Online Supplement

A The Benchmark Case

We examine *a benchmark case* in which we remove the specific traditional retailer-online retailer relationship studied in our original setup (game G).

Setting. We denote the benchmark case as game $\tilde{G}$, where both the traditional retailer and the online retailer are able to sell his/her products directly to consumers. This applies to the case when the online retailer is selling the differentiated product off-line, so that his profit from the differentiated product does not depend on the online retailer's selling channel. Alternatively, this can also be interpreted as the extreme case when the online retailer has zero bargaining power against the traditional retailer on the profit sharing of the differentiated product (could be due to competition among multiple online retailers). Same as in game G, the traditional retailer is able to supply the online retailer her common product. The basic structure of game $\tilde{G}$ is the same as game G, except that there is no profit sharing between the traditional retailer and the online retailer in stage 3:

- *Stage 3 in game $\tilde{G}$.* The traditional retailer chooses quantity q_d for the differentiated product. The traditional retailer and the online retailer sell their products to consumers and profits realize for each party.

Profits. For ease of presentation, we denote $q \equiv \{q_c^t, q_c^w, q_d\}$ as the production quantity vector/profile. We start from the final stage of the game, where the traditional retailer chooses q_d to maximize his profit $\tilde{\pi}_t(w, q) = (a - q_d - \gamma q_c)q_d + w_t q_c^t$. The traditional retailer observes the online retailer's quantity q_c^t but not q_c^w, her order quantity from the competitive fringe. The online retailer chooses her optimal quantity q_c to maximize her profit $\tilde{\pi}_o(w, q) = (a - q_c - \gamma q_d)q_c - w_t q_c^t - w q_c^w$.

According to whether the traditional retailer observes the quantity q_c of the online retailer or not, we first introduce two preliminary cases:

Case 1. When $q_c^t = 0$, the traditional retailer has no information regarding q_c. Their competition results in the Cournot quantities given by $\tilde{q}_d^C = \frac{(2-\gamma)a+\gamma w}{4-\gamma^2}, \tilde{q}_c^C = \frac{(2-\gamma)a-2w}{4-\gamma^2}$.

Note that when $w \geq (1 - \frac{\gamma}{2})a$, the online retailer ends up producing zero quantity in their Cournot competition. It is easily verified that in this case, the traditional retailer's optimal strategy is not to supply the online retailer so that he can reap the monopoly profit. To put their competition in order, we impose the assumption below for our following analysis:

Assumption 1. $w < \bar{w} \equiv (1 - \frac{\gamma}{2})a$.

Case 2. When $q_c^w = 0$, the traditional retailer knows q_c through the observation of q_c^t since $q_c = q_c^t$. They become Stackelberg players with the online retailer the quantity leader. The corresponding equilibrium quantities are $\tilde{q}_c^l = \frac{(2-\gamma)a - 2w_t}{2(2-\gamma^2)}$; $\tilde{q}_d^f = \frac{2(2-\gamma)a - \gamma^2 a + 2\gamma w_t}{4(2-\gamma^2)}$.

Cutoff structure. We find that the game $\tilde{G}(w_t, q_c^t)$ has a unique equilibrium, in which the Cournot quantity $\tilde{q}_c^C$ is the cutoff level of q_c^t for regime changes: if q_c^t is not larger than $\tilde{q}_c^C$, the equilibrium leads to Cournot quantities; otherwise, in equilibrium they supply Stackelberg quantities with the traditional retailer being a Stackelberg follower. In other words, by ordering $q_c^t > \tilde{q}_c^C$, the online retailer achieves an advantage in the ensuing Stackelberg game as she supplies more than the Cournot quantity, whereas the traditional retailer bears a disadvantage for supplying less than the Cournot quantity. Consequently, they have opposite incentives: On the one hand, the online retailer has the incentive to order $q_c^t > \tilde{q}_c^C$ from the traditional retailer for the Stackelberg leader's advantage. Correspondingly, she is willing to pay a premium price $w_t > w$ to the traditional retailer when purchasing the common product. On the other hand, the traditional retailer is unwilling to supply the online retailer because of the Stackelberg follower's disadvantage.

Exclusive sourcing pattern. Such divergence in the traditional retailer's and online retailer's incentive leads to an exclusive sourcing pattern of the online retailer. We find that so long as w_t is not too big, the online retailer orders her common product exclusively from the traditional retailer; otherwise, she orders exclusively from the competitive fringe. There exists a cutoff value of w for the change of the sourcing regime, denoted as $\tilde{w} \in (0, \bar{w})$ and is given by

$$\tilde{w} \equiv \frac{(2-\gamma)a[(2-\gamma)(8-6\gamma^2-\gamma^3) - 4(1-\gamma)(2+\gamma)\sqrt{2(2-\gamma^2)}]}{2(16-8\gamma^2-\gamma^4)}.$$

Our major finding is that in equilibrium, the traditional retailer provides the online retailer's common product only when he is sufficiently more efficient than the competitive fringe. More details are given below.

Lemma A.1. *In equilibrium of game $\tilde{G}$, $q_c^t q_c^w = 0$. I.e., the online retailer orders her common product exclusively either from the competitive fringe or from the traditional retailer. In particular,*

1. *If $w < \tilde{w}$, the online retailer orders exclusively from the competitive fringe and the Cournot outcome is played.*

2. *If $w > \tilde{w}$, the online retailer orders exclusively from the traditional retailer and the Stackelberg outcome is played.*

3. *If $w = \tilde{w}$, there is a continuum of equilibria. In one of these, the online retailer orders exclusively from the traditional retailer and the Stackelberg outcome is played. In the rest of these equilibria the online retailer orders exclusively from the competitive fringe and the Cournot outcome is played.*

By putting a quantity order with her competitor (the traditional retailer), the online retailer forces the latter to become a Stackelberg follower in their ensuing competition and bear the corresponding disadvantage. Such a situation leads to strategic considerations of both parties and impacts the equilibrium sourcing pattern. Same as in game G, the central message of game $\tilde{G}$ is that the traditional retailer may not supply the online retailer's common product even if he enjoys a cost advantage. Below we present a comparison between our findings from the benchmark game $\tilde{G}$ and game G.

Proposition A.1. *Compared to game $\tilde{G}$, game G admits a larger range of w within which the traditional retailer supplies the online retailer's common product.*

The result follows from the fact that $\tilde{w} > w_1$, where w_1 is the cutoff value of w in game G, below which the Cournot outcome arises in equilibrium. To see the intuition to the above result, note that game $\tilde{G}$ is the situation when the traditional retailer gets the whole profit of the differentiated product whereas game G is the situation when the traditional retailer and the online retailer split the profit of the differentiated product. Thus in game G the traditional retailer puts favor on the Stackelberg outcome in which he also profits from supplying the online retailer's common product.

B Detailed derivations of the preliminary results

In this appendix, we provide the detailed derivations of the bargaining outcomes in Section 3.2.

