Appendix

Proof of lemma 2

It is obvious that Π_b^b is a continuous and non-differentiable function and has a non-differential point at $q_b^b = \lambda_s q_s^b$, as the left and right derivatives are not equal, i.e., $\frac{\partial \Pi_{b1}^b}{\partial q_b^b} \neq \frac{\partial \Pi_{b2}^b}{\partial q_b^b}$. The optimal production quantity of the user is obtained by comparing the maximums of the two parts of the profit function separated by this point. We first solve the unconstrained solutions to Π_{b1}^b and Π_{b2}^b. It is simple to show that Π_{b1}^b and Π_{b2}^b are both strictly concave in q_b^b. Solving the first order conditions yields the unconstrained optimal solutions to Π_{b1}^b and Π_{b2}^b, i.e., $q_{b1}^b = \frac{\alpha_b - c_b - r_b - \lambda_b d_b}{2\beta_b}$ and $q_{b2}^b = \frac{\alpha_b - c_b - p_w - \lambda_b d_b}{2\beta_b}$, as $r_b \geq p_w$, $q_{b1}^b \leq q_{b2}^b$.

The constrained optimal solutions to Π_b^b are determined by comparing the unconstrained ones with the separation point $\lambda_s q_s^b$, which are presented as follows.

(i) If $q_{b1}^b > \lambda_s q_s^b$, then q_{b1}^b satisfies the constraint and is the constrained optimal solution to Π_{b1}^b, while q_{b2}^b falls to the right of $\lambda_s q_s^b$ and does not subject to the constraint as $q_{b1}^b \leq q_{b2}^b$. Π_{b2}^b increases as q_b^b approaches $\lambda_s q_s^b$ from the left, thus obtaining the maximum value at $q_b^b = \lambda_s q_s^b$. Since $\Pi_{b2}^b(\lambda_s q_s^b) = \Pi_{b1}^b(\lambda_s q_s^b) < \Pi_{b1}^b(q_{b1}^b)$ by the definition of optimality and continuity, the optimal production quantity that maximizes the user's profit is $q_b^{b*}(q_s^b, p_w, \theta) = q_{b1}^b$.

(ii) If $q_{b2}^b < \lambda_s q_s^b$, then q_{b2}^b subjects to the constraint and is the constrained optimal solution to Π_{b2}^b, while q_{b1}^b falls to the left of $\lambda_s q_s^b$ and is not the constrained optimal solution. Π_{b1}^b increases as q_b^b approaches $\lambda_s q_s^b$ from the right, thus obtaining the maximum value at $q_b^b = \lambda_s q_s^b$. Since $\Pi_{b1}^b(\lambda_s q_s^b) = \Pi_{b2}^b(\lambda_s q_s^b) < \Pi_{b2}^b(q_{b2}^b)$ by the definition of optimality and continuity, the the user get the optimal profit at $q_b^{b*}(q_s^b, p_w, \theta) = q_{b2}^b$.

(iii) If $q_{b1}^b \leq \lambda_s q_s^b \leq q_{b2}^b$, then neither the unconstrained optimal solutions satisfy the constraints. Π_{b1}^b increases as q_b^b approaches $\lambda_s q_s^b$ from the right, and Π_{b2}^b increases as q_b^b approaches $\lambda_s q_s^b$ from the left, thus, the constrained optimal solutions to Π_{b1}^b and Π_{b2}^b both fall at the separated point. Therefore the user maximizes his profit at $q_b^{b*}(q_s^b, p_w, \theta) = \lambda_s q_s^b$.

Proof of Lemma 3

Π_s^b is separated by $q_s^b = \frac{q_{b2}^b}{\lambda_s}$ into two parts, and is continuous but non-differentiable at $q_s^b = \frac{q_{b2}^b}{\lambda_s}$, as the left and right derivatives are not equal. It is simple to show that Π_{s1}^b and Π_{s2}^b are both strictly concave in q_s^b, solving the first order conditions give the unconstrained optimal solutions to Π_{s1}^b and Π_{s2}^b, i.e., $q_{s1}^b = \frac{\alpha_s - c_s - r_s + \lambda_s p_w}{2\beta_s}$ and $q_{s2}^b = \frac{\alpha_s - c_s - r_s - \lambda_s d_s}{2\beta_s}$, respectively, since $p_w \geq -d_s$, $q_{s1}^b \geq q_{s2}^b$.

The constrained optimal solutions to Π_s^b are determined by comparing the unconstrained ones with the separation point $\frac{q_{b2}^b}{\lambda_s}$. Considering $q_{s1}^b \geq q_{s2}^b$ and $q_{b1}^b \leq q_{b2}^b$, there are six scenarios to be analyzed.

(i) If $q_{s2}^b \leq q_{s1}^b < \frac{q_{b1}^b}{\lambda_s} \leq \frac{q_{b2}^b}{\lambda_s}$, then q_{s1}^b satisfies the constraint and is the constrained optimal solution to Π_{s1}^b, while q_{s2}^b falls to the left of $\frac{q_{b2}^b}{\lambda_s}$ and is not the constrained optimal solution. Π_{s2}^b increases as q_s^b approaches $\frac{q_{b2}^b}{\lambda_s}$ from the right and reaches the maximum value at $\frac{q_{b2}^b}{\lambda_s}$. Since Π_s^b is a continuous function, $\Pi_{s1}^b(\frac{q_{b2}^b}{\lambda_s}) = \Pi_{s2}^b(\frac{q_{b2}^b}{\lambda_s})$, and according to the definition of optimality, $\Pi_{s1}^b(q_{s1}^b) > \Pi_{s1}^b(\frac{q_{b2}^b}{\lambda_s})$, so the optimal production quantity that maximizes the generator's profit is $q_s^{b*}(p_w, \theta) = q_{s1}^b$. In response, the optimal production quantity of the user is $q_b^{b*}(p_w, \theta) = q_{b1}^b$.

(ii) If $q_{s2}^b < \frac{q_{b1}^b}{\lambda_s} \leq q_{s1}^b \leq \frac{q_{b2}^b}{\lambda_s}$, then q_{s1}^b satisfies the constraint and is the constrained optimal solution to Π_{s1}^b, while q_{s2}^b falls out of the constrained region and increases as q_s^b approaches $\frac{q_{b2}^b}{\lambda_s}$ from the right, thus obtaining the maximum value at $\frac{q_{b2}^b}{\lambda_s}$. Since $\Pi_{s1}^b(\frac{q_{b2}^b}{\lambda_s}) = \Pi_{s2}^b(\frac{q_{b2}^b}{\lambda_s})$ and $\Pi_{s1}^b(q_{s1}^b) \geq \Pi_{s1}^b(\frac{q_{b2}^b}{\lambda_s})$ by the definition of continuity and optimality, the generator maximizes the profit Π_s^b at $q_s^{b*}(p_w, \theta) = q_{s1}^b$. In response, the optimal production quantity of the generator is $q_s^{b*}(p_w, \theta) = \lambda_s q_{s1}^b$.

