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Multi-Objective Robust Optimisation Model for MDVRPLS in Refined Oil Distribution

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ABSTRACT
At depots with refined oil shortage, arranging a reasonable distribution scheme with limited supply affects operation costs, demand satisfaction rate of gasoline stations (hereafter, "station satisfaction"), and overtime penalty. This study considers the refined oil distribution problem with shortages using a multi-objective optimisation approach from the perspective of decision makers of oil marketing companies. The modelling and solving process involves (i) formulation of a crisp multi-depot vehicle routing model with limited supply (MDVRPLS) which considers station priority and soft time windows, (ii) development of a robust optimisation model (ROM) to manage uncertainty in demand, and (iii) the proposal of a multi-objective particle swarm optimisation (MOPSO)algorithm. Results of numerical experiments show that (i) the crisp model can better balance operation costs, station satisfaction, and overtime penalty, which produces 3.33% and 4.60% incerease in station satisfaction at an increased unit cost and overtime penalty respectively; (ii) ROM successfully addresses uncertainty in demand compared to the crisp model, which requires an additional 8.81% in cost and 12.85% in penalty; and (iii) the MOPSO manages these MDVRPLS models more effectively than other heuristic algorithms. Therefore, applying ROM of refined oil supply shortage to the management significantly improves the efficiency and resists the disturbance caused by external uncertainties, providing scope for efficient distribution of scarce resources.

KEYWORDS
MDVRPLS; refined oil distribution; robust optimisation; multi-objective optimisation; particle swarm optimisation algorithm.

1. Introduction

In everyday life, refined oil shortages often occur in certain countries or regions due to seasonality, natural disasters, government policies, crude oil imports, and other uncertain factors. In general, the refined oil shortages are usually caused by emergencies (e.g. natural disasters or government policies) or regular reasons (e.g. poor business operation, holidays). For the former situation, more than 2,000 gasoline stations in Guangdong, a province in China, were closed owing to refined oil shortages caused by

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a typhoon in 2005.[1] In 2019, refined oil shortages transpired in Michoacán, induced by Mexican government as an attempt to prevent fuel theft.[2] For the latter situation, many gasoline stations of Sinopec in Zhengzhou experience oil shortage, and queuing is observed everywhere, which is caused by oil hoarding in a disguised manner to gain benefits from future price rise.[3] Owing to holidays, such as the Spring Festival, many small gasoline stations can only ration refined oil.[4] Therefore, it is essential that oil marketing companies build scientific oil distribution management systems to guide the oil dispatching centres (ODCs) to distribute refined oil among stations efficiently, so as to ensure that negative impacts (e.g. rising oil prices, travel disruptions, and recession) of refined oil shortages could be reduced to the extent possible.

A refined oil shortage is essentially a multi-depot vehicle routing problem with limited supply (MDVRPLS). Compared to the general MDVRPLS, the refined oil distribution problem has multiple conflicting objectives, which makes it a complex issue. In our surveys of the China National Petroleum Corporation (CNPC) and China Petroleum & Chemical Corporation (Sinopec), we found that ODCs generally assign different priority levels to downstream stations, and stations with higher priority levels are supplied first in the case of an oil shortage. Therefore, the ODC expects to meet the demands of stations with higher levels to the extent possible while minimising operation costs and overtime penalty. The solution of a distribution scheme of vehicle allocation and route planning is focussed on the minimum demands for safe operations and the priority levels associated with stations. To date, existing studies on refined oil distribution mainly focus on the scenario of sufficient supply (Souza, Goldbarg, and Goldbarg 2009; Escobar et al. 2014), and only a few studies consider the scenario of limited supply which concerns either operation costs or operation time (Chakrabortty, Sarker, and Essam 2016). Station satisfaction as an important performance indicator for a distribution scheme with limited supply, has not been considered previously. In this research, a priority-based satisfaction optimisation function is established to mitigate the negative impact of oil shortages.

By considering the uncertainty from demand fluctuations (Ben-Ammar, Bettayeb, and Dolgui 2019), traffic congestion, and weather changes (Adelzadeh, Asl, and Koosha 2014), the optimality and even feasibility of an initial distribution scheme could be eliminated (Xu et al. 2015). We investigated the refined oil distribution of sailing companies from CNPC and Sinopec and found that approximately 25% of initial distribution schemes were infeasible when addressing uncertainty. Therefore, the robustness of a distribution scheme should be considered, which makes the MDVRPLS in refined oil distribution a challenging issue. For a MDVRPLS that is sensitive to uncertainty, robustness has received more attention (Xu et al. 2018), which can ensure the feasibility of a solution in a certain range of variations of uncertain factors (Ben-Tal, El Ghaoui, and Nemirovski 2009). Furthermore, there are various multi-objective robust optimisation models (Ghoddousi et al. 2013), which have been confirmed in dealing with a MDVRPLS effectively (Dan and Trichakis 2014; Chakrabortty, Sarker, and Essam 2016). However, there are limited applications of multi-objective robust optimisation in the field of refined oil distribution. We propose a multi-objective robust model to solve the MDVRPLS in refined oil with uncertain demand.

As a MDVRP is confirmed as a NP-hard problem, classical exact algorithms are unable to solve large-scale problems timeously and efficiently (Dridi et al. 2020). Con-

[1] http://www.china.com.cn/chinese/difang/941592.htm
[2] https://www.sohu.com/a/288026675_115239
[3] https://finance.huanqiu.com/article/9CaKrnJHHRa
[4] http://news.sohu.com/20060219/n241899654.shtml

sequently, many multi-objective heuristic algorithms are proposed and used for managing multi-objective MDVRP (Bo and Qiu 2014; Wang 2013), but only a few, such as multi-objective particle swarm optimisation (MOPSO), have been applied to address a multi-objective MDVRPLS (Tian, Hao, and Gen 2019). In this study, we use MOPSO to solve the proposed multi-objective robust MDVRPLS in refined oil distribution.

With the practical concern about refined oil shortages, this study proposes a multi-objective robust optimisation model for a MDVRPLS in refined oil distribution from the perspective of managers of oil marketing companies, hoping to improve the operating efficiency of the refined oil distribution system. The main contributions are listed as follows: 1) a multi-objective crisp model is established, aimed at minimising operation costs and overtime penalty, and maximising station satisfaction in refined oil distribution with limited supply; 2) a robust model is established on the premise of a multi-objective model to control uncertain demand; 3) a MOPSO algorithm is developed to solve the proposed multi-objective robust optimisation model.

The remainder of the paper is organised as follows. A literature review is provided in Section 2. In Section 3, the problem definition and mathematical descriptions are presented. Then, a crisp multi-objective optimisation model and the robust model of a MDVRPLS are formulated. The MOPSO algorithm is also designed. Section 4 shows the results of numerical experiments and comparative analyses. Finally, the conclusions and scope for future research are provided in Section 5.

2. Literature Review

This study proposes an innovative application of a multi-objective robust MDVRPLS in the refined oil distribution problem. To provide a better understanding, this section reviews the literature on refined oil distribution, multi-objective MDVRP, robust MDVRP, and MDVRP algorithms, respectively.

2.1. *Refined Oil Distribution*

Refined oil distribution from oil depots to gasoline stations is the terminal delivery of the entire distribution network, which has drawn much attention regarding vehicle allocation and route planning (Chan, Shekhar, and Tiwari 2014). In terms of vehicle allocation, Guyonnet, Grant, and Bagajewicz (2009) built an integrated model for daily tanker assignment schemes, which involves unloading, processing, and delivering in the refined oil supply chain. Zhu, Zhang, and Bao (2011) further incorporated tanker allocation into the refined oil logistics network, of which the objectives are meeting more customer needs and reducing distribution costs. With respect to route planning, Shen et al. (2012) built a nonlinear optimisation model, aimed at minimising total driving costs in a specific service level.

In particular, refined oil distribution can be divided into two categories in light of whether they are demand driven, including proactive distribution and reactive distribution. In cases of proactive distribution, ODCs deliver refined oil to gasoline stations based on the supply level of oil depots and recent sales of gasoline stations, without considering the realistic demands of gasoline stations. For example, Gromov, Kuznietzov, and Pigden (2019) studied the gasoline station replenishment problem, in which gasoline stations' output is demand-determined by a prediction method based on daily sales. Tong and Li (2019) established an proactive distribution model under real-time traffic conditions and predicted distribution based on transport and supply. It is found that proactive distribution is usually combined with forecasting. While in cases of reactive distribution, ODCs deliver the refined oil based on gasoline station demands. Brown and Graves (1979) are the first to describe the cost problem in reactive distribution, which involves equitable man and equipment workload, safety standards, and service levels. Cornillier et al. (2008) proposed a gasoline station replenishment problem of multi-period, which studied the multiple types of refined oil allocation of stations, the manner in which to load the oil into tanker compartments, and the routing from depot to stations. Subsequently, the gasoline station replenishment problem with time windows was proposed (Cornillier et al. 2009), aimed at optimising several refined oil types delivery to a number of gasoline stations with the constraints of limited tankers and time. Although the refined oil distribution problem has been widely studied based on different scenarios, to the best of our knowledge, only a few studies focus on limited oil supply (Chakrabortty, Sarker, and Essam 2016). With practical concerns about oil shortages and literature gaps on oil distribution, this study comprehensively considers the importance of station satisfaction, cost, and overtime penalty and aims to examine the oil distribution method with limited supply.

2.2. *Multi-Objective MDVRP*

Refined oil distribution is essentially a vehicle routing problem from multiple oil depots to multiple stations, to satisfy stations with limited vehicle capacity (Luo and Chen 2014). Sumichras and Markham (1995) initially formulated the problem of transporting raw materials from multiple depots to a number of stations as the MDVRP, and designed a heuristic algorithm with the goal of minimising the delivery costs. Sear (1993) focused on a MDVRP in a logistics network consisting of refineries and consumers, and presented a linear programming model for minimising delivery costs with demand limits. With the complexity and flexibility of dispatching decisions, more objectives should be considered in addition to the traditional cost-oriented model (Escobar et al. 2014), such as distance deviation (Xu and Xiao 2015) dynamic transport cost (Hu et al. 2015), vehicle load (Olivera and Viera 2007), carbon emissions (Jabali, Woensel, and Kok 2012), and station satisfaction (Kachitvichyanukul, Sombuntham, and Kunnapapdeelert 2015). Samanlioglu and Funda (2013) developed a multi-objective MDVRP model for hazardous products, in which three objectives were considered: reducing total costs, transportation risk, and population risk. Xu and Xiao (2015) presented a mixed integer linear programming model and a multi-objective genetic clustering algorithm to manage the MDVRP, where the objectives included travel distance imbalance minimisation and workload imbalance minimisation of vehicles. Hu, Li, and Li (2018) formulated a loading-dependent hazmat transportation model, which aims to obtain the optimal equilibrium between transportation risk and cost.

