Throughput-based importance measures of multistate production systems

Many production systems are multistate, with a finite number of performance levels that are between perfect functioning and complete failure. Importance measures are often used in the maintenance planning of complicated systems, to observe the criticalities of components, reveal the system weakness, and thus to guide the allocation of limited maintenance resources. This paper compares several common used importance measures for multistate systems, and investigate their effectiveness and limitations with a simple example. These existing measures focus on the states of a system at some moment, while ignoring the dynamic behaviors in the long-term. For a production system, however, its throughput in a certain period, rather than the instantaneous performance, is the system property of interest. Therefore, two new long-term throughput-based importance measures: total throughput importance measure (TTIM) and maintenance effect importance measure (MEIM) are proposed in this paper, to answer the questions about the criticalities of different components and the long-term effects of successful maintenance activities on the throughput of a production system in a certain period. A case study on an offshore production system is conducted, to illustrate how the new importance measures work and what kind of implications can be provided to the maintenance crew.

Keywords: multistate production system, importance measure, maintenance, performance analysis, total throughput importance measure, maintenance effect importance measure, Markov model

1. Introduction

Maintenances are the activities of keeping an asset in the good working condition, so that the asset may be used to its full productive capacity (Brah and Chong 2004; Gulati and Smith 2009). A production system needs appropriate maintenances to achieve the high productivity under the acceptable costs (Chen and Wu 2007; Xia et al. 2017a).

Considering the balance between the two objectives of high productivity/throughput and low maintenance costs, opportunistic maintenance strategies are often necessary for a production system (Koochaki et al. 2012; Xia et al. 2015, Xia et al. 2017b). One of these strategies is to prioritize the critical components in the maintenance schedule, to minimize the risk of any production loss. A component can be regarded as critical if its failure or degradation in performance is inclined to reduce system throughput in a significant way.

In order to reveal the critical components in a system, researchers have proposed the approach of importance measures. The maintenance crew of a system can rank the components in a system in terms of their contributions to degradation of performance and production loss (Rausand and Høyland 2004; Ramirez-Marquez and Coit 2005; Hague Barker and Ramirez-Marquez 2015).

Numerous importance measures have been developed in the last decades, such as Birnbaum measure, improvement potential measure, risk achievement worth, and risk reduction worth (see Rausand and Høyland 2004; Marugan Marquez and Lev 2017). Early importance measures assume systems as binary ones, where all components only have two states: functioning and failed. However, many real-life production systems degrade continuously or have numbers of performance levels that are between fully functioning and completely failed (Griffith 1980; Lisnianski Frenkel and Ding 2010). The binary assumption may thus imply an oversimplification on system characteristics and deprive decision makers of many opportunities to restore the system before it totally fails.

The release of the binary assumption requires new approaches for analyzing system and component deterioration. In reality, the deterioration can be characterized by multiple discrete states, to provide the maintenance crew with clear symbols for initiating some activities. The examples of systems with components of discrete states include power supply systems (Lisnianski and Levitin, 2003), water distribution systems (Ramirez-Marquez and Coit, 2005), oil & gas production systems (Kawauchi and Rausand, 2002).

For multistate systems, several multistate importance measures have been proposed to evaluate the effects of, e.g., a state transition of a component (Griffith 1980), a component state (Wu and Chan 2003), and the restoration of a component to a certain state (Si et al 2012, 2013; Liu et al 2016), on the system performance.

However, all these measures are evaluating the instantaneous system performance, e.g. the current failure rate or throughput rate, while skipping long-term performances, e.g. the overall throughput in a certain period or average productivity. Although the measures can be very informative for determining which components should be in repair immediately, it is feasible to suspect whether they can provide sufficient information for the schedule of maintenances and the allocation of relevant resources in a longer timeframe. With these concerns, this paper is to propose new importance measures to evaluate the long-term effects of components and their states on the performance of production systems, and the effectiveness of spending maintenance resources.

The rest of the paper is organized as follows: Section 2 briefly reviews the main importance measures for the performance analysis multi-state systems. A small case study is then conducted to compare these measures. In section 3, two new measures are proposed, and an offshore production facility is taken as an example in section 4 to demonstrate how the proposed measures work, and the findings by different measures are discussed. Finally, the summary appears in section 5.

2. Importance measures for multistate systems: Reviews and comparisons

2.1 Notation

The notation of models is used in the following sections:

N	The number of system states
$\phi(\boldsymbol{X})$	The system state function
$\phi(\boldsymbol{X}\|x_i = j)$	The system state when component i is at state j
m_i	The perfectly functioning state of component i
x_i	The state variable of component i
$\boldsymbol{X}$	The vector of the states of all components
δ_k	The output/throughput of a system at state k, $k \geq 0$
Λ_i	The failure transition matrix between different states of component i
Δ_k	The improvement of system performance from state $k-1$ to k, where $k \geq 1$
p_{ij}	The probability that component i is at state j
$p_i(h\|j)$	The probability that component i transits from j to state h
$\Psi(i,\tau)$	The repair matrix of component i in the maintenance time of τ
$T_i(j\|m_i, t)$	The time that component i spends at state j in the period of $[0, t)$ when the starting state is m_i

The following abbreviations are used:

ETR	The expected throughput rate
ETT	The expected total throughput
GIM	Griffith importance measure
GGIM	Generalized Griffith importance measure

IIM	Integrated importance measure
MEIM	Maintenance effect importance measure
TTIM	Total throughput importance measure
WIM	The importance measure proposed by Wu and Chan (2003)

2.2 Multistate systems and performance analysis

Consider a system that consists of n components. For a production system, the components are production equipment, units or devices. Component i can have m_i+1 states, and so $x_i = [0, 1, \dots, j, \dots, m_i]$, where 0 means that component i is completely failed and m_i denotes a state of component i being fully functioning. We set a vector $\boldsymbol{X} = [x_1, \dots, x_i, \dots, x_n]$ as the set of states of all the components in a system. The function $\phi(\boldsymbol{X})$ that takes the states of all the components as an input and returns to a system state, is called the structure function (see Lisnianski Frenkel and Ding 2010; Natvig 2011).