(iii) If $\frac{q_{b1}^b}{\lambda_s} < q_{s2}^b \le q_{s1}^b < \frac{q_{b2}^b}{\lambda_s}$, then q_{s1}^b is the constrained optimal solution to Π_{s1}^b, while q_{s2}^b does not subject to the constraint. Obviously, this scenario is the same as (ii), as a result, the generator maximizes the profit Π_s^b at $q_s^{b*}(p_w,\theta) = q_{s1}^b$, and the user achieves the maximum profit at $q_b^{b*}(p_w,\theta) = \lambda_s q_{s1}^b$.

(iv) If $q_{s2}^b < \frac{q_{b1}^b}{\lambda_s} \le \frac{q_{b2}^b}{\lambda_s} < q_{s1}^b$, then neither the unconstrained optimal solutions to Π_{s1}^b and Π_{s2}^b satisfy the constraints. Therefore, Π_{s1}^b increases as q_s^b approaches $\frac{q_{b2}^b}{\lambda_s}$ from left and Π_{s2}^b increases as q_s^b approaches $\frac{q_{b2}^b}{\lambda_s}$ from right, and both reach the maximum values at $\frac{q_{b2}^b}{\lambda_s}$. Since Π_s^b is a continuous function, $\Pi_{s1}^b(\frac{q_{b2}^b}{\lambda_s}) = \Pi_{s2}^b(\frac{q_{b2}^b}{\lambda_s})$. Hence, the generator gets the optimal profit at $q_s^{b*}(p_w,\theta) = \frac{q_{b2}^b}{\lambda_s}$. In response, the user achieves the optimal profit at $q_b^{b*}(p_w,\theta) = q_{b2}^b$.

(v) If $\frac{q_{b1}^b}{\lambda_s} \le q_{s2}^b \le \frac{q_{b2}^b}{\lambda_s} < q_{s1}^b$, then neither the unconstrained optimal solutions to Π_{s1}^b and Π_{s2}^b satisfy the constraints. The scenario is same as (iv), consequently, the optimal production quantity of the generator is $q_s^{b*}(p_w,\theta) = \frac{q_{b2}^b}{\lambda_s}$, and the optimal production quantity of the user is $q_b^{b*}(p_w,\theta) = q_{b2}^b$.

(vi) If $\frac{q_{b1}^b}{\lambda_s} \le \frac{q_{b2}^b}{\lambda_s} < q_{s2}^b \le q_{s1}^b$, then q_{s2}^b satisfies the constraint and is the constrained optimal solution to Π_{s2}^b, while q_{s1}^b falls out of the constrained region and reaches the maximum value at $\frac{q_{b2}^b}{\lambda_s}$. Since $\Pi_{s1}^b(\frac{q_{b2}^b}{\lambda_s}) = \Pi_{s2}^b(\frac{q_{b2}^b}{\lambda_s})$ and $\Pi_{s2}^b(q_{s2}^b) > \Pi_{s2}^b(\frac{q_{b2}^b}{\lambda_s})$ by the definition of continuity and optimality, the generator maximizes the profit at $q_s^{b*}(p_w,\theta) = q_{s2}^b$. In response, the user reaches the maximum profit at $q_b^{b*}(p_w,\theta) = q_{b2}^b$.

From the above analysis, we can see there are four pairs solutions to Π_s^b and Π_b^b, i.e., (i) When $0 < q_{s1}^b < \frac{q_{b1}^b}{\lambda_s}$, the optimal production quantities of the generator and user are (q_{s1}^b,q_{b1}^b); (ii) When $\frac{q_{b1}^b}{\lambda_s} \le q_{s1}^b \le \frac{q_{b2}^b}{\lambda_s}$, the optimal production quantities of the generator and user are (q_{s1}^b,$\lambda_s q_{s1}^b$); (iii) When $q_{s2}^b \le \frac{q_{b2}^b}{\lambda_s} < q_{s1}^b$, the optimal production quantities of the generator and user are ($\frac{q_{b2}^b}{\lambda_s}$,$q_{b2}^b$) ; (iv) When $q_{s2}^b > \frac{q_{b2}^b}{\lambda_s}$, the optimal production quantities of the generator and user are (q_{s2}^b,q_{b2}^b). The four pairs solutions are characterized by $p_w < p_{w1}$, $p_{w1} \le p_w \le p_{w2}$, $p_{w2} < p_w \le p_{w3}$, and $p_w > p_{w3}$, where p_{w1}, p_{w2}, and p_{w3} are values of p_w such that $q_{s1}^b = \frac{q_{b1}^b}{\lambda_s}$, $q_{s1}^b = \frac{q_{b2}^b}{\lambda_s}$, and $q_{s2}^b = \frac{q_{b2}^b}{\lambda_s}$, respectively. Specifically, $p_{w1} = \frac{\beta_s(\alpha_b - c_b - r_b - \lambda_b d_b) - \beta_b\lambda_s(\alpha_s - c_s - r_s)}{\beta_b\lambda_s^2}$, $p_{w2} = \frac{\beta_s(\alpha_b - c_b - \lambda_b d_b) - \beta_b\lambda_s(\alpha_s - c_s - r_s)}{\beta_s + \beta_b\lambda_s^2}$, and $p_{w3} = \alpha_b - c_b - \lambda_b d_b - \frac{\beta_b\lambda_s(\alpha_s - c_s - r_s - \lambda_s d_s)}{\beta_s}$, respectively.

Since p_w subjects to $[-d_s, r_b]$, separately comparing p_{w1}, p_{w2}, and p_{w3} with the constraints, we find $p_{w1} \ge -d_s$ (i.e., $\beta_s(\alpha_b - c_b - r_b - \lambda_b d_b) \ge \beta_b\lambda_s(\alpha_s - c_s - r_s - \lambda_s d_s)$) and $p_{w3} \le r_b$ (i.e., $\beta_s(\alpha_b - c_b - r_b - \lambda_b d_b) \le \beta_b\lambda_s(\alpha_s - c_s - r_s - \lambda_s d_s)$) are contradictory. Therefore, if $p_{w1} \in [-d_s, r_b]$, i.e., $\beta_b\lambda_s(\alpha_s - c_s - r_s + \lambda_s r_b) \ge \beta_s(\alpha_b - c_b - r_b - \lambda_b d_b) \ge \beta_b\lambda_s(\alpha_s - c_s - r_s - \lambda_s d_s)$, then $p_{w3} > r_b \ge p_{w2} > p_{w1} \ge -d_s$ hold; if $p_{w3} \in [-d_s, r_b]$, i.e., $\beta_s(\alpha_b - c_b - r_b - \lambda_b d_b) \le \beta_b\lambda_s(\alpha_s - c_s - r_s - \lambda_s d_s) \le \beta_s(\alpha_b - c_b + d_s - \lambda_b d_b)$, then $r_b \ge p_{w3} > p_{w2} \ge -d_s > p_{w1}$ hold. Hence, there exist two scenarios with respect to p_w, i.e., $-d_s \le p_{w1} < p_{w2} \le r_b < p_{w3}$ and $p_{w1} < -d_s \le p_{w2} < p_{w3} \le r_b$, and the optimal production decisions of the generator and user are presented according to the two scenarios.

