2}$, it's easy to know $t \in [0, \frac{(a-c)^2}{12}]$. The optimal solution for the unconstraint problem (B.1) is $p_2 = \frac{-c\delta+a(1+\delta)+(1+2\delta)w_2}{2(1+\delta)}$. Constraint (B.2) is satisfied by either of the following conditions: (1) $w_2 \geq \frac{a+c}{2} - \sqrt{\lambda t}$ and $c < p_2 < a$; (2) $c < w_2 < \frac{a+c}{2} - \sqrt{\lambda t}$, and $c < p_2 < \frac{a-c+2w_2-2\sqrt{(w_2-\frac{a+c}{2})^2-\lambda t}}{2}$ or $\frac{a-c+2w_2+2\sqrt{(w_2-\frac{a+c}{2})^2-\lambda t}}{2} \leq p_2 < a$.

For case (1), when $w_2 \geq \frac{a+c}{2} - \sqrt{\lambda t}$, the optimal solution is

$$p_2 = \frac{-c\delta + a\left(1+\delta\right) + \left(1+2\delta\right) w_2}{2\left(1+\delta\right)} \tag{B.3}$$

For case (2), under the condition of $c < w_2 < \frac{a+c}{2} - \sqrt{\lambda t}$, by comparing $\frac{-c\delta+a(1+\delta)+(1+2\delta)w_2}{2(1+\delta)}$ with $\frac{a-c+2w_2-2\sqrt{(w_2-\frac{a+c}{2})^2-\lambda t}}{2}$ and $\frac{a-c+2w_2+2\sqrt{(w_2-\frac{a+c}{2})^2-\lambda t}}{2}$, we get the optimal solutions showed as follows:

$$p_2 = \begin{cases} \dfrac{-c\delta + a\left(1+\delta\right) + \left(1+2\delta\right) w_2}{2\left(1+\delta\right)} & if \quad w^{II} \leq w_2 < \dfrac{a+c}{2} - \sqrt{t\lambda} \\ \dfrac{a - c + 2w_2 - 2\sqrt{(w_2 - \frac{a+c}{2})^2 - \lambda t}}{2} & if \quad c < w_2 < w^{II}, \end{cases} \tag{B.4}$$

where $w^{II} = \frac{2a(1+\delta)^2+c(1+2\delta(2+\delta))-(1+\delta)\sqrt{\left((a-c)^2+4t(3+4\delta(2+\delta))\lambda\right)}}{3+4\delta(2+\delta)}$.

By combining (B.3) and (B.4), we can get the conclusion that when $(\pi_{s,2}-\pi_2)+\lambda(\pi_{s,1}-\pi_1)^+ > 0$, Retailer 2's optimal retail price is given as follows:

$$p_2 = \begin{cases} \dfrac{-c\delta + a(1+\delta) + (1+2\delta)w_2}{2(1+\delta)} & if \quad w^{II} \leq w_2 \\ \dfrac{a-c+2w_2-2\sqrt{(w_2-\frac{a+c}{2})^2-\lambda t}}{2} & if \quad c < w_2 < w^{II} \end{cases} \tag{B.5}$$

When Retailer 2 doesn't experience PF for sympathy, that is, $\pi_{s,2}-\pi_2 \leq -\lambda(\pi_{s,1}-\pi_1)^+$, Retailer 2's utility-maximization problem is given by

$$\max_{p_2} \qquad \pi_2 \tag{B.6}$$

$$s.t. \quad \pi_{s,2}-\pi_2 \leq -\lambda(\pi_{s,1}-\pi_1)^+ \tag{B.7}$$

The optimal solution for the unconstraint problem (B.6) is $p_2 = \frac{a+w_2}{2}$. Constraint (B.7) is satisfied by the conditions of $c < w_2 \leq \frac{a+c}{2} - \sqrt{\lambda t}$ and $\frac{a-c+2w_2-2\sqrt{(w_2-\frac{a+c}{2})^2-\lambda t}}{2} \leq p_2 \leq \frac{a-c+2w_2+2\sqrt{(w_2-\frac{a+c}{2})^2-\lambda t}}{2}$.

When $c < w_2 \leq \frac{a+c}{2} - \sqrt{\lambda t}$, by comparing $\frac{a+w_2}{2}$ with $\frac{a-c+2w_2-2\sqrt{(w_2-\frac{a+c}{2})^2-\lambda t}}{2}$ and $\frac{a-c+2w_2+2\sqrt{(w_2-\frac{a+c}{2})^2-\lambda t}}{2}$, we can get the optimal solutions given as follows:

$$p_2 = \begin{cases} \dfrac{a+w_2}{2} & if \quad w_2 < w^I \\ \dfrac{a-c+2w_2-2\sqrt{(w_2-\frac{a+c}{2})^2-\lambda t}}{2} & if \quad w^I \leq w_2 \leq \dfrac{a+c}{2} - \sqrt{t\lambda}, \end{cases} \tag{B.8}$$

where $w^I = \frac{2a+c-\sqrt{(a-c)^2+12t\lambda}}{3}$.

Because $w^{II} > w^I$ always holds for $\delta > 0$ and $t > 0$, by combining (B.5) and (B.8), we can get Retailer 2's optimal retail price conditional on contract acceptance shown as follows:

$$p_2 = \begin{cases} \dfrac{a+w_2}{2} & if \quad w_2 < w^I \\ \dfrac{a-c+2w_2-2\sqrt{(w_2-\frac{a+c}{2})^2-\lambda t}}{2} & if \quad w^I \leq w_2 < w^{II} \\ \dfrac{-c\delta + a(1+\delta) + (1+2\delta)w_2}{2(1+\delta)} & if \quad w_2 \geq w^{II} \end{cases} \tag{B.9}$$

In the next subsection, we give the the supplier's optimal wholesale prices under different conditions.

APPENDIX B.2. The Supplier's Pricing Decision

APPENDIX B.2.1. The Supplier's Pricing Decision When $p_2 = \frac{a+w_2}{2}$

When $p_2 = \frac{a+w_2}{2}$, applying backward induction, the supplier first maximises $\pi_{s,2}$. Taking the derivative of $\pi_{s,2}$ with respect to w_2, we get that the optimal wholesale price offered to Retailer 2 is w^I. Notice that $t = (\pi_{s,1} - \pi_1)^+$, so the supplier is supposed to charge the optimal wholesale price w_1 to maximise his total profit $\pi_s = \pi_{s,1} + \pi_{s,2}$. The functions of $\pi_{s,1}$ and $\pi_{s,2}$ are given as follows:

$$\pi_{s,1} = \frac{1}{2}(a - w_1)(w_1 - c) \tag{B.10}$$

$$\pi_{s,2} = \frac{1}{18}\left(a - c + \sqrt{(a-c)^2 + 12t\lambda}\right)\left(2a - 2c - \sqrt{(a-c)^2 + 12t\lambda}\right) \tag{B.11}$$

Taking the derivative of π_s with respect to w_1, we get the optimal wholesale price offered to Retailer 1 shown as follows:

$$w_1 = \begin{cases} w_1^{I*} & if \quad 0 < \lambda < \lambda^* \\ w_1^{II*} & if \quad \lambda^* \leq \lambda < 1 \end{cases} \tag{B.12}$$

where we set $\lambda_1 = -3a^4 + 6a^2c^2 - 3c^4 + 17a^4\lambda + 32a^3c\lambda + 33a^2c^2\lambda + 22ac^3\lambda + 4c^4\lambda - 28a^4\lambda^2 - 80a^3c\lambda^2 - 81a^2c^2\lambda^2 - 26ac^3\lambda^2 - c^4\lambda^2 + 16a^4\lambda^3 + 48a^3c\lambda^3 + 36a^2c^2\lambda^3 + 8ac^3\lambda^3$, $\lambda_2 = 12a^3 - 12a^2c - 12ac^2 + 12c^3 - 100a^3\lambda - 162a^2c\lambda - 132ac^2\lambda - 38c^3\lambda + 192a^3\lambda^2 + 402a^2c\lambda^2 + 240ac^2\lambda^2 + 30c^3\lambda^2 - 112a^3\lambda^3 - 216a^2c\lambda^3 - 96ac^2\lambda^3 - 8c^3\lambda^3$, $\lambda_3 = -12a^2 + 24ac - 12c^2 + 231a^2\lambda + 294ac\lambda + 123c^2\lambda - 489a^2\lambda^2 - 642ac\lambda^2 - 165c^2\lambda^2 + 276a^2\lambda^3 + 312ac\lambda^3 + 60c^2\lambda^3$, $\lambda_4 = -252a\lambda - 180c\lambda + 540a\lambda^2 + 324c\lambda^2 - 288a\lambda^3 - 144c\lambda^3$ and $\lambda_5 = 108\lambda - 216\lambda^2 + 108\lambda^3$. For the equation of $\lambda_1 + \lambda_2 x + \lambda_3 x^2 + \lambda_4 x^3 + \lambda_5 x^4 = 0$, we define w_1^{I*} and w_1^{II*} as its third root and first root, respectively. Besides, we define λ^* as the first root of the function $64 - 304x + 364x^2 + 195x^3 - 642x^4 + 195x^5 + 364x^6 - 304x^7 + 64x^8 = 0$.

APPENDIX B.2.2. The Supplier's Pricing Decision When $p_2 = \frac{a-c+2w_2-2\sqrt{(w_2-\frac{a+c}{2})^2-\lambda t}}{2}$

When $p_2 = \frac{a-c+2w_2-2\sqrt{(w_2-\frac{a+c}{2})^2-\lambda t}}{2}$, applying backward induction, the supplier first maximises $\pi_{s,2}$. Taking the derivative of $\pi_{s,2}$ with respect to w_2, we get that the optimal wholesale price offered to Retailer 2 is w^I. So the supplier is supposed to charge the optimal wholesale price w_1 to maximise π_s. Taking the derivative of π_s with respect to w_1, we get the optimal wholesale price offered to Retailer 1 shown as follows:

$$w_1 = \begin{cases} w_1^{I*} & if \quad 0 < \lambda < \lambda^* \\ w_1^{II*} & if \quad \lambda^* \leq \lambda < 1 \end{cases} \tag{B.13}$$

Notice that (B.12) and (B.13) are the same.

APPENDIX B.2.3. The Supplier's Pricing Decision When $p_2 = \frac{-c\delta+a(1+\delta)+(1+2\delta)w_2}{2(1+\delta)}$

When $p_2 = \frac{-c\delta+a(1+\delta)+(1+2\delta)w_2}{2(1+\delta)}$, it needs to satisfy the condition of $w_2 > w^{II}$. Without the constraint, the optimal wholesale prices are $w_1 = \frac{a+c}{2}$ and $w_2 = \frac{a+c+a\delta+3c\delta}{2(1+2\delta)}$. By comparing $\frac{a+c+a\delta+3c\delta}{2(1+2\delta)}$ with w^{II}, we can get: (1) when $\frac{a+c+a\delta+3c\delta}{2(1+2\delta)} \geq w^{II}$, the optimal wholesale price offered to Retailer 2 is $\frac{a+c+a\delta+3c\delta}{2(1+2\delta)}$; (2) when $\frac{a+c+a\delta+3c\delta}{2(1+2\delta)} < w^{II}$, the optimal wholesale price offered to Retailer 2 is w^{II}. We will study the optimal wholesale price offered to Retailer 1 according to the two cases in the below subsections.

The Supplier's Pricing Decision When $w_2 = \frac{a+c+a\delta+3c\delta}{2(1+2\delta)}$

When $\frac{a+c+a\delta+3c\delta}{2(1+2\delta)} \geq w^{II}$, it needs to satisfy one of the following conditions: (1) $0<\lambda<\frac{3}{4}\&0<\delta\leq\frac{1}{2}\&\frac{1}{3}(a+2c)\leq w_1\leq a$; (2) $0<\lambda<\frac{3}{4}\&\frac{1}{2}<\delta\leq\frac{-3-4\lambda}{-6+8\lambda}\&\frac{2a+c}{3}-\frac{1}{6}\sqrt{\frac{(a-c)^2(3+4\lambda+\delta(-6+8\lambda))}{\lambda+2\delta\lambda}}\leq w_1\leq\frac{2a+c}{3}+\frac{1}{6}\sqrt{\frac{(a-c)^2(3+4\lambda+\delta(-6+8\lambda))}{\lambda+2\delta\lambda}}$; (3) $\frac{3}{4}\leq\lambda<1\&0<\delta\leq\frac{1}{2}\&\frac{1}{3}(a+2c)\leq w_1\leq a$; (4) $\frac{3}{4}\leq\lambda<1\&\delta>\frac{1}{2}\&\frac{2a+c}{3}-\frac{1}{6}\sqrt{\frac{(a-c)^2(3+4\lambda+\delta(-6+8\lambda))}{\lambda+2\delta\lambda}}\leq w_1\leq\frac{2a+c}{3}+\frac{1}{6}\sqrt{\frac{(a-c)^2(3+4\lambda+\delta(-6+8\lambda))}{\lambda+2\delta\lambda}}$.

