## Online Appendix

## Proof of Lemma 1:

Proof. The profits of the social enterprise and the farmer cooperative are $\pi_{E}^{N}=\frac{\mu_{y}^{2}}{16 S^{2}}\left(\mu_{m}-\right.$ $c)^{2}, \pi_{F}^{N N}=\frac{\mu_{y}^{2}}{8 S^{2}}\left(\mu_{m}-c\right)^{2}$, where $S^{2}=\sigma_{y}^{2}+\mu_{y}^{2}$. It is easy to see that both $\pi_{E}^{N}$ and $\pi_{F}^{N N}$ are increasing in $\mu_{m}$ and decreasing with $\sigma_{y}$. By noting that $\frac{d S^{2}}{d \mu_{y}}=2 \mu_{y}$, we have

$$
\frac{d \pi_{E}^{N}}{d \mu_{y}}=\frac{\left(\mu_{m}-c\right)^{2}}{16} \frac{S^{2} \frac{d \mu_{y}^{2}}{d \mu_{y}}-\mu_{y}^{2} \frac{d S^{2}}{d \mu_{y}}}{\left(S^{2}\right)^{2}}=\frac{\left(\mu_{m}-c\right)^{2}}{16} \frac{2 S^{2} \mu_{y}-\mu_{y}^{2} 2 \mu_{y}}{S^{4}}=\frac{\left(\mu_{m}-c\right)^{2}}{8} \frac{\mu_{y} \sigma_{y}^{2}}{S^{4}}>0,
$$

and

$$
\frac{d \pi_{F}^{N N}}{d \mu_{y}}=\frac{\left(\mu_{m}-c\right)^{2}}{8} \frac{S^{2} \frac{d \mu_{y}^{2}}{d \mu_{y}}-\mu_{y}^{2} \frac{d S^{2}}{d \mu_{y}}}{\left(S^{2}\right)^{2}}=\frac{\left(\mu_{m}-c\right)^{2}}{8} \frac{2 S^{2} \mu_{y}-\mu_{y}^{2} 2 \mu_{y}}{S^{4}}=\frac{\left(\mu_{m}-c\right)^{2}}{4} \frac{\mu_{y} \sigma_{y}^{2}}{S^{4}}>0 .
$$

The proof then completes.

## Proof of Lemma 2:

Proof. For the social enterprise, the profit is $\pi_{E}^{Y}=\frac{\mu_{y}^{2}}{16 S^{2}}\left[\left(\mu_{m}-c\right)^{2}+4 \rho^{2} \sigma_{m}^{2}\right]$, where $S^{2}=$ $\sigma_{y}^{2}+\mu_{y}^{2}$. So, it is easy to see that $\pi_{E}^{Y}$ is increasing with $\rho^{2}, \mu_{m}, \sigma_{m}$ and decreasing with $\sigma_{y}$. By noting that $\frac{d S^{2}}{d \mu_{y}}=2 \mu_{y}$, we have
$\frac{d \pi_{E}^{Y}}{d \mu_{y}}=\frac{\left(\mu_{m}-c\right)^{2}+4 \rho^{2} \sigma_{m}^{2}}{16} \frac{S^{2} \frac{d \mu_{y}^{2}}{d \mu_{y}}-\mu_{y}^{2} \frac{d S^{2}}{d \mu_{y}}}{\left(S^{2}\right)^{2}}=\frac{\left(\mu_{m}-c\right)^{2}+4 \rho^{2} \sigma_{m}^{2}}{16} \frac{2 S^{2} \mu_{y}-\mu_{y}^{2} 2 \mu_{y}}{S^{4}}=\frac{\left(\mu_{m}-c\right)^{2}+4 \rho^{2} \sigma_{m}^{2}}{8} \frac{\mu_{y} \sigma_{y}^{2}}{S^{4}}>0$.
In such a case, the profit of the upstream farmer cooperative is the same as the profit in Case 1. Therefore, it should possess the same properties as stated in Lemma 1. The proof then completes.