Proof of corollary 2

(i) When $-d_s \le p_{w1} < p_{w2} \le r_b < p_{w3}$, for $-d_s \le p_w \le p_{w2}$, $\Pi_s^{b*}(p_w,\theta) + \theta F - \Pi_s^{N*} = \frac{(\alpha_s - c_s - r_s + \lambda_s p_w)^2}{4\beta_s} - \frac{(\alpha_s - c_s - r_s - \lambda_s d_s)^2}{4\beta_s}$, since $p_w \ge -d_s$, $\Pi_s^{b*}(p_w,\theta) + \theta F \ge \Pi_s^{N*}$. For $p_{w2} < p_w \le r_b$, $\Pi_s^{b*}(p_w,\theta) + \theta F - \Pi_s^{N*} = \frac{(\alpha_b - c_b - p_w - \lambda_b d_b)[2\beta_b\lambda_s(\alpha_s - c_s - r_s + \lambda_s p_w) - \beta_s(\alpha_b - c_b - p_w - \lambda_b d_b)]}{4\beta_b^2\lambda_s^2} - \frac{(\alpha_s - c_s - r_s - \lambda_s d_s)^2}{4\beta_s} > \beta_b\lambda_s(\alpha_s - c_s - r_s - \lambda_s d_s) * \frac{[\beta_s(\alpha_b - c_b - p_w - \lambda_b d_b) - \beta_b\lambda_s(\alpha_s - c_s - r_s - \lambda_s d_s)]}{4\beta_s\beta_b^2\lambda_s^2} + \frac{(\alpha_b - c_b - p_w - \lambda_b d_b)[\beta_b\lambda_s(\alpha_s - c_s - r_s + \lambda_s p_w) - \beta_s(\alpha_b - c_b - p_w - \lambda_b d_b)]}{4\beta_b^2\lambda_s^2}$, since $q_{s1}^b > \frac{q_{b2}^b}{\lambda_s} > q_{s2}^b$, $\Pi_s^{b*}(p_w,\theta) + \theta F > \Pi_s^{N*}$. In summarize, $\Pi_s^{b*}(p_w,\theta) + \theta F - \Pi_s^{N*} \ge 0$ regardless of p_w. For $-d_s \le p_w < p_{w1}$, $\Pi_b^{b*}(p_w,\theta) + (1-\theta)F - \Pi_b^{N*} = \frac{\lambda_s(r_b - p_w)(\alpha_s - c_s - r_s + \lambda_s p_w)}{2\beta_s}$, as $r_b \ge p_w$, $\Pi_b^{b*}(p_w,\theta) + (1-\theta)F \ge \Pi_b^{N*}$. For $p_{w1} \le p_w \le p_{w2}$, $\Pi_b^{b*}(p_w,\theta) + (1-\theta)F - \Pi_b^{N*} = \frac{\lambda_s(\alpha_s - c_s - r_s + \lambda_s p_w)[2\beta_s(\alpha_b - c_b - p_w - \lambda_b d_b) - \beta_b\lambda_s(\alpha_s - c_s - r_s + \lambda_s p_w)]}{4\beta_s^2} - \frac{(\alpha_b - c_b - r_b - \lambda_b d_b)^2}{4\beta_b} > \beta_s(\alpha_b - c_b - r_b - \lambda_b d_b) * \frac{[\beta_b\lambda_s(\alpha_s - c_s - r_s + \lambda_s p_w) - \beta_s(\alpha_b - c_b - r_b - \lambda_b d_b)]}{4\beta_b\beta_s^2} + \frac{\lambda_s(\alpha_s - c_s - r_s + \lambda_s p_w)[\beta_s(\alpha_b - c_b - p_w - \lambda_b d_b) - \beta_b\lambda_s(\alpha_s - c_s - r_s + \lambda_s p_w)]}{4\beta_s^2}$, as $\frac{q_{b1}^b}{\lambda_s} \le$

$q_{s1}^{b} \leq \frac{q_{b2}^{b}}{\lambda_s}$, $\Pi_b^{b*}(p_w,\theta)+(1-\theta)F > \Pi_b^{N*}$. For $p_{w2} < p_w \leq r_b$, $\Pi_b^{b*}(p_w,\theta)+(1-\theta)F-\Pi_b^{N*} = \frac{(\alpha_b-c_b-p_w-\lambda_b d_b)^2}{4\beta_b} - \frac{(\alpha_b-c_b-r_b-\lambda_b d_b)^2}{4\beta_b}$, since $r_b \geq p_w$, $\Pi_b^{b*}(p_w,\theta)+(1-\theta)F \geq \Pi_b^{N*}$. Therefore, $\Pi_b^{b*}(p_w,\theta)+\theta F \geq \Pi_b^{N*}$ regardless of p_w.

(ii) When $p_{w1} < -d_s \leq p_{w2} < p_{w3} \leq r_b$, for $-d_s \leq p_w \leq p_{w2}$, we can see from above that $\Pi_s^{b*}(p_w,\theta)+\theta F \geq \Pi_s^{N*}$ and $\Pi_b^{b*}(p_w,\theta)+(1-\theta)F > \Pi_b^{N*}$. For $p_{w2} < p_w \leq p_{w3}$, similar to the results described above, $\Pi_s^{b*}(p_w,\theta)+\theta F > \Pi_s^{N*}$ and $\Pi_b^{b*}(p_w,\theta)+(1-\theta)F \geq \Pi_b^{N*}$ hold. For $p_{w3} < p_w \leq r_b$, $\Pi_s^{b*}(p_w,\theta)+\theta F-\Pi_s^{N*} = \frac{(p_w+d_s)(\alpha_b-c_b-p_w-\lambda_b d_b)}{2\beta_b} > 0$, so $\Pi_s^{b*}(p_w,\theta)+\theta F > \Pi_s^{N*}$. $\Pi_b^{b*}(p_w,\theta)+(1-\theta)F-\Pi_b^{N*} = \frac{(\alpha_b-c_b-p_w-\lambda_b d_b)^2}{4\beta_b} - \frac{(\alpha_b-c_b-r_b-\lambda_b d_b)^2}{4\beta_b}$, since $r_b \geq p_w$, $\Pi_b^{b*}(p_w,\theta)+(1-\theta)F \geq \Pi_b^{N*}$. Therefore, $\Pi_s^{b*}(p_w,\theta)+\theta F \geq \Pi_s^{N*}$ and $\Pi_b^{b*}(p_w,\theta)+\theta F \geq \Pi_s^{N*}$ regardless of p_w.