Bargaining with Stackelberg outcome. Given any q_c^t, the best response of the traditional retailer is given by $q_d^b(q_c^t) = \frac{1}{2}(a - \gamma q_c^t)$. By (2) and $q_c = q_c^t$, the Nash bargaining solution is

$$v(q_c^t, q_d^b(q_c^t)) = \begin{cases} \frac{a+\gamma q_c^t}{2(a-\gamma q_c^t)} & \text{if} \quad q_c^t \leq \frac{a}{3\gamma} \\ 1 & \text{if} \quad q_c^t > \frac{a}{3\gamma}. \end{cases}$$

In this case, the online retailer becomes a Stackelberg leader and the traditional retailer becomes a

Stackelberg follower. At any given q_c^t, the traditional retailer's profit is

$$\pi_t^f(w_t,q_c^t) \equiv \pi_t(w_t, v(q_c^t, q_d^b(q_c^t)), q_c^t, q_d^b(q_c^t))$$
$$= \begin{cases} \frac{a^2}{8} + (w_t - \frac{a\gamma}{2})q_c^t + \frac{3\gamma^2}{8}(q_c^t)^2 & \text{if } \; q_c^t \leq \frac{a}{3\gamma} \\ w_t q_c^t & \text{if } \; q_c^t > \frac{a}{3\gamma}; \end{cases}$$

and the online retailer's profit is

$$\pi_o^l(w_t,q_c^t) \equiv \pi_o(w_t, v(q_c^t, q_d^b(q_c^t)), q_c^t, q_d^b(q_c^t))$$
$$= \begin{cases} \frac{a^2}{8} + (a - \frac{\gamma a}{2} - w_t)q_c^t + (\frac{3\gamma^2}{8} - 1)(q_c^t)^2 & \text{if } \; q_c^t \leq \frac{a}{3\gamma} \\ \frac{a^2}{4} + (a - \gamma a - w_t)q_c^t + (\frac{3\gamma^2}{4} - 1)(q_c^t)^2 & \text{if } \; q_c^t > \frac{a}{3\gamma}. \end{cases}$$

Bargaining with Cournot outcome. In this case, $v(q_c^w, q_d) = \frac{a-q_d}{2(a-q_d-\gamma q_c^w)}$ is their bargaining solution, given by (2) at $q_c^t = 0$. The solution that jointly solves (3) and (4), denoted by $\{q_d^C, q_c^C\}$, is given by

$$q_c^C = \frac{(2-\gamma)a - 2w}{4-\gamma^2}, \text{ and } q_d^C = \frac{(2-\gamma)a + \gamma w}{4-\gamma^2}. \tag{6}$$

Inserting (6) into (2), the negotiated revenue sharing is:

$$v^C \equiv v(q_c^C, q_d^C) = \frac{(2+\gamma-\gamma^2)a - \gamma w}{2[(2-\gamma)a + \gamma w]}. \tag{7}$$

Correspondingly, π_t^C and π_o^C are the traditional retailer's and online retailer's profits under these quantity and revenue sharing decisions:

$$\pi_o^C = \frac{(3+\gamma)(2-\gamma)^2a^2 - (8-\gamma^2)(2-\gamma)aw + (8-\gamma^2)w^2}{2(2-\gamma)^2(2+\gamma)^2}, \pi_t^C = \frac{[(2-\gamma)a + \gamma w][(2-3\gamma+\gamma^2)a + 3\gamma w]}{2(2-\gamma)^2(2+\gamma)^2}.$$

C Proofs of main results

In this appendix, we provide the technical proofs of our main results.

Proof of Lemma 3.1. For each game $G(w_t, q_c^t)$, a perfect Bayesian equilibrium (PBE) is specified by the triplet $\{\tilde{v}, \tilde{q}_c^w, \tilde{q}_d\}$ and a belief system. We adopt the degenerate belief system that the traditional retailer assigns probability 1 to $q_c^w = \tilde{q}_c^w$ and the online retailer assigns probability 1 to $q_d = \tilde{q}_d$. Given this belief, we can conveniently use $\tilde{v}$ given by (2) as it coincides with what the traditional retailer and the online retailer will settle on. In any PBE, the Nash bargaining solution is

$$\tilde{v} \equiv v(q_c^t + \tilde{q}_c^w, \tilde{q}_d) = \frac{a - \tilde{q}_d}{2(a - \tilde{q}_d - \gamma q_c^t - \gamma \tilde{q}_c^w)}. \tag{8}$$

The decisions of q_c^w by the online retailer and q_d by the traditional retailer are tantamount to a simultaneous-move game. Thus, at optimality the traditional retailer chooses $\tilde{q}_d$ such that:

$$\begin{aligned}\tilde{q}_d &= \arg\max_{q_d \geq 0} \pi_d(v, \tilde{q}_c^w, q_d) \\ &= \arg\max_{q_d \geq 0} \begin{cases} (1-v)(a - q_d - \gamma q_c^t - \gamma \tilde{q}_c^w) q_d + w_t q_c^t & \text{if} \quad \tilde{q}_c^w > 0 \\ (1-v)(a - q_d - \gamma q_c^t) q_d + w_t q_c^t & \text{if} \quad \tilde{q}_c^w = 0. \end{cases}\end{aligned} \tag{9}$$

Foreseeing the bargaining outcome $v = \tilde{v}$, the online retailer chooses $\tilde{q}_c^w$ such that

$$\begin{aligned}\tilde{q}_c^w &= \arg\max_{q_c^w \geq 0} \pi_c(\tilde{v}, q_c^w, \tilde{q}_d) \\ &= \arg\max_{q_c^w \geq 0} \{\tilde{v}(a - \tilde{q}_d - \gamma q_c^t - \gamma q_c^w)\tilde{q}_d + (a - q_c^t - q_c^w - \gamma \tilde{q}_d)(q_c^t + q_c^w) - w_t q_c^t - w q_c^w\}.\end{aligned} \tag{10}$$

Solving (9) and (10) together gives rise to the following solution:

$$\tilde{q}_d = \begin{cases} q_d^C & \text{if} \quad q_c^t \leq q_c^C \\ q_d^b(q_c^t) & \text{if} \quad q_c^t > q_c^C \end{cases}, \text{ and } \tilde{q}_c^w = \begin{cases} q_c^C - q_c^t & \text{if} \quad q_c^t \leq q_c^C \\ 0 & \text{if} \quad q_c^t > q_c^C \end{cases}.$$

By (8), in the PBE

$$\tilde{v} = \begin{cases} v^C & \text{if} \quad q_c^t \leq q_c^C \\ v(q_c^t, q_d^b(q_c^t)) & \text{if} \quad q_c^t > q_c^C. \end{cases}$$

Note that $q_c^C < \frac{a}{3\gamma}$. Thus, q_c^C is the threshold of q_c^t that separates the Cournot outcome and the Stackelberg outcome in which the traditional retailer is a follower. □

Proof of Proposition 3.1. Before we carry out the proof, we first give the explicit expressions of the quantities, the value of the revenue sharing, and the payoffs. The Stackelberg leader's quantity of the online retailer, denoted by $q_c^L(w_t)$, is given below:

$$q_c^L(w_t) = \begin{cases} q_c^{Lc}(w_t) = \frac{a}{3\gamma} & \text{if} \quad w_t < \hat{w}_t \\ q_c^{L*}(w_t) = \frac{2[(2-\gamma)a - 2w_t]}{8-3\gamma^2} & \text{if} \quad w_t \in [\hat{w}_t, \bar{w}] \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