Although the multi-objective MDVRP has been studied extensively, there are limited applications in the refined oil distribution problem, and none consider station satisfaction. This paper presents multi-objective MDVRP models and algorithms for refined oil distribution problem, in which station satisfaction, operation costs, and overtime penalty are considered. In Section 4.2, we compare the proposed model with a single-objective MDVRP model using cost minimisation (Escobar et al. 2014), to prove the advantages of the multi-objective model in balancing the operation costs, station satisfaction, and overtime penalty.

2.3. *Robust MDVRP*

Considering the uncertainties caused by demand fluctuations, the uncertain MDVRP has been studied extensively and applied to various industrial fields (Dolgui and Prodhon 2007). Dolgui and Proth (2010) gave a particular attention to the bullwhip effect caused by stochastic demands and highlighted some robust models (e.g., Newsboy model) to reduce this undesirable phenomenon. Robust optimisation, which as one of the most effective methods in developing models and making plans that are insensitive to uncertainty (Hazir and Dolgui 2019), has been developed and proved successful in dealing with uncertain MDVRPs with high performance. In general, most early studies focus on single objective robust optimisation (Yang, Liu, and Yang 2020). For example, by focusing on the uncertain demand for cargo services in airline allocation and planning problems, Mulvey, Vanderbei, and Zenios (1995) proposed a demand scenario based on the robust MDVRP model, which aimed to minimise the penalty costs of overage or underage supply. Jafari-Eskandari et al. (2010) considered an uncertain supply in a milk-run system, added a multi-scenario with weight to MDVRP, which are constructed as deviations of an expected minimum supply value, and proposed a robust counterpart model with the minimum total cost.

In recent years, the research on MDVRP with uncertainty shifted from a single objective to multiple objectives. Bahri, Amor, and Talbi (2016) defined the uncertain customer requirements as a triangular fuzzy number and used the β-robustness approach to solve the multi-depot vehicle scheduling and route planning problem, with the objective to minimise the total travelled distance and total tardiness. Men et al. (2020) set up an uncertain set containing 32 potential incident scenarios with transportation risk parameters and developed two versions of robust criterion to transform a hazardous material MDVRP with time windows into robust models, pursuing the balance between the number of vehicles and the transportation risk.

Research on robust MDVRP has attracted increasing attention, but only few studies focus on the refined oil distribution problem. Inspired by Bertsimas and Sim (2004), our paper proposes a multi-objective MDVRP (MOMDVRP) model, which aims to reduce the negative effects caused by demand uncertainty.

2.4. *Algorithms for MDVRP*

MOMDVRP as a multi-objective optimisation problem is NP-hard, in which obtaining the optimal solution in polynomial time is difficult or infeasible. As an effective solving algorithm applied for NP-hard problems, multi-objective heuristic algorithms (MOHAs) are developed and proved successful in the practice of MOMDVRP. Jemai, Zekri, and Mellouli (2012) focused on a bi-objective MDVRP in green logistics and applied NSGA-II aimed at the shortest travel distance and the minimum CO_2 emission. Liu and Kachitvichyanukul (2015) studied MOMDVRP focused on the minimum total cost and the maximum customer demand served and designed two MOPSO including different coding to obtain Pareto solutions. Rabbani, Taheri, and Ravanbakhsh (2016) considered travel distance, vehicle capacity, and service satisfaction, compared the efficiency and sensitivity of different algorithms on MOMDVRP, and found that MOPSO are more efficient than NSGA-II. In addition, other MOHAs as MOABC and MOACO have been applied to manage MOMDVRP (Jia et al. 2013; Hu et al. 2018; Xu, Hao, and Zheng 2020).

Among these MOHAs, MOPSO has proved successful in managing uncertain optimisation problems with high performance. Yang and Xu (2008) considered random travel time and fuzzy demand in refined oil distribution, proposed a chance-constrained model for MOMDVRP, and developed a hybrid MOPSO based on strength Pareto evolutionary algorithm (SPEA2) to solve the equivalent model with LR-fuzzy random coefficients. Validi, Bhattacharya, and Byrne (2014) referred to total cost, CO_2 emission, and travel distance and proposed a DoE-guided robust optimisation solution based on MOPSO to address MOMDVRP. Guo et al. (2018) incorporated random customer demands into a dynamic MOMDVRP; built a two-phase dynamic programming model aimed at the minimisation of carbon emission, waiting time, and vehicle quantity; and proposed a robust optimisation based on MOPSO that can generate and insert virtual routes into executing static routes.

The algorithms for MOMDVRP have been studied extensively, which provide valuable references for us to design the robust optimisation solution for MDVRPLS. Consequently, we incorporate the robustness into the multi-objective refined oil distribution problem with uncertain demands and propose an improved MOPSO algorithm to ensure a satisfactory solution.

Based on the researches presented above, a more detail classification of the literature is illustrated in Table 1. Compared with existing studies, our study takes more factors into account, including limited supply, demand uncertainty, time, satisfaction, cost. Based on the above content, we build a multi-objective robust optimization model for refined oil distribution under limited supply, and design MOPSO to solve the problem.

Table 1. A Summary of Excisting Studies

Studies	Elements					Models			Algorithms	
	Limited supply	Demand uncertainty	Time	Satisfaction	Cost	Single objective	Multi objective	Robust	Single objective	Multi objective
Dolgui and Prodhon (2007)		*	*					*		
Yang and Xu (2008)		*	*				*			*
Souza, Goldbarg, and Goldbarg (2009)			*				*		*	
Cornillier et al. (2009)					*	*			*	
Jafari-Eskandari et al. (2010)			*			*		*	*	
Zhu, Zhang, and Bao (2011)				*	*	*			*	
Jabali, Woensel, and Kok (2012)			*		*		*		*	
Ghoddousi et al. (2013)	*		*		*		*	*		*
Samanlioglu and Funda (2013)					*		*			*
Adelzadeh, Asl, and Koosha (2014)			*		*		*		*	
Bo and Qiu (2014)				*		*			*	
Chan, Shekhar, and Tiwari (2014)				*	*		*			*
Escobar et al. (2014)					*	*			*	
Luo and Chen (2014)			*			*			*	
Kachitvichyanukul, Sombuntham, and Kunnapapdeelert (2015)			*	*			*		*	
Liu and Kachitvichyanukul (2015)				*	*		*			*
Xu and Xiao (2015)					*		*			*
Chakrabortty, Sarker, and Essam (2016)	*		*			*			*	
Bahri, Amor, and Talbi (2016)		*	*				*	*		*
Rabbani, Taheri, and Ravanbakhsh (2016)			*	*			*			*
Hu et al. (2018)				*	*		*		*	
Hu, Li, and Li (2018)		*			*	*			*	
Guo et al. (2018)		*	*				*	*		*
Xu et al. (2018)		*	*		*		*		*	
Ben-Ammar, Bettayeb, and Dolgui (2019)		*	*		*	*		*	*	
Dridi et al. (2020)			*			*		*	*	
Men et al. (2020)			*				*	*		*
Xu, Hao, and Zheng (2020)			*		*		*			*
Our work	√	√	√	√	√		√	√		√

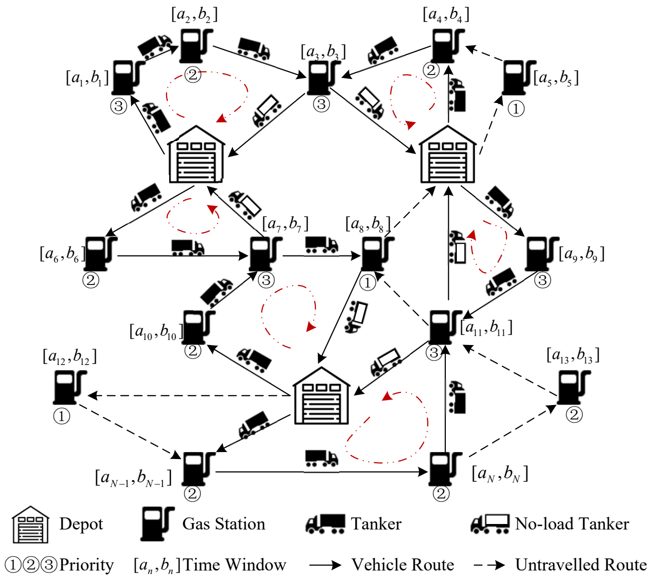


Figure 1. MDVRPLS in Refined Oil Distribution Networks

3. Materials and Methods

3.1. *Problem Description*

Based on the background of limited supply of refined oil, optimization systems that comprehensively considers the station satisfaction, operation cost and overtime penalty are built from the perspective of decision makers of oil marketing companies. We hope to use the multi-objective optimization models to help oil marketing companies balance the relationship between service quality, time reputation and cost when the refined oil supply is in short supply, and guide ODCs to complete the distribution plan scientifically and efficiently.

In a refined oil distribution problem with limited supplies, the priority of the gasoline station is an important factor that should be considered. Prioritising the stations will help ODCs improve station satisfaction. On this basis, the ODC should ensure a more effective vehicle scheduling by considering station priority to distribute limited refined oil.

First, stations are prioritised by ODCs according to low, medium, and high levels in terms of geographic location, demand emergency, and station density. Stations that are able to meet more customer needs in the market, are a higher priority, while others are considered a lower priority. Based on different priorities, weights are set for stations. The higher the priority, the higher the weight. In case of limited supply, the demands of high-priority stations will be met as much as possible, but low-priority stations may be granted with only the minimum demands for safe operation. In this study, we assume that the priorities of stations are known.

Second, a MDVRPLS is applied to allocate vehicles from multiple depots to numerous stations and to optimise travel routes to minimise the total cost and overtime penalty. In a distribution network as Fig. 1, there are multiple depots and stations. Each depot is equipped with multi-compartment tankers to transport various types of refined oil to stations at a time. Each circle represents one closed loop and each tanker departs with refined oil and returns without a load. Solid lines with arrows represent a vehicle route, and dotted lines represent an untravelled route due to supply shortage.

In distribution, the ODC focuses several primary tasks, including refined oil distribution, multi-compartment tanker scheduling, and soft time window matching. Each station requires different types and quantities of refined oil; hence, the ODC distributes limited refined oil based on station priority to maximise station satisfaction. Correspondingly, the tankers can carry different types of oil in multiple compartments. Additionally, the ODC needs to arrange customers for each tanker according to the type and quantity of oil it carries and optimises routes to minimise the cost. Soft time windows in distribution can be divided into desired service time and acceptable service time in multi-objective MDVRPLS. If stations receive the oil within a desired service time, there is no overtime penalty. The penalty is proportional to the time spent outside of the desired service time window.