For case (1), the optimal wholesale price offered to Retailer 1 is $w_1=\frac{a+c}{2}$; for case (2), comparing $\frac{a+c}{2}$ with $\frac{2a+c}{3}-\frac{1}{6}\sqrt{\frac{(a-c)^2(3+4\lambda+\delta(-6+8\lambda))}{\lambda+2\delta\lambda}}$ and $\frac{2a+c}{3}+\frac{1}{6}\sqrt{\frac{(a-c)^2(3+4\lambda+\delta(-6+8\lambda))}{\lambda+2\delta\lambda}}$, we know when $0<\lambda<\frac{3}{4}\&\frac{1}{2}<\delta<\frac{-1-\lambda}{-2+2\lambda}$, $w_1^*=\frac{a+c}{2}$ and when $0<\lambda<\frac{3}{4}\&\frac{-1-\lambda}{-2+2\lambda}\leq\delta\leq\frac{-3-4\lambda}{-6+8\lambda}$, $w_1^*=\frac{2a+c}{3}-\frac{1}{6}\sqrt{\frac{(a-c)^2(3+4\lambda+\delta(-6+8\lambda))}{\lambda+2\delta\lambda}}$; for case (3), the optimal wholesale price offered to Retailer 1 is $w_1^*=\frac{a+c}{2}$; for case (4), comparing $\frac{a+c}{2}$ with $\frac{2a+c}{3}-\frac{1}{6}\sqrt{\frac{(a-c)^2(3+4\lambda+\delta(-6+8\lambda))}{\lambda+2\delta\lambda}}$ and $\frac{2a+c}{3}+\frac{1}{6}\sqrt{\frac{(a-c)^2(3+4\lambda+\delta(-6+8\lambda))}{\lambda+2\delta\lambda}}$, we know when $\frac{1}{2}<\delta<\frac{7}{2}\&\frac{3}{4}\leq\lambda<1$ or $\delta\geq\frac{7}{2}\&\frac{-1+2\delta}{1+2\delta}<\lambda<1$, $w_1^*=\frac{a+c}{2}$ and when $\delta>\frac{7}{2}\&\frac{3}{4}\leq\lambda<\frac{-1+2\delta}{1+2\delta}$, $w_1^*=\frac{2a+c}{3}-\frac{1}{6}\sqrt{\frac{(a-c)^2(3+4\lambda+\delta(-6+8\lambda))}{\lambda+2\delta\lambda}}$.

By combining case (1), (2), (3) and (4), we know that when $0<\delta\leq\frac{1}{2}\&0<\lambda<1$ or $\delta>\frac{1}{2}\&\frac{-1+2\delta}{1+2\delta}\leq\lambda<1$, the optimal wholesale price offered to Retailer 1 is

$$w_1=\frac{a+c}{2} \tag{B.14}$$

And when $\delta>\frac{1}{2}\&\frac{-3+6\delta}{4+8\delta}<\lambda<\frac{-1+2\delta}{1+2\delta}$, the optimal wholesale price offered to Retailer 1 is

$$w_1=\frac{2a+c}{3}-\frac{1}{6}\sqrt{\frac{(a-c)^2\left(3+4\lambda+\delta\left(-6+8\lambda\right)\right)}{\lambda+2\delta\lambda}} \tag{B.15}$$

Eventually, by combining (B.14) and (B.15), we can conclude that when $w_2=\frac{a+c+a\delta+3c\delta}{2(1+2\delta)}$, the optimal wholesale price offered to Retailer 1 is

$$w_1=\begin{cases}\frac{a+c}{2} & if\ \ 0<\delta<\frac{1}{2}\&0<\lambda<1\ \ or\ \ \delta\geq\frac{1}{2}\&\frac{-1+2\delta}{1+2\delta}\leq\lambda<1\\ \frac{2a+c}{3}-\frac{1}{6}\sqrt{\frac{(a-c)^2\left(3+4\lambda+\delta\left(-6+8\lambda\right)\right)}{\lambda+2\delta\lambda}} & if\ \ \delta\geq\frac{1}{2}\&\frac{-3+6\delta}{4+8\delta}\leq\lambda<\frac{-1+2\delta}{1+2\delta}\end{cases} \tag{B.16}$$

The Supplier's Pricing Decision When $w_2=w^{II}$

When $\frac{a+c+a\delta+3c\delta}{2(1+2\delta)}<w^{II}$, it needs to satisfy one of the following conditions: (1) $0<\lambda<\frac{3}{4}\&\frac{1}{2}<\delta<\frac{-3-4\lambda}{-6+8\lambda}\&\frac{a+2c}{3}\leq w_1<\frac{a+2c}{3}-\frac{1}{6}\sqrt{\frac{(a-c)^2(3+4\lambda+\delta(-6+8\lambda))}{\lambda+2\delta\lambda}}$; (2) $0<\lambda<\frac{3}{4}\&\frac{1}{2}<\delta<\frac{-3-4\lambda}{-6+8\lambda}\&\frac{a+2c}{3}+\frac{1}{6}\sqrt{\frac{(a-c)^2(3+4\lambda+\delta(-6+8\lambda))}{\lambda+2\delta\lambda}}<w_1<a$; (3) $0<\lambda<\frac{3}{4}\&\delta\geq\frac{-3-4\lambda}{-6+8\lambda}\&\frac{1}{3}(a+2c)\leq w_1\leq a$; (4) $\frac{3}{4}\leq\lambda<1\&\delta>\frac{1}{2}\&\frac{a+2c}{3}\leq w_1<\frac{a+2c}{3}-\frac{1}{6}\sqrt{\frac{(a-c)^2(3+4\lambda+\delta(-6+8\lambda))}{\lambda+2\delta\lambda}}$; (5) $\frac{3}{4}\leq\lambda<1\&\delta>\frac{1}{2}\&\frac{a+2c}{3}+\frac{1}{6}\sqrt{\frac{(a-c)^2(3+4\lambda+\delta(-6+8\lambda))}{\lambda+2\delta\lambda}}<w_1<a$. Because the optimal wholesale price offered to Retailer 2 is the function of w_1, so the supplier is supposed to maximise his total profit $\pi_s=\pi_{s,1}+\pi_{s,2}$ by choosing the optimal wholesale price w_1. Taking the derivative of the supplier's total profit with respect to w_1, we give the optimal wholesale price w_1 under different conditions.

For case (1), the optimal wholesale price offered to Retailer 1 is given as below:

When $\frac{1}{2} < \delta \leq \delta^{1o}\&\frac{-3+6\delta}{4+8\delta} < \lambda < \frac{-1+2\delta}{1+2\delta}$, the optimal wholesale price offered to Retailer 1 is

$$w_1 = w_1^{\vee} \tag{B.17}$$

where $\lambda^{1o} = 4(2a+c-3x)^2(a+2c-3x)(a-x)(1+\delta)^2(1+2\delta)$, $\lambda^{2o} = 4c^3+4c^3\delta+7cx^2(1+2\delta)(3+2\delta)^2-6x^3(1+2\delta)(3+2\delta)^2-30c^2x-8c^2x\delta(9+\delta(7+2\delta))$, $\lambda^{3o} = -(a-c)^2(a+c-2x)^2(3+2\delta)$, $\lambda^{4o} = a^3(17+2\delta(19+2\delta(7+2\delta)))+-66a^2x+24\delta a^2x(8+\delta(7+2\delta))+3a^2c(5+2\delta(13+2\delta(7+2\delta)))$, $\lambda^{5o} = 2ac^2(1+2\delta)(3+2\delta)^2+11ax^2(1+2\delta)(3+2\delta)^2-2acx(33+2\delta(69+10\delta(7+2\delta)))$, $\lambda^{6o} = -c^3(-1+\delta+2\delta^2)+28cx^2(3+4\delta(2+\delta))-24x^3(3+4\delta(2+\delta))$, $\lambda^{7o} = 2a^3(7+3\delta(5+2\delta))-27c^2x-61\delta c^2x-26\delta c^2x\delta+3a^2c(11+3\delta(11+6\delta))-3a^2x(25+63\delta+30\delta^2)$ and $\lambda^{8o} = 8ac^2(3+4\delta(2+\delta))+44ax^2(3+4\delta(2+\delta))-2acx(57+\delta(163+86\delta)$. For the equation of $\lambda^{1o}+(\lambda^{2o}+\lambda^{4o}+\lambda^{5o})(a+c-2x)\lambda+\lambda^{3o}+(\lambda^{6o}+\lambda^{7o}+\lambda^{8o})(2a+c-3x)(1+\delta)\lambda^2 = 0$, we define $w_1^{\wedge}$ and $w_1^{\vee}$ as its first and third root, respectively. Meanwhile, we define δ^{1o} as the second root of the equation $-3303-5106x+5477x^2+13660x^3+8816x^4+4656x^5+2224x^6 = 0$.

Besides, when $\frac{1}{2} < \delta < \delta^{1o}\&\frac{-1+2\delta}{1+2\delta} < \lambda < \frac{3}{4}$, the optimal wholesale price offered to Retailer 1 is

$$w_1 = \frac{2a+c}{3} - \frac{1}{6}\sqrt{\frac{(a-c)^2(3+4\lambda+\delta(-6+8\lambda))}{\lambda+2\delta\lambda}} \tag{B.18}$$

.

When $\delta^{1o} < \delta < \delta^{2o}\&\frac{-3+6\delta}{4+8\delta} < \lambda \leq \frac{-1+2\delta}{1+2\delta}$, the optimal wholesale price offered to Retailer 1 is

$$w_1 = w_1^{\vee} \tag{B.19}$$

where we set δ^{2o} as the second root of equation $-3888-14256x-8422x^2+24603x^3+31770x^4-584x^5-10128x^6+3696x^7+4128x^8 = 0$.

When $\delta^{1o} < \delta < \delta^{2o}\&\frac{-1+2\delta}{1+2\delta} \leq \lambda < \frac{3}{4}$, the optimal wholesale price offered to Retailer 1 is

$$w_1 = \frac{2a+c}{3} - \frac{1}{6}\sqrt{\frac{(a-c)^2(3+4\lambda+\delta(-6+8\lambda))}{\lambda+2\delta\lambda}} \tag{B.20}$$

.

When $\delta^{2o} < \delta < \delta^{3o}\&\frac{-3+6\delta}{4+8\delta} < \lambda \leq \lambda^1$, the optimal wholesale price offered to Retailer 1 is

$$w_1 = w_1^{\vee} \tag{B.21}$$

where we set $\delta^{1*} = (-6480-60048\delta+12672\delta^2+20736\delta^3+9984\delta^4+1536\delta^5)x+1728+3456\delta+2304\delta^2+512\delta^3$, $\delta^{2*} = 1620-47736\delta-139968\delta^2-125184\delta^3-18432\delta^4+27648\delta^5+13056\delta^6+1536\delta^7$, $\delta^{3*} = 15093+103842\delta+91476\delta^2-315864\delta^3-716736\delta^4-567168\delta^5-183488\delta^6-9600\delta^7+5376\delta^8+512\delta^9$, $\delta^{4*} = -12069+15228\delta+470367\delta^2+1399038\delta^3+1549152\delta^4+334560\delta^5-719616\delta^6-645888\delta^7-208320\delta^8-23424\delta^9$, $\delta^{5*} = -12069-182574\delta-759834\delta^2-1030896\delta^3+691239\delta^4+3708654\delta^5+4676808\delta^6+2875632\delta^7+880752\delta^8+106080\delta^9$, $\delta^{6*} = 15093+101952\delta+26631\delta^2-1468486\delta^3-5416641\delta^4-9386988\delta^5-9240759\delta^6-5291526\delta^7-1643076\delta^8-213704\delta^9$, $\delta^{7*} = 1620+78192\delta+696636\delta^2+2853720\delta^3+6633468\delta^4+9477216\delta^5+8508996\delta^6+4690872\delta^7+1453104\delta^8+193824\delta^9$, $\delta^{8*} = -6480-89856\delta-525744\delta^2-1720224\delta^3-3492720\delta^4-4587840\delta^5-3915216\delta^6-2100384\delta^7-644544\delta^8-86400\delta^9$ and $\delta^{9*} = 1728+20736\delta+$

$108864\delta^2+328320\delta^3+627264\delta^4+787968\delta^5+651456\delta^6+342144\delta^7+103680\delta^8+13824\delta^9$. For the equation of $\delta^{1*}+\delta^{2*}x^2+\delta^{3*}x^3+\delta^{4*}x^4+\delta^{5*}x^5+\delta^{6*}x^6+\delta^{7*}x^7+\delta^{8*}x^8+\delta^{9*}x^9=0$, we define λ^1 is its second root. Meanwhile, we define δ^{3o} as the first root of the equation $-1913571-3219318x-1267146x^2-6134672x^3-4441059x^4+14697486x^5+14751720x^6-295056x^7-247536x^8+1839456x^9=0$.