Accordingly, the follower's quantity (supplied by the traditional retailer) and the equilibrium value of the revenue sharing v are:

$$q_d^F(w_t) = \begin{cases} q_d^{Fc}(w_t) = \frac{a}{3} & \text{if} \quad w_t < \hat{w}_t \\ q_d^{F*}(w_t) = \frac{(8-4\gamma-\gamma^2)a + 4\gamma w_t}{2(8-3\gamma^2)} & \text{if} \quad w_t \in [\hat{w}_t, \bar{w}] \\ \frac{a}{2} & \text{otherwise,} \end{cases} \tag{12}$$

$$v^S(w_t) = \begin{cases} 1 & \text{if} \quad w_t < \hat{w}_t \\ v(q_c^{L*}(w_t), q_d^{F*}(w_t)) = \frac{(8+4\gamma-5\gamma^2)a-4\gamma w_t}{2[(8-4\gamma-\gamma^2)a+4\gamma w_t]} & \text{if} \quad w_t \in [\hat{w}_t, \bar{w}] \\ \frac{1}{2} & \text{if} \quad w_t > \bar{w}. \end{cases} \tag{13}$$

The corresponding online retailer's and traditional retailer's profits in this leader-follower relationship are

$$\pi_o^L(w_t) = \begin{cases} \pi_o^{Lc}(w_t) = \frac{a(3\gamma a - a - 3\gamma w_t)}{9\gamma^2} & \text{if} \quad w_t < \hat{w}_t \\ \pi_o^{L*}(w_t) = \frac{(24+\gamma^2-16\gamma)a^2+(16\gamma-32)aw_t+16w_t^2}{8(8-3\gamma^2)} & \text{if} \quad w_t \in [\hat{w}_t, \bar{w}] \\ \frac{a^2}{8} & \text{if} \quad w_t > \bar{w}, \end{cases}$$

$$\pi_t^F(w_t) = \begin{cases} \pi_t^{Fc}(w_t) = \frac{aw_t}{3\gamma} & \text{if} \quad w_t < \hat{w}_t \\ \pi_t^{F*}(w_t) = \frac{(8-12\gamma+3\gamma^2)(8-4\gamma-\gamma^2)a^2-16(16-12\gamma^2+3\gamma^3)aw_t+(144\gamma^2-256)w_t^2}{8(8-3\gamma^2)^2} & \text{if} \quad w_t \in [\hat{w}_t, \bar{w}] \\ \frac{a^2}{8} & \text{if} \quad w_t > \bar{w}. \end{cases}$$

Now we prove the proposition. First, when $q_c^t \le q_c^C$, by Lemma 3.1, the online retailer orders $q_c^w = q_c^C - q_c^t$ and her profit is $\pi_o^C + (w - w_t)q_c^t$. Thus, there are three cases. (i) If $w < w_t$, the online retailer sets $q_c^t = 0, q_c^w = q_c^C$, and her profit is π_o^C. (ii) If $w > w_t$, the online retailer sets $q_c^t = q_c^C, q_c^w = 0$, and her profit is $\pi_o^C + (w - w_t)q_c^C$. (iii) If $w = w_t$, the online retailer sets $q_c^t \in [0, q_c^C]$ and $q_c^w = q_c^C - q_c^t$, and her profit is π_o^C. We denote the scenario when $q_c^t \le q_c^C$ as the *Cournot regime*.

On the other hand, when $q_c^t > q_c^C$, by Lemma 3.1, the online retailer orders $q_c^w = 0$ and becomes a Stackelberg leader in the subsequent competition with the traditional retailer. Thus, she should choose q_c^t to maximize $\pi_o^l(w_t, q_c^t)$. The optimal q_c^t is solved as $q_c^L(w_t)$ and the online retailer's optimal profit is $\pi_o^L(w_t)$. We denote the scenario when $q_c^t > q_c^C$ as the *Stackelberg regime*.

Therefore, for any given w_t, the optimal q_c^t is determined by the online retailer through comparing her equilibrium profit in the Cournot regime and in the Stackelberg regime. There are two cases, depending on whether $w_t < w$ or $w_t \ge w$. In what follows, we divide our analysis into these two cases.

Case 1. $w_t < w$.

In this case, $w_t < \bar{w}$. In the Cournot regime, the online retailer's profit is $\pi_o^C + (w - w_t)q_c^C$. In the Stackelberg regime, her profit depends on the value of w_t. We compare the online retailer's profits in the two subcases:

Subcase 1a. $w_t < \hat{w}_t$**.**

Since $\hat{w}_t > 0$ for $\gamma > 0.845$, the scenario $w_t < \hat{w}_t$ can arise only when $\gamma > 0.845$. In the Cournot regime, the online retailer's profit is $\pi_o^C + (w - w_t)q_c^C$. On the other hand, in the Stackelberg regime, the online retailer becomes a monopolist and her profit is $\pi_o^{Lc}(w_t)$. It is verifiable that $\phi \equiv \pi_o^{Lc}(w_t) - [\pi_o^C + (w - w_t)q_c^C]$ strictly decreases in w_t, and $\phi|_{w_t=\hat{w}_t} > 0$. We conclude that for $w_t < \hat{w}_t$, the online retailer will set $q_c^t = q_c^L(w_t) > q_c^C$ for the Stackelberg regime.

Subcase 1b. $w_t \in [\hat{w}_t, w)$**.**

In the Cournot regime, the online retailer's profit is $\pi_o^C + (w - w_t)q_c^C$; and in the Stackelberg regime, the online retailer's profit is $\pi_o^{L*}(w_t)$. It is verifiable that $\pi_o^{L*}(w_t) - [\pi_o^C + (w - w_t)q_o^C]$ strictly decreases in w_t, and at $w_t = w$, $\pi_o^{L*}(w_t) - [\pi_o^C + (w - w_t)q_c^C] > 0$. Thus $\pi_o^{L*}(w_t) > \pi_o^C + (w - w_t)q_c^C$. We conclude that for $w_t \in [\hat{w}_t, w)$, the online retailer sets $q_c^t = q_c^{L*}(w_t) > q_c^C$ for the Stackelberg regime.

Case 2. $w_t \geq w$.

Again there are two subcases:

Subcase 2a. $w_t \in [w, \hat{w}_t]$**.**

This is relevant only when $\gamma > 0.845$. In the Cournot regime, the online retailer's profit is π_o^C; and in the Stackelberg regime, the online retailer's profit is $\pi_o^{Lc}(w_t)$. Define a strictly increasing function $\bar{w}_t'(w) : [0, \hat{w}] \to [w, \hat{w}_t]$ as:

$$\bar{w}_t'(w) = \frac{3\gamma(8-\gamma^2)[a(2-\gamma) - w]w}{2a(2+\gamma)^2(2-\gamma)^2} - \frac{a(1-\gamma)(8-8\gamma-3\gamma^2)}{6\gamma(2+\gamma)^2}.$$

We find that $\pi_o^{Lc}(w_t) \gtreqless \pi_o^C \Leftrightarrow w_t \lesseqgtr \bar{w}_t'(w)$.