In this study, we assume that the multistate production systems are monotone and coherent (see Griffith 1980; Natvig 2011 for the definitions). In addition, all the components in a system are assumed independent from each other, meaning that the failure of one component has no effect on the others.

The performance of a production system can be assessed as the probability distribution of system states and the mean system throughput rate during a period of time (Aven 1993), e.g. the production rate of dehydrated gas from an offshore facility in the following case. Several stochastic approaches have been applied in capturing the probability distribution of system states, e.g. the approach by Lisnianski (2007), combining reliability block diagram (modelling the system structure), the Markov method (modelling the behaviour of each component), and the universal generating function (calculating the probability distribution of the system performance). Monte Carlo simulation is also often used to assess multistate systems (e.g. Zio and Podofillini 2003; Sherer and Ramkrishna 2008).

2.3 Importance measures for multi-state systems

2.3.1 Griffith importance measure

Griffith (1980) proposed the first important measures for multistate systems by generalizing the Birnbaum measure for binary systems. Griffith importance measure (GIM) calculates the effect of improving a component on the overall system performance. Consider a system of n components, and we define:

$$I_{jk}^{\mathrm{GIM}}(i) = \Pr[\phi(\boldsymbol{X}|x_i = j) \geq k] - \Pr[\phi(\boldsymbol{X}|x_i = j-1) \geq k] \tag{1}$$

where $\phi(\boldsymbol{X}|x_i = j)$ means the system state when component i is at state j ($j \geq 1$), and $I_{jk}^{\mathrm{GIM}}(i)$ represents the change of probability that the system state is equal to or higher than k when component i is improved from state $j-1$ to state j.

Let $\delta_0 \leq \delta_1 \leq \cdots \leq \delta_k \leq \cdots \leq \delta_N$ represent the throughput of the system at different states from 0 to N, and we can set $\Delta_k = \delta_k - \delta_{k-1}$ to represent the improvement of system throughput from one level to the next. We have:

$$\begin{aligned} I_j^{\mathrm{GIM}}(i) &= \textstyle\sum_{k=1}^{N} \Delta_k I_{jk}^{\mathrm{GIM}}(i) \\ &= \textstyle\sum_{k=1}^{N} (\delta_k - \delta_{k-1})[\Pr[\phi(\boldsymbol{X}|x_i = j) \geq k] - \Pr[\phi(\boldsymbol{X}|x_i = j-1) \geq k]] \end{aligned} \tag{2}$$

where $I_j^{\mathrm{GIM}}(i)$ considers all system states and also measures the increase in the expected system performance when component i moves from $j-1$ to j. Then, we can consider all states of component i, and so the importance measure of component i is a vector

$$I^{\mathrm{GIM}}(i) = [I_0^{\mathrm{GIM}}(i), \ldots, I_j^{\mathrm{GIM}}(i), \ldots, I_{m_i}^{\mathrm{GIM}}(i)] \tag{3}$$

In simplification, the importance measure of component i can be calculated as the sum of the measures of all the states:

$$I^{\mathrm{GIM}}(i) = \sum_{j=0}^{m_i} I_j^{\mathrm{GIM}}(i) \tag{4}$$

2.3.2 Wu's Importance measure

Wu and Chan (2003) have improved GIM, with suggesting WIM for evaluating the importance of component i in state j:

$$I_j^{\mathrm{WIM}}(i) = \sum_{k=1}^{N} \delta_k \cdot \Pr[\phi(\boldsymbol{X}|x_i = j) = k] \tag{5}$$

$I_j^{\mathrm{WIM}}(i)$ measures the actual contribution of state j of component i in the current expected performance. WIM cannot measure the importance of components, since all components have the same WIM values.

2.3.3 Integrated importance measure

Si et al. (2012) have proposed integrated importance measure (IIM) based on the rate of performance loss of a system state as:

$$I_j^{\mathrm{IIM}}(i) = p_{ij} \cdot \lambda_{j,0}^{i} \cdot \sum_{k=1}^{N} \delta_k [\Pr[\phi(\boldsymbol{X}|x_i = j) = k] - \Pr[\phi(\boldsymbol{X}|x_i = 0) = k]] \tag{6}$$

where p_{ij} is the probability of component i being in state j, and $\lambda_{j,0}^{i}$ is the rate of component i from state j directly to state 0. The measure is obtained by multiplying the probability of a component being at a certain state, with the failure rate from this state j to state 0, and the expected performance degradation due to this failure.

The measure considers the complete failures, but ignores other types of deteriorations. Therefore, Si et al. (2013) have improved IIM with including transitions from state j to all the worse states:

$$I_j^{\mathrm{IIM}}(i) = p_{ij} \cdot \sum_{l=0}^{j-1} \{\lambda_{j,l}^i \cdot \sum_{k=1}^{N} \delta_k [\Pr[\phi(\boldsymbol{X}|x_i = j) = k] - \Pr[\phi(\boldsymbol{X}|x_i = l) = k]]\} \quad (7)$$

where $j \geq 1$, $l \geq 0$, and $\lambda_{j,l}^i$ is the transition rate of component i from state j to state l. Note that IIM is also only for measuring the criticality of a state.