The ODC decides on the quantity and type of refined oil according to a station's priority, demand, location, time window, and other parameters. Considering limited supply, the quantity delivered to each station is not a fixed amount, but an interval value. The lower limit is the minimum demand for maintaining a safe operation, and the upper limit is the actual demand of the station. Station satisfaction will decline when supply fails to meet the demand. In particular, the higher the priority of a station, the higher the station satisfaction. Subsequently, the ODC should form a distribution scheme by considering the maximum station satisfaction and minimum operation cost and overtime penalty.

In addition, the oil demand of a station fluctuates within a certain range due to uncertain factors. In reality, the ODC can usually predict demand based on historical data. However, when there is a deviation between actual values and predicted values, the scheme which is not robust will be directly affected or become impossible to execute. Since accurate demand cannot be predicted in advance and the cost loss caused by the remaining supply cannot be ignored in MDVRPLS, we should consider the fluctuation of demand and apply robust optimisation methods to improve the resilience of the distribution scheme. Hence, the robust distribution scheme is expected to preserve the fluctuation of uncertain factors, reduce total costs and overtime penalty, and improve satisfaction.

Based on the above problem description, a multi-objective robust optimisation model is proposed below for a MDVRPLS in refined oil distribution to pursue the maximum station satisfaction and minimum operation costs and overtime penalty. The mathematical description of Fig. 1 is as follows: distribution network $\mathbb{G} = (\mathbb{A}, \mathbb{R})$ is composed of D depots, N stations with priority level set λ^n, where $\mathbb{A} = (\mathbb{D} \cup \mathbb{N})$, $\mathbb{D} = \{1, 2, ..., D\}$, $\mathbb{N} = \{1, 2, ..., N\}$, $\mathbb{R} = \{r^{ij}\}$. The parameter r^{ij} represents the distance in Amap[5] between node i and node j, $(i,\ j \in \mathbb{A})$. There are K_d multi-compartment oil tankers, each with a maximum load of Q at depot $d \in \mathbb{D}$. In addition, $\mathbb{K}_d = \{1, 2, ..., K_d\}$ is the set of tankers in depot d, $\mathbb{P} = \{1, 2, ..., P\}$ is the set of types of refined oil in all stations, and $\mathbb{U} = \{1, 2, ..., U\}$ is the set of compartment of each multi-compartment tanker.

To simplify the problem, we discuss the process of refined oil distribution within a specific time period. There are K_d oil tankers from depot d carrying several types of refined oil separately in U compartments with capacity Q/U. The demand of the *p-th* oil type at station n is δ^{pn}, and the soft time window is $[a^n,\ b^n]$ (i.e. desired service time). Tanker k departs from depot d and transports the *p-th* type of refined oil with load h^p_{dk}. It travels a distance of r^{ij} with an average speed of v, arrives at

[5]https://ditu.amap.com/

station n at arrival time T_{dk}^{n}, and spends a period of time t_s to service the station. It is necessary to consider the penalty rules of advance or delayed time. After completing the distribution task at station n, the tanker k drives to the next station until all the oil loaded is distributed and returns to depot d.

Throughout the distribution process, the ODC decides in advance on the routes of the tankers, the quantity of refined oil distributed to each station, and the timetable for each tanker to arrive at each station. When the effective supply of all depots are less than the total demand of all stations (i.e. $\sum_{d=1}^{D} \varsigma_{d}^{p} \leq \sum_{n=1}^{N} \delta^{pn}$), this will inevitably lead to the result wherein the total amount of p-th type of oil distributed to station n by all tankers from different depots (i.e. $\sum_{d=1}^{D} \sum_{k\in\mathbb{K}_d} z_{dk}^{pn}$) is less than the demand δ^{pn} required at some stations. In conclusion, when refined oil supply is limited, stations should be prioritised first, ensuring the minimum demand for safe operation at stations to the extent possible. On this basis, vehicle allocation and route planning can be implemented to achieve the maximum station satisfaction, the minimum operation cost, and overtime penalty.

3.2. *Models*

In this section, we first design a crisp model of MDVRPLS in refined oil distribution, in which operation costs, station satisfaction and overtime penalty are the objectives. Then, we further develop a robust optimisation model of MDVRPLS against uncertainty factors, induced by station demands, in order to assist ODC managers to make efficient refined oil distribution plan. To be specific, the model mainly answers the following three questions:

(1) Which route should a tanker take;
(2) Which quantity of each type of refined oil should a tanker transport for a gasoline station;
(3) When will a tanker arrive at a gasoline station with a soft time window.

To answer these questions, we establish a multi-objective optimisation model for MDVRPLS and pursue the lowest operation cost distribution and the highest demand station satisfaction. The parameters and variables referred to in this paper are defined in Table 2.

3.2.1. Station Satisfaction

In case of limited supply, the demand of gasoline stations should be met to the extent possible to improve the station satisfaction. Here, we use the priority level of the station as weight coefficient and measure the total station satisfaction as the weighted sum of deviations between demand and supply; that is,

$$S(\mathbf{z}) = \sum_{n=1}^{N} \sum_{p=1}^{P} \lambda^{n} \left(\delta^{pn} - \sum_{d=1}^{D} \sum_{k\in\mathbb{K}_d} z_{dk}^{pn} \right). \tag{1}$$

3.2.2. Operation Cost

The operation cost consists of transportation cost and fixed cost. In Eq.(2), the first part is the total transportation cost of the tankers, which is proportional to the distance travelled, and the second part is the total dispatching cost of the tankers, which is a

Table 2. Sets, Parameters, and Decision Variables

Sets	Description
$\mathbb{G}$	Set of distribution networks, $\mathbb{G} = (\mathbb{A}, \mathbb{R})$;
$\mathbb{A}$	Set of all nodes, $\mathbb{A} = (\mathbb{D} \cup \mathbb{N})$;
$\mathbb{R}$	Distance matrix travelled by tankers, $\mathbb{R} = \{r^{ij}\}$,$i, j \in \mathbb{A}$;
$\mathbb{D}$	Set of oil depots, $\mathbb{D} = \{1, 2, ..., D\}$, $d \in \mathbb{D}$;
$\mathbb{N}$	Set of gasoline stations, $\mathbb{N} = \{1, 2, ..., N\}$, $n \in \mathbb{N}$;
$\mathbb{P}$	Set of all types of refined oil, $\mathbb{P} = \{1, 2, ..., P\}$, $p \in \mathbb{P}$;
$\mathbb{K}_d$	Set of multi-compartment tankers of depot d, $\mathbb{K}_d = \{1, 2, ..., K_d\}$, $k \in \mathbb{K}_d$;
$\mathbb{U}$	Set of compartments of each tanker, $\mathbb{U} = \{1, 2, ..., U\}$, $u \in \mathbb{U}$;
Λ	Set of priorities of stations, $\Lambda = \{\lambda^n\}$.
Parameters	**Description**
Q	Maximum load of a tanker;
ς_d^p	Available supply of the p-th type of oil of depot d;
Δ^p	Total demand of the p-th type of oil in the market;
δ^{pn}	Demand of the p-th type of oil at station n;
$[a^n, b^n]$	Soft time window, where a^n, b^n are the lower limit and upper limit of desired service time;
h_{dk}^p	Amount of the p-th type of oil carried by tanker k , which starts from depot d;
t_s	Fixed service time taken to load or unload the oil;
c_t	Transportation cost per kilometer;
c_f	Fixed dispatch cost of each time;
v	Average speed of a tanker.
Decision varibles	**Description**
x_{dk}^{ij}	Binary variable, which is equal to 1 if tanker k travels from node i to j and otherwise, 0;
z_{dk}^{pn}	Amount of the p-th type of oil distributed to station n by tanker k from depot d;
T_{dk}^n	Arriving time at which tanker k arrives at station n.

fixed cost generated by tankers per trip.

$$C(\mathbf{x}) = \sum_{d=1}^{D} \sum_{k \in \mathbb{K}_d} \sum_{i,j \in \mathbb{A}} (c_t r^{ij} + c_f) x_{dk}^{ij}. \tag{2}$$

3.2.3. Overtime Penalty

The gasoline stations in this study are with the soft time window, which specifies the distribution time of tankers. When a tanker arrives at the station outside of the soft time window, some penalty must be imposed. To make it easier to understand, we define an expression $f^+ = \max(f, 0)$. To ensure that the model is linear, we use the following equations to replace the max notation; that is,

$$\begin{cases} f^+ \geq f \\ f^+ \geq 0 \\ f^+ \leq f + M(1-\rho_1) \\ f^+ \leq M(1-\rho_2) \\ \rho_1 + \rho_2 \geq 1 \\ \rho_1, \rho_2 \in \{0,1\}, \end{cases} \tag{3}$$

where M is an infinite constant and ρ_1 and ρ_2 are the binary variables. Then, the overtime penalty can be calculated by

$$P(\mathbf{T}) = \sum_{d=1}^{D}\sum_{n=1}^{N}\sum_{k\in\mathbb{K}_d}(a^n - T^n_{dk})^+ + \sum_{d=1}^{D}\sum_{n=1}^{N}\sum_{k\in\mathbb{K}_d}(T^n_{dk} - b^n)^+. \tag{4}$$

3.2.4. Time Constraints

The time constraints indicate that the tanker needs to complete the refuelling service, continue on the route, and wait for the start time to proceed to next station between the arrival time of two adjacent nodes:

$$T^n_{dk} + r^{nm}/v + t_s + (a^m - T^m_{dk})^+ - M\left(1 - x^{nm}_{dk}\right) \leq T^m_{dk}, \forall n, m \in \mathbb{N}, k \in \mathbb{K}_d, d \in \mathbb{D}, \tag{5}$$

where T^n_{dk} and T^m_{dk} are the arrival times for tanker k at two adjacent stations n and m, r^{nm}/v is the travelling time, t_s is the service time, $(a^m - T^m_{dk})^+$ is the potential waiting time, M is an infinite constant, and x^{nm}_{dk} determines whether the tanker goes through the path from n to m.