Subcase 2b. $w_t \in (\hat{w}_t, \bar{w}]$**.**

In this case, the marginal cost of the online retailer is w in the Cournot regime and it is w_t in the Stackelberg regime. In the Cournot regime, her profit is π_o^C; In the Stackelberg regime, her profit is $\pi_o^{L*}(w_t)$. Define a strictly increasing function $\bar{w}_t(w) : [\hat{w}, \bar{w}] \to [\hat{w}_t, \bar{w}]$ as:

$$\bar{w}_t(w) = \frac{a(2-\gamma)}{2} - \frac{[(2-\gamma)a - 2w]\sqrt{(8-3\gamma^2)(8-\gamma^2)}}{4(2+\gamma)(2-\gamma)}.$$

Note that $\bar{w}_t(\bar{w}) = \bar{w}$. We find that $\pi_o^{L*}(w_t) \gtreqless \pi_o^C \Leftrightarrow w_t \lesseqgtr \bar{w}_t(w)$. Here $\hat{w}$ is defined as

$$\hat{w} = \frac{a[3A\gamma(2-\gamma) - 3\gamma^4 + 20\gamma^2 - 32]}{6A\gamma}$$

with $A = \sqrt{(8-\gamma^2)(8-3\gamma^2)}$. Note that $\hat{w} > 0$ only for $\gamma > 0.851$. It holds true that $\bar{w}_t(w) > w, \bar{w}'_t(w) > w$; $\bar{w}_t(w) = \bar{w}'_t(w) = \hat{w}_t$ at $w = \hat{w}$. We obtain the results as stated in the proposition. Note that in both regimes, $q_c^t q_c^w = 0$; thus, exclusive sourcing always occurs in equilibrium. □

Proof of Theorem 3.1. We organize the proof as follows. First, in step 1, we provide the closed-form expressions of some critical wholesale price cutoffs and the corresponding quantities and payoffs. Following this, we in step 2 establish the cutoff structure of the equilibrium as stated in the theorem. Finally, in step 3 we meticulously refine the equilibria to show that there does not exist other equilibria than those stated in the theorem.

Step 1) Definitions and explicit expressions.

Let us focus on Stackelberg regime first. First, when the traditional retailer sets the wholesale price at this cutoff level, the resulting order quantity from the online retailer is

$$q_c^{L*} \equiv q_c^{L*}(\bar{w}_t(w)) = \frac{[(2-\gamma)a - 2w]\sqrt{(8-3\gamma^2)(8-\gamma^2)}}{(4-\gamma^2)(8-3\gamma^2)}.$$

Accordingly, the traditional retailer's follower quantity of the differentiated product is set at:

$$q_d^{F*} \equiv q_d^{F*}(\bar{w}_t(w)) = \frac{1}{2} - \frac{\sqrt{(8-3\gamma^2)(8-\gamma^2)}a}{2(2+\gamma)(8-3\gamma^2)} + \frac{\sqrt{(8-3\gamma^2)(8-\gamma^2)}\gamma w}{(4-\gamma^2)(8-3\gamma^2)}.$$

On the other hand, if the traditional retailer is able to maximize his Stackelberg follower's profit $\pi_t^{F*}(w_t)$, let us define $w_t^* \in (0, \bar{w})$ as

$$w_t^* = \frac{a(16+3\gamma^3-12\gamma^2)}{2(16-9\gamma^2)},$$

which is the interior solution of the optimal wholesale price in the Stackelberg regime. Under this wholesale price, $w_t^* \leq \bar{w}_t(w)$ holds so that the online retailer complies with the traditional retailer's plan to play the Stackelberg outcome, and the traditional retailer's payoff is maximized. The Stackelberg quantities of the online retailer and the traditional retailer at $w_t = w_t^*$ are

$$q_c^{L*}(w_t^*) = \frac{4a(1-\gamma)}{16-9\gamma^2}, \quad q_d^{F*}(w_t^*) = \frac{a(16-4\gamma-5\gamma^2)}{2(16-9\gamma^2)}.$$

We further define w_{1x} as the critical value such that the following is satisfied:

$$\pi_t^{F*}(\bar{w}_t(w)) \gtreqless \pi_t^C \Leftrightarrow w \gtreqless w_{1x}. \tag{14}$$

where $\pi_t^{F*}(\bar{w}_t(w))$ is the traditional retailer's maximum follower profit in the Stackelberg regime if he sets the boundary wholesale price that induces the online retailer to order from him, and

π_t^C is the traditional retailer's profit in the Cournot regime. We can express a precise formula for $w_{1x} \in (0, \bar{w})$ as

$$w_{1x} = \frac{a}{32(2+\gamma)}[(2-\gamma)(16-12\gamma^2-3\gamma^3) - 2(1-\gamma)(2+\gamma)\sqrt{(8-\gamma^2)(8-3\gamma^2)}].$$

Similarly, we define another cutoff value w_{1y} such that

$$\pi_t^{Fc}(\bar{w}_t'(w)) \gtreqless \pi_t^C \Leftrightarrow w \gtreqless w_{1y}.$$

where $\pi_t^{Fc}(\bar{w}_t'(w))$ is the traditional retailer's maximum follower profit in the Stackelberg regime given that $v = 1$ in their profit sharing, if he sets the boundary wholesale price that induces the online retailer to order from him. Since $\bar{w}_t'(w)$ is defined only for $w \leq \hat{w}$ and $\hat{w} > 0$ only for $\gamma > 0.851$, we can express a precise formula for $w_{1y} \in (0, \bar{w})$ for $\gamma > 0.851$ as

$$w_{1y} = \frac{a(2-\gamma)[3\gamma(2-\gamma) - (2+\gamma)\sqrt{3\gamma^2 - 10\gamma^2 + 12\gamma - 4}]}{3\gamma(4+\gamma^2)}.$$

It holds that $w_{1x}|_{\gamma=0} = 0, w_{1y}|_{\gamma=1} = 0$. For $\gamma > 0.851$, it holds that $w_{1x} \gtreqless w_{1y} \Leftrightarrow \gamma \gtreqless 0.888$. We define

$$w_1 = \begin{cases} w_{1x} & \text{if} \quad \gamma \leq 0.888 \\ w_{1y} & \quad \text{o.w.} \end{cases}.$$

Note that $\bar{w}_t'(w) < w_t^*$ for $\gamma > 0.888$. For $\gamma \leq 0.888$, we have $\bar{w}_t(w) \gtreqless w_t^* \Leftrightarrow w \gtreqless w_2$, with this cutoff level w_2 given below:

$$w_2 = \frac{a(2-\gamma)}{2}[1 - \frac{4(1-\gamma)(2+\gamma)(8-3\gamma^2)}{(16-9\gamma^2)\sqrt{(8-\gamma^2)(8-3\gamma^2)}}].$$

Thus the cutoff level $w_2 \in (w_1, \bar{w})$ separates two regions. When the fringe is not very competitive ($w \geq w_2$), the traditional retailer can easily set the wholesale price to beat the fringe in the common product sourcing. Thus, the optimal wholesale price is set at the interior solution w_t^*. The corresponding Stackelberg profits are $\pi_t^{F*}(w_t^*) = \frac{a^2(32-32\gamma+7\gamma^2)}{8(16-9\gamma^2)}$ for the traditional retailer, and $\pi_o^{L*}(w_t^*) = \frac{a^2(384-256\gamma-208\gamma^2+96\gamma^3+33\gamma^4)}{8(16-9\gamma^2)^2}$ for the online retailer. On the other hand, when the fringe is rather competitive ($w < w_2$) in the Stackelberg regime, the traditional retailer intends to increase the wholesale price, but it is upper bounded by the cutoff level $\bar{w}_t(w)$. Thus, he will simply set at the upper bound.