Proof of Theorem 2

When $-d_s \leq p_{w1} < p_{w2} \leq r_b < p_{w3}$, for $-d_s \leq p_w \leq p_{w2}$, $\frac{\partial q_s^{b*}(p_w,\theta)}{\partial p_w} = \frac{\lambda_s}{2\beta_s} > 0$, for $p_{w2} < p_w \leq r_b$, $\frac{\partial q_s^{b*}(p_w,\theta)}{\partial p_w} = -\frac{1}{2\beta_b\lambda_s} < 0$. For $-d_s \leq p_w < p_{w1}$, $\frac{\partial q_b^{b*}(p_w,\theta)}{\partial p_w} = 0$, for $p_{w1} \leq p_w \leq p_{w2}$, $\frac{\partial q_b^{b*}(p_w,\theta)}{\partial p_w} = \frac{\lambda_s^2}{2\beta_s} > 0$, and for $p_{w2} < p_w \leq r_b$, $\frac{\partial q_b^{b*}(p_w,\theta)}{\partial p_w} = \frac{-1}{2\beta_b} < 0$.

When $p_{w1} < -d_s \leq p_{w2} < p_{w3} \leq r_b$, for $-d_s \leq p_w \leq p_{w2}$, $\frac{\partial q_s^{b*}(p_w,\theta)}{\partial p_w} = \frac{\lambda_s}{2\beta_s} > 0$, for $p_{w2} < p_w \leq p_{w3}$, $\frac{\partial q_s^{b*}(p_w,\theta)}{\partial p_w} = -\frac{1}{2\beta_b\lambda_s} < 0$, for $p_{w3} < p_w \leq r_b$, $\frac{\partial q_s^{b*}(p_w,\theta)}{\partial p_w} = 0$. For $-d_s \leq p_w \leq p_{w2}$, $\frac{\partial q_b^{b*}(p_w,\theta)}{\partial p_w} = \frac{\lambda_s^2}{2\beta_s} > 0$, for $p_{w2} < p_w \leq r_b$, $\frac{\partial q_b^{b*}(p_w,\theta)}{\partial p_w} = \frac{-1}{2\beta_b} < 0$.

Proof of Theorem 3

When $-d_s \leq p_{w1} < p_{w2} \leq r_b < p_{w3}$, $\Delta\Pi_T^b$ is a continuous function of p_w, since values of $\Delta\Pi_T^b$ at the separated points $p_w = p_{w1}$ and $p_w = p_{w2}$ are equal, respectively. For $-d_s \leq p_w < p_{w1}$, $\frac{\partial\Delta\Pi_T^b}{\partial p_w} = \frac{\lambda_s^2(r_b-p_w)}{2\beta_s} > 0$, $\frac{\partial^2\Delta\Pi_T^b}{\partial p_w^2} = \frac{-\lambda_s^2}{2\beta_s} < 0$, $\frac{\partial\Delta\Pi_T^b}{\partial r_b} = \frac{\lambda_s(\alpha_s-c_s-r_s+\lambda_s p_w)}{2\beta_s} > 0$, and $\frac{\partial\Delta\Pi_T^b}{\partial d_s} = \frac{\lambda_s(\alpha_s-c_s-r_s-\lambda_s d_s)}{2\beta_s} > 0$, so $\Delta\Pi_T^b$ is an increasingly concave function of p_w and an increasing function of r_b and d_s, respectively. For $p_{w1} \leq p_w \leq p_{w2}$, $\frac{\partial\Delta\Pi_T^b}{\partial p_w} = \frac{\lambda_s^2(\alpha_b-c_b-p_w-\lambda_b d_b)}{2\beta_s} - \frac{\beta_b\lambda_s^3(\alpha_s-c_s-r_s+\lambda_s p_w)}{2\beta_s^2} > 0$, $\frac{\partial^2\Delta\Pi_T^b}{\partial p_w^2} = \frac{-\lambda_s^2(\beta_s+\beta_b\lambda_s^2)}{2\beta_s} < 0$, $\frac{\partial\Delta\Pi_T^b}{\partial r_b} = \frac{\alpha_b-c_b-r_b-\lambda_b d_b}{2\beta_b} > 0$, and $\frac{\partial\Delta\Pi_T^b}{\partial d_s} = \frac{\lambda_s(\alpha_s-c_s-r_s-\lambda_s d_s)}{2\beta_s} > 0$, so $\Delta\Pi_T^b$ is an increasingly concave function of p_w and an increasing function of r_b and d_s, respectively. For $p_{w2} < p_w \leq r_b$, $\frac{\partial\Delta\Pi_T^b}{\partial p_w} = \frac{\beta_s(\alpha_b-c_b-p_w-\lambda_b d_b)-\lambda_s\beta_b(\alpha_s-c_s-r_s+\lambda_s p_w)}{2\lambda_s^2\beta_b^2} < 0$, $\frac{\partial^2\Delta\Pi_T^b}{\partial p_w^2} = \frac{-(\beta_s+\beta_b\lambda_s^2)}{2\lambda_s^2\beta_b^2} < 0$, $\frac{\partial\Delta\Pi_T^b}{\partial r_b} = \frac{\alpha_b-c_b-r_b-\lambda_b d_b}{2\beta_b} > 0$, and $\frac{\partial\Delta\Pi_T^b}{\partial d_s} = \frac{\lambda_s(\alpha_s-c_s-r_s-\lambda_s d_s)}{2\beta_s} > 0$, so $\Delta\Pi_T^b$ is a decreasingly concave function of p_w and an increasing function of r_b and d_s, respectively.

When $p_{w1} < -d_s \leq p_{w2} < p_{w3} \leq r_b$, $\Delta\Pi_T^b$ is a continuous function of p_w since values of $\Delta\Pi_T^b$ at the separated points $p_w = p_{w2}$ and at $p_w = p_{w3}$ are equal, respectively. For $-d_s \leq p_w \leq p_{w2}$, from above, we can see that $\Delta\Pi_T^b$ is an increasingly concave function of p_w, r_b and d_s, respectively. For $p_{w2} < p_w \leq p_{w3}$, form above, we can see that $\Delta\Pi_T^b$ is a decreasingly concave function of p_w and an increasingly concave function of r_b and d_s, respectively. For $p_{w3} < p_w \leq r_b$, $\frac{\partial\Delta\Pi_T^b}{\partial p_w} = \frac{-(p_w+d_s)}{2\beta_b} < 0$, $\frac{\partial^2\Delta\Pi_T^b}{\partial p_w^2} = \frac{-1}{2\beta_b} < 0$, $\frac{\partial\Delta\Pi_T^b}{\partial r_b} = \frac{\alpha_b-c_b-r_b-\lambda_b d_b}{2\beta_b} > 0$, and $\frac{\partial\Delta\Pi_T^b}{\partial d_s} = \frac{\alpha_b-c_b-p_w-\lambda_b d_b}{2\beta_b} > 0$, so $\Delta\Pi_T^b$ is a decreasingly concave function of p_w and an increasing function of r_b and d_s, respectively.