When $\delta^{2o}<\delta<\delta^{3o}$ & $\lambda^1<\lambda<\frac{-1+2\delta}{1+2\delta}$, the optimal wholesale price offered to Retailer 1 is

$$w_1=w_1^{\wedge} \tag{B.22}$$

When $\delta^{2o}<\delta<\delta^{3o}$ & $\frac{-1+2\delta}{1+2\delta}\leq\lambda<\frac{3}{4}$, the optimal wholesale price offered to Retailer 1 is

$$w_1=\frac{2a+c}{3}-\frac{1}{6}\sqrt{\frac{(a-c)^2\left(3+4\lambda+\delta\left(-6+8\lambda\right)\right)}{\lambda+2\delta\lambda}} \tag{B.23}$$

.

When $\delta^{3o}\leq\delta\leq\frac{7}{2}\&\frac{-3+6\delta}{4+8\delta}<\lambda<\frac{-1+2\delta}{1+2\delta}$, the optimal wholesale price offered to Retailer 1 is

$$w_1=w_1^{\wedge} \tag{B.24}$$

When $\delta^{3o}\leq\delta\leq\frac{7}{2}\&\frac{-1+2\delta}{1+2\delta}\leq\lambda<\frac{3}{4}$, the optimal wholesale price offered to Retailer 1 is

$$w_1=\frac{2a+c}{3}-\frac{1}{6}\sqrt{\frac{(a-c)^2\left(3+4\lambda+\delta\left(-6+8\lambda\right)\right)}{\lambda+2\delta\lambda}} \tag{B.25}$$

.

When $\delta>\frac{7}{2}\&\frac{-3+6\delta}{4+8\delta}<\lambda<\frac{3}{4}$, the optimal wholesale price offered to Retailer 1 is

$$w_1=w_1^{\wedge} \tag{B.26}$$

By combining (B.18), (B.20), (B.23) and (B.25), we can conclude that when $\frac{1}{2}<\delta<\frac{7}{2}\&\frac{-1+2\delta}{1+2\delta}\leq\lambda<\frac{3}{4}$, the optimal wholesale price offered to Retailer 1 is

$$w_1=\frac{2a+c}{3}-\frac{1}{6}\sqrt{\frac{(a-c)^2\left(3+4\lambda+\delta\left(-6+8\lambda\right)\right)}{\lambda+2\delta\lambda}} \tag{B.27}$$

By combining (B.17),(B.19),(B.21),(B.22),and (B.24), we can conclude that when $\frac{1}{2}<\delta<\frac{7}{2}\&\frac{-3+6\delta}{4+8\delta}\leq\lambda<\frac{-1+2\delta}{1+2\delta}$, the optimal wholesale price offered to Retailer 1 is

$$w_1=w_1^{\wedge}\quad or\quad w_1^{\vee} \tag{B.28}$$

For case (2), the optimal wholesale price is

$$w_1=\frac{2a+c}{3}+\frac{1}{6}\sqrt{\frac{(a-c)^2\left(3+4\lambda+\delta\left(-6+8\lambda\right)\right)}{\lambda+2\delta\lambda}} \tag{B.29}$$

We find the supplier's total profit π_s will be larger if he chooses $w_1=\frac{2a+c}{3}-\frac{1}{6}\sqrt{\frac{(a-c)^2(3+4\lambda+\delta(-6+8\lambda))}{\lambda+2\delta\lambda}}$ other than $w_1=\frac{2a+c}{3}+\frac{1}{6}\sqrt{\frac{(a-c)^2(3+4\lambda+\delta(-6+8\lambda))}{\lambda+2\delta\lambda}}$. This means that case (2) needs not to be discussed.

For case (3), the optimal wholesale price w_1 is given as below.

When $\frac{1}{2} < \delta < \delta^{3o} \& 0 < \lambda < \frac{-3+6\delta}{4+8\delta}$, the optimal wholesale price offered to Retailer 1 is

$$w_1 = w_1^{\vee} \tag{B.30}$$

When $\delta \geq \delta^{3o}$ & $0 < \lambda < \lambda^1$, the optimal wholesale price offered to Retailer 1 is

$$w_1 = w_1^{\vee} \tag{B.31}$$

when $\delta \geq \delta^{3o}$ & $\lambda^1 < \lambda < \frac{-3+6\delta}{4+8\delta}$, the optimal wholesale price offered to Retailer 1 is

$$w_1 = w_1^{\wedge} \tag{B.32}$$

We can conclude that when $\delta > \frac{1}{2} \& 0 < \lambda < \frac{-3+6\delta}{4+8\delta}$, the optimal wholesale price offered to Retailer 1 is $w_1 = w_1^{\wedge}\ or\ w_1^{\vee}$.

For case (4), the optimal wholesale price w_1 is given as below:

When $\frac{1}{2} < \delta \leq \frac{7}{2} \& \frac{3}{4} \leq \lambda < 1$, the optimal wholesale price offered to Retailer 1 is

$$w_1 = \frac{2a+c}{3} - \frac{1}{6}\sqrt{\frac{(a-c)^2\left(3+4\lambda+\delta\left(-6+8\lambda\right)\right)}{\lambda+2\delta\lambda}} \tag{B.33}$$

When $\delta > \frac{7}{2} \&\& \frac{3}{4} \leq \lambda < \frac{-1+2\delta}{1+2\delta}$, the optimal wholesale price offered to Retailer 1 is

$$w_1 = w_1^{\wedge} \tag{B.34}$$

When $\delta > \frac{7}{2} \&\& \frac{-1+2\delta}{1+2\delta} \leq \lambda < 1$, the optimal wholesale price offered to Retailer 1 is

$$w_1 = \frac{2a+c}{3} - \frac{1}{6}\sqrt{\frac{(a-c)^2\left(3+4\lambda+\delta\left(-6+8\lambda\right)\right)}{\lambda+2\delta\lambda}} \tag{B.35}$$

By combining (B.33) and (B.35), we can conclude that when $\frac{3}{4} \leq \lambda < 1 \& \frac{1}{2} < \delta \leq \frac{-1-\lambda}{-2+2\lambda}$, the optimal wholesale price offered to Retailer 1 is

$$w_1 = \frac{2a+c}{3} - \frac{1}{6}\sqrt{\frac{(a-c)^2\left(3+4\lambda+\delta\left(-6+8\lambda\right)\right)}{\lambda+2\delta\lambda}} \tag{B.36}$$

For case (5), the optimal wholesale price w_1 is given as below:

$$w_1 = \frac{2a+c}{3} + \frac{1}{6}\sqrt{\frac{(a-c)^2\left(3+4\lambda+\delta\left(-6+8\lambda\right)\right)}{\lambda+2\delta\lambda}} \tag{B.37}$$

Note the situation of case (5) is similar with that of case (2), so case (5) needs not to be discussed.

By combining case (1), (2), (3), (4) and (5), we can conclude that when $w_2 = w^{II}$, the supplier's optimal wholesale price offered to Retailer 1 is given as follows:

$$w_1 = \begin{cases} w_1^{\wedge}\ \ or\ \ w_1^{\vee} & if\ \delta > \frac{1}{2} \& 0 < \lambda < \frac{-1+2\delta}{1+2\delta} \\ \frac{2a+c}{3} - \frac{1}{6}\sqrt{\frac{(a-c)^2(3+4\lambda+\delta(-6+8\lambda))}{\lambda+2\delta\lambda}} & if\ \delta > \frac{1}{2} \& \frac{-1+2\delta}{1+2\delta} \leq \lambda < 1 \end{cases} \tag{B.38}$$

APPENDIX B.3. The Equilibrium Prices

From (B.12), (B.16) and (B.38), we know that the supplier has three options of the optimal wholesale prices given as follows:

Option 1:

$$w_1 = \begin{cases} w_1^{I*} & if \quad 0 < \lambda < \lambda^* \\ w_1^{II*} & if \quad \lambda^* \leq \lambda < 1 \end{cases} \tag{B.39}$$

$$w_2 = \frac{2a + c - \sqrt{(a-c)^2 + 12t\lambda}}{3}, p_2 = \frac{a + w_2}{2} \tag{B.40}$$

Option 2:

$$w_1 = \begin{cases} \frac{a+c}{2} & if \quad 0 < \delta \leq \frac{1}{2} \& 0 < \lambda < 1 \quad or \quad \delta > \frac{1}{2} \& \frac{-1+2\delta}{1+2\delta} \leq \lambda < 1 \\ \frac{2a+c}{3} - \frac{1}{6}\sqrt{\frac{(a-c)^2 (3 + 4\lambda + \delta(-6+8\lambda))}{\lambda + 2\delta\lambda}} & if \quad \delta > \frac{1}{2} \& \frac{-3+6\delta}{4+8\delta} < \lambda < \frac{-1+2\delta}{1+2\delta} \end{cases} \tag{B.41}$$

$$w_2 = \frac{a + c + a\delta + 3c\delta}{2(1+2\delta)}, \ p_2 = \frac{-c\delta + a(1+\delta) + (1+2\delta) w_2}{2(1+\delta)} \tag{B.42}$$

Option 3:

$$\mathrm{w}_1 = \begin{cases} w_1^{\wedge} \quad or \quad w_1^{\vee} & if \ \delta > \frac{1}{2} \& 0 < \lambda < \frac{-1+2\delta}{1+2\delta} \\ \frac{2a+c}{3} - \frac{1}{6}\sqrt{\frac{(a-c)^2(3+4\lambda+\delta(-6+8\lambda))}{\lambda+2\delta\lambda}} & if \ \delta > \frac{1}{2} \& \frac{-1+2\delta}{1+2\delta} \leq \lambda < 1 \end{cases} \tag{B.43}$$

$$w_2 = w^{II}, p_2 = \frac{-c\delta + a(1+\delta) + (1+2\delta) w_2}{2(1+\delta)} \tag{B.44}$$

APPENDIX C. Peer-regarding Fairness Model For Schadenfreude

APPENDIX C.1. Retailer 2's Best Response Function

If Retailer 2 doesn't experience PF for schadenfreude, that is, $\pi_{s,2} - \pi_2 \leq \eta(\pi_{s,1} - \pi_1)^+$. It requires $p_2^I \leq p_2 < p_2^{II}$, where we use p_2^I and p_2^{II} to denote $\frac{(a-c+2w_2)-\sqrt{(a+c-2w_2)^2+4t\eta}}{2}$ and $\frac{(a-c+2w_2)+\sqrt{(a+c-2w_2)^2+4t\eta}}{2}$, respectively.

Retailer 2's utility-maximization problem upon acceptance is given by

$$\max_{p_2} \quad \pi_2 \tag{C.1}$$

$$s.t. \quad p_2^I \leq p_2 < p_2^{II} \tag{C.2}$$

Without constraint (B.2), Retailer 2's optimal retail price is $p_2 = \frac{a+w_2}{2}$.

If Retailer 2 experiences PF for schadenfreude, that is, $\pi_{s,2} - \pi_2 > \eta t$. It requires $p_2 < p_2^I$ or $p_2 \geq p_2^{II}$. But $p_2^{II} > a$, so we give up the case of $p_2 \geq p_2^{II}$ and only consider the case of $p_2 < p_2^I$.

Retailer 2's utility-maximization problem upon acceptance is given by

$$\max_{p_2} \quad \pi_2 - \delta\left((\pi_{s,2} - \pi_2) - \eta(\pi_{s,1} - \pi_1)^+\right) \tag{C.3}$$

$$s.t. \quad p_2 < p_2^I \tag{C.4}$$

Without constraint (B.4), Retailer 2's optimal retail price is $p_2 = \frac{-c\delta+a(1+\delta)+(1+2\delta)w_2}{2(1+\delta)}$.