Next, we provide some structural properties of the wholesale prices and the profit functions that are useful for the subsequent analysis. As these results follow from straightforward calculations. We omit the details and state them in a lemma.

Lemma C.1. *The following statements are true.*

1. $\pi_t^{F*}(w_t)$ strictly increases for $w_t < w_t^*$ and strictly decreases for $w_t > w_t^*$. It is maximized at $w_t = w_t^*$.

2. π_t^C strictly increases in w.

3. $\bar{w}_t(w) \lesseqqgtr w_t^* \Leftrightarrow w \lesseqqgtr w_2$.

4. $\bar{w}_t(w_{1x}) \gtreqqless \hat{w}_t \Leftrightarrow \gamma \lesseqqgtr 0.888$; $\bar{w}_t'(w_{1y}) \gtreqqless \hat{w}_t \Leftrightarrow \gamma \lesseqqgtr 0.888$.

Step 2) Cutoff structure.

Now we establish the cutoff structure that distinguishes the two regimes. Specifically, our goal is to prove that for $\gamma \le 0.888$,

1. If $w \le w_{1x}$, then $w_t \ge \bar{w}_t(w)$ followed by the Cournot outcome is in SPNE.

2. If $w_{1x} \le w \le w_2$, then $w_t = \bar{w}_t(w)$ followed by the Steckelberg outcome is in SPNE.

3. If $w > w_2$, then $w_t = w_t^*$ followed by the Steckelberg outcome is in SPNE.

And for $\gamma > 0.888$,

1. If $w \le w_{1y}$, then $w_t \ge \bar{w}_t'(w)$ followed by the Cournot outcome is in SPNE.

2. If $w > w_{1y}$, then $w_t = \bar{w}_t'(w)$ followed by the Steckelberg outcome is in SPNE.

We provide proof for the case $\gamma \le 0.888$; the argument for $\gamma > 0.888$ is similar and hence omitted. To prove the above cutoff structure, we divide our analysis into three cases: (i) $w \le w_1$, (ii) $w_1 \le w \le w_2$, and (iii) $w > w_2$.

Case (i) $w \le w_1$.

If $w_t \ge \bar{w}_t(w)$, by Proposition 3.1, there exists an SPNE in stages 2 and 3 where the Cournot outcome is played. In this SPNE, the traditional retailer's profit is π_t^C. To sustain such an equilibrium, we need to prove that in stage one the traditional retailer has no incentive to deviate. Suppose that the traditional retailer deviates to a wholesale price $w_t' \ge \bar{w}_t(w)$. By Proposition 3.1, the Cournot outcome follows and his profit is still π_t^C. Thus the traditional retailer has no incentive to deviate. Now suppose that the traditional retailer deviates to choose $w_t' < \bar{w}_t(w)$. By Proposition 3.1, the Stackelberg outcome follows. If $w_t' \ge \hat{w}_t$, the traditional retailer's profit is $\pi_t^{F*}(w_t')$.

By Lemma C.1, we obtain that $w_t' < \bar{w}_t(w) \leq \bar{w}_t(w_1) < w_t^*$. Additionally, the shape of π_t^{F*} (from Lemma C.1) indicates that $\pi_t^{F*}(w_t') < \pi_t^{F*}(\bar{w}_t(w)) \leq \pi_t^C$. The traditional retailer is worse off by this deviation. Instead if $w_t' < \hat{w}_t$, the traditional retailer's profit is $\pi_t^{Fc}(w_t')$. By Lemma C.1, we obtain that $w_t' < \bar{w}_t'(w_1)$. Since $\pi_t^{Fc}(w_t)$ strictly increases in w_t, we have $\pi_t^{Fc}(w_t') < \pi_t^{Fc}(\bar{w}_t'(w)) \leq \pi_t^C$. The traditional retailer is again worse off by this deviation.

Case (ii) $w_1 \leq w \leq w_2$.

Given that $w_t = \bar{w}_t(w)$, by Proposition 3.1, there exists an SPNE in stages 2 and 3 where the Stackelberg outcome is played. Since $\bar{w}_t(w) > \hat{w}_t$ (Lemma C.1), the Stackelberg follower's profit of the traditional retailer is $\pi_t^{F*} = \pi_t^{F*}(\bar{w}_t(w))$. We need to prove that in stage 1 the traditional retailer has no incentive to deviate. First, let the traditional retailer deviate to choose $w_t' < \bar{w}_t(w)$. According to Proposition 3.1, the Stackelberg outcome follows. In this case, there are two scenarios: a) $w_t' \geq \hat{w}_t$ and b) $w_t' < \hat{w}_t$. In scenario a) where $w_t' \geq \hat{w}_t$, the traditional retailer's profit is $\pi_t^{F*}(w_t') < \pi_t^{F*}(\bar{w}_t(w))$ by the monotonicity of $\pi_t^{F*}(w_t)$ and the fact that $w_t' < \bar{w}_t(w) \leq w_t^*$ for $w \leq w_2$. Thus, the traditional retailer is worse off and will not deviate to $w_t' \geq \hat{w}_t$. In scenario b) where $w_t' < \hat{w}_t$, the traditional retailer's profit is $\pi_t^{Fc}(w_t') < \pi_t^{Fc}(\hat{w}_t)$. It is verifiable that $\pi_t^{F*}(\bar{w}_t(w_{1x})) > \pi_t^{Fc}(\hat{w}_t)$ for $\gamma < 0.888$. Thus, the traditional retailer is again worse off and will not deviate.

Second, suppose that the traditional retailer deviates to choose $w_t' > \bar{w}_t(w)$. In this case, the Cournot outcome follows (by Proposition 3.1) and the traditional retailer gets a profit π_t^C. Because $\pi_t^C \leq \pi_t^{F*}$ by (14), the traditional retailer again has no incentive to deviate.

Case (iii) $w > w_2$.

According to Lemma C.1, $w_t^* < \bar{w}_t(w)$. Given that $w = w_t^*$, Proposition 3.1 implies that in SPNE of stages 2 and 3, the Stackelberg outcome is played and the traditional retailer's profit is $\pi_t^{F*}(w_t^*)$. Let us now show that in stage 1 the traditional retailer has no incentive to deviate. First, let the traditional retailer deviate to $w_t' < w_t^*$. Proposition 3.1 then implies that the Stackelberg outcome follows. There are two scenarios: a) $w_t' \geq \hat{w}_t$ and b) $w_t' < \hat{w}_t$. If $w_t' \geq \hat{w}_t$, the traditional retailer's profit is $\pi_t^{F*}(w_t') < \pi_t^{F*}(w_t^*)$ by Lemma C.1. This implies that the traditional retailer is worse off and will not deviate. If $w_t' < \hat{w}_t$, the traditional retailer's profit is $\pi_t^{Fc}(w_t')$ satisfying $\pi_t^{Fc}(w_t') < \pi_t^{Fc}(\hat{w}_t) < \pi_t^{F*}(\bar{w}_t(w)) < \pi_t^{F*}(w_t^*)$. Thus, this deviation is not profitable. Second, let the traditional retailer deviate to choose $w_t' \in (w_t^*, \bar{w}_t(w)]$. By Proposition 3.1, the Stackelberg outcome follows and the traditional retailer's profit is $\pi_t^{F*}(w_t') < \pi_t^{F*}(w_t^*)$ by Lemma C.1. This again leads to an unprofitable deviation. Third, suppose that the traditional retailer deviates to choose $w_t' \geq \bar{w}_t(w)$. By Proposition 3.1, the Cournot outcome follows and the traditional retailer gets $\pi_t^C < \pi_t^{F*}(\bar{w}_t(w)) < \pi_t^{F*}(w_t^*)$ by (14) and Lemma C.1. Collectively, we conclude that no deviation is profitable.