2.3.4 Generalized Griffith importance measure

Liu et al. (2016) have proposed a generalized Griffith importance measure (GGIM) to evaluate the effect when a component transits from a specific state to the better states. In addition, GGIM considers the probability of achieving a successful repair during a limited maintenance time. For the state j of component i, we have

$$I_j^{\mathrm{GGIM}}(i) = \sum_{h=j}^{m_i} \Pr(h|j) \cdot \sum_{k=1}^{N} \delta_k [\Pr[\phi(\boldsymbol{X}|x_i = h) = k] - \Pr[\phi(\boldsymbol{X}|x_i = j) = k]] \quad (8)$$

And then, for component i

$$I^{\mathrm{GGIM}}(i) = \sum_{j=1}^{m_i} p_{ij} \sum_{h=j}^{m_i} \{p_i(h|j) \cdot \sum_{k=1}^{N} \delta_k [\Pr[\phi(\boldsymbol{X}|x_i = h) = k] - \Pr[\phi(\boldsymbol{X}|x_i = j) = k]]\} \quad (9)$$

where $h \geq j$. $p_i(h|j)$ is the probability that component i actually transits from j to state h during a maintenance, and it can be calculated based on the restoration rates and the maintenance time. Component i has a matrix $\Psi(i,\tau)$ that describes its transition probabilities during the maintenance of τ:

$$\Psi(i,\tau) = \begin{pmatrix} p_i(0|0) & p_i(1|0) & \dots & p_i(m_i|0) \\ 0 & p_i(1|1) & \dots & p_i(m_i|1) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & p_i(m_i|m_i) \end{pmatrix} \quad (10)$$

2.4 Comparison of Importance Measures

Here we use an example with three multistate components to compare the above measures. In a small production system, components 1 and 2 are in parallel to form a subsystem A, with a throughput as the sum of throughputs of the two components. In terms of probability distribution of throughputs of such a parallel structure consisted of two components, we have

$$\Pr(Z = z) = \sum_{x,y;x+y=z} \Pr(X = x)\Pr(Y = y) \tag{11}$$

where $\Pr(Z = z)$ denotes the probability that the throughput of the parallel structure is equal to z, and $\Pr(X = x)$ and $\Pr(Y = y)$ mean that the throughput of component X is equal to x, and the throughput of component Y is equal to y.

Component 3 forms subsystem B by its own, connected in series with subsystem A. The system throughput is the minimum of the throughputs of the two subsystems A and B. For the probability distribution of throughputs of such a series structure, we have

$$\Pr(Z = z) = \Pr(X = z)\Pr(Y \geq z) + \Pr(Y = z)\Pr(X \geq z) \tag{12}$$

In this section and the follows, we use throughput rate (instantaneous or average) to reflect system performance. All the three components in the system have five performance levels (states) in terms of throughput rate. State 4 is the perfectly functioning and state 0 is the completely failed. Table 1 shows the throughput rates of the components at different states with numbers before the parentheses. The system throughput δ_k can be [0, 20, 25, 40, 50, 60, 75, 80, 100].

Table 1. The throughput rates of components at different states and their steady-state probabilities, adopted from Zio and Podofillini (2003)

Component	State 0	State 1	State 2	State 3	State 4
1	0 (0.1309)	20 (0.0977)	40 (0.1232)	60 (0.2148)	80 (0.4333)
2	0 (0.0863)	20 (0.1522)	40 (0.1876)	60 (0.2494)	80 (0.3246)
3	0 (0.0002)	25 (0.0033)	50 (0.0132)	75 (0.0937)	100 (0.8895)

We assume that all the three components are independent, and the transitions between different states follow the exponential distribution. The transition rate matrices (Λ_i) for the three components are as follows without losing generality, from the study of Zio and Podofillini (2003):

$$\Lambda_1 = \begin{bmatrix} - & 5\cdot10^{-3} & 0 & 0 & 1\cdot10^{-2} \\ 5\cdot10^{-3} & - & 5\cdot10^{-3} & 0 & 5\cdot10^{-3} \\ 0 & 5\cdot10^{-3} & - & 5\cdot10^{-3} & 5\cdot10^{-3} \\ 0 & 0 & 5\cdot10^{-3} & - & 5\cdot10^{-3} \\ 5\cdot10^{-4} & 6\cdot10^{-3} & 8\cdot10^{-3} & 8\cdot10^{-3} & - \end{bmatrix}$$

$$\Lambda_2 = \begin{bmatrix} - & 5\cdot10^{-3} & 0 & 0 & 1\cdot10^{-2} \\ 5\cdot10^{-3} & - & 5\cdot10^{-3} & 0 & 5\cdot10^{-3} \\ 0 & 5\cdot10^{-3} & - & 5\cdot10^{-3} & 5\cdot10^{-3} \\ 0 & 0 & 5\cdot10^{-3} & - & 5\cdot10^{-3} \\ 1.5\cdot10^{-3} & 2\cdot10^{-3} & 3\cdot10^{-3} & 4\cdot10^{-3} & - \end{bmatrix}$$

$$\Lambda_3 = \begin{bmatrix} - & 5\cdot10^{-3} & 0 & 0 & 1\cdot10^{-1} \\ 5\cdot10^{-4} & - & 5\cdot10^{-3} & 0 & 5\cdot10^{-2} \\ 0 & 5\cdot10^{-4} & - & 5\cdot10^{-3} & 5\cdot10^{-2} \\ 0 & 0 & 5\cdot10^{-4} & - & 5\cdot10^{-2} \\ 5\cdot10^{-5} & 6\cdot10^{-5} & 7\cdot10^{-5} & 8\cdot10^{-5} & - \end{bmatrix}$$

In the matrices, λ_{jl} means the transition rate from state *j* to state *l*. For example in Λ_1, the transition rate λ_{40} is $5 \cdot 10^{-4}$, meaning that the rate of component 1 degrading from state 4 (with the throughput rate of 80) to state 0 is $5 \cdot 10^{-4}$. In some cases, degradation is not regarded as emergent as complete failure, and maintenance is not arranged immediately. Thus, in the matrices, it is not impossible that the repair rate at state 0 is higher than those at other states. In this study, the probability distributions of components are calculated with the Markov method (the Markov graph is skipped since it can be easily drawn based on Table 1). For more information about the Markov method, see Rausand and Høyland (2004). The numbers in the parentheses of Table 1 are the steady-state probabilities of components sojourning in the different states.

Table 2 provides the values of GIM, WIM, IIM and GGIM of the three components obtained with the steady-state probabilities in Table 1. For GIM and GGIM, we can calculate component importance and rank components based on Eqs. (4) and (9).