Proof of theorem 4

When $-d_s \leq p_{w1} < p_{w2} \leq r_b < p_{w3}$, for $-d_s \leq p_w < p_{w1}$, $\frac{\partial\Delta E_T^b}{\partial p_w} = \frac{-[e_r+e_p(1-\delta)]\lambda_s^2}{2\beta_s} < 0$, so ΔE_T^b decreases with p_w, and get the minimum value at $p_w = p_{w1}$. For $p_{w1} \leq p_w \leq p_{w2}$, $\frac{\partial\Delta E_T^b}{\partial p_w} = \frac{\delta e_p\lambda_s^2}{2\beta_s} > 0$, so ΔE_T^b increases with p_w, and get the minimum value at $p_w = p_{w1}$. For $p_{w2} < p_w \leq r_b$, $\frac{\partial\Delta E_T^b}{\partial p_w} = \frac{-\delta e_p}{2\beta_b} < 0$, so ΔE_T^b decreases with p_w, and get the minimum value when $p_w = r_b$.

When $p_{w1} < -d_s \leq p_{w2} < p_{w3} \leq r_b$, we can see form above that for $-d_s \leq p_w \leq p_{w2}$, ΔE_T^b increases with p_w, for $p_{w2} < p_w \leq p_{w3}$,

ΔE_T^b decreases with p_w. For $p_{w3} < p_w \le r_b$, $\frac{\partial \Delta E_T^b}{\partial p_w} = \frac{e_d - \delta e_p}{2\beta_b}$, if $e_d - \delta e_p < 0$, then ΔE_T^b decreases with p_w, and get the minimum value when $p_w = r_b$, otherwise, ΔE_T^b increases with p_w, and get the minimum value when $p_w = p_{w3}$.

Proof of theorem 5

When $-d_s \le p_{w1} < p_{w2} \le r_b \le p_{w3}$, for $-d_s \le p_w < p_{w1}$, $\frac{\partial \Delta E_T^b}{\partial d_s} = \frac{e_d \lambda_s^2}{2\beta_s} > 0$, $\frac{\partial \Delta E_T^b}{\partial r_b} = 0$, for $p_{w1} \le p_w \le p_{w2}$, $\frac{\partial \Delta E_T^b}{\partial d_s} = \frac{e_d \lambda_s^2}{2\beta_s} > 0$, $\frac{\partial \Delta E_T^b}{\partial r_b} = \frac{e_r + e_p}{2\beta_b} > 0$, for $p_{w2} < p_w \le r_b$, $\frac{\partial \Delta E_T^b}{\partial d_s} = \frac{e_d \lambda_s^2}{2\beta_s} > 0$, $\frac{\partial \Delta E_T^b}{\partial r_b} = \frac{e_r + e_p}{2\beta_b} > 0$.

When $p_{w1} < -d_s \le p_{w2} < p_{w3} \le r_b$, for $-d_s \le p_w \le p_{w2}$, $\frac{\partial \Delta E_T^b}{\partial d_s} = \frac{e_d \lambda_s^2}{2\beta_s} > 0$, $\frac{\partial \Delta E_T^b}{\partial r_b} = \frac{e_r + e_p}{2\beta_b} > 0$, for $p_{w2} < p_w \le p_{w3}$, $\frac{\partial \Delta E_T^b}{\partial d_s} = \frac{e_d \lambda_s^2}{2\beta_s} > 0$, $\frac{\partial \Delta E_T^b}{\partial r_b} = \frac{e_r + e_p}{2\beta_b} > 0$, for $p_{w3} < p_w \le r_b$, $\frac{\partial \Delta E_T^b}{\partial d_s} = 0$, $\frac{\partial \Delta E_T^b}{\partial r_b} = \frac{e_r + e_p}{2\beta_b} > 0$.

Proof of theorem 6

When $-d_s \le p_{w1} < p_{w2} \le r_b < p_{w3}$, from theorem 3, we see that $\Delta \Pi_T^b$ is an increasing concave function for $p_w \in [-d_s, p_{w2}]$, and a decreasing concave function for $p_w \in (p_{w2}, r_b]$. Therefore, $\Delta \Pi_T^b$ achieves the maximum value at $p_w = p_{w2}$, and the minimum value at either $p_w = -d_s$ or $p_w = r_b$. Since $\Delta \Pi_T^b(r_b) = \Delta \Pi_T^b(p_{w1}) > \Delta \Pi_T^b(-d_s)$, so $\Delta \Pi_T^b$ obtains the minimum value at $p_w = -d_s$.

When $p_{w1} < -d_s \le p_{w2} < p_{w3} \le r_b$, $\Delta \Pi_T^b$ is an increasing concave function for $p_w \in [-d_s, p_{w2}]$, and a decreasing concave function for $p_w \in (p_{w2}, r_b]$. Therefore, $\Delta \Pi_T^b$ achieves the maximum value at $p_w = p_{w2}$, and the minimum value at either $p_w = -d_s$ or $p_w = r_b$. Since $\Delta \Pi_T^b(-d_s) = \Delta \Pi_T^b(p_{w3}) > \Delta \Pi_T^b(r_b)$, so $\Delta \Pi_T^b$ obtains the minimum value at $p_w = r_b$.

Proof of Lemma 4

Considering the complexity of the Nash product in our model, we calculate the waste trading price p_w and share of the fixed investment cost θ by maximizing the logarithm of $G(p_w, \theta)$. Let $\ln G(p_w, \theta) = \ln G_1$ when $-d_s \le p_w < p_{w1}$, $\ln G(p_w, \theta) = \ln G_2$ when $p_{w1} \le p_w \le p_{w2}$ or $-d_s \le p_w \le p_{w2}$, $\ln G(p_w, \theta) = \ln G_3$ when $p_{w2} < p_w \le r_b$ or $p_{w2} < p_w \le p_{w3}$, and $\ln G(p_w, \theta) = \ln G_4$ when $p_{w3} < p_w \le r_b$.