3.2.5. Capacity Constraints

Eq.(6) shows that the quantity of *p-th* type of refined oil delivered from depot d to the stations should not exceed the maximum supply of depot d; that is,

$$\sum_{n=1}^{N}\sum_{k\in\mathbb{K}_d} z^{pn}_{dk} \leq \varsigma^p_d, \forall d \in \mathbb{D}, p \in \mathbb{P}, \tag{6}$$

where the left side is the total distribution quantity of the *p-th* type of refined oil delivered from depot d and ς^p_d is the available supply of the *p-th* type of refined oil at depot d. Eq.(7) ensures that the total delivered quantity of refined oil distributed by tanker k (i.e. $\sum_{n=1}^{N} z^{pn}_{dk}$) is no more than the total quantity carried h^p_{dk}.

$$\sum_{n=1}^{N} z^{pn}_{dk} \leq h^p_{dk}, \forall k \in \mathbb{K}_d, d \in \mathbb{D}, p \in \mathbb{P} \tag{7}$$

3.2.6. Restriction of VRP

Eq.(8) is applied to avoid the formation of a loop between two nodes:

$$x_{dk}^{ij} + x_{dk}^{ji} \leq 1, \forall i, j \in \mathbb{A}, k \in \mathbb{K}_d, d \in \mathbb{D}, \tag{8}$$

where x_{dk}^{ij} and x_{dk}^{ji} are binary variables, indicating whether tanker k passes through the specified route. Eq.(9) ensures that an equal number of tankers arrive at or leave a station:

$$\sum_{d=1}^{D} \sum_{k \in \mathbb{K}_b} \sum_{i \in \mathbb{A}} x_{dk}^{in} = \sum_{d=1}^{D} \sum_{k \in \mathbb{K}_b} \sum_{j \in \mathbb{A}} x_{dk}^{nj}, \forall n \in \mathbb{N}. \tag{9}$$

Eq.(10) ensures that tanker k from depot d will not drive directly to another depot c, but to a gasoline station that needs to be served.

$$x_{dk}^{dc} = 0, \forall d, c \in \mathbb{D}, k \in \mathbb{K}_d, d \in \mathbb{D} \tag{10}$$

Eq.(11) represents that a tanker departs from one depot and returns to it, to ensure a closed-loop operation.

$$x_{dk}^{dn} = x_{dk}^{md}, \forall n, m \in \mathbb{N}, k \in \mathbb{K}_d, d \in \mathbb{D}, \tag{11}$$

where the left side determines whether the tanker k starts from depot d. If the tanker departs from depot d, the value of the left side is 1; otherwise, the value is 0. Similarly, the right side determines whether tanker k arrives at the depot d after distributing refined oil. Eq.(12)-(14) define the ranges of decision variables.

$$x_{dk}^{ij} \in \{0, 1\}, \forall d \in \mathbb{D}, i, j \in \mathbb{A}, k \in \mathbb{K}_d, \tag{12}$$

$$z_{dk}^{pn} \geq 0, \forall d \in \mathbb{D}, n \in \mathbb{N}, k \in \mathbb{K}_d, p \in \mathbb{P}, \tag{13}$$

$$T_{dk}^{n} \geq 0, \forall d \in \mathbb{D}, n \in \mathbb{N}, k \in \mathbb{K}_d. \tag{14}$$

3.2.7. Crisp Model on MDVRPLS

Based on the abovementioned equations, the crisp MDVRPLS model defines station satisfaction, operation cost and overtime penalty as the objectives, and considers the time window, capacity, and VRP constraints, which is established as follows:

$$\begin{cases} \min & \{S(\mathbf{z}), C(\mathbf{x}), P(\mathbf{T})\} \\ \text{s.t.} & Constraints\ (5) - (14), \end{cases} \tag{15}$$

where $\mathbf{z} = \{z_{dk}^{pn}, \forall p \in \mathbb{P}, n \in \mathbb{N}, d \in \mathbb{D}, k \in \mathbb{K}_d\}$, $\mathbf{x} = \{x_{dk}^{ij}, \forall i, j \in \mathbb{A}, d \in \mathbb{D}, k \in \mathbb{K}_d\}$, $\mathbf{T} = \{T_{dk}^{n}, n \in \mathbb{N}, d \in \mathbb{D}, k \in \mathbb{K}_d\}$ are the decision variables.

To demonstrate the effect of the proposed model, we set up a simple case. In Fig. 2, one depot and three stations are located on the four corners of a square with sides

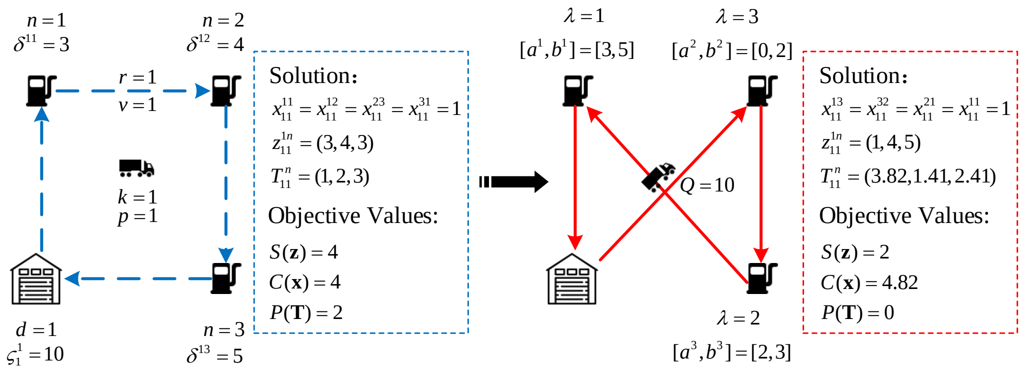


Figure 2. Illustration of Simple Case

of 1. One tanker $k = 1$ is departing from depot $d = 1$ to deliver 1-*th* type of oil to stations $n = 1, 2, 3$. The supply of the 1-*th* type of oil of depot 1 is $\varsigma_1^1 = 10$, and the total demand is $\sum_{n=1}^{N} \sum_{p=1}^{P} \delta^{pn} = 12$. Other parameters are also set in the figure (unmarked parameter values are set to 1). If we do not consider the limited supply, station priority, and soft time window, but only optimise the driving cost, the solution is shown on the left. In this case, the cost and the station satisfaction are lower, but the overtime penalty is higher. However, when we solve it using the proposed crisp model, the solution changes. If limited supply, priorities, and time window are considered, the distribution scheme is no longer with the shortest route, but it is optimised by considering the operation cost and overtime penalty simultaneously. Thus, the right solution is superior to the left one in two dimensions, which also displays the effect the proposed crisp model on MDVRPLS.

3.2.8. Robust Model on MDVRPLS

The model is used to resolve the MDVRPLS in a refined oil distribution in the case that demand is determined. In practice, the demand cannot be predicted accurately, but rather fluctuates within a range. Therefore, the boundary values of $\tilde{\delta}^{pn}$ in a range can be expressed as follows:

$$\tilde{\delta}^{pn} \in [\tilde{\delta}^{pn-}, \tilde{\delta}^{pn+}], \forall n \in \mathbb{N}, p \in \mathbb{P}, \tag{16}$$

where $\tilde{\delta}^{pn}$ are the fluctuating parameters that are induced by uncertain demand and $\tilde{\delta}^{pn-}$ and $\tilde{\delta}^{pn+}$ are the lower and upper bounds of demand for the *p-th* type of refined oil at station n.

Considering the uncertain parameters present in the objective function that make it difficult to solve, we are inspired by the study of Bertsimas and Sim (2004), in which uncertain parameters are transformed into constraint equations with dual theory. The robust model is intended to replace δ^{pn} in the crisp model with uncertain parameters $\tilde{\delta}^{pn}$. Finally, a computable linear robust model is constructed. Considering demand parameters, which only appear in Eq.(1) of the crisp model, we revise this formulation and rebuild a new robust model.

After the replacement with uncertain parameters, Eq.(1) is shown as

$$S(\mathbf{z}) = \sum_{n=1}^{N} \sum_{p=1}^{P} \lambda^{n} \left(\tilde{\delta}^{pn} - \sum_{d=1}^{D} \sum_{k \in \mathbb{K}_d} z_{dk}^{pn} \right). \tag{17}$$

We can regard the above formulas as maximising $S(\mathbf{z})$ and then minimising the outcome. Therefore, Eq.(1) can be formulated as

$$\min_{z_{dk}^{pn}} \left(\max_{\tilde{\delta}^{pn}} \left(\sum_{n=1}^{N} \sum_{p=1}^{P} \lambda^{n} \left(\tilde{\delta}^{pn} - \sum_{d=1}^{D} \sum_{k \in \mathbb{K}_d} z_{dk}^{pn} \right) \right) \right). \tag{18}$$

To eliminate the impact of uncertainty, here $\tilde{\delta}^{pn}$ can be temporarily treated as the decision variable, and z_{dk}^{pn} is regarded as a constant in Eq.(18). Then, Eq.(18) can be further transferred as Eq.(19), while Eq.(16) can be considered a constraint. The simplified model can be expressed as follows:

$$\begin{cases} \max & \sum\limits_{n=1}^{N} \sum\limits_{p=1}^{P} \lambda^{n} \tilde{\delta}^{pn} \\ \text{s.t.} & \tilde{\delta}^{pn} \geq \tilde{\delta}^{pn-}, \forall n \in \mathbb{N}, p \in \mathbb{P} \\ & \tilde{\delta}^{pn} \leq \tilde{\delta}^{pn+}, \forall n \in \mathbb{N}, p \in \mathbb{P} \\ & \sum\limits_{n=1}^{N} \tilde{\delta}^{pn} \leq \Delta^{p}, \forall p \in \mathbb{P} \\ & \tilde{\delta}^{pn} \geq 0, \forall n \in \mathbb{N}, p \in \mathbb{P}. \end{cases} \tag{19}$$

Here, the uncertain demand $\tilde{\delta}^{pn}$ in the robust model is used to replace δ^{pn} in the crisp model, and the total uncertain demand of p-th type of refined oil does not exceed the total demand in the market (i.e. Δ^{p}). To obtain a feasible solution, $\tilde{\delta}^{pn}$ in the objective function will be transferred into constraint based on dual theory, and the simplified model will be converted into the min-min form. Thus, the dual model of the linear model is proposed as follows:

$$\begin{cases} \min & \sum\limits_{n=1}^{N} \sum\limits_{p=1}^{P} \left(\tilde{\delta}^{pn-} \alpha^{pn} + \tilde{\delta}^{pn+} \beta^{pn} \right) + \sum\limits_{p=1}^{P} \Delta^{p} \gamma^{p} \\ \text{s.t.} & \alpha^{pn} + \beta^{pn} + \gamma^{p} \geq \lambda^{n}, \forall n \in \mathbb{N}, p \in \mathbb{P} \\ & \alpha^{pn} \leq 0, \beta^{pn} \geq 0, \gamma^{p} \geq 0, \forall n \in \mathbb{N}, p \in \mathbb{P}, \end{cases} \tag{20}$$

where α^{pn}, β^{pn}, and γ^{p} are the dual variables. The objective of station satisfaction (i.e. $S(\mathbf{z})$) is converted into the new robust objective $\Phi(\mathbf{z})$ as follows:

$$\Phi(\mathbf{z}) = \sum_{n=1}^{N} \sum_{p=1}^{P} \left(\tilde{\delta}^{pn-} \alpha^{pn} + \tilde{\delta}^{pn+} \beta^{pn} \right) + \sum_{p=1}^{P} \Delta^{p} \gamma^{p} - \sum_{n=1}^{N} \sum_{p=1}^{P} \lambda^{n} \left(\sum_{d=1}^{D} \sum_{k \in \mathbb{K}_d} z_{dk}^{pn} \right). \tag{21}$$