Table 2. Importance measures of the three components

GIM Component	State 4	State 3	State 2	State 1	State 0	Importance (Rank)
1	4.3640	7.9672	12.7722	19.0373	-	44.14 (2)

2	4.2515	6.6593	10.7321	18.8377	-	40.48 (3)
3	17.3498	21.4336	23.5086	24.5763	-	86.89 (1)
WIM						Importance
Component	State 4	State 3	State 2	State 1	State 0	(Rank)
1	41.2496	19.5113	10.2093	6.8507	6.6835	84.51 (-)
2	30.6392	22.4807	15.6608	11.0722	4.6516	84.51 (-)
3	77.2694	6.5167	0.6366	0.0817	0	84.51 (-)
IIM$(\times 10^{-3})$						
Component	State 4	State 3	State 2	State 1	State 0	
1	19.0176	59.8210	39.2919	14.8847	-	
2	26.6101	26.3752	26.7085	11.4684	-	
3	11.5798	1.3956	0.2002	0.0007	-	
GGIM						Importance
Component	State 4	State 3	State 2	State 1	State 0	(Rank)
1	0	1.3193	2.6319	4.2949	30.7712	5.06 (1)
2	0	1.2853	2.2308	3.6106	28.4263	3.74 (2)
3	0	5.2453	7.3675	8.3070	67.1495	0.63 (3)

All the measures are informative for the maintenances of production systems. GIM shows that component 3 is generally the most important. The measure observes how much improvement is in the system throughput when a component is improved. For example, if all the three components are at state 3, it is desired to improve the performance of component 3. While the three components are completely failed, the repairs to restore any of them to state 1 have similar effects on the system. GIM is helpful to illustrate the maintenances on which component are more effective to improve production. However, it notes that GIM only considers transitions between neighbor states.

WIM shows which state of which component has the highest contribution to overall throughput. In this case, state 4 of component 3 has the highest importance. In WIM, the sum of the contributions of the states of each component is always equal to the expected system performance (here the sum is 84.5), and so there is no rank for components. WIM can give the maintenance crew an indication about the criticality of different states on the expected performance of the system, but it does not provide information about the changes after maintenances.

IIM does not rank components either. In this case, IIM is from Eq. (7), and shows that the criticality of component 3 significantly increases when it is at a state with higher throughput. IIM can inform the maintenance crew with the potential loss when a component is at a certain state. An integrated consideration of GIM and IIM is also helpful, e.g. in this case, GIM illustrates that an improvement of component 3 from state 3 to state 4 can elevate the overall throughput, while IIM shows that such an improvement also increases in loss probability. It is beneficial to estimate the sojourn time of component 3 in state 4, to determine whether the maintenance on component from state 3 to state 4 is profitable or not.

All the above three measures are based on the instantaneous throughput rates or simplify the production process with using the average throughput rates. If the behaviors of components are dynamic, meaning that the components can degrade after maintenances and maintenances can be conducted iteratively, the throughput will change with time, and the effectiveness of these measures will be challenged.

Different from the previous three measures, GGIM requires not only the transition rates between states, but the probabilities of successful maintenances. Maintenance time (τ) is set as 72 hours. Suppose no failure occurring during maintenances, we ignore all the transitions from states with higher throughputs to those with lower throughputs in matrices of Λ_i. The transition probabilities related to the maintenance activities (i.e. only upward transitions during the maintenance) are calculated according to Eq. (10) as follows:

$$\Psi(1{,}72) = \Psi(2{,}72) = \begin{pmatrix} 0.1653 & 0.1331 & 0.1626 & 0.1665 & 0.3725 \\ 0 & 0.6977 & 0.2512 & 0.0452 & 0.0060 \\ 0 & 0 & 0.6977 & 0.2512 & 0.0512 \\ 0 & 0 & 0 & 0.6977 & 0.3023 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\Psi(3,72)=\begin{pmatrix} 1.5\mathrm{E}-8 & 0.1424 & 0.1907 & 0.1990 & 0.4680 \\ 0 & 0.6977 & 0.2512 & 0.0452 & 0.0060 \\ 0 & 0 & 0.6977 & 0.2512 & 0.0512 \\ 0 & 0 & 0 & 0.6977 & 0.3023 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

where $\Psi(i,72)$ is the transition matrix of component i during the maintenance of 72 hours.

According to Table 2, GGIM shows that it is in general preferable to spend more maintenance resources on component 1, but this is not true in every state. For example, if all components are in state 3, it is worth allocating more maintenances on component 3, to increase the instantaneous system throughput from 75 to 100. Such kind of ranking is dependent on the probabilities of successful maintenances, and it is valid to assume that the priorities of components will change if new approaches are introduced to improve a certain maintenances. Therefore, GGIM also plays as a guidance on the selection of maintenance techniques and policies.

However, GGIM ignores the behaviors of components after maintenances. If a component is ranked lowly in GGIM (e.g. component 3 in the case), it will be allocated with fewer maintenance resources. The failure of such kind of component can nevertheless result in huge loss of system performance (e.g. component 3 degrades from state 4), especially given that the component has a higher probability to totally fail from the fully functioning state. Therefore, we will propose new importance measures in the next section, to observe the effects of components after maintenances and their contributions to the expected system performance in a period.

3. New Importance Measures for Multistate Production Systems

A general objective of proposing new importance measures is to provide guidance on opportunistic maintenances to maximize the throughput of a production system in a given period. Specifically, we expect to answer the following two questions:

- Which components affects the overall throughput in the period of t most significantly in consideration their dynamic behaviors?
- Which maintenance activities (on which states of which components) can lead to maximum increases of overall throughput in the period of t?