To ensure that $\Pi_s^{b*}(p_w, \theta) - \Pi_s^N > 0$ and $\Pi_b^{b*}(p_w, \theta) - \Pi_b^N > 0$, we assume $(\alpha_s - c_s - r_s + \lambda_s p_w)^2 - (\alpha_s - c_s - r_s - \lambda_s d_s)^2 + 2\lambda_s(r_{b-p_w})(\alpha_s - c_s - r_s + \lambda_s p_w) - 4\beta_s F > 0$. Taking the first-order and second-order partial derivatives of $\ln G_1$ with regard to p_w and θ, we have

$$\frac{\partial \ln G_1}{\partial p_w} = \frac{2\xi_s \lambda_s(\alpha_s - c_s - r_s + \lambda_s p_w)}{(\alpha_s - c_s - r_s + \lambda_s p_w)^2 - (\alpha_s - c_s - r_s - \lambda_s d_s)^2 - 4\beta_s \theta F} + \frac{\xi_b[\lambda_s^2(r_b - p_w) - \lambda_s(\alpha_s - c_s - r_s + \lambda_s p_w)]}{\lambda_s(r_b - p_w)(\alpha_s - c_s - r_s + \lambda_s p_w) - 2\beta_s(1-\theta)F},$$

$$\frac{\partial^2 \ln G_1}{\partial p_w^2} = \frac{-2\xi_s \lambda_s^2[(\alpha_s - c_s - r_s + \lambda_s p_w)^2 + (\alpha_s - c_s - r_s - \lambda_s d_s)^2 + 4\beta_s \theta F]}{[(\alpha_s - c_s - r_s + \lambda_s p_w)^2 - (\alpha_s - c_s - r_s - \lambda_s d_s)^2 - 4\beta_s \theta F]^2} - \frac{2\xi_b \lambda_s^2}{\lambda_s(r_b - p_w)(\alpha_s - c_s - r_s + \lambda_s p_w) - 2\beta_s(1-\theta)F} - \frac{\xi_b \lambda_s^2[\lambda_s(r_b - p_w) - (\alpha_s - c_s - r_s + \lambda_s p_w)]^2}{[\lambda_s(r_b - p_w)(\alpha_s - c_s - r_s + \lambda_s p_w) - 2\beta_s(1-\theta)F]^2},$$

$$\frac{\partial \ln G_1}{\partial \theta} = \frac{-4\xi_s \beta_s F}{(\alpha_s - c_s - r_s + \lambda_s p_w)^2 - (\alpha_s - c_s - r_s - \lambda_s d_s)^2 - 4\beta_s \theta F} + \frac{2\xi_b \beta_s F}{\lambda_s(r_b - p_w)(\alpha_s - c_s - r_s + \lambda_s p_w) - 2\beta_s(1-\theta)F},$$

$$\frac{\partial^2 \ln G_1}{\partial \theta^2} = \frac{-16\xi_s\beta_s^2F^2}{\left[(\alpha_s-c_s-r_s+\lambda_s p_w)^2-(\alpha_s-c_s-r_s-\lambda_s d_s)^2-4\beta_s\theta F\right]^2} - \frac{4\xi_b\beta_s^2F^2}{[\lambda_s(r_b-p_w)(\alpha_s-c_s-r_s+\lambda_s p_w)-2\beta_s(1-\theta)F]^2},$$

$$\frac{\partial^2 \ln G_1}{\partial \theta \partial p_w} = \frac{8\xi_s\lambda_s\beta_s F(\alpha_s-c_s-r_s+\lambda_s p_w)}{[(\alpha_s-c_s-r_s+\lambda_s p_w)^2-(\alpha_s-c_s-r_s-\lambda_s d_s)^2-4\beta_s\theta F]^2} - \frac{2\xi_b\lambda_s\beta_s F[\lambda_s(r_b-p_w)-(\alpha_s-c_s-r_s+\lambda_s p_w)]}{[\lambda_s(r_b-p_w)(\alpha_s-c_s-r_s+\lambda_s p_w)-2\beta_s(1-\theta)F]^2}.$$

Let $A=\alpha_s-c_s-r_s+\lambda_s p_w$, $B=\alpha_s-c_s-r_s-\lambda_s d_s$, $C=\alpha_b-c_b-p_w-\lambda_b d_b$, $D=\alpha_b-c_b-r_b-\lambda_b d_b$.

It is obvious $\frac{\partial^2 \ln G_1}{\partial p_w^2}<0$ and $\frac{\partial^2 \ln G_1}{\partial \theta^2}<0$, then $\ln G_1$ is a strictly concave function both in p_w for a given θ, and in θ for a given p_w. Therefore, we can first calculate the optimal θ for a given p_w and then substitute θ, which is a function of p_w, into $\ln G_1$ to search the optimal p_w. Solving $\frac{\partial \ln G_1}{\partial \theta}=0$, we can get $\theta_1(p_w)=\frac{\xi_b(A^2-B^2)-2\xi_s[\lambda_s(r_b-p_w)A-2\beta_s F]}{4\beta_s F(\xi_s+\xi_b)}$.

Substituting $\theta_1(p_w)$ into $\frac{d\ln G_1}{dp_w}$, we can get $\frac{d\ln G_1}{dp_w}=\frac{2\lambda_s^2(\xi_s+\xi_b)(r_b-p_w)}{A^2-B^2+2[\lambda_s(r_b-p_w)A-2\beta_s F]}$, then $\frac{d\ln G_1}{dp_w}>0$ is always true according to the assumption. Therefore, $\ln G_1$, constrained by $-d_s\le p_w<p_{w1}$, is an increasing function in p_w, and reaches the maximum value at $p_w^*=p_{w1}$, correspondingly, the optimal share of cost is $\theta^*=\theta_1(p_{w1})$.

Taking the first-order and second-order partial derivatives of $\ln G_2$ with regard to p_w and θ, we can see that $\ln G_2$ is a strictly concave function both in p_w for a given θ, and in θ for a given p_w, because $\frac{\partial^2 \ln G_2}{\partial p_w^2}<0$ and $\frac{\partial^2 \ln G_2}{\partial \theta^2}<0$. Therefore, just like above, we can first calculate the optimal θ for a given p_w and then substitute θ, which is a function of p_w, into $\ln G_2$ to search the optimal p_w. Solving $\frac{\partial \ln G_2}{\partial \theta}=0$, we can get $\theta_2(p_w)=\frac{\xi_b\beta_s\beta_b(A^2-B^2)-\xi_s[2\lambda_s\beta_s\beta_b AC-\lambda_s^2\beta_b^2A^2-\beta_s^2D^2-4\beta_b\beta_s^2F]}{4\beta_b\beta_s^2F(\xi_s+\xi_b)}$.

Substituting $\theta_2(p_w)$ into $\frac{d\ln G_2}{dp_w}$, we can get $\frac{d\ln G_2}{dp_w}=\frac{2\beta_b(\xi_s+\xi_b)\lambda_s^2(\beta_s C-\lambda_s\beta_b A)}{\beta_s\beta_b(A^2-B^2)+2\lambda_s\beta_s\beta_b AC-\lambda_s^2\beta_b^2A^2-\beta_s^2D^2-4\beta_b\beta_s^2F}$, then $\frac{d\ln G_2}{dp_w}>0$ is constantly true, since $\mathit{\Pi}_s^{b*}(p_w,\theta)>\mathit{\Pi}_s^{N*}$ and $\mathit{\Pi}_b^{b*}(p_w,\theta)>\mathit{\Pi}_b^{N*}$ hold, and $\beta_s C>\lambda_s\beta_b A$ when $p_{w1}\le p_w\le p_{w2}$. Therefore, $\ln G_2$ increases with p_w, and reaches the maximum value at $p_w^*=p_{w2}$, at which $\theta^*=\theta_2(p_{w2})$.