## Proof of Proposition 1:

Proof. In Case 4, the profits of the social enterprise and the farmer cooperative are respectively $\pi_{E}^{Y}=\frac{\mu_{y}^{2}}{16 S^{2}}\left[\left(\mu_{m}-c\right)^{2}+\rho^{2} \sigma_{m}^{2}\right]$ and $\pi_{F}^{Y N}=\frac{\mu_{y}^{2}}{8 S^{2}}\left[\left(\mu_{m}-c\right)^{2}+\rho^{2} \sigma_{m}^{2}\right]$, where $S^{2}=\sigma_{y}^{2}+\mu_{y}^{2}$. It is easy to see that both $\pi_{E}^{Y}$ and $\pi_{F}^{Y N}$ are increasing with $\sigma_{m}, \mu_{m}$ and decreasing with $\sigma_{y}$. By noting that $\frac{d S^{2}}{d \mu_{y}}=2 \mu_{y}$, we have

$$
\frac{d \pi_{E}^{Y}}{d \mu_{y}}=\frac{\left(\mu_{m}-c\right)^{2}+\rho^{2} \sigma_{m}^{2}}{16} \frac{S^{2} \frac{d \mu_{y}^{2}}{d \mu_{y}}-\mu_{y}^{2} \frac{d S^{2}}{d \mu_{y}}}{\left(S^{2}\right)^{2}}=\frac{\left(\mu_{m}-c\right)^{2}+\rho^{2} \sigma_{m}^{2}}{16} \frac{2 S^{2} \mu_{y}-\mu_{y}^{2} 2 \mu_{y}}{S^{4}}=\frac{\left(\mu_{m}-c\right)^{2}+\rho^{2} \sigma_{m}^{2}}{8} \frac{\mu_{y} \sigma_{y}^{2}}{S^{4}}>0,
$$

and
$\frac{d \pi_{F}^{Y N}}{d \mu_{y}}=\frac{\left(\mu_{m}-c\right)^{2}+\rho^{2} \sigma_{m}^{2}}{8} \frac{S^{2} \frac{d \mu_{y}^{2}}{d \mu_{y}}-\mu_{y}^{2} \frac{d S^{2}}{d \mu_{y}}}{\left(S^{2}\right)^{2}}=\frac{\left(\mu_{m}-c\right)^{2}+\rho^{2} \sigma_{m}^{2}}{8} \frac{2 S^{2} \mu_{y}-\mu_{y}^{2} 2 \mu_{y}}{S^{4}}=\frac{\left(\mu_{m}-c\right)^{2}+\rho^{2} \sigma_{m}^{2}}{4} \frac{\mu_{y} \sigma_{y}^{2}}{S^{4}}>0$.
The proof then completes.

## Proof of Proposition 2:

Proof. By the farmer cooperative's profits with agricultural advice listed in Table 3, his profit is strictly improved only in Case 8 wherein when both players can access to market information. In Case 8, the profits of the social enterprise and the farmer cooperative are respectively $\pi_{E}^{Y}=\frac{\gamma^{2} \mu_{y}^{2}}{16 S^{\prime 2}}\left[\left(\alpha \mu_{m}-\beta c\right)^{2}+\rho^{2} \sigma_{m}^{2}\right]$ and $\pi_{F}^{Y}=\frac{\gamma^{2} \mu_{y}^{2}}{8 S^{\prime 2}}\left[\left(\alpha \mu_{m}-\beta c\right)^{2}+\rho^{2} \sigma_{m}^{2}\right]$. Recalling that $S^{\prime 2}=\sigma_{y}^{2}+\gamma^{2} \mu_{y}^{2}$, we have $\pi_{E}^{Y}$ and $\pi_{F}^{Y Y}$ are increasing with $\sigma_{m}, \alpha, \mu_{m}$ and decreasing with $\sigma_{y}, \beta$. By noting that $\frac{d S^{\prime 2}}{d \mu_{y}}=2 \gamma^{2} \mu_{y}$ and $\frac{d S^{\prime 2}}{d \gamma}=2 \gamma \mu_{y}^{2}$, we have