3.1 Total throughput importance measure

Let $T_i(j|m_i, t)$ as the time that component i spends at state j in the period of [0, t), given that the component is in state m_i when $t = 0$. The matrix $\mathbb{T}_i(t)$ contains the times that component i spends at different states with [0, t) when the component starts from different states.

$$\mathbb{T}_i(t) = \begin{pmatrix} T_i(0|0,t) & \cdots & T_i(j|0,t) & \cdots & T_i(m_i|0,t) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ T_i(0|j,t) & \dots & T_i(j|j,t) & \dots & T_i(m_i|j,t) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ T_i(0|m_i,t) & \dots & T_i(j|m_i,t) & \dots & T_i(m_i|m_i,t) \end{pmatrix} \quad (13)$$

The first row in the matrix describes how long component i is expected to spend in every state in the period when this component starts in state 0. The sums of the times in different rows are equal. The expected system throughput rate (ETR) is:

$$\mathrm{ETR} = \sum_{k=1}^{N} \delta_k \Pr[\phi(\boldsymbol{X}) = k] \quad (14)$$

The expected total throughput (ETT) during t is:

$$\mathrm{ETT}(t) = t \cdot \sum_{k=1}^{N} \delta_k \Pr[\phi(\boldsymbol{X}) = k] \quad (15)$$

When component i is at state j, the system ETT in the period of t is:

$$\mathrm{ETT}(t|x_i = j) = \sum_{l=0}^{m_i} \sum_{k=1}^{N} T_i(l|j,t)\, \delta_k \Pr[\phi(x_i = l, \boldsymbol{X}) = k] \quad (16)$$

We propose a new total throughput importance measure (TTIM) of state j of component i by calculating the difference between $\mathrm{ETT}(t|x_i = j)$ and the expected throughput of the system in the period of t:

$$I_j^{\mathrm{TTIM}}(i) = \sum_{l=0}^{m_i} \sum_{k=1}^{N} T_i(l|j,t)\, \delta_k \Pr[\phi(x_i = l, \boldsymbol{X}) = k] - t \cdot \sum_{k=1}^{N} \delta_k \Pr[\phi(\boldsymbol{X}) = k] \quad (17)$$

In a certain period, if the system throughput given that the component starts from state j will be higher than the expected throughput, $I_j^{\mathrm{TTIM}}(i)$ is positive. Otherwise, the measure can be negative or zero. For example, if in the period the system is at state 0 for maintenance, the system produces nothing, and $I_0^{\mathrm{TTIM}}(i)$ is negative.

Higher absolute value of $I_j^{\mathrm{TTIM}}(i)$ implies that sojourning at the state j is more important for the system. The TTIM of component i can be calculated by considering all the $I_j^{\mathrm{TTIM}}(i)$ values at all states and the corresponding sojourn probabilities:

$$I^{\mathrm{TTIM}}(i) = \sum_{j=0}^{m_i} p_{ij} \cdot \left| I_j^{\mathrm{TTIM}}(i) \right| \quad (18)$$

3.2 Maintenance effect importance measure

Based on the TTIM and its consideration for the total throughput in a period, we can propose a maintenance effect importance measure (MEIM) with taking account the expected system throughput changes due to maintenances:

$$I_j^{\mathrm{MEIM}}(i) = \sum_{l=j}^{m_i} p_i(h|j) \cdot \sum_{k=1}^{N} \big(\mathrm{ETT}(t|x_i = h) - \mathrm{ETT}(t|x_i = j) \big) \quad (19)$$

MEIM calculates, at first, the difference between the expected total throughput when component i starts in state j and the expected total throughput when it starts in state h ($h \geq j$). The result is, then, multiplied by the probability of this transition during the available maintenance time. MEIM implies the overall effects of successful maintenances on component i when it is at state j on the system throughput. To some degree, MEIM considers the maintainability of components when evaluating their criticalities.

3.3 Numerical example for TTIM and MEIM

The example of three components in section is employed here again to illustrate the two new measures. The operational period of t is set as 8760 hours (one year). For each component, we use the Markov model to determine the transition matrix $\mathbb{T}_i(t)$. The calculation has been carried out using the programs Matlab and GRIF workshop (see *TOTAL* 2016).

In Table 3, TTIMs of states are calculated according to Eq. (17), and then the three components are ranked according to Eq. (18). In the period of one year, component 3 is the least important, since it is rather reliable compared with the others, and keeps itself at the states with high performance in most time. For the maintenance crew, given that reliability or degradation rates of the components are fixed, component 1 can be prioritized to receive more resources. However, in terms of states, the maintenance crew have to pay much attention to states 1 or 2 of component 3, where significant throughput loss may occur. Such a finding can be explained by the fact that component 3 has the highest probability to continue to deteriorate to state 0 from states 1 or 2, and to decrease the system throughput to 0.

Compared with WIM and IIM, TTIM can rank not only component states but also components. Moreover, in comparison with GIM, TTIM observes the performance changes of the components along with time. GIM regards component 3 as most important, which is true when the system is put into operation. If the time t considered in TTIM is rather short, we can find that, e.g. $\sum_{j=0}^{m_i} T_i(j|m_i,t) \approx T_i(m_i|m_i,t)$, and TTIM will lead to the same conclusion with GIM. However, if we lengthen the time framework, all the three components have higher probabilities sojourning at some states different from the initiating one. The sojourning probabilities are dependent on the degradation rates and restoration rates, but these factors are not taken into account by GIM.

Table 3. TTIMs and MEIMs of the three components in Figure 1

TTIM						
Component	State 4	State 3	State 2	State 1	State 0	Rank
1	1678.2	-138.4	-1546.4	-2647.1	-1533.2	1 (1406.8)
2	1592.6	457.1	-1136.7	-2478.3	-1218.0	2 (1326.5)
3	603	-3288	-11690	-23425	-7777	3 (1077.6)
MEIM						
Component	State 4	State 3	State 2	State 1	State 0	Rank
1	0	549.2	518.6	415.6	1278.0	-
2	0	343.3	540.0	493.9	1171.3	-
3	0	1176.4	2739.3	4000.8	1840.4	-

MEIM, on the other hand, indicates that state 1 of component 3 is most important, while GGIM shows the significance of state 0 of component 3. The difference results from the emphasis of GGIM on the instantaneous effect of the maintenance, but assuming that the system will keep itself at the state after the restoration. MEIM explores which maintenance has the long-term positive influence, and can keep the system sojourning at a higher level longer. In this case, if we only restore the component 3 from state 0 to state 1 according to the guidance from GGIM, the component has a higher probability to fail completely again in the near future. However, MEIM can tell the crew that the maintenances before the occurrence of a complete failure are more helpful.