Step 3) Equilibrium refinement.

We focus on the case $\gamma \leq 0.888$. The last step is the equilibrium refinement. We again first divide our analysis into three cases: (i) $w < w_1$, (ii) $w_1 < w \leq w_2$, and (iii) $w > w_2$. Following this, we then discuss the boundary case (iv) $w = w_1$.

Case (i) $w < w_1$.

First, we show that for $w_t \geq \bar{w}_t(w)$, in SPNE it must be the Cournot outcome followed in stage 2 and stage 3. If $w_t > \bar{w}_t(w)$, this follows immediately by Proposition 3.1. If $w_t = \bar{w}_t(w)$, in stage 2 the online retailer is indifferent between the Cournot outcome and the Stackelberg outcome. We need to show that it cannot be the Stackelberg outcome in SPNE. Suppose not. Then the traditional retailer's profit is $\pi_t^F(\bar{w}_t(w))$, which is given by $\pi_t^{F*}(\bar{w}_t(w))$ since $\bar{w}_t(w) > \hat{w}_t$. By (14), $\pi_t^{F*}(\bar{w}_t(w)) < \pi_t^C$. The traditional retailer is better off deviating to $w_t' > \bar{w}_t(w)$, since by deviating he gets the Cournot outcome and his profit is π_t^C, which leads to a contradiction. Therefore, the traditional retailer will deviate, meaning that the Stackelberg outcome cannot be in SPNE.

Second, we show that in any SPNE, it must be that $w_t \geq \bar{w}_t(w)$. Suppose not. Then we obtain that $w_t < \bar{w}_t(w)$. From Proposition 3.1, the traditional retailer's profit is $\pi_t^{F*}(w_t)$. By Lemma C.1 and (14), $\pi_t^{F*}(w_t) < \pi_t^{F*}(\bar{w}_t(w)) < \pi_t^C$. The traditional retailer is better off deviating to $w_t > \bar{w}_t(w)$ for the Cournot profit π_t^C, which leads to a contradiction.

Case (ii) $w_1 < w \leq w_2$.

To show that there does not exist any other SPNE, we first prove that at $w_t = \bar{w}_t(w)$, in SPNE it must be the Stackelberg outcome followed in stages 2 and 3. In this case, the online retailer is indifferent between the Cournot outcome and the Stackelberg outcome, but the traditional retailer prefers the Stackelberg outcome as $\pi_t^C < \pi_t^{F*}(\bar{w}_t(w))$. If the Cournot outcome were played, the traditional retailer would deviate to choose $w_t' = \bar{w}_t(w) - \epsilon$ where ϵ is an arbitrarily small positive number such that the Stackelberg outcome results according to Proposition 3.1. With an infinitesimally small ϵ, the traditional retailer would be strictly better off.

Second, we show that in SPNE, it must be $w_t = \bar{w}_t(w)$. To this end, we in the following rule out the two possibilities (i) $w_t > \bar{w}_t(w)$ and (ii) $w_t < \bar{w}_t(w)$. In scenario (i) $w_t > \bar{w}_t(w)$, by

Proposition 3.1, the Cournot outcome follows and the traditional retailer gets π_t^C. Now consider the possibility that the traditional retailer deviates to choose $w_t' = \bar{w}_t(w) - \epsilon$, with ϵ an arbitrarily small positive number. By Proposition 3.1, the Stackelberg outcome follows and the traditional retailer gets $\pi_t^{F*}(w_t') > \pi_t^C$ by (14), which implies that this is a profitable deviation. In scenario (ii) where $w_t < \bar{w}_t(w)$, by Proposition 3.1, the traditional retailer's profit is $\pi_t^{F*}(w_t)$ for $w_t \geq \hat{w}_t$ and $\pi_t^{Fc}(w_t)$ for $w_t < \hat{w}_t$. Suppose $w_t \geq \hat{w}_t$. By Lemma C.1, the traditional retailer is better off deviating to $w_t' = w_t + \epsilon < \bar{w}_t(w)$ since $\pi_t^{F*}(w_t') > \pi_t^{F*}(w_t)$, a contradiction. On the other hand, suppose $w_t < \hat{w}_t$. By Lemma C.1, the traditional retailer, by deviating to $w_t' = w_t + \epsilon$, can strictly improve his profit; this again leads to a contradiction. Thus it must be that $w_t = \bar{w}_t(w)$ in SPNE.

Case (iii) $w > w_2$.

We again rule out any other SPNE. Note that at $w_t = w_t^* < \bar{w}_t(w)$, by Proposition 3.1, it follows that it must be the Stackelberg outcome in stages 2 and 3. There are several scenarios to be ruled out. First, suppose $w_t \in (w_t^*, \bar{w}_t(w)]$ or $w_t \in [\hat{w}_t, w_t^*)$. From Proposition 3.1, the traditional retailer gets the Stackelberg follower's profit $\pi_t^{F*}(w_t)$, which is less than $\pi_t^{F*}(w_t^*)$ by Lemma C.1. Thus, the traditional retailer is better off deviating to $w_t' = w_t^*$, a contradiction. Second, suppose that $w_t \geq \bar{w}_t(w)$. From Proposition 3.1, the traditional retailer gets the Cournot profit π_t^C. Since $\pi_t^C < \pi_t^{F*}(\bar{w}_t(w)) < \pi_t^{F*}(w_t^*)$ by Lemma C.1 and (14), the traditional retailer is better off deviating to $w_t' = w_t^*$ to get $\pi_t^{F*}(w_t^*)$. Finally, suppose that $w_t < \hat{w}_t$. Then according to Lemma C.1, the traditional retailer is better off deviating to $w_t' = w_t + \epsilon < \hat{w}_t$, again a contradiction. We conclude that it must be $w_t = w_t^*$.

Case (iv) $w = w_1$.

In this case, the traditional retailer's profit is π_t^C in the Cournot outcome and it is $\pi_t^{F*}(\bar{w}_t(w_1))$ in the Stackelberg outcome. By (14), $\pi_t^{F*}(\bar{w}_t(w_1)) = \pi_t^C$. Thus, both outcomes can emerge as an equilibrium. It is then straightforward to verify that there does not exist any other SPNE.