Based on the above, the robust model for multi-objective MDVRPLS in refined oil

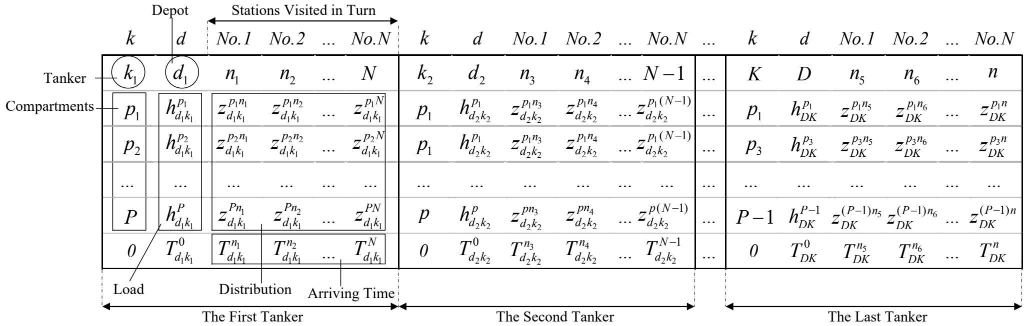


Figure 3. Illustration of Particle

distribution can be formulated as

$$\begin{cases} \min & \{\Phi(\mathbf{z}), C(\mathbf{x}), P(\mathbf{T})\} \\ \text{s.t.} & \alpha^{pn} + \beta^{pn} + \gamma^{p} \geq \lambda^{n}, \forall n \in \mathbb{N}, p \in \mathbb{P} \\ & \alpha^{pn} \leq 0, \beta^{pn} \geq 0, \gamma^{p} \geq 0, \forall n \in \mathbb{N}, p \in \mathbb{P} \\ & Constraints\ (5)-(14). \end{cases} \tag{22}$$

3.3. *MOPSO Algorithm*

In general, the analytical algorithm cannot handle the large-scale MDVRPLS efficiently, while heuristic algorithms can solve them effectively (Lahyani, Gouguenheim, and Coelho 2019). Among them, particle swarm optimisation (PSO) has demonstrated an efficient and wide applicability on continuous spaces search, compared to other evolutionary algorithms, such as genetic algorithm (GA) (Padhye, Branke, and Mostaghim 2009; Ma, Guan, and Wang 2020). Based on PSO, the MOPSO algorithm was first proposed by Coello and Lechuga (2002), which can easily be implemented and requires less parameter tuning. MOPSO obtains the non-dominated solutions in terms of particle fitness. Compared to other MOHAs, MOPSO is suitable to fewer parameters and faster convergence. In this study, the particle structure of general MOPSO was innovated, which is multi-segment and multi-layered. In addition, the usual two-dimensional solution space is extended to three-dimensional space; that is, the solutions obtained can form a Pareto surface based on three objectives for non-dominant sorting. In this study, MOPSO is adopted to solve both the crisp and robust models.

3.3.1. Particle Structure

In MOPSO, each particle represents a feasible solution to the problem. MOPSO aims to establish a one-to-one mapping between the solution space and the particle space and to facilitate subsequent population update steps. The particle structure is shown in Fig. 3, which is divided into $\sum_{d=1}^{D} K_d$ segments based on the number of vehicles dispatched.

In each travelled segment, the first line represents the vehicle number k, the depot number d, and gasoline stations visited in turn. Correspondingly, each subsequent row successively represents the p-th type of refined oil loaded by tanker k, loading capacity h_{dk}^{p} of the p-th type of refined oil, and quantity of the p-th type of refined oil delivered to each station. The last line, starting from the second column, shows the departure time of tanker k from the depot and the arrival time at the gasoline station.

3.3.2. Initialisation

We first generated a swarm of particles, initialising them as described above. Then, we calculated the fitness values $S(\mathbf{z})$ (or $\Phi(\mathbf{z})$), $C(\mathbf{x})$ and $P(\mathbf{T})$ of each particle. We initialised $\mathbf{X}$ and $\mathbf{V}$ for each particle, where $\mathbf{X}$ is the position and $\mathbf{V}$ is the velocity of the particle. Both $\mathbf{X}$ and $\mathbf{V}$ are vectors in the particle space.

There are two concepts in MOPSO: personal best and global best. At this stage, all initialised particles are personal best, and the global best particle is the best one of personal best particles in the Pareto set. The Pareto set generation process is described in the following section.

3.3.3. Pareto Set Selection

The Pareto set was obtained by comparing three fitness values. The particles in the selected Pareto set are superior to other particles against the three fitness dimensions, while those in the Pareto set are not inferior to each other. The Pareto set selection process is summarised as follows:

Step 1 Compare the fitness of any two particles. If all objective values of one particle are not less than that of the others, the particle is considered superior;
Step 2 Record the superior particle and its position;
Step 3 Repeat Steps 1–2 until all particles in the population are compared;
Step 4 Record the Pareto set with all superior particles.

3.3.4. Update and Optimisation

The iteration process of MOPSO depends on the update of velocity and position, such as Eq.(23)-(24):

$$\mathbf{V}' = \omega\mathbf{V} + c_1r_1(\mathbf{P_{pb}} - \mathbf{X}) + c_2r_2(\mathbf{P_{gb}} - \mathbf{X}), \tag{23}$$

$$\mathbf{X}' = \mathbf{X} + \mathbf{V}', \tag{24}$$

where P_{pb} is the position of the personal best particle, P_{gb} is the location of global optimal particle, $\mathbf{V}$ is the velocity of a particle, $\mathbf{V}'$ is the updated velocity, $\mathbf{X}$ is the position of a particle, and $\mathbf{X}'$ is the updated position. The dynamic inertia weight ω is adopted, as shown in Eq.(25), where $Maxgen$ is the maximum evolutionary algebra. The parameters ω_{max} and ω_{min} are the initial and minimum inertia values, respectively, and the value range is [0.2, 1.2]. ω shows linear decrement with the progress of the iteration process. The performance of MOPSO will be significantly improved as ω decreases. The learning factors c_1 and c_2 are usually equal, and the value range is [0, 4]. In Eq.(23) and (24), each updated particle is reordered in which the ω value is calculated in Eq.(25). Then, the new particle population is generated.

$$\omega = \omega_{max} - (\omega_{max} - \omega_{min}) * k/Maxgen. \tag{25}$$

After the new particle population is generated, the fitness values of the particle are calculated again. Then, the fitness values of the new particle and that of the prior personal best particle are compared. If the fitness values of the cost, penalty, and satisfaction functions of the new particle are less than or equal to the previous personal best, the new particle is better; otherwise, the previous personal best particle is better.

We placed new particles into a non-inferior solution set, if they were not dominated by other particles and the current non-inferior solution set particles.

The pseudo-code of the proposed MOPSO is shown in Algorithm 1.

```
Input: The information of MDVRPLS
Initialise particle population;
Calculate the fitness of first generation;
Screening for Pareto set;
for iter = 1 to Maxgen do
    Update the particle swarm;
    Calculate fitness of each generation;
    Update pbest and gbest;
    Update the Pareto set;
end
Output: Pareto set
```

Algorithm 1: MOPSO Process

4. Results

In this section, we first propose a dataset. Then, we establish a crisp multi-objective MDVRPLS model based on the dataset and validate the superiority compared to the classical model. Furthermore, the robust model is tested and compared to the crisp model. The MOHAs are coded in Matlab R2019b, with the following running environment: a Windows 10 platform with processor speed 1.60 GHz and memory 2 GB. Prior to MOPSO operation, we set $population\_size = 100$, $\omega_{max} = 1.2$, $\omega_{min} = 0.2$, $c_1 = c_2 = 2$, $Maxgen = 200$.

4.1. *Dataset*

First, a case of limited supply from CNPC is taken. The detailed information of oil depot (No.A-C) and gasoline station (No.1-16) is shown in Table. 3, which includes node location (longitude and latitude), soft time windows, station demand, depot supply, and demand intervals. Here, the priority reflects the importance and emergency of a gasoline station and can be divided into three levels: low, medium, and high, scored one, two, and three, respectively. In this study, the priority setting needs to determine whether the following conditions are met:

(1) The gasoline station is located near the main road;
(2) The total demand for refined oil at one gasoline stations exceeds 4 kl;
(3) There are other gasoline stations within a radius of 5 km.

If all the above conditions are met, then it is high level. If it satisfies two of them, then it is medium level; Otherwise, the level is low. In addition, the operating time of each depot is 8 h, and the tanker used in this case has three compartments in which the effective volume is 10 kl. The service time at each station t_s is 30 min, the

Table 3. Dataset Information

Nodes	Coordinate	Priority	Soft Time Window	Supply/Demand (kl)			Demand Intervals		
				92#	95#	98#	92#	95#	98#
A	(113.59,23.09)	-	[0,480]	13.56	17.10	7.65	-	-	-
B	(113.57,22.95)	-	[0,480]	12.34	14.50	5.35	-	-	-
C	(113.88,22.51)	-	[0,480]	7.20	6.70	4.00	-	-	-
1	(113.82,23.12)	2	[0,300]	-	4.00	-	-	[3.69,4.31]	-
2	(113.82,23.08)	3	[0,240]	2.60	4.28	1.44	[2.40,2.80]	[3.95,4.61]	[1.33,1.55]
3	(113.71,22.80)	2	[61,360]	3.72	2.88	1.44	[3.43,4.01]	[2.66,3.10]	[1.33,1.55]
4	(113.70,22.95)	2	[301,480]	-	3.88	3.88	-	[3.58,4.18]	[3.58,4.18]
5	(113.89,22.96)	2	[241,480]	-	2.88	1.44	-	[2.66,3.10]	[1.33,1.55]
6	(113.63,23.03)	2	[0,180]	5.82	1.94	-	[5.37,6.27]	[1.79,2.09]	-
7	(114.18,22.96)	1	[61,300]	1.94	1.94	-	[1.79,2.09]	[1.79,2.09]	-
8	(113.77,23.03)	1	[181,480]	1.94	3.88	1.94	[1.79,2.09]	[3.58,4.18]	[1.79,2.09]
9	(113.83,22.87)	2	[61,240]	1.94	-	1.94	[1.79,2.09]	-	[1.79,2.09]
10	(113.82,22.86)	3	[0,360]	2.60	2.00	3.44	[2.40,2.80]	[1.84,2.16]	[3.17,3.71]
11	(113.98,23.02)	2	[0,480]	-	2.00	2.80	-	[1.84,2.16]	[2.58,3.02]
12	(113.96,22.94)	2	[181,360]	4.50	3.54	-	[4.15,4.85]	[3.27,3.81]	-
13	(113.89,23.07)	2	[241,360]	7.82	-	-	[7.21,8.43]	-	-
14	(114.17,22.84)	2	[61,420]	3.16	3.44	1.44	[2.92,3.4]	[3.17,3.71]	[1.33,1.55]
15	(113.68,23.10)	3	[120,480]	3.26	3.58	1.44	[3.01,3.51]	[3.30,3.86]	[1.33,1.55]
16	(113.67,23.09)	1	[120,480]	1.94	1.94	-	[1.79,2.09]	[1.79,2.09]	-