The objective of Retailer 2 is to set his retail price to maximise his utility. That's to say, Retailer 2 is supposed to choose his optimal retail price from $\frac{-c\delta+a(1+\delta)+(1+2\delta)w_2}{2(1+\delta)}$, p_2^I and $\frac{a+w_2}{2}$. Notice $\frac{a+w_2}{2} < \frac{-c\delta+a(1+\delta)+(1+2\delta)w_2}{2(1+\delta)}$ always holds for $w_2 \in (c, a)$. We study Retailer 2's optimal pricing decision according to the relationships among $\frac{a+w_2}{2}$, $\frac{-c\delta+a(1+\delta)+(1+2\delta)w_2}{2(1+\delta)}$ and p_2^I. We can get three cases shown below:

Case 1. Retailer 2's Pricing Decision when $\frac{a+w_2}{2} < \frac{-c\delta+a(1+\delta)+(1+2\delta)w_2}{2(1+\delta)} \leq p_2^I$

This case requires that $0 < \eta < \frac{(a-c)^2}{12t+32t\delta+16t\delta^2} \& w^{III} \leq w_2 \leq w^{IV}$, where we set $w^{III} = \frac{2a(1+\delta)^2+c(1+2\delta(2+\delta))-\sqrt{(1+\delta)^2\left((a-c)^2-4t\eta(3+4\delta(2+\delta))\right)}}{3+4\delta(2+\delta)}$ and $w^{IV} = \frac{2a(1+\delta)^2+c(1+2\delta(2+\delta))+\sqrt{(1+\delta)^2\left((a-c)^2-4t\eta(3+4\delta(2+\delta))\right)}}{3+4\delta(2+\delta)}$.

When Retailer 2's utility function is $u_2 = \pi_2$, the interval of retail price is $p_2 \in [p_2^I, p_2^{II})$, and Retailer 2's optimal retail price is $p_2 = p_2^I$; when Retailer's utility function is $u_2 = \pi_2 - \delta\left((\pi_{s,2} - \pi_2) - \eta(\pi_{s,1} - \pi_1)^+\right)$, the interval of retail price is $p_2 < p_2^I$, and Retailer 2's optimal retail price is $p_2 = \frac{-c\delta+a(1+\delta)+(1+2\delta)w_2}{2(1+\delta)}$.

Comparing the two choices, we find Retailer 2 will choose the second choice because it makes his utility larger.

Case 2. Retailer 2's Pricing Decision When $\frac{a+w_2}{2} \leq p_2^I < \frac{-c\delta+a(1+\delta)+(1+2\delta)w_2}{2(1+\delta)}$

The limiting conditions of this case are given as below:

(1) $0 < \eta \leq \frac{(a-c)^2}{12t+32t\delta+16t\delta^2} \& \frac{2a+c-\sqrt{(a-c)^2-12t\eta}}{3} \leq w_2 < w_2^{III}$; (2) $0 < \eta \leq \frac{(a-c)^2}{12t+32t\delta+16t\delta^2} \& w^{IV} < w_2 \leq \frac{2a+c+\sqrt{a^2-2ac+c^2-12t\eta}}{3}$; (3) $\frac{a^2-2ac+c^2}{12t+32t\delta+16t\delta^2} < \eta < \frac{a^2-2ac+c^2}{12t} \& \frac{2a+c-\sqrt{a^2-2ac+c^2-12t\eta}}{3} \leq w_2 \leq \frac{2a+c+\sqrt{a^2-2ac+c^2-12t\eta}}{3}$.

If Retailer 2's utility function is $u_2 = \pi_2$, his optimal retail price will be $p_2 = p_2^I$; if it is $u_2 = \pi_2 - \delta((\pi_{s,2} - \pi_2) - \eta(\pi_{s,1} - \pi_1)^+)$, his optimal retail price will be $p_2 = p_2^I$ as well. Thus, for case 2, Retailer 2 will choose $p_2 = p_2^I$.

Case 3. Retailer 2's Pricing Decision When $p_2^I < \frac{a+w_2}{2} < \frac{-c\delta+a(1+\delta)+(1+2\delta)w_2}{2(1+\delta)}$

The limiting conditions of this case are given as follows:

(1)$0 < \eta \leq \frac{(a-c)^2}{12t} \& c < w_2 < \frac{2a+c-\sqrt{a^2-2ac+c^2-12t\eta}}{3}$; (2)$0 < \eta \leq \frac{(a-c)^2}{12t} \& \frac{2a+c+\sqrt{a^2-2ac+c^2-12t\eta}}{3} < w_2 < a$; (3)$\eta > \frac{(a-c)^2}{12t} \& c < w_2 < a$.

If Retailer 2's utility function is $u_2 = \pi_2$, his optimal retail price will be $p_2 = \frac{a+w_2}{2}$; if Retailer 2's utility function is $u_2 = \pi_2 - \delta\left((\pi_{s,2} - \pi_2) - \eta(\pi_{s,1} - \pi_1)^+\right)$, his optimal retail price will be $p_2 = p_2^I$. It's easy to find that Retailer 2's optimal retail price will be $p_2 = \frac{a+w_2}{2}$.

By combining Cases 1, 2 and 3, we can get Retailer 2's optimal retail price additional on the profit inequality between the supplier and Retailer 1, and the acceptance of wholesale price offer w_2 given as follows:

$$p_2 = \begin{cases} \frac{-c\delta+a(1+\delta)+(1+2\delta)w_2}{2(1+\delta)} & if \quad \eta \in (0,\Theta)\&w_2 \in [w^{III}, w^{IV}) \\ \frac{a-c+2w_2-2\sqrt{\left(w_2-\frac{a+c}{2}\right)^2+t\eta}}{2} & if \quad \eta \in (0,\Theta)\&w_2 \in [w_1^A, w^{III}) \cup [w^{IV}, w_1^B) \\ & or \quad \eta \in [\Theta, \frac{(a-c)^2}{12t})\&w_2 \in [w_1^A, w_1^B) \\ \frac{a+w_2}{2} & if \quad \eta \in (0, \frac{(a-c)^2}{12t})\&w_2 \in (c, w_1^A) \cup [w_1^B, a) \\ & or \quad \eta \in [\frac{(a-c)^2}{12t}, \infty)\&w_2 \in (c,a), \end{cases} \tag{C.5}$$

where $\Theta = \frac{(a-c)^2}{12t+32t\delta+16t\delta^2}$, $w_1^A = \frac{2a+c-\sqrt{(a-c)^2-12t\eta}}{3}$, $w_1^B = \frac{2a+c+\sqrt{(a-c)^2-12t\eta}}{3}$.

APPENDIX C.2. The Supplier's Pricing Decision

APPENDIX C.2.1. The Supplier's Pricing Decision When $p_2 = \frac{a+w_2}{2}$

When Retailer 2 sets his retail price as $\frac{a+w_2}{2}$, we divide the the limiting conditions into two subsections bellow according to (C.5).

2.1.1 The Supplier's Pricing Decision When $\eta \in (0, \frac{(a-c)^2}{12t})\&w_2 \in (c, w_1^A) \cup [w_1^B, a)$

Without the limiting conditions, the supplier's optimal wholesale price is $w_1^* = w_2^* = \frac{a+c}{2}$. By comparing $\frac{a+c}{2}$ with w_1^A and w_1^B under the conditions of $\eta \in (0, \frac{(a-c)^2}{12t})$, we can get: (1) when $\frac{a+c}{2} \leq w_1^A$, the supplier's optimal wholesale price is $w_2 = \frac{a+c}{2}$, and the case needs to be satisfied by $\eta \in [\frac{(a-c)^2}{16t}, \frac{(a-c)^2}{12t})$; (2) when $w_1^A < \frac{a+c}{2} < w_1^B$, the supplier's optimal wholesale price offered to Retailer 2 is $w_2 = w_1^A$, and the case needs to be satisfied by $\eta \in (0, \frac{(a-c)^2}{16t})$; (3) $\frac{a+c}{2} \geq w_1^B$ never holds under the given conditions.

Next, the supplier decides his optimal wholesale price offered to Retailer 1 to maximise his total profit.

2.1.1.1 The Supplier's Pricing Decision When $w_2 = \frac{a+c}{2}$

Given the conditions of $\eta \in [\frac{(a-c)^2}{16t}, \frac{(a-c)^2}{12t})$, the supplier maximises $\pi_{s,1}$ by offering the optimal wholesale price to Retailer 1. We can get the supplier's optimal wholesale price offered to Retailer 1 given as follows:

$$w_1 = \begin{cases} \frac{2a+c}{3} - \frac{1}{6}\sqrt{\frac{-3(a-c)^2+4\eta(a-c)^2}{\eta}} & if \quad \frac{3}{4} \leq \eta < 1 \\ \frac{a+c}{2} & if \quad 1 \leq \eta < \frac{4}{3} \\ \frac{2a+c}{3} - \frac{1}{3}\sqrt{\frac{-(a-c)^2+\eta(a-c)^2}{\eta}} & if \quad \eta \geq \frac{4}{3} \end{cases} \tag{C.6}$$

2.1.1.2 The Supplier's Pricing Decision When $w_2 = w_1^A$

Given the conditions of $\eta \in (0, \frac{(a-c)^2}{16t})$, we know $\pi_{s,2}$ is the function of w_1. So the supplier maximises his total profit by choosing the optimal wholesale price offered to Retailer 1. Take the derivative of π_s with respect to w_1 and we get it shown as follows:

$$w_1 = \begin{cases} \mathrm{w}_1^{\#} & if \quad 0 < \eta < 1 \\ \frac{2a+c}{3} - \frac{1}{6}\sqrt{\frac{-3(a-c)^2+4\eta(a-c)^2}{\eta}} & if \quad \eta \geq 1 \end{cases} \tag{C.7}$$

where we set $\eta_1 = 3a^4-6a^2c^2+3c^4+17a^4\eta+32a^3c\eta+33a^2c^2\eta+22ac^3\eta+4c^4\eta+28a^4\eta^2+80a^3c\eta^2+81a^2c^2\eta^2+26ac^3\eta^2+c^4\eta^2+16a^4\eta^3+48a^3c\eta^3+36a^2c^2\eta^3+8ac^3\eta^3$, $\eta_2 =$

$-12a^3+12a^2c+12ac^2-12c^3-100a^3\eta-162a^2c\eta-132ac^2\eta-38c^3\eta-192a^3\eta^2-402a^2c\eta^2-240ac^2\eta^2-30c^3\eta^2-112a^3\eta^3-216a^2c\eta^3-96ac^2\eta^3-8c^3\eta^3$, $\eta_3 = 12a^2-24ac+12c^2+231a^2\eta+294ac\eta+123c^2\eta+489a^2\eta^2+642ac\eta^2+165c^2\eta^2+276a^2\eta^3+312ac\eta^3+60c^2\eta^3$, $\eta_4 = -252a\eta-180c\eta-540a\eta^2-324c\eta^2-288a\eta^3-144c\eta$ and $\eta_5 = 108\eta+216\eta^2+108\eta^3$. For the equation of $\eta_1+\eta_2x+\eta_3x^2+\eta_4x^3+\eta_5x^4=0$, we define $w_1^{\#}$ as its first root.

2.1.2 The Supplier's Pricing Decision When $\eta \in [\frac{(a-c)^2}{12t}, \infty)$ $\& w_2 \in (c,a)$

The optimal wholesale price offered to Retailer 2 is $w_2 = \frac{a+c}{2}$. Thus, to maximise his total profit, the supplier just needs to choose the optimal wholesale price offered to Retailer 1. Take the derivative of $\pi_{s,1}$ with respect to w_1 and we get it shown as follows:

$$w_2 = \begin{cases} \dfrac{2a+c}{3} - \dfrac{1}{3}\sqrt{\dfrac{-(a-c)^2+\eta(a-c)^2}{\eta}} & if \quad 1 \le \eta < \dfrac{4}{3} \\ \dfrac{a+c}{2} & if \quad \eta \ge \dfrac{4}{3} \end{cases} \tag{C.8}$$

Given (C.6), (C.7) and (C.8), compare the supplier's total profit and we can get the supplier's optimal wholesale prices when $p_2 = \frac{a+w_2}{2}$. They are shown as follows:

when $0 < \eta < 1$,

$$w_1 = w_1^{\#}, w_2 = \frac{2a+c}{3} - \frac{\sqrt{(a-c)^2 - 12t\eta}}{3}; \tag{C.9}$$

when $\eta \ge 1$,

$$w_1 = w_2 = \frac{a+c}{2} \tag{C.10}$$

APPENDIX C.2.2. The Supplier's Pricing Decision When $p_2 = \frac{a-c+2w_2-2\sqrt{\left(w_2-\frac{a+c}{2}\right)^2+t\eta}}{2}$

Applying backward induction, to maximise his total profit, the supplier first decides the optimal wholesale price offered to Retailer 2. Taking the derivative of $\pi_{s,2}$ with respect to w_2, we can get the optimal wholesale price offered to Retailer 2 is $w_2 = \frac{2a+c}{3} - \frac{\sqrt{(a-c)^2-12t\eta}}{3}$. We find the optimal solutions of C.2.2 and 2.1.1.2 are the same. So this subsection needs not to be discussed.