When the maintenance strategy is determined, meaning that all restoration rates in $\Psi(i, \tau)$ are fixed, MEIM is helpful for practices in consideration that the maintenance resources are limited. For example, if it is detected that all the three components are at state 2 in the previous case, the component 3 should prioritized in the plan.

4. A Case Study of an Offshore Production System

In this section, a case study is conducted to illustrate the usefulness of the proposed

TTIM and MEIM and to compare them with the existing measures. The case is an offshore production/processing system and a similar system can be found in Kawauchi and M. Rausand (2002).

4.1 System Description

The offshore production/processing system consists of three subsystems: one for separation, one for dehydration and one for compression, see Figure 1. Each subsystem consists of two or three equipment/components in parallel. The throughput of each subsystem is the sum of its equipment's throughputs. The total system throughput is the minimum of subsystems' throughputs.

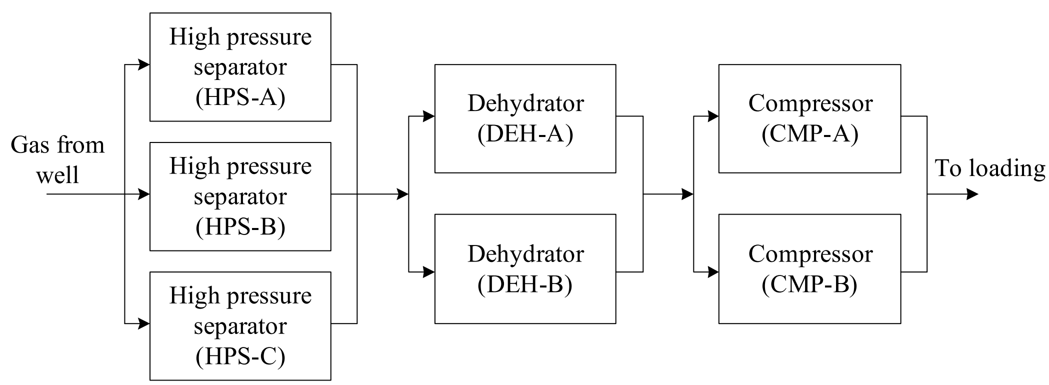

Figure 1. An offshore production/processing system

The separation subsystem consists of three high pressure separators (HPS), each with a maximum throughput rate estimated to be 55% of the highest possible throughput rate of the system. Each separator has three performance levels [55%, 23%, 0%]. Maintenances are assumed perfect and thus the degraded or failed components always can be restored to the state with the maximum throughput. HPS-A and HPS-B are identical. On the other hand, when HPS-C is at the degradation state, it is less prone to completely fail and easier to be restored, as shown in Table 4. In the highest state, HPS equipment will expose to more random total failures because of the higher stresses during with high throughput.

The dehydration subsystem consists of two dehydrators (DEH) in parallel, each with 65% of the required system throughput rate and four performance levels [65%, 45%, 20%, 0%]. There are two types of failures for dehydrators:

- Complete failures: These failures can occur in all states and result in 0% of throughput. The repair for such failures is minimal. It only brings the component to the previous state.
- Partial failures: Due to aging or other causes, the component deteriorate to the next lower state. When it deteriorates to the lowest state, a new equipment will replace the failed one.

The compression subsystem consists of two parallel compressors (CMP), and each compressor has four performance states [52%, 35%, 15%, 0%] in terms of their throughput rate to the system requirement. Similar with the dehydrators, the compressors also have complete failures and partial failures. Repairs are always minimal, only restoring the deteriorated/failed component to the next higher state.

Table 4. Transitions and transition rates between different states

Component	**Transition**	**Rate (/h)**
HPS-A	Failure	$8.91 \cdot 10^{-5}$
	Restoration	$2.54 \cdot 10^{-3}$
HPS-B	Failure	$8.91 \cdot 10^{-5}$
	Restoration	$2.54 \cdot 10^{-3}$
HPS-C	Failure from state 3	$3.56 \cdot 10^{-4}$
	Failure from state 2	$2.23 \cdot 10^{-5}$
	Restoration from state 1	$6.35 \cdot 10^{-4}$
	Restoration from state 2	$1.30 \cdot 10^{-3}$
DEH-A	Complete failure	$3.11 \cdot 10^{-5}$
	Restoration from a complete failure	$3.95 \cdot 10^{-3}$
	Partial failure	$2.69 \cdot 10^{-5}$
	Restoration from a partial failure	$3.95 \cdot 10^{-4}$
DEH-B	Complete failure	$3.11 \cdot 10^{-5}$
	Restoration from a complete failure	$3.95 \cdot 10^{-3}$
	Partial failure	$2.69 \cdot 10^{-5}$
	Restoration from a partial failure	$3.95 \cdot 10^{-4}$
CMP-A	Failure	$3.50 \cdot 10^{-5}$
	Restoration	$5.14 \cdot 10^{-3}$

CMP-B	Failure Restoration	$3.50 \cdot 10^{-5}$ $5.14 \cdot 10^{-3}$

Figure 2 is a Markov driven block diagram. It is noticeable that the models for dehydrators have seven states, ensuring that each degradation state corresponds to a complete failure state, and a repair from such a state brings the component to its previous state. Note that in Figure 2, *F* or *R* just denotes that an edge is for failure or repair, rather than the transition rate. The grey circles in Figure 2 are completely faulty states of components, and TP: 0.23, e.g. means the throughput of a state.