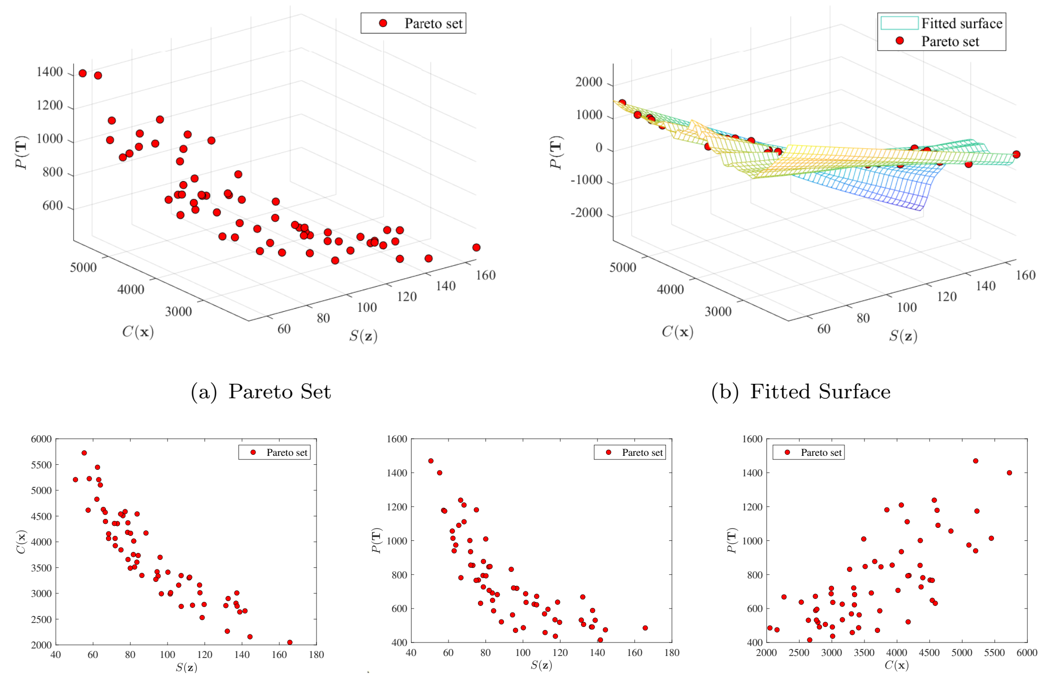
transportation cost c_t is 4 Yuan/km, the fixed dispatch cost c_f is 30 Yuan/vehicle each time, and the average vehicle speed v is 70 km/h.

Second, to verify the robustness of the proposed model, we assume that the demand from the gasoline station is an interval variable. Suppose that the demand of the p-th type of refined oil in station n is $\tilde{\delta}^{pn} \in [\tilde{\delta}^{pn-}, \tilde{\delta}^{pn+}]$. The minimum demand of all stations equals the maximum supply of all depots (i.e. $\sum_{d=1}^{D} \tilde{\varsigma}_d^{p+} = \sum_{n=1}^{N} \tilde{\delta}^{pn-}$), which is the threshold of the limited supply. The demand intervals for the robust model is presented in Table. 3.

4.2. *Illustration on Crisp Model*

We first developed the crisp model of the case and compared it with the traditional model without considering satisfaction and penalty, solved by MOPSO. Subsequently, two other MOHAs were also applied to solve the crisp model, while comparing the algorithm performance with MOPSO. The Pareto solution ($S(\mathbf{z})$, $C(\mathbf{x})$, $P(\mathbf{T})$) can be obtained and shown in Fig. 4; each point in the coordinates corresponds to a distribution scheme shown in Table. 4.

The obtained Pareto set is located in the three-dimensional space, of which each solution is superior to the others in at least one dimension. To observe the relationships between each two objectives, we present Fig. 4(c)-(e). With respect to the objective minimisation of station satisfaction, the operation cost and overtime penalty will decrease. If companies aim to save their cost, the station satisfaction will suffer. Simultaneously, the increased cost implies that tankers travel longer distances to distribute to more stations, and the complicated routes increases the risk of overtime, resulting in increasing penalty. The scheme selection decision is highly dependent on business objectives set by the oil marketing company. For example, the station satisfaction of scheme 1 is the highest, indicating the minimal impact of limited supply. Meanwhile, it also leads to a high total cost (i.e. 5207.01 Yuan) and maximum overtime penalty (i.e. 1469.67 min) in all schemes. Although scheme 62 saves over 60% of the costs and reduces over 65% of the penalty when compared to scheme 1, station satisfaction declines, of which $S(\mathbf{z})$ increases from 50.54 to 165.68. That is, station satisfaction is lower, while the supply of depots is in surplus in case of limited supply, which is not a feasible scheme in reality. The extreme conditions of the Pareto solution are rarely adopted by companies, while marketing companies prefer to spend more to improve the station satisfaction and reduce the overtime penalty.



(a) Pareto Set (b) Fitted Surface

(c) Projection on $S(\mathbf{z})$-$C(\mathbf{x})$ Plane (d) Projection on $S(\mathbf{z})$-$P(\mathbf{T})$ Plane (e) Projection on $C(\mathbf{x})$-$P(\mathbf{T})$ Plane

Figure 4. The Pareto Set and Fitted Surface of the Crisp Model

The details of several route schemes are presented in Table. 5. Evidently, if the satisfaction of gasoline stations is ignored, the total cost and time can be significantly reduced, while many stations may be not serviced or the distribution is limited. For example, the distribution of scheme 1 accounts for 88.60% of the total supply and 74.86% of the total demand, but high satisfaction implies higher cost and penalty. In contrast, the cost and penalty of scheme 62 is lower, and the resource utilisation rate is only 34.61%. Therefore, with respect to limited supply, oil marketing companies generally prioritise schemes with higher satisfaction and pay less attention to cost and penalty.

To verify the effectiveness and superiority of the multi-objective model in solving a MDVRP with limited supply, we used a traditional MDVRP case from Escobar et al. (2014) in contrast. To compare the results, we adjusted the objective of the traditional MDVRP model into $C(\mathbf{x})$.

After 200 iterations, the optimisation result is shown in Fig. 5(a). When satisfaction is not considered, the total cost of the traditional model is 1958.84, which is lower than the schemes obtained by the crisp model. Meanwhile, the average satisfaction of stations is a mere 43.18%, as shown in Fig. 5(b). When the cost, station satisfaction, and overtime penalty are considered the objectives of the crisp model in this study, we found that a small increase in cost can lead to a significant increase in satisfaction; that is, each additional unit of cost will result in a 3.33% increase in station satisfaction at most, and each additional unit of overtime penalty will result in a 4.60% increase in satisfaction at most. Therefore, considering the total cost, station satisfaction, and overtime penalty is preferable in the crisp model. The advantage is that it not only considers station satisfaction with limited supply, but it also effectively balances cost, satisfaction and overtime penalty to the extent possible. Consequently, the Pareto

Table 4. Objective Values of the Pareto Set of the Crisp Model

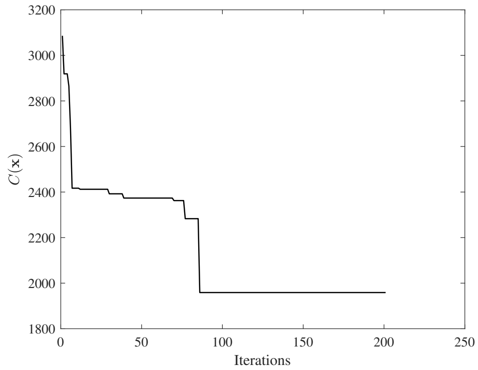
Schemes	$S(\mathbf{z})$	$C(\mathbf{x})$	$P(\mathbf{T})$	Schemes	$S(\mathbf{z})$	$C(\mathbf{x})$	$P(\mathbf{T})$
1	50.54	5207.01	1469.67	32	84.22	3735.11	586.67
2	55.25	5725.30	1399.67	33	86.20	3348.56	682.33
3	57.37	4614.53	1178.67	34	88.41	4171.04	521.67
4	57.96	5226.14	1174.00	35	93.71	3272.51	831.00
5	62.03	4826.72	1056.33	36	94.38	3419.19	562.67
6	62.39	5447.24	1014.00	37	94.94	3335.58	721.33
7	63.04	5206.06	940.00	38	95.99	3699.74	471.67
8	63.90	5102.87	974.00	39	96.67	2992.87	719.00
9	65.51	4628.80	1090.33	40	100.19	3411.16	487.00
10	66.52	4571.71	1238.33	41	101.43	2989.05	687.33
11	66.66	4395.60	781.67	42	101.66	3014.21	636.33
12	68.33	4065.89	1209.67	43	105.98	3159.32	625.67
13	68.37	4155.96	1111.33	44	107.30	3345.64	622.00
14	71.47	4356.72	1000.67	45	107.35	2747.15	672.00
15	71.84	4066.41	935.00	46	111.58	3299.91	568.67
16	71.93	3925.75	855.67	47	111.88	3313.37	458.67
17	73.08	4353.66	854.33	48	113.40	2769.56	595.67
18	74.82	4540.52	766.00	49	117.19	3161.14	534.00
19	74.97	3843.79	1181.33	50	117.40	3011.82	437.00
20	76.04	4507.58	767.67	51	118.59	2530.78	637.67
21	77.22	4586.79	631.00	52	119.66	2785.23	519.00
22	78.52	4184.34	794.67	53	131.31	2762.79	532.00
23	78.72	4368.98	727.33	54	132.10	2265.53	668.33
24	78.76	3657.71	877.33	55	132.55	2900.07	506.67
25	79.99	3489.06	1009.67	56	136.76	2808.79	491.67
26	80.12	4165.84	792.67	57	137.20	3007.66	491.33
27	81.65	3753.36	845.67	58	137.38	2751.32	588.67
28	81.84	4014.76	707.00	59	138.78	2639.61	531.00
29	82.30	3511.13	847.67	60	141.59	2660.07	415.00
30	83.52	3605.45	691.67	61	144.27	2158.32	474.67
31	83.66	4540.79	648.33	62	165.68	2049.38	485.67