APPENDIX C.2.3. The Supplier's Pricing Decision When $p_2 = \frac{-c\delta+a(1+\delta)+(1+2\delta)w_2}{2(1+\delta)}$

The limiting conditions are $\eta \in (0,\Theta)\& w_2 \in [w^{III}, w^{IV})$. Without the constraint, the supplier's optimal wholesale price is $w_2 = \frac{a+c+a\delta+3c\delta}{2(1+2\delta)}$. Comparing $\frac{a+c+a\delta+3c\delta}{2(1+2\delta)}$ with w^{III} and w^{IV}, we can get: (1) if $\frac{a+c+a\delta+3c\delta}{2(1+2\delta)} \le w^{III}$, the optimal wholesale price is $w_2 = w^{III}$, and the case needs to satisfy that $0 < \delta \le \frac{1}{2} \& \frac{(a-c)^2-2a^2\delta+4ac\delta-2c^2\delta}{16t+32t\delta} < \eta < \frac{(a-c)^2}{12t+32t\delta+16t\delta^2}$ or $\delta > \frac{1}{2} \& 0 < \eta < \frac{(a-c)^2}{12t+32t\delta+16t\delta^2}$; (2) if $w^{III} < \frac{a+c+a\delta+3c\delta}{2(1+2\delta)} < w^{IV}$, the supplier's optimal wholesale price is $w_2 = \frac{a+c+a\delta+3c\delta}{2(1+2\delta)}$, and the case needs to satisfy that $0 < \delta < \frac{1}{2} \& 0 < \eta < \frac{a-c)^2-2\delta(a-c)^2}{16t+32t\delta}$; (3) given the conditions, $\frac{a+c+a\delta+3c\delta}{2(1+2\delta)} < w^{IV}$ always hold.

By combining case (1), (2) and (3), we can get

$$w_2 = \begin{cases} \dfrac{a+c+a\delta+3c\delta}{2(1+2\delta)} & if \quad 0<\delta<\dfrac{1}{2}\&0<\eta<\dfrac{(a-c)^2(1-2\delta)}{16t+32t\delta} \\ w_1^{III} & if \quad 0<\delta\leq\dfrac{1}{2}\&\dfrac{(a-c)^2(1-2\delta)}{16t+32t\delta}\leq\eta<\dfrac{(a-c)^2}{12t+32t\delta+16t\delta^2} \\ & or \quad \delta>\dfrac{1}{2}\&0<\eta<\dfrac{(a-c)^2}{12t+32t\delta+16t\delta^2} \end{cases} \tag{C.11}$$

Applying backward induction, the supplier is supposed to determine the optimal wholesale price w_1 to maximise his total profit. We divide this process into two parts shown as follows.

2.3.1 The Supplier's Pricing Decision When $w_2 = \frac{a+c+a\delta+3c\delta}{2(1+2\delta)}$

When $w_2 = \frac{a+c+a\delta+3c\delta}{2(1+2\delta)}$, the limiting conditions are $0<\delta<\frac{1}{2}\&0<\eta<\frac{(a-c)^2-2\delta(a-c)^2}{16t+32t\delta}$. Because $\pi_{s,2}$ is only the function of w_2, so the supplier just needs to maximise $\pi_{s,1}$. Without the limiting conditions, the supplier's optimal wholesale price offered to Retailer 1 is $w_1 = \frac{a+c}{2}$. By comparing $\frac{a+c}{2}$ with the interval of w_1, we can get the supplier's optimal wholesale price w_1 shown as follows:

$$w_1 = \begin{cases} \dfrac{a+c}{2} & if\ 0<\delta<\dfrac{1}{2}\&0<\eta<\dfrac{1-2\delta}{1+2\delta} \\ \dfrac{2a+c}{3}-\dfrac{1}{6}\sqrt{\dfrac{(a-c)^2(-3+6\delta+\eta(4+8\delta))}{\eta+2\eta\delta}} & if\ 0<\delta<\dfrac{1}{2}\&\eta\geq\dfrac{1-2\delta}{1+2\delta} \end{cases} \tag{C.12}$$

2.3.2 The Supplier's Pricing Decision When $w_2 = w^{III}$

The profit of the supplier in Supply Chain 2 is the function of w_1. Thus, the supplier gives the optimal wholesale price w_1 to maximise his total profit. According to the restricted conditions, we divide this process into two subsections as below.

2.3.2.1 The supplier's pricing decision when $0<\delta\leq\frac{1}{2}\&\frac{(a-c)^2-2\delta(a-c)^2}{16t+32t\delta}\leq\eta<\frac{(a-c)^2}{12t+32t\delta+16t\delta^2}$

We take the derivative of π_s with respect to w_1 and get the optimal wholesale price w_1 given as follows:

$$w_1 = \begin{cases} \dfrac{2a+c}{3}-\dfrac{1}{6}\sqrt{\dfrac{(a-c)^2(-3+6\delta+\eta(4+8\delta))}{\eta+2\eta\delta}} & if \quad 0<\delta\leq\dfrac{1}{2}\&\dfrac{3-6\delta}{4+8\delta}<\eta\leq\dfrac{1-2\delta}{1+2\delta} \\ \mathrm{w}_1^{\exists} & if \quad 0<\delta\leq\dfrac{1}{2}\&\eta>\dfrac{1-2\delta}{1+2\delta}, \end{cases} \tag{C.13}$$

where we define $\delta_1 = -c^4(1+\eta+\eta\delta)(-3-2\delta+\eta(-1+\delta+2\delta^2))+33a^2c^2\eta+4a^3c\eta\left(8+12\eta^2(1+\delta)^2(1+2\delta)+\eta(1+\delta)(20+57\delta+30\delta^2)+\delta(29+4\delta(7+2\delta))\right)+$

$2ac^3\eta\left(11+4\eta^2(1+\delta)^2(1+2\delta)+4\delta(8+\delta(7+2\delta))+\eta(1+\delta)(13+\delta(31+14\delta))\right)+$

$a^4\eta\left(17+16\eta^2(1+\delta)^2(1+2\delta)+4\eta(1+\delta)(7+3\delta(5+2\delta))+2\delta(19+2\delta(7+2\delta))\right)+$

$a^2c^2\eta\left(36\eta^2(1+\delta)^2(1+2\delta)+2\delta(69+10\delta(7+2\delta))+\eta(1+\delta)(81+\delta(227+118\delta))\right)$ $+(3+2\delta)a^4-(6+4\delta)a^2c^2$, $\delta_2 = \eta^2\left(15+44\delta+39\delta^2+10\delta^3\right)c^3+4\eta^3(1+\delta)^2(1+2\delta)c^3+\eta\left(19+40\delta+28\delta^2+8\delta^3\right)c^3+56\eta^3a^3(1+\delta)^2(1+2\delta)+2a^3\eta\left(25+67\delta+56\delta^2+16\delta^3\right)+6c^3+4\delta c^3+6a^3+4\delta a^3+6a^3\eta^2\left(16+55\delta+57\delta^2+18\delta^3\right)+312\delta a^2c\eta+308\delta^2a^2c\eta+$

$88\delta^3a^2c\eta-6ac^2-4\delta ac^2+40ac^2\eta^2\left(3+11\delta+12\delta^2+4\delta^3\right)-6a^2c-4\delta a^2c+201\eta^2a^2c+770\delta\eta^2a^2c+108\eta^3(1+\delta)^2\left(1+2\delta\right)a^2c+81a^2c\eta+2ac^2\eta\left(33+117\delta+112\delta^2+32\delta^3\right)+867\delta^2\eta^2a^2c+298\delta^3\eta^2a^2c+48ac^2\eta^3(1+\delta)^2\left(1+2\delta\right)$, $\delta_3=60\eta^3c^2(1+\delta)^2\left(1+2\delta\right)+\eta^2c^2\left(1+\delta\right)\left(165+\delta\left(407+190\delta\right)\right)-8ac\left(3+2\delta\right)+2\delta\eta c^2\left(177+22\delta\left(7+2\delta\right)\right)+294\eta ac+312\eta^3ac(1+\delta)^2\left(1+2\delta\right)+4\delta\eta ac\left(273+38\delta\left(7+2\delta\right)\right)+2\delta\eta a^2\left(357+46\delta\left(7+2\delta\right)\right)+12a^2+8\delta a^2+231\eta a^2+276\eta^3a^2(1+\delta)^2\left(1+2\delta\right)+a^2\eta^2\left(1+\delta\right)\left(489+\delta\left(1271+622\delta\right)\right)+12c^2+8\delta c^2+123\eta c^2+2ac\eta^2\left(1+\delta\right)\left(321+\delta\left(889+458\delta\right)\right)$, $\delta_4=-252a\eta-180c\eta-540a\eta^2-324c\eta^2-288a\eta^3-144c\eta^3-840a\eta\delta-600c\eta\delta-1980a\eta^2\delta-1188c\eta^2\delta-1152a\eta^3\delta-576c\eta^3\delta-784a\eta\delta^2-560c\eta\delta^2-2160a\eta^2\delta^2-1296c\eta^2\delta^2-1440a\eta^3\delta^2-720c\eta^3\delta^2-224a\eta\delta^3-160c\eta\delta^3-720a\eta^2\delta^3-432c\eta^2\delta^3-576a\eta^3\delta^3-288c\eta^3\delta^3$ and $\delta_5=108\eta+216\eta^2+108\eta^3+360\eta\delta+792\eta^2\delta+432\eta^3\delta+336\eta\delta^2+864\eta^2\delta^2+540\eta^3\delta^2+96\eta\delta^3+288\eta^2\delta^3+216\eta^3\delta^3$. For the equation of $\delta_1+2\delta_2x+\delta_3x^2+\delta_4x^3+\delta_5x^4=0$, we define $w_1^{\exists}$ as its first root.

2.3.2.2 The supplier's pricing decision when $\delta>\frac{1}{2}\&0<\eta<\frac{(a-c)^2}{12t+32t\delta+16t\delta^2}$

Take the derivative of π_s with respect to w_1 and we get the optimal wholesale price offered to Retailer 1 given as follows: (1)when $\delta>\frac{1}{2}\&0<\eta\leq\frac{3}{3+8\delta+4\delta^2}$, $w_1=w_1^{\exists}$; (2) when $\delta>\frac{1}{2}\&\eta\geq\frac{3+2\delta}{-3+3\delta+6\delta^2}$, $w_1=\frac{a+2c}{3}$; (3) when $\delta>\frac{1}{2}\&\frac{3}{3+8\delta+4\delta^2}<\eta\leq\frac{3+2\delta}{-3+3\delta+6\delta^2}$, $w_1=w_1^{\exists}$.

By combining (1),(2) and (3), we can get the optimal wholesale price offered to the first retailer given as follows:

$$w_1=\begin{cases} w_1^{\exists} & if \quad \delta>\dfrac{1}{2}\&0<\eta<\dfrac{3+2\delta}{-3+3\delta+6\delta^2} \\ \dfrac{a+2c}{3} & if \quad \delta>\dfrac{1}{2}\&\eta\geq\dfrac{3+2\delta}{-3+3\delta+6\delta^2} \end{cases} \tag{C.14}$$

The supplier compares (C.12) with (C.13) to find the optimal wholesale prices to maximise his total profit in the distribution channel. The results are shown as follows:

(1) when $0<\delta\leq\frac{1}{2}\&0<\eta\leq\frac{1-2\delta}{1+2\delta}$,

$$w_1=\frac{a+c}{2},w_2=\frac{a+c+a\delta+3c\delta}{2\left(1+2\delta\right)}; \tag{C.15}$$

(2) when $0<\delta\leq\frac{1}{2}\&\eta>\frac{1-2\delta}{1+2\delta}$,

$$w_1=w_1^{\exists},w_2=w^{III}, \tag{C.16}$$

where $i=1,2$.

APPENDIX D. Proofs

APPENDIX D.1. Proof of Proposition 1

Proof. The proof of the proposition can be seen in (B.9) by combining Retailer 2's best-response price functions (B.5) and (B.8). In particular, $w^I=\frac{2a+c-\sqrt{(a-c)^2+12t\lambda}}{3}$ and $w^{II}=\frac{2a(1+\delta)^2+c(1+2\delta(2+\delta))-(1+\delta)\sqrt{\left((a-c)^2+4t(3+4\delta(2+\delta))\lambda\right)}}{3+4\delta(2+\delta)}$. □

APPENDIX D.2. Proof of Proposition 2

Proof. The supplier charges his optimal wholesale prices w_1^* and w_2^* to maximise his total profit $\pi_s = \pi_{s,1} + \pi_{s,2}$ in the distribution channel. Firstly, we trade off Option 1 and Option 3 in *Appendix B.3* to exclude the suboptimal choice. Compare the supplier's total profit in Option 1 and Option 3 under the conditions of $\delta > 0$, $0 < \lambda < 1$ and $w_1 \in (c, a)$. We find that the supplier's profit in Option 3 is no superior to that in Option 1. Thus, Option 3 is supposed to be abandoned.