$$
\begin{aligned}
\frac{d \pi_{E}^{Y}}{d \mu_{y}} & =\frac{\left(\alpha \mu_{m}-\beta c\right)^{2}+\rho^{2} \sigma_{m}^{2}}{16} \frac{S^{\prime 2} \frac{d \gamma^{2} \mu_{y}^{2}}{d \mu_{y}}-\gamma^{2} \mu_{y}^{2} \frac{d S^{\prime 2}}{d \mu_{y}}}{\left(S^{\prime 2}\right)^{2}}=\frac{\left(\alpha \mu_{m}-\beta c\right)^{2}+\rho^{2} \sigma_{m}^{2}}{16} \frac{2 S^{\prime 2} \gamma^{2} \mu_{y}-\gamma^{2} \mu_{y}^{2} 2 \mu_{y} \gamma^{2}}{S^{\prime 4}} \\
& =\frac{\left(\alpha \mu_{m}-\beta c\right)^{2}+\rho^{2} \sigma_{m}^{2}}{8} \frac{\gamma^{2} \mu_{y} \sigma_{y}^{2}}{S^{4}}>0, \\
\frac{d \pi_{E}^{Y}}{d \gamma} & =\frac{\left(\alpha \mu_{m}-\beta c\right)^{2}+\rho^{2} \sigma_{m}^{2}}{16} \frac{S^{\prime 2} \frac{d \gamma^{2} \mu_{y}^{2}}{d \gamma}-\gamma^{2} \mu_{y}^{2} \frac{d S^{\prime 2}}{d \gamma}}{\left(S^{\prime 2}\right)^{2}}=\frac{\left(\alpha \mu_{m}-\beta c\right)^{2}+\rho^{2} \sigma_{m}^{2}}{16} \frac{2 S^{\prime 2} \gamma \mu_{y}^{2}-\gamma^{2} \mu_{y}^{2} 2 \gamma \mu_{y}^{2}}{S^{\prime 4}} \\
& =\frac{\left(\alpha \mu_{m}-\beta c\right)^{2}+\rho^{2} \sigma_{m}^{2}}{8} \frac{\gamma \mu_{y}^{2} \sigma_{y}^{2}}{S^{\prime 4}>0,} \\
\frac{d \pi_{F}^{Y Y}}{d \mu_{y}} & =\frac{\left(\alpha \mu_{m}-\beta c\right)^{2}+\rho^{2} \sigma_{m}^{2}}{8} \frac{S^{\prime 2} \frac{d \gamma^{2} \mu_{y}^{2}}{d \mu_{y}}-\gamma^{2} \mu_{y}^{2} \frac{d S^{\prime 2}}{d \mu_{y}}}{\left(S^{\prime 2}\right)^{2}}=\frac{\left(\alpha \mu_{m}-\beta c\right)^{2}+\rho^{2} \sigma_{m}^{2}}{8} \frac{2 S^{\prime 2} \gamma^{2} \mu_{y}-\gamma^{2} \mu_{y}^{2} 2 \mu_{y} \gamma^{2}}{S^{\prime 4}} \\
& =\frac{\left(\alpha \mu_{m}-\beta c\right)^{2}+\rho^{2} \sigma_{m}^{2}}{4} \frac{\gamma^{2} \mu_{y} \sigma_{y}^{2}}{S^{4}}>0,
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{d \pi_{F}^{\gamma Y}}{d \gamma} & =\frac{\left(\alpha \mu_{m}-\beta c\right)^{2}+\rho^{2} \sigma_{m}^{2}}{8} \frac{S^{\prime 2} \frac{d \gamma^{2} \mu_{y}^{2}}{d \gamma}-\gamma^{2} \mu_{y}^{2} \frac{d S^{\prime 2}}{d \gamma}}{\left(S^{\prime 2}\right)^{2}}=\frac{\left(\alpha \mu_{m}-\beta c\right)^{2}+\rho^{2} \sigma_{m}^{2}}{8} \frac{2 S^{\prime 2} \gamma \mu_{y}^{2}-\gamma^{2} \mu_{y}^{2} 2 \gamma \mu_{y}^{2}}{S^{\prime 4}} \\
& =\frac{\left(\alpha \mu_{m}-\beta c\right)^{2}+\rho^{2} \sigma_{m}^{2}}{4} \frac{\gamma \mu_{y}^{2} \sigma_{y}^{2}}{S^{\prime 4}}>0 .
\end{aligned}
$$

The proof then completes.

## Proof of Proposition 4:

Proof. (i) By the equilibrium results in Table 4, only providing market information to the farmer cooperative does not benefit the social enterprise, the farmer cooperative as well as the whole supply chain. That is, the information failure effect still occurs when market information is only available to the farmer cooperative regardless of the farmer cooperative's adoption of agricultural advice.
(ii) We compare the equilibrium results under different cases in Table 4. To do so, we use $\pi_{E i}$ and $\pi_{F j}$, where $i, j=a, \ldots, h$, to denote the profits of social enterprise and farmer cooperative in Case $i$ and $j$, respectively. Then, we have

$$
\pi_{E b}-\pi_{E a}=\pi_{E d}-\pi_{E c}=\frac{\mu_{y}^{2}}{4 S^{2}}\left[\left(\mu_{m}-w\right)^{2}+\rho^{2} \sigma_{m}^{2}\right]-\frac{\mu_{y}^{2}}{4 S^{2}}\left(\mu_{m}-w\right)^{2}=\frac{\mu_{y}^{2}}{4 S^{2}} \rho^{2} \sigma_{m}^{2}>0,
$$

and

$$
\pi_{F a}=\pi_{F b}=\pi_{F c}=\pi_{F d}=\frac{\mu_{y}^{2}}{2 S^{2}}\left(\mu_{m}-c\right)(w-c)
$$