Table 5. Distribution Schemes of the Crisp Model

Schemes	Tankers	Routes	Arriving Time	Distribution(kl)
1	1	A-4-2-5-7-1-3-8-12-A	61-78-148-231-299-305-368-448	4.73,4.94,1.98,2.17,1.61,2.53,1.50,2.55
	2	A-2-3-1-6-5-A	70-152-198-242-297	1.03, 4.38, 1.78, 3.09, 1.09
	3	B-2-4-6-13-12-15-B	45-108-144-195-284-327	1.46, 1.65, 4.60, 3.21, 3.42, 3.24
	4	B-11-9-10-12-14-B	40-90-139-212-257	2.83, 2.04, 3.19, 1.62, 2.42
	5	C-8-10-15-16-14-C	98-126-163-196-265	4.24, 3.15, 3.69, 1.57, 2.09
13	1	A-4-2-5-3-A	82-140-187-210	6.52, 6.88, 2.51, 3.41
	2	A-3-2-1-6-8-A	69-130-202-259-349	3.80, 1.13, 2.66, 3.59, 1.09
	3	B-1-10-8-5-B	42-64-115-162	1.11, 4.24, 3.46, 1.18
	4	B-8-12-14-15-B	114-162-241-297	1.88, 4.05, 4.95, 3.47
	5	C-7-13-16-15-11-12-C	88-159-249-327-417-495	2.92, 3.73, 1.46, 1.04, 1.75, 1.11
	6	C-11-14-C	54-112	2.21, 1.37
25	1	A-6-4-7-2-10-12-A	45-110-177-230-272-316	2.92, 4.61, 2.38, 4.30, 4.23, 1.37
	2	A-8-2-10-11-A	61-139-199-282	3.74, 2.53, 1.82, 1.01
	3	B-6-10-12-14-15-16-B	46-87-120-145-182-228	2.74, 1.16, 3.28, 3.35, 3.03, 1.07
	4	B-4-11-13-B	20-101-142	1.64, 1.65, 4.99
	5	C-12-1-8-3-C	88-130-187-253	2.24, 1.08, 2.85, 5.86
37	1	A-5-2-1-3-A	53-139-225-292	3.25, 4.91, 3.39, 3.50
	2	A-3-4-8-2-12-A	63-100-157-199-249	3.34, 3.18, 1.18, 2.56, 1.43
	3	B-6-10-12-14-B	56-99-136-181	3.09, 6.70, 6.39, 4.14
	4	C-4-6-9-8-7-16-C	39-83-129-176-236-287	3.64, 3.05, 2.95, 2.28, 1.47, 1.09
49	1	A-1-2-3-8-5-4-A	86-101-186-220-244-282	2.17, 4.93, 2.22, 1.57, 1.08, 1.69
	2	B-1-6-11-8-5-7-12-4-B	25-65-110-123-146-176-217-261	1.72, 5.03, 1.39, 2.65, 1.37, 2.91, 3.80, 1.00
	3	B-6-4-13-15-B	54-95-140-211	2.33, 1.96, 1.20, 1.39
	4	C-3-2-12-5-8-C	5-44-96-180-220	3.55, 2.09, 2.78, 1.16, 1.54
62	1	A-4-7-6-2-8-9-A	30-87-131-177-238-289	2.58, 1.59, 1.26, 2.40, 3.23, 1.51
	2	B-6-5-10-12-B	73-144-170-241	3.33, 1.01, 4.10, 3.89
	3	C-10-3-C	69-149	1.23, 4.57

solutions obtained by the crisp model can reflect pertinence and superiority, which cannot be obtained using the traditional MDVRP model.

To demonstrate the effectiveness of the MOPSO algorithm, two other heuristic



(a) Iteration

Station	Satisfaction	Station	Satisfaction
1	60.25%	9	53.20%
2	48.03%	10	46.17%
3	54.75%	11	44.58%
4	34.54%	12	28.76%
5	40.65%	13	32.10%
6	40.98%	14	31.65%
7	42.37%	15	54.83%
8	44.85%	16	33.25%

(b) Station Satisfaction

Figure 5. Result of the Traditional Model

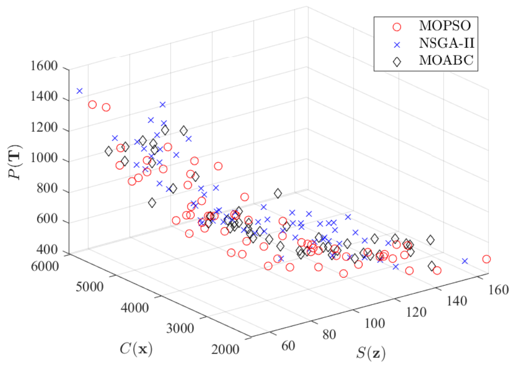


Figure 6. Pareto Sets of the Crisp Model Obtained by Three Algorithms

algorithms are selected for differentiation. For NSGA-II, we set $population\_size = 100$, $Maxgen = 200$, $p_{cross} = 0.8$, $p_{mutation} = 0.1$. For MOABC, we set $population\_size = 100$, $Maxgen = 200$, $limit = 30$, $foodnumber = 50$. The results are presented in Fig. 6.

For more scientific evaluation, several metrics (Li, Wang, and Liu 2008; Wu, Chien, and Gen 2012) are selected to compare the performance of these three MOHAs:

(1) ONVG: Overall nondominated vector generation (ONVG) is the number of distinct nondominated solutions in the set (Veldhuizen 1999);

(2) CM: A C metric reflect dominance relationship between solutions in two Pareto sets:

$$C(Set_1, Set_2) = |\{\varepsilon_2 \in Set_2 | \exists : \varepsilon_1 \in Set_1, \varepsilon_2 \preceq \varepsilon_1\}|/|Set_2|; \tag{26}$$

(3) D_{av}, D_{max}: The two distance metrics are used to measure the performance of the Pareto set relative to a reference set R, which is formed by all the nondominated solutions from all sets:

$$D_{av} = \sum_{\varepsilon_R \in R} \min_{\varepsilon \in Set_1} d(\varepsilon, \varepsilon_R)/|R|, \tag{27}$$

Table 6. Comparison of Algorithms for Solving the Crisp Model

Metrics	MOPSO	NSGA-II	MOABC
ONVG	62	65	49
C(MOPSO,·)	-	0.7846	0.6531
C(NSGA-II,·)	0.0645	-	0.3469
C(MOABC,·)	0.0000	0.3846	-
D_{av}	180.1900	181.1741	200.6119
D_{max}	560.3834	606.6086	821.0185
TS	9.2124	11.0686	10.3023
RT	177.0266	210.5965	173.2038

$$D_{max} = \max_{\varepsilon_R \in R} \left\{ \min_{\varepsilon \in Set_1} d(\varepsilon, \varepsilon_R) \right\}. \tag{28}$$

Here, D_{av} is the average distance from a solution $\varepsilon_R \in R$ to its closest solution in Set_1 (Piotr et al. 1998), and D_{max} is the maximum of the minimum distance from a solution $\varepsilon_R \in R$ to any solution in Set_1 (Ulungu, Teghem, and Ost 1998).

(4) TS: This spacing metric is used to measure how evenly the solutions are distributed.

$$TS = \sqrt{\frac{1}{|Set_1|} \sum_{i=1}^{|Set_1|} (D_i - \overline{D})^2 / \overline{D}}, \tag{29}$$

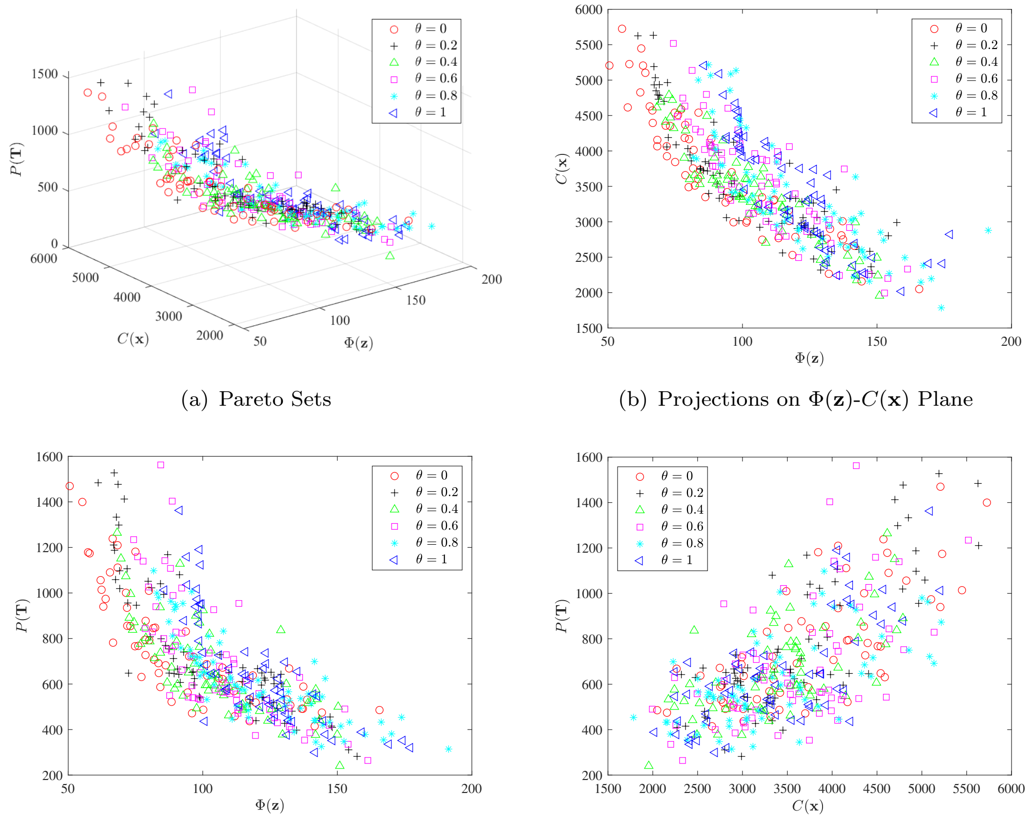
where $\overline{D} = \sum_{i=1}^{Set_1} D_i / |Set_1|$ and D_i is the Euclid distance in objective space between solution i and its nearest solution (Tan et al. 2006);

(5) RT: The running time (RT) is used as a key metric to reflect the efficiency of MOHAs.