Taking the first-order and second-order partial derivatives of $\ln G_3$ with regard to p_w and θ, we can see that $\ln G_3$ is a strictly concave function both in p_w for a given θ, and in θ for a given p_w, since $\frac{\partial^2 \ln G_3}{\partial p_w^2}<0$ and $\frac{\partial^2 \ln G_3}{\partial \theta^2}<0$. Therefore, we can first calculate the optimal θ for a given p_w and then substitute θ, which is a function of p_w, into $\ln G_3$ to search the optimal p_w. Solving $\frac{\partial \ln G_3}{\partial \theta}=0$, we can get $\theta_3(p_w)=\frac{\xi_b(2\lambda_s\beta_s\beta_b AC-\beta_s^2C^2-\lambda_s^2\beta_b^2D^2)-\xi_s\beta_s\beta_b\lambda_s^2(C^2-D^2-4\beta_b F)}{4\beta_s\beta_b^2\lambda_s^2(\xi_s+\xi_b)F}$.

Substituting $\theta_3(p_w)$ into $\frac{d\ln G_3}{dp_w}$, we can get $\frac{d\ln G_3}{dp_w}=\frac{2\beta_s(\xi_s+\xi_b)(\beta_s C-\lambda_s\beta_b A)}{2\lambda_s\beta_s\beta_b AC-\beta_s^2C^2-\lambda_s^2\beta_b^2B^2+\beta_s\beta_b\lambda_s^2(C^2-D^2-4\beta_b F)}$, then $\frac{d\ln G_3}{dp_w}<0$ is constantly true, as $\mathit{\Pi}_s^{b*}(p_w,\theta)>\mathit{\Pi}_s^{N*}$ and $\mathit{\Pi}_b^{b*}(p_w,\theta)>\mathit{\Pi}_b^{N*}$ hold, and $\beta_s C<\lambda_s\beta_b A$ when $p_{w2}<p_w\le r_b$. Therefore, $\ln G_3$ decreases with p_w, and reaches the maximum value at $p_w^*=p_{w2}$, correspondingly, the optimal share of cost is $\theta^*=\theta_3(p_{w2})$.

Taking the first-order and second-order partial derivatives of $\ln G_4$ with regard to p_w and θ, we can see that $\ln G_4$ is a strictly concave function both in p_w for a given θ, and in θ for a given p_w, since $\frac{\partial^2 \ln G_4}{\partial p_w^2}<0$ and $\frac{\partial^2 \ln G_4}{\partial \theta^2}<0$. Therefore, we can first calculate the optimal θ for a given p_w and then substitute θ, which is a function of p_w, into $\ln G_4$ to search the optimal p_w. Solving $\frac{\partial \ln G_4}{\partial \theta}=0$, we can get $\theta_4(p_w)=\frac{2\xi_b(p_w+d_s)C-\xi_s(C^2-D^2-4\beta_b F)}{4\beta_b(\xi_s+\xi_b)F}$.

Substituting $\theta_4(p_w)$ into $\frac{d\ln G_4}{dp_w}$, we can get $\frac{d\ln G_4}{dp_w}=\frac{-2(\xi_b+\beta_s\xi_s)(p_w+d_s)}{2(p_w+d_s)C+C^2-D^2-4\beta_b F}$, then $\frac{d\ln G_4}{dp_w}<0$ is constantly true according to the assumption. Therefore, $\ln G_4$, constrained by $p_{w3}<p_w\le r_b$, is a decreasing function of p_w, and reaches the maximum value at $p_w^*=p_{w3}$, at which $\theta^*=\theta_4(p_{w3})$.

Proof of theorem 7

As p_w is independent of ξ_s and ξ_b, we investigate the effects of ξ_s and ξ_b on θ by using $\theta(p_w)$. When $-d_s \le p_{w1} < p_{w2} \le r_b < p_{w3}$, for $-d_s \le p_w < p_{w1}$, $\frac{\partial\theta_1(p_w)}{\partial\xi_s} = \frac{-\xi_b[A^2-B^2+2\lambda_s(r_b-p_w)A-4\beta_s F]}{4\beta_s F(\xi_s+\xi_b)^2} < 0$ and $\frac{\partial\theta_1(p_w)}{\partial\xi_b} = \frac{\xi_s[A^2-B^2+2\lambda_s(r_b-p_w)A-4\beta_s F]}{4\beta_s F(\xi_s+\xi_b)^2} > 0$ according to $\Pi_s^{b*}(p_w,\theta) > \Pi_s^N$ and $\Pi_b^{b*}(p_w,\theta) > \Pi_b^N$. For $p_{w1} \le p_w \le p_{w2}$, $\frac{\partial\theta_2(p_w)}{\partial\xi_s} = \frac{-\xi_b[\beta_s\beta_b(A^2-B^2)+2\beta_s\beta_b\lambda_s AC-\beta_b^2\lambda_s^2A^2-\beta_s^2D^2-4\beta_s^2\beta_b F]}{4\beta_s^2\beta_b F(\xi_s+\xi_b)^2} < 0$ and $\frac{\partial\theta_2(p_w)}{\partial\xi_b} = -\frac{\xi_s}{\xi_b} * \frac{\partial\theta_2(p_w)}{\partial\xi_s} > 0$ according to $\Pi_s^{b*}(p_w,\theta) > \Pi_s^N$ and $\Pi_b^{b*}(p_w,\theta) > \Pi_b^N$. For $p_{w2} < p_w \le r_b$, $\frac{\partial\theta_3(p_w)}{\partial\xi_b} = -\frac{\xi_s}{\xi_b} * \frac{\partial\theta_3(p_w)}{\partial\xi_s} > 0$ and $\frac{\partial\theta_3(p_w)}{\partial\xi_s} = \frac{-\xi_b[\beta_s\beta_b\lambda_s^2(C^2-D^2-4\beta_b F)+2\beta_s\beta_b\lambda_s AC-\beta_s^2C^2-\beta_b^2\lambda_s^2D^2]}{4\beta_s\beta_b^2\lambda_s^2 F(\xi_s+\xi_b)^2} < 0$ according to $\Pi_s^{b*}(p_w,\theta) > \Pi_s^N$ and $\Pi_b^{b*}(p_w,\theta) > \Pi_b^N$.