First, for $w_t > \bar{w}_t(w_1)$, by Proposition 3.1, in SPNE it must be the Cournot outcome in stage 2 and stage 3. Second, for $w_t = \bar{w}_t(w_1)$, by Proposition 3.1, either the Cournot outcome or the Stacekelberg outcome can arise in SPNE in stage 2 and stage 3. Third, we show that it cannot be $w_t < \bar{w}_t(w_1)$ in any SPNE. Suppose not. By Proposition 3.1, the traditional retailer's profit is $\pi_t^{F*}(w_t)$ for $w_t \geq \hat{w}_t$ and $\frac{w_t(a-w_t)}{2}$ for $w_t < \hat{w}_t$. We consider the two scenarios separately: (a) Suppose $w_t \geq \hat{w}_t$. By Lemma C.1, the traditional retailer is better off deviating to $w_t' = w_t + \epsilon < \bar{w}_t(w)$ since $\pi_t^{F*}(w_t') > \pi_t^{F*}(w_t)$, a contradiction. (b) Suppose that $w_t < \hat{w}_t$. Lemma C.1 then suggests that the traditional retailer, by deviating to $w_t' = w_t + \epsilon < \hat{w}_t$, can strictly improve his profit. Thus, it cannot be $w_t < \bar{w}_t(w_1)$ in SPNE. $\square$

Proof of Corollaries 3.1-3.2. The proof follows the proof of Theorem 3.1 and is omitted.

Proof of Corollary 3.3. It is verifiable that $\frac{dw_1}{da} > 0$ and $\frac{dw_1}{d\gamma} > 0$, which subsequently give rise to the statements in the corollary. $\square$

Proof of Corollary 3.4. For $w > w_2$, straightforward algebra shows that

$$\frac{dq_c^{L*}(w_t^*)}{d\gamma} < 0; \qquad \frac{dq_d^{F*}(w_t^*)}{d\gamma} \gtreqless 0 \Leftrightarrow \gamma \gtreqless 0.602.$$

$\square$

Proof of Corollary 3.5. We first consider the Cournot regime where $v = v^C$. In this case, we obtain that $\text{sign}(\frac{dv^C}{d\gamma}) = \text{sign}((2-\gamma)^2 a - (4+\gamma^2)w)$, which is positive for $w \leq w_1$.

In the Stackelberg regime with $w > w_2$, $v = v^S(w_m^*)$ in equilibrium. This gives rise to the following: $\frac{dv^S(w_t^*)}{d\gamma} \gtreqless 0 \Leftrightarrow \gamma \lesseqgtr 0.602$. When $w \in (w_1, w_2]$, $v = v^S(\bar{w}_t)$ and $\text{sign}(\frac{dv^S(\bar{w}_t)}{d\gamma}) = \text{sign}(x)$, with $x \equiv 3a\gamma^6 - 3w\gamma^6 - 4a\gamma^5 - 36a\gamma^4 + 36w\gamma^4 + 96a\gamma^3 - 256a\gamma + 256a - 256w$. It holds that $\frac{dv^S(\bar{w}_t)}{d\gamma})|_{w=w_1} > 0$ and $\frac{dv^S(\bar{w}_t)}{d\gamma})|_{w=w_2} \gtreqless 0 \Leftrightarrow \gamma \lesseqgtr 0.581$. These collectively lead to the corollary. $\square$

Proof of Proposition 4.1. We first specify the timing of this alternative game. Stages 1 and 2 are kept the same as in the baseline model. In stage 3, the traditional retailer decides his quantity q_d alone; afterwards, in stage 4, the online retailer and the traditional retailer bargain over v as the online retailer's revenue sharing proportion from the differentiated product.

To characterize the equilibrium, we again use backward induction to solve for the subgame perfect Nash equilibrium. We restrict our attention to the range of parameters in which there is actual selling on both the differentiated and common products, i.e., q_d and q_c are both strictly positive. Note that in stage 4, the revenue sharing v from the Nash bargaining for any pair of q_d and q_c has been given in Section 3 as follows:

$$v(q_c, q_d) = \begin{cases} \frac{a-q_d}{2(a-q_d-\gamma q_c)} & \text{if} \quad q_c \leq \frac{a-q_d}{2\gamma} \\ 1 & \text{otherwise.} \end{cases}$$

Confining the following analysis to $q_c \leq \frac{a-q_d}{2\gamma}$, we now return to stage 3 in which both parties determine their (ultimate) quantities. Let us first consider two preliminary cases separately: Cournot case (i.e., $q_c^t = 0$) and Stackelberg case (i.e., $q_c^w = 0$).

Case (1) Cournot case.

Since the traditional retailer does not observe q_c^w, the quantity decisions in stage 2 and stage 3 are tantamount to the simultaneous-move game in which both parties anticipate the bargaining result $v(q_c, q_d)$ in stage 4. Thus, the traditional retailer chooses q_d to maximize

$$\pi_t(q_c^w, q_d) = \frac{1}{2}(a - q_d - 2\gamma q_c^w)q_d,$$

and the online retailer chooses q_c^w to maximize $\pi_o(q_c^w, q_d) = \frac{1}{2}(a - q_d)q_d - (a - q_c^w - \gamma q_d - w)q_c^w$. The Cournot quantities are solved as

$$q_c^C = \frac{a(2-\gamma) - 2w}{2(2-\gamma^2)}, \text{ and } q_d^C = \frac{(1-\gamma)a + \gamma w}{2-\gamma^2}.$$

Case (2) Stackelberg case.

In this case, the quantity $q_c^t > 0$ acquires the commitment power and establishes the online retailer as a Stackelberg leader. In stage 3, the traditional retailer maximizes

$$\pi_t(q_c^t, q_d) = \frac{1}{2}(a - q_d - 2\gamma q_c^t)q_d,$$

which gives rise to the following best response: $q_d^b(q_c^t) = \frac{a}{2} - \gamma q_c^t$.Anticipating $q_d = q_d^b(q_c^t)$ and $v = v(q_c, q_d)$ in the future, in stage 2 the online retailer chooses q_c^t to maximize

$$\pi_c(q_c^t, q_d^b(q_c^t)) = \frac{a^2}{8} + (a - \frac{\gamma}{2}a + \frac{\gamma^2}{2}q_c^t - q_c^t - w_t)q_c^t.$$

Her Stackelberg leader's quantity is $q_c^L = \frac{a(2-\gamma)-2w_t}{2(2-\gamma^2)}$. Correspondingly, the traditional retailer supplies the Stackelberg follower's quantity of the differentiated product, given by $q_d^F = q_d^b(q_c^L) = \frac{(1-\gamma)a+\gamma w_t}{2-\gamma^2}$. Clearly, the Stackelberg quantities are given by the Cournot quantities with the unit cost of the common product as w_t.