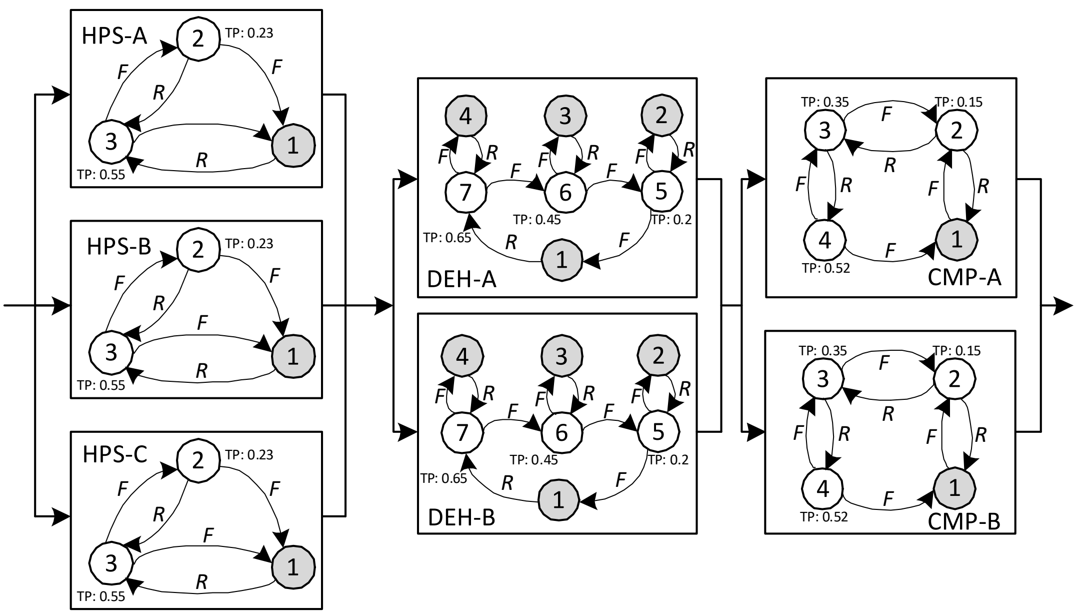

Figure 2. A Markov driven block diagram for the offshore production system

4.2 Analysis with New Importance Measures

The analysis starts at the beginning of a planned maintenance. Assume that no failure occurs in the maintenance, and the duration of maintenance is 72 hours. The maintenance crew need to decide where to put more maintenance hours, in consideration that the next planned maintenance will take place after one year.

We can identify that system has 21 states in total considering the throughput rates, and the sojourn probabilities of the system at these states can be calculated (skipped in the paper due to the limitation of tables). With these probabilities, the expected throughput rate of the system is 76.59%.

TTIMs of components are calculated based on Eqs. (13) - (18). Considering that HPS-A and HPS-B are identical, DEH-A and DEH-B and identical and CMP-A and CMP-B are identical, only four types of components are list in Table 5.

Table 5. TTIMs, MEIMs and GGIMs of 4 types of components

TTIM	State						
Component (Rank)	1	2	3	4	5	6	7
HPS-A (4)	-3715	-1199	175.4	-	-	-	-
HPS-C (2)	-1534	882.8	801.2	-	-	-	-
DEH-A (1)	-4871	-1343E2	1103E1	9803E1	-1287E2	1997E1	1097E2
CMP-A (3)	-9212	-3758.3	-641.5	190.4	-	-	-
MEIM	**State**						
Component (Rank)	1	2	3	4	5	6	7
HPS-A (4)	650.1	229.7	0	-	-	-	-
HPS-C (3)	104.4	-7.124	0	-	-	-	-
DEH-A (1)	3231	1941	2542	2893.0	4204.9	2516.4	0
CMP-A (2)	1862	1009	257.3	0	-	-	-
GGIM	**State**						
Component (Rank)	1	2	3	4	5	6	7
HPS-A (4)	1.730	0.625	0	-	-	-	-
HPS-C (3)	0.1032	0.0583	0	-	-	-	-
DEH-A (1)	1.370	4.844	9.816	12.10	0.5715	0.2621	0
CMP-A (2)	4.456	3.944	1.404	0	-	-	-

TTIM shows that the dehydrators are the most critical, and all states of these components affect, positively or negatively, the system if we consider the throughput in the future year. It is noticeable that state 4 even as a failed state has a higher positive effect than state 6 as a degraded state, because at state 4 the failed component due to a random failure can be restored to state 7, but while the component is at state 6, it only continues to deteriorate.

Following Eq. (10) and utilizing the Markov models in Figure 2 (only considering the repair transitions); we can obtain the probabilities of all the successful maintenances. The calculations and transition matrices are skipped here for due to the length limitation. Table 5 is helpful to show that the dehydrators are also the most rewarding components in terms of maintenances. Furthermore, we can rank the states of the component considering their criticalities: [5-1-4-3-6-2-7]. The state 5 is highest ranked since a repair at such a moment can stop and reverse the degradation process and bring the component to a state with higher throughput.

4.3 Comparisons and Discussions

Here we compare the implications of GGIM and MEIM on the decision-making for maintenances. GGIM values for all the states of the four types of components are obtained based on the same transition matrix for MEIM, as listed in Table 5.

Different from MEIM, GGIM implies to prioritize the state 4 of the DEH, while regards state 6 insignificant. GGIM only monitors the immediate increase in system performance, with considering 1) the probability of a failure is fixed, and 2) the instantaneous throughput rate after fixing. However, as Figure 3 illustrates, the expected throughput rate and the corresponding impact of a dehydrator are time-dependent in the following year from a planned maintenance. When the component starts from state 6, its expected throughput rate will continue to increase after a small decrease. GGIM only observes the starting point, and therefore allocates very low importance to state 6. However, if the maintenance crew pay more attention to this degraded state, and stop the degradation, the overall throughput of the system is expected to be much higher a longer term. While such kind of importance in the potential improvement can be identified by MEIM.