When $p_{w1} < -d_s \le p_{w2} < p_{w3} \le r_b$, for $-d_s \le p_w \le p_{w2}$ and $p_{w2} < p_w \le p_{w3}$, $\theta_2(p_w)$ and $\theta_3(p_w)$ change with ξ_s and ξ_b in the same direction as in the case $-d_s \le p_{w1} < p_{w2} \le r_b < p_{w3}$. For $p_{w3} < p_w \le r_b$, $\frac{\partial\theta_4(p_w)}{\partial\xi_s} = \frac{-\xi_b[C^2-D^2-4\beta_b F+2(p_w+d_s)]}{4\beta_b F(\xi_s+\xi_b)^2} < 0$ and $\frac{\partial\theta_4(p_w)}{\partial\xi_b} = -\frac{\xi_s}{\xi_b} * \frac{\partial\theta_4(p_w)}{\partial\xi_s} > 0$ according to $\Pi_s^{b*}(p_w,\theta) > \Pi_s^N$ and $\Pi_b^{b*}(p_w,\theta) > \Pi_b^N$. Since the optimal profits of the generator and user decrease and increase with θ, respectively, it is easy to get the effect of ξ_s and ξ_b on Π_s^{b*} and Π_b^{b*}.

Proof of theorem 8

When $-d_s \le p_{w1} < p_{w2} \le r_b < p_{w3}$, from theorem 3, 4, we see that $E_T^{b*}(p_w,\theta)$ decreases and $\Pi_T^{b*}(p_w,\theta)$ increases with p_w for $-d_s \le p_w < p_{w1}$, thus the optimal economic and environmental performance are simultaneously achieved at $p_w = p_{w1}$, and the two goals align. For $p_{w1} \le p_w \le r_b$, $E_T^{b*}(p_w,\theta)$ and $\Pi_T^{b*}(p_w,\theta)$ both first increase then decrease with p_w, hence, the optimal economic performance and worst environmental performance are both achieved at $p_w = p_{w2}$, and there exist conflicts between the two goals.

When $p_{w1} < -d_s \le p_{w2} < p_{w3} \le r_b$, from theorem 3, 4, we see that for $p_{w1} \le p_w \le p_{w3}$, both $E_T^{b*}(p_w,\theta)$ and $\Pi_T^{b*}(p_w,\theta)$ first increase then decrease with p_w, thus the optimal economic and environmental goals can not be achieved simultaneously. For $p_{w3} < p_w \le r_b$, $\Pi_T^{b*}(p_w,\theta)$ decreases with p_w while $E_T^{b*}(p_w,\theta)$ increases with p_w when $e_d > \delta e_p$, then the optimal economic and environmental performance align if $e_d > \delta e_p$, otherwise, the two goals conflict.

Proof of theorem 9

When $-d_s \le p_{w1} < p_{w2} \le r_b < p_{w3}$, for $-d_s \le p_w < p_{w1}$, the optimal waste trading price is $p_w = p_{w1}$, $E_T^{b*}(p_{w1}) - E_T^{N*} = \frac{(\delta-1)e_p(\alpha_b-c_b-r_b-\lambda_b d_b)}{2\beta_b} - \frac{\lambda_s e_d(\alpha_s-c_s-r_s-\lambda_s d_s)}{2\beta_s} - \frac{e_r(\alpha_b-c_b-r_b-\lambda_b d_b)}{2\beta_b} < 0$, as $0 < \delta < 1$, therefore, the interfirm waste utilization is more environmentally preferable than the benchmark case. For $p_{w1} \le p_w \le p_{w2}$ and $p_{w2} < p_w \le r_b$, the optimal waste trading price is $p_w = p_{w2}$, $E_T^{b*}(p_{w2}) - E_T^{N*} = \frac{e_p[\delta\lambda_s[\alpha_s-c_s-r_s+\lambda_s(\alpha_b-c_b-\lambda_b d_b)]-\beta_s(\beta_s+\beta_b\lambda_s^2)(\alpha_b-c_b-r_b-\lambda_b d_b)]}{2\beta_s\beta_b(\beta_s+\beta_b\lambda_s^2)} - \frac{e_r(\alpha_b-c_b-r_b-\lambda_b d_b)}{2\beta_b} - \frac{\lambda_s e_d(\alpha_s-c_s-r_s-\lambda_s d_s)}{2\beta_s}$, if $\delta < \frac{\beta_s(\beta_s+\beta_b\lambda_s^2)(\alpha_b-c_b-r_b-\lambda_b d_b)}{\lambda_s[\alpha_s-c_s-r_s+\lambda_s(\alpha_b-c_b-\lambda_b d_b)]}$, then $E_T^{b*}(p_{w2}) < E_T^{N*}$, otherwise, the interfirm waste utilization is environmentally superior to the benchmark case if $e_p < \frac{(\beta_s+\beta_b\lambda_s^2)[\beta_b\lambda_s e_d(\alpha_s-c_s-r_s-\lambda_s d_s)+\beta_s e_r(\alpha_b-c_b-r_b-\lambda_b d_b)]}{\delta\lambda_s[\alpha_s-c_s-r_s+\lambda_s(\alpha_b-c_b-\lambda_b d_b)]-\beta_s(\beta_s+\beta_b\lambda_s^2)(\alpha_b-c_b-r_b-\lambda_b d_b)} = \widetilde{\Omega}(e_d,e_r)$.

When $p_{w1} < -d_s \le p_{w2} < p_{w3} \le r_b$, from above we can see that for $p_{w1} \le p_w \le p_{w2}$ and $p_{w2} < p_w \le p_{w3}$, the interfirm waste utilization is more environmentally preferable than the benchmark case if $\delta < \frac{\beta_s(\beta_s+\beta_b\lambda_s^2)(\alpha_b-c_b-r_b-\lambda_b d_b)}{\lambda_s[\alpha_s-c_s-r_s+\lambda_s(\alpha_b-c_b-\lambda_b d_b)]}$, otherwise, it is environmentally superior if $e_p < \widetilde{\Omega}(e_d,e_r)$. For $p_{w3} < p_w \le r_b$, the optimal waste trading price is $p_w = p_{w3}$, $E_T^{b*}(p_{w3}) - E_T^{N*} = \frac{e_p[\delta\beta_b\lambda_s(\alpha_s-c_s-r_s-\lambda_s d_s)-\beta_s(\alpha_b-c_b-r_b-\lambda_b d_b)]}{2\beta_s\beta_b} - \frac{\lambda_s e_d(\alpha_s-c_s-r_s-\lambda_s d_s)}{2\beta_s} - \frac{e_r(\alpha_b-c_b-r_b-\lambda_b d_b)}{2\beta_b}$, if $\delta < \frac{\beta_s(\alpha_b-c_b-r_b-\lambda_b d_b)}{\beta_b\lambda_s(\alpha_s-c_s-r_s-\lambda_s d_s)}$, then $E_T^{b*}(p_{w3}) < E_T^{N*}$, otherwise, it is environmentally superior to the benchmark case if $e_p < \frac{\beta_b\lambda_s e_d(\alpha_s-c_s-r_s-\lambda_s d_s)+\beta_s e_r(\alpha_b-c_b-r_b-\lambda_b d_b)}{\delta\beta_b\lambda_s(\alpha_s-c_s-r_s-\lambda_s d_s)-\beta_s(\alpha_b-c_b-r_b-\lambda_b d_b)} = \widehat{\Omega}(e_d,e_r)$.