Also,
$\pi_{E f}-\pi_{E e}=\pi_{E h}-\pi_{E g}=\frac{\gamma^{2} \mu_{y}^{2}}{4 S^{\prime 2}}\left[\left(\alpha \mu_{m}-w\right)^{2}+\rho^{2} \sigma_{m}^{2}\right]-\frac{\gamma^{2} \mu_{y}^{2}}{4 S^{\prime 2}}\left(\alpha \mu_{m}-w\right)^{2}=\frac{\gamma^{2} \mu_{y}^{2}}{4 S^{\prime 2}} \rho^{2} \sigma_{m}^{2}>0$, and

$$
\pi_{F e}=\pi_{F f}=\pi_{F g}=\pi_{F h}=\frac{\gamma^{2} \mu_{y}^{2}}{2 S^{\prime 2}}\left(\alpha \mu_{m}-w\right)(w-\beta c)
$$

That means when the government intervenes in pricing of the agricultural products, the market information provided by the government will only increase the social enterprise's profit but not the farmer cooperative's profit.
(iii) From Table 4, the profits of the social enterprise in Case b and Case d are both $\frac{\mu_{y}^{2}}{4 S^{2}}\left[\left(\mu_{m}-w\right)^{2}+\rho^{2} \sigma_{m}^{2}\right]$. Then, we immediately have

$$
\pi_{E d}-\pi_{E b}=\frac{\mu_{y}^{2}}{4 S^{2}}\left[\left(\mu_{m}-w\right)^{2}+\rho^{2} \sigma_{m}^{2}\right]-\frac{\mu_{y}^{2}}{4 S^{2}}\left[\left(\mu_{m}-w\right)^{2}+\rho^{2} \sigma_{m}^{2}\right]=0
$$

The profits of the farmer cooperative in Case b and Case d are both $\frac{\mu_{y}^{2}}{2 S^{2}}\left(\mu_{m}-c\right)(w-c)$. Then, we immediately have

$$
\pi_{F d}-\pi_{F b}=\frac{\mu_{y}^{2}}{2 S^{2}}\left(\mu_{m}-c\right)(w-c)-\frac{\mu_{y}^{2}}{2 S^{2}}\left(\mu_{m}-c\right)(w-c)=0 .
$$

Also, the profits of the social enterprise in Case f and Case h are both $\frac{\gamma^{2} \mu_{y}^{2}}{4 S^{\prime 2}}\left[\left(\alpha \mu_{m}-\right.\right.$ $\left.w)^{2}+\rho^{2} \sigma_{m}^{2}\right]$. Then, we immediately have

$$
\pi_{E h}-\pi_{E f}=\frac{\gamma^{2} \mu_{y}^{2}}{4 S^{\prime 2}}\left[\left(\alpha \mu_{m}-w\right)^{2}+\rho^{2} \sigma_{m}^{2}\right]-\frac{\gamma^{2} \mu_{y}^{2}}{4 S^{\prime 2}}\left[\left(\alpha \mu_{m}-w\right)^{2}+\rho^{2} \sigma_{m}^{2}\right]=0
$$

The profits of the farmer cooperative in Case $f$ and Case $h$ are both $\frac{\gamma^{2} \mu_{y}^{2}}{2 S^{\prime 2}}\left(\alpha \mu_{m}-w\right)(w-\beta c)$. Then, we immediately have

$$
\pi_{F h}-\pi_{F f}=\frac{\gamma^{2} \mu_{y}^{2}}{2 S^{\prime 2}}\left(\alpha \mu_{m}-w\right)(w-\beta c)-\frac{\gamma^{2} \mu_{y}^{2}}{2 S^{\prime 2}}\left(\alpha \mu_{m}-w\right)(w-\beta c)=0
$$

That means, when the government intervenes in pricing of the agricultural products, sharing information with the farmer cooperative does not affect both the social enterprise's and the farmer cooperative's profits, regardless of the farmer cooperative's agricultural advice adoption.

The proof then completes.

## Proof of Proposition 6:

Proof. (i) The results can be immediately obtained following the same manner in Proposition 4 and hence we omit the details.
(ii) The results regarding impacts of the social enterprise's social responsibility concern can be quickly drawn from the equilibrium profits for each role in the supply chain as listed in Table 6.