Then, we compare Option 1 and Option 2 to get the supplier's optimal choice. Firstly, we compare Option 1 with Option 2 under the conditions of $0 < \delta < \frac{1}{2}$ and $0 < \lambda < 1$. We use π_s^1 and π_s^2 to separately denote the supplier's total profit in the distribution channel under the condition of $0 < \lambda < \lambda^*$ or $\lambda^* \leq \lambda < 1$. In Option 2, when $0 < \delta < \frac{1}{2}$ and $0 < \lambda < 1$, the supplier's total profit is $\pi_s^{\theta 1} = \frac{(a-c)^2(1+\delta)}{8+16\delta} + \frac{(a-c)^2}{8}$. Comparing it with π_s^1 under the condition of $0 < \lambda < \lambda^*$, we find that when $\delta \geq \frac{1}{3}$, $\pi_s^{\theta 1} > \frac{(a-c)^2(1+\delta)}{8+16\delta} + \frac{(a-c)^2}{8}$. And when $0 < \delta \leq \frac{1}{7}$, $\pi_s^{\theta 1} < \frac{(a-c)^2(1+\delta)}{8+16\delta} + \frac{(a-c)^2}{8}$. So we can conclude there exists $\delta_1^*(\lambda) \in (\frac{1}{7}, \frac{1}{3})$ which satisfies that when $0 < \lambda < \lambda^*$ and $0 < \delta < \delta_1^*(\lambda)$ the supplier charges the optimal wholesale price as $w_1^* = \frac{a+c}{2}$, and when $\delta_1^*(\lambda) \leq \delta < \frac{1}{2}$, the supplier charges the optimal wholesale price as $w_1^* = w_1^I$. Compare π_s^2 with $\pi_s^{\theta 1}$ under the condition of $\lambda^* \leq \lambda < 1$. The case is similar with the above and we can conclude that there exists $\delta_2^*(\lambda) \in (\frac{1}{4}, \frac{2}{5})$ which satisfies that under the conditions of $\lambda^* \leq \lambda < 1$ and $0 < \delta < \delta_2^*(\lambda)$, the supplier charges optimal wholesale price as $w_1^* = \frac{a+c}{2}$, and when $\delta_1^*(\lambda) \leq \delta < \frac{1}{2}$, the supplier charges optimal wholesale price as $w_1^* = w_1^{II}$. It's easy to know that when $0 < \delta < \frac{1}{2}$ and $\frac{-1+2\delta}{1+2\delta} \leq \lambda < 1$, the supplier's optimal wholesale price is $w_1^* = w_1^{II}$. For convenience of expression, we use $\delta^*(\lambda)$ to denote $\delta_1^*(\lambda)$ under the condition of $0 < \lambda < \lambda^*$ and $\delta_2^*(\lambda)$ under the condition of $\lambda^* \leq \lambda < 1$. Therefore, there exists $\delta^*(\lambda) \in (\frac{1}{7}, \frac{2}{5})$ which satisfies that when $\delta \in (0, \delta^*(\lambda))$ and $0 < \lambda < 1$, the equilibrium results of the game are $w_1^* = \frac{a+c}{2}, p_1^* = \frac{3a+c}{4}$, $w_2^* = \frac{a+c}{2} - \frac{\delta(a-c)}{2(1+2\delta)}$ and $p_2^* = \frac{3a+c}{4}$; when $\delta \in [\delta^*(\lambda), \frac{1}{2})$ and $0 < \lambda < 1$, or $\delta \geq \frac{1}{2}$ and $\frac{-3+6\delta}{4+8\delta} \leq \lambda < \frac{-1+2\delta}{1+2\delta}$, the equilibrium results of the game are $w_1^* = w_1^{II}$ *or* w_1^{II*}, $w_2^* = \frac{2a+c-\sqrt{(a-c)^2+12t\lambda}}{3}, p_2 = \frac{a+w_2}{2}$.

Now we investigate the relationships between $\delta^*(\lambda)$ and λ. When $0 < \lambda < \lambda^*$, π_s^1 is decreasing with λ; when $\lambda^* \leq \lambda < 1$, π_s^2 is decreasing with λ. $\pi_s^{\theta 1}$ is decreasing with δ. So when $0 < \lambda < \lambda^*$, to make $\pi_s^1 = \pi_s^{\theta 1}$, higher λ means higher $\delta_1^*(\lambda)$; when $\lambda^* \leq \lambda < 1$, to make $\pi_s^2 = \pi_s^{\theta 1}$, higher λ means higher $\delta_2^*(\lambda)$. $\lim\limits_{\lambda \to \lambda^{*-}} \pi_s^1 > \lim\limits_{\lambda \to \lambda^{*+}} \pi_s^2$, so $\lim\limits_{\lambda \to \lambda^{*-}} \delta_1^*(\lambda) < \lim\limits_{\lambda \to \lambda^{*+}} \delta_2^*(\lambda)$. So we can conclude that $\delta^*(\lambda)$ increases with the sympathy parameter.

At last, we compare the supplier's profits in Option 1 and Option 2 under the conditions of $\delta \geq \frac{1}{2}$ and $\frac{-3+6\delta}{4+8\delta} \leq \lambda < \frac{-1+2\delta}{1+2\delta}$. In Option 2, the supplier's total profit is $\pi_s^{\theta} = \frac{(a-c)^2\left(-3+13\lambda + \delta(6 + 17\lambda) + 2\sqrt{3-6\delta + 4\lambda + 8\delta\lambda}\right)}{72(\lambda + 2\delta\lambda)}$. The conditions of $\delta \geq \frac{1}{2}$ and $\frac{-3+6\delta}{4+8\delta} \leq \lambda < \frac{-1+2\delta}{1+2\delta}$ are equivalent to the conditions that: (1) $0 < \lambda < \frac{3}{4}$ and $\frac{-1-\lambda}{-2+2\lambda} \leq \delta < \frac{-3-4\lambda}{-6+8\lambda}$, and (2) $\frac{3}{4} \leq \lambda < 1$ and $\delta \geq \frac{-1-\lambda}{-2+2\lambda}$. When $0 < \lambda < \lambda^*$ and $\frac{-1-\lambda}{-2+2\lambda} \leq \delta < \frac{-3-4\lambda}{-6+8\lambda}$, $\frac{1}{10}(a-c)^2 + \frac{(a-c)^2}{8} > \frac{(a-c)^2\left(-3+13\lambda+\delta(6+17\lambda)+2\sqrt{3-6\delta+4\lambda+8\delta\lambda}\right)}{72(\lambda+2\delta\lambda)}$ and $\pi_s^1 > \frac{1}{10}(a-c)^2 + \frac{(a-c)^2}{8}$, so $\pi_s^1 > \frac{(a-c)^2\left(-3+13\lambda+\delta(6+17\lambda)+2\sqrt{3-6\delta+4\lambda+8\delta\lambda}\right)}{72(\lambda+2\delta\lambda)}$; when $\lambda^* \leq \lambda <$

$\frac{3}{4}$ and $\frac{-1-\lambda}{-2+2\lambda} < \delta < \frac{-3-4\lambda}{-6+8\lambda}$, $\frac{3}{32}(a-c)^2 + \frac{(a-c)^2}{8} > \frac{(a-c)^2\left(-3+13\lambda+\delta(6+17\lambda)+2\sqrt{3-6\delta+4\lambda+8\delta\lambda}\right)}{72(\lambda+2\delta\lambda)}$ and $\pi_s^2 > \frac{3}{32}(a-c)^2 + \frac{(a-c)^2}{8}$, so $\pi_s^2 > \frac{(a-c)^2\left(-3+13\lambda+\delta(6+17\lambda)+2\sqrt{3-6\delta+4\lambda+8\delta\lambda}\right)}{72(\lambda+2\delta\lambda)}$; when $\frac{3}{4} \leq \lambda < 1$ and $\delta > \frac{-1-\lambda}{-2+2\lambda}$, $\frac{3}{32}(a-c)^2 + \frac{(a-c)^2}{8} > \frac{(a-c)^2\left(-3+13\lambda+\delta(6+17\lambda)+2\sqrt{3-6\delta+4\lambda+8\delta\lambda}\right)}{72(\lambda+2\delta\lambda)}$ and $\pi_s^2 > \frac{3}{32}(a-c)^2 + \frac{(a-c)^2}{8}$, so $\pi_s^2 > \frac{(a-c)^2\left(-3+13\lambda+\delta(6+17\lambda)+2\sqrt{3-6\delta+4\lambda+8\delta\lambda}\right)}{72(\lambda+2\delta\lambda)}$. Therefore, when $\delta \in [\frac{1}{2}, \infty)$, the equilibrium results of the game are $w_1^* = w_1^{I*}$ *or* w_1^{II*}, $w_2^* = \frac{2a+c-\sqrt{(a-c)^2+12t\lambda}}{3}, p_2 = \frac{a+w_2}{2}$.

For convenience, we use w_1^{o*} to denote w_1^{I*} *or* w_1^{II*} under corresponding conditions. In general, there exists $\delta^*(\lambda) \in (\frac{1}{7}, \frac{2}{5})$ which satisfies that, if $\delta \in (0, \delta^*(\lambda))$, the equilibrium prices are $w_1^* = \frac{a+c}{2}, w_2^* = \frac{a+c}{2} - \frac{\delta(a-c)}{2(1+2\delta)}; p_i^* = \frac{3a+c}{4}$; if $\delta \in [\delta^*(\lambda), \infty)$, the equilibrium prices are $w_1^* = w_1^{o*}, w_2^* = \frac{2a+c}{3} - \frac{\sqrt{(a-c)^2+12\lambda t}}{3}; p_i^* = \frac{a+w_i^*}{2}, i = 1, 2.$ □

APPENDIX D.3. Proof of Corollary 1

Proof. Based on the proof of Proposition 2, we know that when $\delta \in [\delta^*(\lambda), \infty)$, if $0 < \lambda < \lambda^*$ and $\delta \geq \delta_1^*(\lambda)$, $w_1^* = w_1^{I*}$; if $\lambda^* \leq \lambda < 1$ and $\delta \geq \delta_2^*(\lambda)$, $w_1^* = w_1^{II*}$. Taking the derivative of w_1^* with respect to λ, we can get that w_1^{I*} decreases with λ. Retailer 1's profit function is $\pi_1 = \frac{(a-w_1)^2}{4}$, which decreases with w_1. When $0 < \lambda < 1$ and $\delta \in [\delta^*(\lambda), \infty)$, $w_1^* < \frac{a+c}{2}$, so Retailer 1 gets more profits. Retailer 1's share of the channel surplus $\frac{p_1-w_1}{p_1-c} = \frac{a-w_1}{a-2c+w_1}$ decreases with w_1. Thus, Retailer 1 gets a higher share of the channel surplus. $\frac{d\pi_1}{dw_1} = -\frac{a-w_1}{2} < 0$, so π_1 increases with λ when $\lambda \in (0,1)$. The profit of Supply Chain 1 is $\pi_1^p = (a-p_1)(p_1-c) = \frac{(a-w_1)(a+w_1-2c)}{2}$. Taking the derivative of it with respect to w_1, we can get that $\frac{d\pi_1^p}{dw_1} = \frac{c-w_1}{2} < 0$. So The performance of Supply Chain 1 increases with λ when $\lambda \in (0,1)$. As a result, we can conclude that when $\lambda \in (0,1)$, w_1^* decreases with λ, but $\frac{p_1^*-w_1^*}{p_1^*-c}$, π_1, and π_1^p increase with λ.