For the results presented in Table. 6, MOPSO performs better on most metrics than NSGA-II and MOABC. From the ONVG metric, it can be observed that NSGA-II obtains the most solutions, whereas MOPSO obtains fewer. In terms of CM, few solutions obtained by MOPSO are dominated by those obtained by NSGA-II, and none is dominated by those obtained by MOABC. In contrast, the solutions obtained by NSGA-II and MOABC are easier to be dominated by those obtained by MOPSO. In summary, the MOPSO produces a better Pareto set. With regards to D_{av} and D_{max}, the values of MOPSO is smaller than two other algorithms, which implies that MOPSO is able to obtain solutions that are closer to the optimal Pareto set. Further, the TS values of MOPSO are smaller than NSGA-II and MOABC, which indicates the Pareto set obtained by MOPSO have a more uniform distribution. Finally, MOPSO consumes 177.0266 s to solve the program, only about 4 s longer than MOABC. In summary, MOPSO is superior to NSGA-II and MOABC in four out of six metrics. Therefore, we believe MOPSO has advantages in solving the problems in this study.

4.3. *Illustration on Robust Model*

In this section, we define the disturbance indictor of uncertainty as θ. If $\theta = 1$, then the disturbance achieves the maximum. If $\theta = 0$, then there is no uncertainty in the system, and the schemes in Section 4.2 become feasible. We discuss five different types



(a) Pareto Sets

(b) Projections on $\Phi(\mathbf{z})$-$C(\mathbf{x})$ Plane

(c) Projections on $\Phi(\mathbf{z})$-$P(\mathbf{T})$ Plane

(d) Projections on $C(\mathbf{x})$-$P(\mathbf{T})$ Plane

Figure 7. Pareto Sets of Robust Models with Different θ

of θ. The robust sets are shown in Fig. 7.

In terms of the fluctuation induced by the uncertainty of demand, the distribution of robust Pareto sets have changed. As we can observed, Pareto sets move to the right on the $\Phi(\mathbf{z})$ dimension with the increase in θ. This implies that the uncertain demand has a stronger effect on station satisfaction. From a data perspective, the $\Phi(\mathbf{z})$ will increase 5.28% for every 0.2 increase in θ on average. As the uncertainty increases, achieving the same customer satisfaction rate requires an average additional cost of 8.81% and additional penalty of 12.85%. However, the impact of demand uncertainty on $C(\mathbf{x})$ and $P(\mathbf{T})$ is not evident. With every 0.2 increase in θ, the cost $C(\mathbf{x})$ and penalty $P(\mathbf{T})$ increase 0.59% and 0.72% on average respectively, which can be ignored owing to the state of the data. For $\Phi(\mathbf{z})$, the fluctuation in demand will aggravate the mismatch between supply and demand and the entire Pareto set tends to shift to the right with the increase of θ. For $C(\mathbf{x})$ and $P(\mathbf{T})$, the influence of uncertain demands is not strong, which can also be observed from the Eq.(2) and (4) that $\tilde{\delta}_n^p$ does not directly act on these two objectives.

In general, the numerical experiments showed the advantages of the robust model through the numerical experiment; hence, the larger θ, the more inclusive the model. For example, when $\theta = 1$, the scheme can accept any fluctuation in demand in the uncertainty interval, remaining unchanged. This shows strong stability in terms of the robust model, which can be realised through robust optimisation, avoiding the problem of frequent change of distribution schemes. Therefore, the robust model is

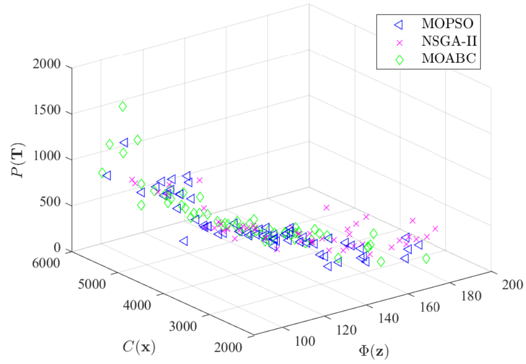


Figure 8. The Pareto Sets of the Robust Model Obtained by Three Algorithms

Table 7. Comparison of Algorithms for the Solving Robust Model

Metrics	MOPSO	NSGA-II	MOABC
ONVG	62	55	57
C(MOPSO,·)	-	0.6000	0.5439
C(NSGA-II,·)	0.1129	-	0.4035
C(MOABC,·)	0.2742	0.4000	-
D_{av}	166.3477	199.4334	168.2624
D_{max}	1056.4364	1210.3045	603.3461
TS	11.9609	9.7836	10.9299
RT	229.0334	240.6536	203.2354

more valuable than the crisp model in practical applications.

Furthermore, to prove the validity of the algorithm selection, we compared MOPSO with the other two MOHAs (NSGA-II and MOABC) again. Let $\theta = 1$; the Pareto sets of the robust model obtained by the three algorithms are shown in Fig. 8. The performance indicators are shown in Table. 7. All three algorithms can obtain a Pareto set. In terms of ONVG, MOPSO obtains the most solutions compared to NSGA-II and MOABC. With regards to CM metrics, the solution in Pareto set obtained by MOPSO dominates the solutions in other two Pareto sets, indicating the superiority of the Pareto set obtained by MOPSO. The MOPSO obtains the minimum D_{av}, which indicates that the Pareto set obtained by MOPSO is closer to the optimal one. However, MOPSO also has the maximum D_{max}, which shows that the Pareto set obtained by MOPSO is not smooth and sufficiently uniform. Correspondingly, the TS metrics also confirms this view. From running time, the NSGA-II is better than MOPSO. In summary, the MOPSO is superior to two other algorithms in more than half of the targets when solving the robust model.

4.4. *Managerial Insights*

Based on the above experiments, it is obvious that both the multi-objective robust optimization model and the improved MOPSO algorithm proposed in this paper are effective and superior for MDVRPLS in refined oil distribution, which is of practical significance. Combined with the above analysis, several management insights are summarized for decision makers of oil marketing companies:

(1) In the case of limited supply, oil marketing companies should not only focus on cost or time, but also pay more attention to whether the gasoline station demand is fully satisfied. The crisp model proposed can better balance operation costs, station satisfaction, and overtime penalty, which produces 3.33% and 4.60% incereases in station satisfaction at an increased unit cost and overtime penalty respectively. Considering the supply shortage, the station satisfaction directly determines the safety of oil supply. Therefore, it is necessary to increase operation costs and distribution time in exchange for greater station satisfaction for oil marketing companies in a situation of supply shortage.

(2) Oil marketing companies should fully consider the influence caused by the uncertain factors in the distribution system. It can be seen that the higher uncertainty indictor θ, the more inclusive the robust scheme. When θ reaches about 0.8, the marginal robustness increases the fastest. To ensure that distribution scheme is feasible and optimal, the oil distribution management system should be upgraded to a more robust version to fully withstand the impact caused by uncertainty. In this way can the oil marketing companies could eliminate the negative impact of uncertainty and ensure the effective implementation of the distribution scheme.

(3) The oil marketing companies should consider to apply MOPSO when build the management system for refined oil distribution. MOPSO shows an efficient and wide applicability on continuous spaces search. Through algorithm comparison experiment, we found that MOPSO performs better in terms of the number of solutions, the evenness of Pareto sets, the probability of non-inferior solutions, and the computing time. The calculation results of MOPSO can provide more excellent options for decision makers of oil marketing companies, which is more suitable for building an oil distribution planning platform.

5. Discussion

In this study, both oil shortage and uncertain demand have been considered in the distribution planning, vehicle scheduling, and route optimisation for oil distribution networks. A multi-objective robust optimisation model has been designed to address the problem, which maximises station satisfaction and minimises operation costs. The model shows strong stability in handling the demand fluctuation, ensuring a balance between station satisfaction and cost. In addition, this study examined an oil marketing company as a case and applied MOPSO to solve the robust model. Considering the comparative analysis, the conclusions drawn are as follows:

(1) Considering the uncertain factors that affect demand, a robust model has been proposed to improve scheme robustness while decreasing the total cost and overtime penalty and increasing gasoline satisfaction. Among the three objectives, $\Phi(\mathbf{z})$ is the most affected, which increases 5.28% on average when the uncertainty of demand increases. If the satisfaction rate is constant, the cost increases to 8.81%, and over tie penalty increases to 12.85% on average. The superiority of the robust model is evident in how it resolves a distribution optimisation problem with low cost, high time rate, and high station satisfaction in refined oil distribution networks with uncertain demand, and how it effectively minimise loss of cost and the waste consumed and reduces station satisfaction caused by uncertain factors.

(2) MOPSO used in this study is more suitable than other MOHAs for multi-objective MDVRPLS. In terms of Pareto set, MOPSO obtains more solutions than

NSGA-II and MOABC, especially in handling the robust model. The solutions obtained by MOPSO has a higher probability to dominate the solutions obtained by other MOHAs. Regarding the optimality of the Pareto set, MOPSO also performs better. Generally, the Pareto set obtained by MOPSO is more suitable than that obtained by other MOHAs, in terms of optimisation performance and solution quality. MOPSO has more advantages in solving the robust model than the crisp model.

The proposed approach in this paper can be widely applied in reality. In addition to the distribution of refined oil and other hazardous products, the robust MDVRPLS model and MOPSO algorithm can also be applied for other daily goods transportation under the supply shortage. Moreover, the robust model also plays a role in other supply-demand scenarios, including supply-demand balances and oversupply, by modifying constraints (e.g., removing Eq.(6)).

In addition, the distribution scope division has also some limitations. In particular, the boundaries of the distribution scope is fixed. In reality, depots usually distribute refined oil jointly, meaning that the distribution scope of each depot is elastic (Xu, Lin, and Zhu 2020), of which the size can be adjusted and coverage changes based on various uncertainties. In this situation, the total distribution path is relatively short, which will help save costs, improve the use efficiency of vehicles, and make it easier for each ODC to manage vehicles and improve station satisfaction.

In the future, we will continue to conduct more in-depth research on the basis of this study. In addition to the elastic distribution boundary problem mentioned previously, the application field of the model in this paper can also be expanded. For example, the problem of resource distribution in emergency situations (e.g. COVID-19), which is very popular recently, can also be solved using the robust MDVRPLS model presented in this paper. In addition, the model in this paper can be applied to the closed-loop supply chain (Liu et al. 2019), corporate social responsibilities (Bian et al. 2021), and other fields.

Acknowledgements

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