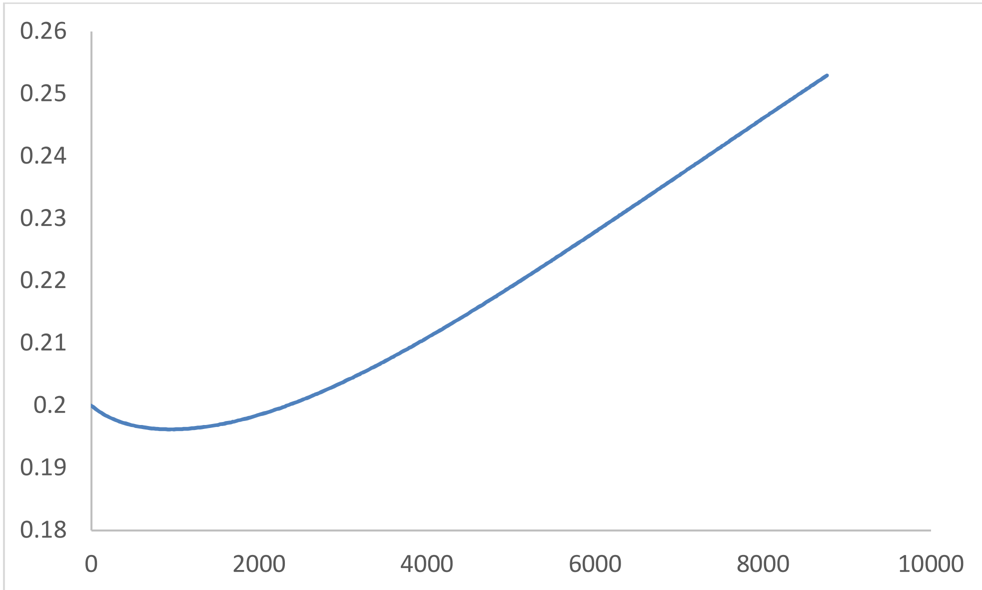

Figure 3. The expected throughput rate with time of a dehydrator when it starts from state 6.

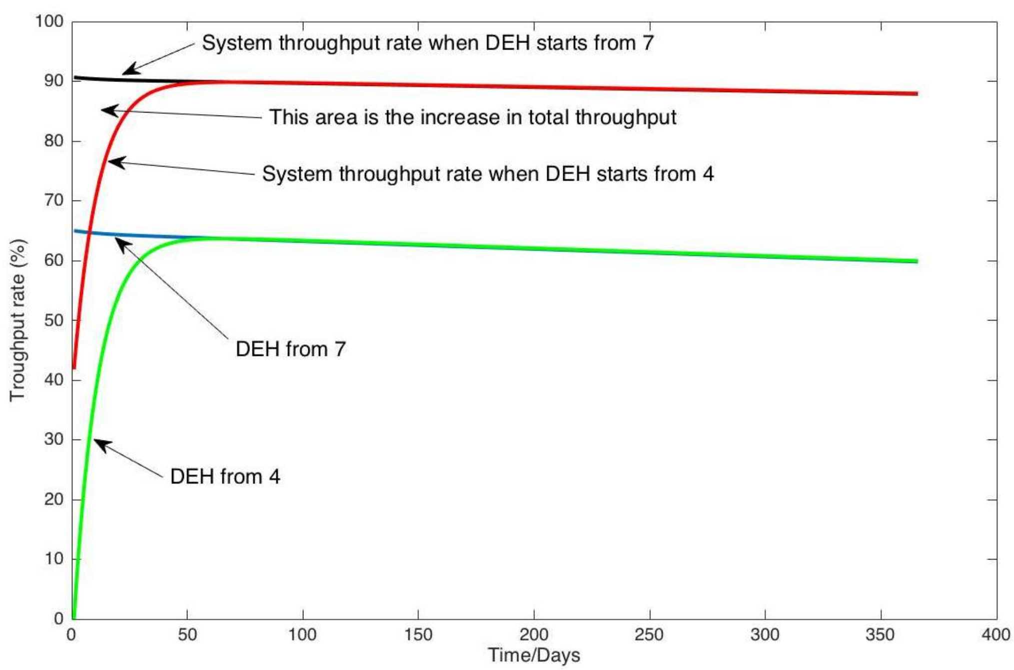

Figure 4. The effects of different initiating states on system throughputs

Figure 4 illustrates the time-dependent throughputs of the dehydrator and the overall system when the component starts from different states. It can be found that no matter the dehydrator starts from state 4 (completely failed) or state 7 (fully functional), the expected throughput rates will coincide after some time (around 60 days in this case). In other words, the expected throughput rates of both the component and the system in a longer term (e.g. one year) will approach a fixed value, and thus MEIM is not as sensitive as GGIM to the current states of components. However, such a conclusion is only reached with the assumption that the restoration from the failure is not very slow (around 10 days in this case). If the restoration durations are relatively long in comparison with the total period, they may have more impacts on the expected throughputs and the values of the importance measure.

The negative value of MEIM for HPS-C in Table 5 is noticeable, since it means that maintenances at this state cannot ensure the improvement of system throughput in the next one year. Such a negative value is resulted from the fact that HPS-C outputs higher at state 3, but its failure rate at this state is also higher than that at state 2. The managerial implication is that we should select equipment that is more reliable, and otherwise, the maintenances on HPS-C are less meaningful. It is another evidence that MEIM focuses on identifying system weaknesses in a longer time framework.

5. Summary and conclusion

In this study, two new importance measures, TTIM and MEIM are proposed for multistate production systems, based on the review and numerical comparisons of the existing importance measures. TTIM and MEIM can be used to answer the questions about the effect of a component on the total throughput of the system in a certain period, and the long-term effect of a maintenance action.

A case from an offshore production/processing system is introduced to illustrate the usefulness of the two new measures. Maintenance crew can obtain some clues on the allocation of maintenance resources to achieve higher probability of maintenance successes and higher performance in a relatively long term, so that the overall production system can have to ensure higher throughput in a specific period.

For future studies, the effects of predictive and preventive maintenances should be well considered, since this paper only focuses on the restorations or corrective maintenances in cases of failures and degradations. Periodic preventive maintenances and condition-based predictive maintenances can keep assets healthy and eliminate potential failures before they result in production loss. The failure and degradation rates of components may change with predictive and preventive maintenances, and thus new approaches are needed to evaluate the varied contributions of components to system performance and their criticalities. Moreover, since importance measures are developed to evaluate components in a fixed structure, more researches on the effects of structural changes on the system performance are very helpful.

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