Collectively, the online retailer's sourcing decision in stage 2 is purely driven by efficiency comparison. As long as $w_t < w$, she exclusively sources from the traditional retailer. If $w_t = w$, the quantity procured by each party does not vary across the Stackelberg regime and the Cournot regime. In this scenario, the online retailer is indifferent between sourcing from the traditional retailer or from the competitive fringe because her profit is the same. However, in stage 1 the traditional retailer strictly prefers the Stackelberg regime because in this regime he also gets the positive margin w through supplying the online retailer. In equilibrium, the traditional retailer sets his price w_t infinitesimally close to w from below, and the online retailer exclusively sources from the traditional retailer for the common product. Taking $w_t = w$, the corresponding equilibrium profits are

$$\pi_c^L = \frac{(3-2\gamma)a^2 + 2w(a\gamma - 2a + w)}{4(2-\gamma^2)},$$
$$\pi_d^F = \frac{(1-\gamma)^2 a^2 + 4aw(1-\gamma^2) + aw\gamma^2 - (4-3\gamma^2)w^2}{2(2-\gamma^2)^2}. \square$$

D Numerical Examples in Section 4

D.1 Numerical Examples in Section 4.1

Example 1. Suppose $a = 10, b = 8, \alpha = 0.5$. As long as $w < 2.5$, the online retailer supplies a positive quantity for the common product when she orders exclusively from the competitive fringe. In this Cournot case, $v^C = \frac{20-w}{2(15+w)}$. Instead, in the Stackelberg case, $v^S(w_t) = \frac{75-4w_t}{2(55+4w_t)}$, which is larger than v^C at $w_t = w$. Comparing profits of the online retailer in the two cases shows that she is willing to pay a price premium to the traditional retailer for the Stackelberg case. The maximum value of w_t she is willing to pay is given by $\bar{w}_t(w) = \frac{70-5\sqrt{195}}{28} + \frac{195}{14}w$. On the other hand, the traditional retailer's profit in the Stackelberg case strictly increases in w_t for $w_t \le \bar{w}_t(w)$. Thus when supplying the online retailer, the optimal profit of the traditional retailer is achieved at $w_t = \bar{w}_t(w)$. The traditional retailer will choose the Stackelberg case only when $w > 0.13$; otherwise, it is optimal for the traditional retailer to quote a w_t high enough to ward off the online retailer and therefore have the Cournot result in their sequential competition. Therefore, with vertically differentiated products, we identify the same disincentive for the traditional retailer to supply the online retailer due to the disadvantage he faces as a Stackelberg follower. The traditional retailer will supply the online retailer only when the efficiency gain is substantial, which occurs if the traditional retailer has a large enough cost advantage relative to the competitive fringe.

Example 2. Suppose again $a = 10, b = 8, \alpha = 0.5$. The traditional retailer's unit procurement cost for the differentiated product is $c = 2$, and for the common product is 0; whereas the unit procurement cost of the competitive fringe for the common product is $w \in [0, 2)$. Again w measures the cost advantage of the traditional retailer with the common product relative to the competitive fringe. The central result is that the traditional retailer supplies the online retailer only when $w > 0.086$. Note that this cutoff value of w at $c = 0$ is 0.13. Thus when the traditional retailer bears a cost disadvantage with the high-quality differentiated product, he becomes more favorable of supplying the online retailer. As a result, the range of w, within which the traditional retailer has a cost advantage for the common product but chooses not to supply the online retailer, shrinks when c gets larger.

D.2 Numerical Example in Section 4.2

Consider $a = 10, \gamma = 0.5$. To start, consider a pre-fixed value of v at $v = 0$. We are in the benchmark case game $\tilde{G}$ (see Appendix A), wherein the traditional retailer appropriates the whole profit from selling the differentiated product. Lemma A.1 then indicates that the Cournot outcome, in which the online retailer orders exclusively from the competitive fringe, is the unique sourcing mode when the fringe is not too inefficient relative to the traditional retailer. By continuity, the result will hold for given $v > 0$ so long as w is not too big. To see this, consider $w = 0$, i.e., the fringe and the traditional retailer are equally efficient in procuring the common product. It turns out that the traditional retailer, as long as his profit sharing with the differentiated product, measured by $1 - v$, is not very small (calculated as $v < 0.910$), will favor his Cournot profit and will set w_t high to induce the Cournot result. To verify that it must be the Cournot outcome in equilibrium, it suffices to see that in equilibrium, $v^C = 0.726$ when the two parties anticipate the Cournot outcome, whereas $v^S = 0.741$ when they anticipate the Stackelberg outcome. Therefore, at $w = 0$, it must be $v = 0.726$ and the Cournot outcome is played in equilibrium. By continuity, the theme of this result will hold with w positive but close to zero. For instance, when $w = 0.02$, the traditional retailer strictly prefers the Stackelberg outcome for $v < 0.763$. Nonetheless, $v^C = 0.725$ and $v^S = 0.740$. Thus in equilibrium $v = 0.725$ and the Cournot outcome is played. If we continue to increase w, the Cournot regime gradually diminishes. For instance, at $w = 1$, the traditional retailer always prefers the Stackelberg result regardless of the value of v.

D.3 Numerical Example in Section 4.3

Consider $a = 10, \gamma = 0.5$. We assume that $w < 7.143$ to guarantee that both firms are active when the online retailer orders exclusively from the fringe her common product. We find that so long as $w > 0.251$, the game has a unique equilibrium, in which the online retailer orders her common product exclusively from the traditional retailer. In addition, the traditional retailer quotes prices $w_t = \frac{35}{4} - \frac{1}{8}\sqrt{5000 - 1100w + 65w^2}, \tau = 5 - \frac{1}{30}\sqrt{5000 - 1100w + 65w^2}$. Correspondingly, the online retailer's quantity of the common product is $q_c^t = \frac{1}{15}\sqrt{5000 - 1100w + 65w^2}$ and the quantity of the differentiated product is $q_d = \frac{5}{2} - \frac{1}{60}\sqrt{5000 - 1100w + 65w^2}$.

D.4 Numerical Example in Section 4.4

Consider $a = 10, \gamma = 0.5, w = 1$. In the Cournot regime the traditional retailer's quantity is $q_d^C = 4.13$. Instead in the Stackelberg regime, it is calculated that $w_1 = 0.05, w_2 = 4.86$. Thus at $w = 1$, the unconstrained total quantity is $Q^{S*} = q_c^{L*} + q_d^{F*} = 7.69$.

Now we assume that the traditional retailer's procurement capacity is K with $K \in [4.13, 7.69)$. Thus in the Stackelberg regime, the traditional retailer is unable to obtain the total quantity Q^{S*}. At a given w_t, the online retailer now chooses q_c^t while understanding that the traditional retailer will supply $q_d = K - q_c^t$ and the online retailer's profit sharing is $v^S(K) = \frac{10+q_c^t-K}{20+q_c^t-2K}$, which strictly increases in K. Correspondingly, the optimal quantity of the online retailer is $q_c^{L*}(K) = \frac{K}{4} - \frac{w_t}{2} + \frac{5}{2}$.

By contrasting the online retailer's Stackelberg profit to her Cournot profit, we find the cutoff value of w_t is given by $\bar{w}_t(K) = 5 + \frac{K}{2} - \frac{1}{15}\sqrt{450K^2 - 4500K + 21728}$, such that the online retailer strictly prefers the Stackelberg regime to the Cournot regime for $w_t < \bar{w}_t(K)$. In addition, for $K > 5.88$, $\bar{w}_t(K) > \bar{w}_t(w)$, showing that the online retailer is with a stronger incentive to order from the traditional retailer when the traditional retailer is capacity constrained. On the traditional retailer's side, we contrast $\pi_t^{F*}(\bar{w}_t(K))$, which is his optimal profit in the Stackelberg regime, to his Cournot profit. The result indicates that as long as $K > 4.66$, in equilibrium the traditional retailer sets $w_t = \bar{w}_t(K)$ and the Stackelberg outcome is played.