If Retailer 2 only has distributional fairness concern, when $\delta \in [\delta^*(\lambda), \infty)$, his wholesale price is $\frac{a+2c}{3}$. If Retailer 2 has PF for sympathy, he gets a wholesale price $w_2^* = \frac{2a+c}{3} - \frac{\sqrt{(a-c)^2+12\lambda t}}{3} < \frac{a+2c}{3}$ and sets the retail price as $p_2^* = \frac{a+w_2^*}{2}$. Retailer 2's profit function is $\pi_2 = \frac{(a-w_2)^2}{4}$. Following the example of Retailer 1, we can get the conclusion that when $\delta \in [\delta^*(\lambda), \infty)$, Retailer 2 gets more profit and enjoys a larger share of the channel surplus. Taking the derivative of Retailer 2's profit function with respect to w_2, we know that π_2 decreases with w_2 when $w_2 \in (c, a)$. It's easy to know that w_2^* decreases with λt, so π_2 increases with λt. Take the derivative of λt with respect to λ and we can get: (1) when $0 < \lambda < \lambda^*$, $\frac{dt\lambda}{d\lambda} > 0$; (2) when $\lambda^* \leq \lambda < \lambda^o$, $\frac{dt\lambda}{d\lambda} > 0$; (3)when $\lambda^o \leq \lambda < 1$, $\frac{dt\lambda}{d\lambda} < 0$. Here, we set $s(x) = 48 - 275x + 66x^2 + 883x^3 - 548x^4 - 717x^5 + 834x^6 - 19x^7 - 208x^8 + 64x^9$, and λ^o denotes the third root of $s(x) = 0$. Because $\lim_{\lambda \to \lambda^{*-}} \frac{dt\lambda}{d\lambda} < \lim_{\lambda \to \lambda^{*+}} \frac{dt\lambda}{d\lambda}$, so π_2 increases with λ when $\lambda \in (0, \lambda^o)$ and decreases with λ when $\lambda \in [\lambda^o, 1)$. Besides, $\frac{d\frac{p_2-w_2}{p_2-c}}{dw_2} < 0$ for $w_2 \in (c, a)$, so π_2^* and $\frac{p_2^*-w_2^*}{p_2^*-c}$ increases with λ when $\lambda \in (0, \lambda^o)$ and decreases with λ when $\lambda \in [\lambda^o, 1)$. Taking the derivative of the profit of Supply Chain 2 π_2^p with respect to w_2, we can get that $\frac{d\pi_2^p}{dw_2} = \frac{c-w_2}{2} < 0$. So π_2^p increases with λ when $\lambda \in (0, \lambda^o)$ and decreases with λ when $\lambda \in [\lambda^o, 1)$. □

APPENDIX D.4. Proof of Proposition 3

Proof. By combining case 1, case 2 and case 3 in *Appendix C.1*, we can get Retailer 2's best-response price as (C.5). Thus, Proposition 3 can be proved. $\square$

APPENDIX D.5. Proof of Proposition 4

Proof. Firstly, We consider the supplier's optimal wholesale prices under the conditions of $0<\delta\leq\frac{1}{2}$ and $0<\eta<1$. Based on (C.15) and (C.16), we know that: (1) if $0<\delta\leq\frac{1}{2}$ and $0<\eta\leq\frac{1-2\delta}{1+2\delta}$, $w_1^*=\frac{a+c}{2}, w_2^*=\frac{a+c+a\delta+3c\delta}{2(1+2\delta)}$; (2) if $0<\delta\leq\frac{1}{2}$ and $\eta>\frac{1-2\delta}{1+2\delta}$, $w_1^*=w_1^{\exists}, w_2^*=w^{III}$. The conditions in (1) are equivalent to $0<\eta<1\&0<\delta<\frac{1-\eta}{2+2\eta}$. The conditions in (2) are equivalent to $0<\eta<1$ and $\frac{1-\eta}{2+2\eta}\leq\delta<\frac{1}{2}$ or $\eta\geq1$ and $0<\delta<\frac{1}{2}$. Compare case (1) and (2) with (C.9) to get the supplier's optimal wholesale pricing decisions under different conditions.

Thus, the problem can be described as below: 1. when $0<\eta<1$ *and* $0<\delta<\frac{1-\eta}{2+2\eta}$, $w_1^*=w_1^{\#}, w_2^*=\frac{2a+c}{3}-\frac{\sqrt{(a-c)^2-12\eta t}}{3}$ or $w_1^*=\frac{a+c}{2}, w_2^*=\frac{a+c+a\delta+3c\delta}{2(1+2\delta)}$, what are the wholesale prices the supplier chooses to maximise his total profit in the distribution channel? 2. when $0<\eta<1$ *and* $\frac{1-\eta}{2+2\eta}\leq\delta<\frac{1}{2}$, $w_1^*=w_1^{\#}, w_2^*=\frac{2a+c}{3}-\frac{\sqrt{(a-c)^2-12\eta t}}{3}$ or $w_1^*=w_1^{\exists}, w_2^*=w^{III}$, what are the wholesale prices the supplier chooses to maximise his total profit in the distribution channel?

For question 1, the supplier's total profit in the distributional channel is $\frac{(a-c)^2(1+\delta)}{8+16\delta}+\frac{(a-c)^2}{8}$ when he chooses $w_1^*=\frac{a+c}{2}$, $w_2^*=\frac{a+c+a\delta+3c\delta}{2(1+2\delta)}$. π_s^1 is used to denote the supplier's total profit when he chooses $w_1^*=w_1^{\#}, w_2^*=\frac{2a+c}{3}-\frac{\sqrt{(a-c)^2-12\eta t}}{3}$. Compare π_s^1 with $\frac{(a-c)^2(1+\delta)}{8+16\delta}+\frac{(a-c)^2}{8}$ under the conditions of $0<\eta<1$ and $0<\delta<\frac{1-\eta}{2+2\eta}$. Given $\eta\in(0,\frac{1-2\delta}{1+2\delta})$, π_s^1 is increasing with η. $\frac{(a-c)^2(1+\delta)}{8+16\delta}+\frac{(a-c)^2}{8}$ decreases with δ. When $\delta=0$, $\pi_s^1(\eta)<\frac{(a-c)^2(1+\delta)}{8+16\delta}+\frac{(a-c)^2}{8}$. And when $\delta=\frac{1}{7}$, $\pi_s^1(\eta)>\frac{(a-c)^2(1+\delta)}{8+16\delta}+\frac{(a-c)^2}{8}$. So we can conclude that, given $\eta\in(0,\frac{1-2\delta}{1+2\delta})$, there exists $\delta^*(\eta)\in(0,\frac{1}{7})$ which satisfies that when $\delta\in(0,\delta^*(\eta))$, the supplier chooses $w_1^*=\frac{a+c}{2}$, $w_2^*=\frac{a+c+a\delta+3c\delta}{2(1+2\delta)}$; when $\delta\in[\delta^*(\eta),\frac{1}{2})$, the supplier chooses $w_1^*=w_1^{\#}, w_2^*=\frac{2a+c}{3}-\frac{\sqrt{(a-c)^2-12\eta t}}{3}$.

For question 2, π_s^2 is used to denote the supplier's total profit when the optimal wholesale prices are $w_1^*=w_1^{\exists}, w_2^*=w^{III}$. Compare π_s^1 with π_s^2 under the conditions of $0<\eta<1$ and $\frac{1-\eta}{2+2\eta}\leq\delta<\frac{1}{2}$. π_s^{1x} is used to denote the supplier's total profit when the optimal wholesale price offered to Retailer 2 is $w_2=w^{III}$, and π_s^{2x} is used to denote the supplier's total profit when the optimal wholesale price offered to Retailer 2 is $w_2^*=\frac{2a+c}{3}-\frac{\sqrt{(a-c)^2-12\eta t}}{3}$. Compare π_s^{1x} with π_s^{2x} under the conditions of $0<\eta<1$, $\frac{1-\eta}{2+2\eta}\leq\delta<\frac{1}{2}$ and $c<w_1<a$. We can find that $\pi_s^{1x}<\pi_s^{2x}$ always hold. That's to say, $\pi_s^{1x}(w_1^{\exists})<\pi_s^{2x}(w_1^{\exists})$. Under the conditions of $0<\eta<1$, $\frac{1-\eta}{2+2\eta}\leq\delta<\frac{1}{2}$ and $c<w_1<a$, $w_1^{\#}$ maximises π_s^{2x}, so $\pi_s^{2x}(w_1^{\#})>\pi_s^{1x}(w_1^{\exists})$. Thus, under the conditions of $0<\eta\leq1$ and $\frac{1-\eta}{2+2\eta}\leq\delta<\frac{1}{2}$, the supplier chooses $w_1^*=w_1^{\#}, w_2^*=\frac{2a+c}{3}-\frac{\sqrt{(a-c)^2-12\eta t}}{3}$.

Now we consider the supplier's optimal pricing decisions under the conditions of $0<\lambda<1$ and $\delta\geq\frac{1}{2}$. In (C.14), we can get: (1) if $0<\eta<1$ and $\frac{1}{2}<\delta<\frac{2-3\eta}{12\eta}+\frac{1}{12}\sqrt{\frac{4+60\eta+81\eta^2}{\eta^2}}$, $w_1^*=w_1^{\exists}, w_2^*=w^{III}$; (2) if $0<\eta<1$ and $\delta\geq\frac{2-3\eta}{12\eta}+$

$\frac{1}{12}\sqrt{\frac{4+60\eta+81\eta^2}{\eta^2}}$, $w_1^* = \frac{a+2c}{3}, w_2^* = w^{III}$. π_s^{1y} and π_s^{2y} are used to represent the supplier's total profit in the two cases, respectively. In (C.9), when $0 < \eta < 1$, $w_1^* = w_1^{\#}, w_2^* = \frac{2a+c}{3} - \frac{\sqrt{(a-c)^2-12\eta t}}{3}$. $\pi_s^{1z}(w_1)$ is used to denote the supplier's total profit when he chooses $w_2^* = w^{III}$, and $\pi_s^{2z}(w_1)$ is used when he chooses $w_2^* = \frac{2a+c}{3} - \frac{\sqrt{(a-c)^2-12\eta t}}{3}$. $\pi_s^{1z}(w_1) < \pi_s^{2z}(w_1)$ always holds for $0 < \eta < 1$ and $\frac{1}{2} \leq \delta < \frac{2-3\eta}{12\eta} + \frac{1}{12}\sqrt{\frac{4+60\eta+81\eta^2}{\eta^2}}$. That's to say, $\pi_s^{1z}(w_1^{\exists}) < \pi_s^{2z}(w_1^{\exists})$. $\pi_s^{2z}(w_1^{\exists}) < \pi_s^{2z}(w_1^{\#})$, so the supplier will choose $w_1^* = w_1^{\#}, w_2^* = \frac{2a+c}{3} - \frac{\sqrt{(a-c)^2-12\eta t}}{3}$ under the conditions of $0 < \eta < 1$ and $\frac{1}{2} \leq \delta < \frac{2-3\eta}{12\eta} + \frac{1}{12}\sqrt{\frac{4+60\eta+81\eta^2}{\eta^2}}$. $\pi_s^{1z}(\frac{a+2c}{3})$ is decreasing with δ when $\delta \geq \frac{1}{2}$. When $\delta = \frac{1}{2}$, $\pi_s^{2z}(w_1^{\#}) > \pi_s^{1z}(\frac{a+2c}{3})$, so the supplier choose $w_1^* = w_1^{\#}, w_2^* = \frac{2a+c}{3} - \frac{\sqrt{(a-c)^2-12\eta t}}{3}$ when $0 < \eta < 1$ and $\delta \geq \frac{2-3\eta}{12\eta} + \frac{1}{12}\sqrt{\frac{4+60\eta+81\eta^2}{\eta^2}}$.

In addition, we know $\delta^*(\eta)$ makes $\pi_s^1 = \frac{(a-c)^2}{8} + \frac{(a-c)^2(1+\delta)}{8+16\delta}$. $\frac{(a-c)^2}{8} + \frac{(a-c)^2(1+\delta)}{8+16\delta}$ decreases with δ. Notice that π_s^1 is increasing with η when $0 < \eta < 1$, so we can conclude that $\delta^*(\eta) \in (0, \frac{1}{7})$ is decreasing with η when $\eta \in (0, 1)$.

As a result, the equilibrium prices are $w_1^* = \frac{a+c}{2}$, $w_2^* = \frac{a+c+a\delta+3c\delta}{2(1+2\delta)}$ and $p_i^* = \frac{a+3c}{4}$ when $\delta \in (0, \delta^*(\eta))$ and $\eta \in (0, \frac{1-2\delta}{1+2\delta})$. And when $\delta \in (0, \delta^*(\eta))$ and $\eta \in [\frac{1-2\delta}{1+2\delta}, 1)$ or $\delta \in [\delta^*(\eta), \infty)$ and $\eta \in (0, 1)$, the equilibrium prices are $w_1^* = w_1^{\#}(\eta), w_2^* = \frac{2a+c}{3} - \frac{\sqrt{(a-c)^2-12t\eta}}{3}$ and $p_i = \frac{a+w_i}{2}$. Besides, $\delta^*(0) = \frac{1}{7}$ and $w_1^{\#}(0) = \frac{a+c}{2}$. □

APPENDIX D.6. Proof of Corollary 2

Proof. Taking the derivative of $w_1^{\#}(\eta)$ with respect to η, we can get that w_1 increases with η when $\eta \in (0, \eta^*)$ and decreases with η when $\eta \in [\eta^*, 1)$, where we set $t(x) = -64 - 208x + 19x^2 + 834x^3 + 717x^4 - 548x^5 - 883x^6 + 66x^7 + 275x^8 + 48x^9$ and define η^* as the third root of $t(x) = 0$. Besides, $w_1^*(0) = w_1^*(1) = \frac{a+c}{2}$. $w_2^* = \frac{2a+c}{3} - \frac{\sqrt{(a-c)^2-12t\eta}}{3}$, taking the derivative of it with respect to η, we can get $\frac{\mathrm{d}w_2}{\mathrm{d}\eta} = \frac{\mathrm{d}w_2}{\mathrm{d}\eta t}\frac{\mathrm{d}\eta t}{\mathrm{d}\eta}$. For $\eta \in (0, 1)$, $\frac{\mathrm{d}w_2}{\mathrm{d}\eta t} > 0$ and $\frac{\mathrm{d}\eta t}{\mathrm{d}\eta} = \frac{t\mathrm{d}\eta + \eta \mathrm{d}t}{\mathrm{d}\eta} = t + \frac{\eta \mathrm{d}t}{\mathrm{d}\eta} > 0$, so $\frac{\mathrm{d}w_2}{\mathrm{d}\eta} > 0$. Besides, $p_i^* = \frac{a+w_i^*}{2}$, so Corollary 2 is proved. □

APPENDIX D.7. Proof of Corollary 3

Proof. When $\delta \in (0, \delta^*(\eta))$ and $\eta \in [\frac{1-2\delta}{1+2\delta}, 1)$ or $\delta \in [\delta^*(\eta), \infty)$ and $\eta \in (0, 1)$, $w_1^* = w_1^{\#}, w_2^* = \frac{2a+c}{3} - \frac{\sqrt{(a-c)^2-12t\eta}}{3}$. Make a direct comparison between p_1^*, p_2^* and $\frac{2a+c}{3}$, $\frac{3a+c}{4}$, and it's easy to get that: (1) when $\delta \in (0, \delta^*(\eta))$ and $\eta \in [\frac{1-2\delta}{1+2\delta}, 1)$ or $\delta \in [\delta^*(\eta), \frac{1}{7})$ and $\eta \in (0, 1)$, $p_2^* < \frac{3a+c}{4} < p_1^*$; (2) when $\delta \in [\frac{1}{7}, \infty)$ and $\eta \in (0, 1)$, $\frac{2a+c}{3} < p_2^*$, $\frac{3a+c}{4} < p_1^*$. □

APPENDIX D.8. Proof of Corollary 4

Proof. When $\delta \in (0, \delta^*(\eta))$ and $\eta \in [\frac{1-2\delta}{1+2\delta}, 1)$ or $\delta \in [\delta^*(\eta), \infty)$ and $\eta \in (0, 1)$, the supplier offers the wholesale prices as $w_1^* = w_1^{\#}, w_2^* = \frac{2a+c}{3} - \frac{\sqrt{(a-c)^2-12t\eta}}{3}$. Then, the retailers set their retail price as $p_i^* = \frac{a+w_i^*}{2}, i = 1, 2$. Retailer i's profit is $\pi_i^* = \frac{(a-w_i^*)^2}{4}$. Retailer i's profit is $\frac{(a-c)^2}{16}$ in Benchmark Model 1. Compare π_1^* with $\frac{(a-c)^2}{16}$ and we can get that $\pi_1{}^* < \frac{(a-c)^2}{16}$.

Based on the proof of Proposition 4, we can get that, when $\delta \in (0, \delta^*(\eta))$ and $\eta \in [\frac{1-2\delta}{1+2\delta}, 1)$ or $\delta \in [\delta^*(\eta), \infty)$ and $\eta \in (0, 1)$, (i) ${\pi_1}^*$ decreases with η when $\eta \in (0, \eta^*)$ and increases with it when $\eta \in [\eta^*, 1)$; (ii) ${\pi_2}^*$ decreases with η when $\eta \in (0, 1)$. $\square$

APPENDIX D.9. Proof of Proposition 5

Proof. Consider the supplier's optimal pricing decision if $\eta \geq 1$. Based on (C.10), (C.14) and (C.16), the supplier decides his optimal wholesale prices if $\eta \geq 1$. When $w_2 = w^{III}$, the supplier's total profit is $\pi_s = \frac{(a-w_1)(w_1-c)}{2} + \frac{(a-w^{III})(w^{III}-c)}{2}$. By comparing it with $\frac{(a-c)^2}{4}$ under the conditions of $\eta \geq 1$ and $c < w_1 < a$, we can find that $\pi_s < \frac{(a-c)^2}{4}$ always hold. So the equilibrium prices of the game are $w_1^* = w_2^* = \frac{a+c}{2}, p_i^* = \frac{a+w_i^*}{2}$. $\square$

APPENDIX D.10. Proof of Proposition 6

Proof. Based on Proposition 2 and Proposition 4, we know that in Model PF1, when $\delta \in (0, \delta^*(\lambda))$, where $\delta^*(\lambda) \in (\frac{1}{7}, \frac{2}{5})$, $w_1^* = \frac{a+c}{2}, p_1^* = \frac{3a+c}{4}, w_2^* = \frac{a+c}{2} - \frac{\delta(a-c)}{2(1+2\delta)}$; in Model PF2, if $\eta \in (0, \frac{1-2\delta}{1+2\delta})$ and $\delta \in (0, \delta^*(\eta))$, where $\delta^*(\eta) \in (0, \frac{1}{7})$, $w_1^* = \frac{a+c}{2}, p_1^* = \frac{3a+c}{4}, w_2^* = \frac{a+c}{2} - \frac{\delta(a-c)}{2(1+2\delta)}$. So we can conclude that compared to the counterpart in Model PF1, when the peer-regarding fairness parameter and the the schadenfreude parameter is small (i.e., $\delta \in (0, \delta^*(\eta))$ and $\eta \in (0, \frac{1-2\delta}{1+2\delta})$), Retailer i in Model PF2 accepts the same wholesale price and gets the same profit. In Model PF1, when $\delta \in [\delta^*(\lambda), \infty)$, $w_1^* = w_1^{o*}(\lambda), w_2^* = \frac{2a+c}{3} - \frac{\sqrt{(a-c)^2+12\lambda t}}{3}$; in Model PF2, when $\eta \in [\frac{1-2\delta}{1+2\delta}, 1)$ and $\delta \in (0, \delta^*(\eta))$, or $\eta \in (0,1)$ and $\delta \in [\delta^*(\eta), \infty)$, $w_1^* = w_1^{\#}(\eta)$, $w_2^* = \frac{2a+c}{3} - \frac{\sqrt{(a-c)^2-12\eta t}}{3}$. It's easy to know that when $\delta \in (0, \delta^*(\eta)]$ and $\eta \in [\frac{1-2\delta}{1+2\delta}, 1)$, or $\delta \in (\delta^*(\eta), \delta^*(\lambda))$ and $\eta \in (0, 1)$, $\frac{a+c}{2} < w_1^{\#}(\eta)$, but it's difficult to make a comparison between $\frac{a+c}{2} - \frac{\delta(a-c)}{2(1+2\delta)}$ and $\frac{2a+c}{3} - \frac{\sqrt{(a-c)^2-12\eta t}}{3}$. When $\delta \in [\delta^*(\lambda), \infty)$ and $\eta \in (0, 1)$, $w_1^{o*}(\lambda) < \frac{a+c}{2} < w_1^{\#}(\eta)$ and $\frac{2a+c}{3} - \frac{\sqrt{(a-c)^2+12\lambda t}}{3} < \frac{a+2c}{3} < \frac{2a+c}{3} - \frac{\sqrt{(a-c)^2-12\eta t}}{3}$. Thus, we can conclude that when $\delta \in [\delta^*(\lambda), \infty)$ and $\eta \in (0, 1)$, Retailer i in Model PF2 accepts a higher wholesale price. At last, in Model PF2, when $\eta > 1$, $w_1^* = w_2^* = \frac{a+c}{2}$. Note that in Model PF1 and Model PF2, $p_i^* = \frac{a+w_i^*}{2}, i = 1, 2$. So $\pi_i^* = \frac{(a-w_i^*)}{4}$. This shows that Retailer i with a higher wholesale price will get less profit. Thus, Proposition 6 is proved. $\square$

APPENDIX D.11. Proof of Proposition 7

Proof. Based on Proposition 6, we know when the peer-regarding fairness parameter and the schadenfreude parameter is small, Retailer i in Model PF2 accepts the same wholesale price and gets the same profit compared to the counterparts in Model PF2. So the supplier's total profit in the distribution channel in Model PF1 is the same as the counterpart in Model PF2. If $\eta \geq 1$, in Model PF2, $w_1^* = w_2^* = \frac{a+c}{2}$, the supplier's total profit is $\frac{(a-c)^2}{4}$. In Model PF1, when $\delta \in (0, \delta^*(\lambda))$, $w_1^* = \frac{a+c}{2}, w_2^* = \frac{a+c}{2} - \frac{\delta(a-c)}{2(1+2\delta)}$, $p_i^* = \frac{3a+c}{4}$, $\pi_s^* < \frac{(a-c)^2}{4}$; when $\delta \in [\delta^*(\lambda), \infty)$, $w_1^* = w_1^{I*}$ *or* $w_1^{II*} < \frac{a+c}{2}, w_2^* = \frac{2a+c}{3} - \frac{\sqrt{(a-c)^2+12\lambda t}}{3} < \frac{a+2c}{3}$, $\pi_s^* < \frac{(a-c)^2}{4}$. So we can conclude that $\pi_{s,PF1}^* < \pi_{s,PF2}^*$ if $\eta \geq 1$. In Model PF2, when $\delta \in (0, \delta^*(\eta))$ and $\eta \in [\frac{1-2\delta}{1+2\delta}, 1)$ or $\delta \in [\delta^*(\eta), \infty)$ and $\eta \in (0, 1)$, $w_{1,PF1}^* = w_1^{\#}(\eta), w_{2,PF1}^* = \frac{2a+c}{3} - \frac{\sqrt{(a-c)^2-12\eta t}}{3}$. It's difficult to give a comparison between the supplier's profits in the two models when $\delta \in (0, \delta^*(\eta))$ and $\eta \in [\frac{1-2\delta}{1+2\delta}, 1)$ or $\delta \in [\delta^*(\eta), \delta^*(\lambda))$ and $\eta \in (0, 1)$. When $\delta \in [\delta^*(\lambda), \infty)$ and $\eta \in (0, 1)$, $p^*_{i,PF1} = p^*_{i,PF2} = \frac{a+w_i^*2}{,} i = 1, 2$. So we can get that $\pi^*_{s,PF1} < \frac{17(a-c)^2}{72} < \pi^*_{s,PF2}$. Thus, Proposition 7 is proved. □

APPENDIX D.12. Proof of Proposition 8

Proof. The distribution channel profit is $\pi^* = (a-p_1)(p_1-c)+(a-p_2)(p_2-c)$. π^*_{PF1} and π^*_{PF2} are defined as the distribution channel profit in Model PF1 and Model PF2, respectively. In Model PF1, when $\delta \in (0, \delta^*(\lambda))$, $\pi^*_{PF1} = \frac{3(a-c)^2}{8}$; when $\delta \in [\delta^*(\lambda), \infty)$, $w_1^* = w_1^{I*}$ *or* $w_1^{II*} < \frac{a+c}{2}, w_2^* = \frac{2a+c}{3} - \frac{\sqrt{(a-c)^2+12\lambda t}}{3} < \frac{a+2c}{3}$, $p_i^* = \frac{a+w_i}{2}$, so $\pi^*_{PF1} > \frac{3(a-c)^2}{16} + \frac{2(a-c)^2}{9}$. In Model PF2, when $\delta \in (0, \delta^*(\eta))$ and $\eta \in (0, \frac{1-2\delta}{1+2\delta})$, $\pi^*_{PF2} = \frac{3(a-c)^2}{8}$; when $\delta \in (0, \delta^*(\eta))$ and $\eta \in [\frac{1-2\delta}{1+2\delta}, 1)$ or $\delta \in [\delta^*(\eta), \infty)$ and $\eta \in (0, 1)$, $w_1^* = w_1^{\#} > \frac{a+c}{2}, w_2^* = \frac{2a+c}{3} - \frac{\sqrt{(a-c)^2-12\eta t3}}{>}\frac{a+2c}{3}$ and $p_i^* = \frac{a+w_i}{2}$, it's easy to get that $\frac{3(a-c)^2}{8} < \pi^*_{PF2} < \frac{3(a-c)^2}{16} + \frac{2(a-c)^2}{9}$; when $\eta \in [1, \infty)$ and $\delta \in (0, \infty)$, $\pi^*_{PF2} = \frac{3(a-c)^2}{8}$. Thus, Proposition 8 is proved. □