Supplementary Materials:

A hybrid learning-based meta-heuristic algorithm for scheduling of an additive manufacturing system consisting of parallel SLM machines

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S.1. Data related to the problem with 10 parts and 2 machines

Table ST.1. Part parameters related to a problem with 10 parts and 2 machines

i	HP_i-cm	AP_i-cm^2	VP_i-cm^3	MT_i	D_i-hr	TP_i-\$
1	7.123	233.830	1718.883	1	285	3
2	26.374	184.800	2394.820	1	142	3
3	8.213	275.277	717.197	2	185	6
4	11.638	639.580	1933.091	2	285	8
5	27.479	71.908	2240.960	2	300	3
6	18.645	383.452	920.842	1	270	2
7	15.784	741.616	380.811	1	110	7
8	15.472	439.936	1476.417	1	155	7
9	7.153	736.592	519.702	1	50	5
10	7.857	571.947	2511.035	1	156	5

Table ST.2. Machine parameters related to a problem with 10 parts and 2 machines

	$\delta_m^{1,1}$	$\delta_m^{1,2}$	$\delta_m^{2,1}$	$\delta_m^{2,2}$	δ_m^{01}	δ_m^{02}	Γ_m^1	Γ_m^2	Ω_m^1	Ω_m^2	BA_m	BH_m
$m1$	1.271	1.146	1.517	1.449	1.314	1.142	0.035	0.034	0.7	0.7	1000	33.374
$m2$	1.616	1.238	1.529	1.605	1.043	1.051	0.031	0.036	0.7	0.7	900	38.746

S.2. Overfitting

Choosing the best number of training epochs is a major challenge in training artificial neural networks. Low training epochs lead to underfitting on the training sets, and too much training leads to overfitting with a weak performance on the test data set and high accuracy on the training data set. The early stopping method is a simple and effective method that is widely used to control overfitting. This method stops training epochs when the model improvement on the validation dataset stops and the prediction error for the validation dataset is getting worse. In this paper, the early stopping method is used to stop the training procedure before the occurrence of the overfitting. 20% of the training data set is used as a validation dataset. A comparison of the training procedure without and with the early stopping method is elaborated in Figure SF.1a and b, respectively. Figure SF.1a indicates an increase in the mean squared error for the validation dataset after a certain number of iteration while this index is decreasing

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on the training data set. In Figure SF.1b, the training approach is stopped faster using the early stopping method.

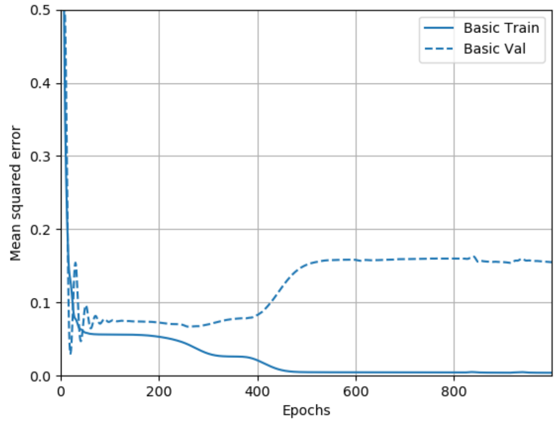

Figure SF.1a Caption: Without the early stopping method

Figure SF.1a Alt text: Increase in the mean squared error for the validation dataset after a certain number of iteration while this index is decreasing on the training data set.

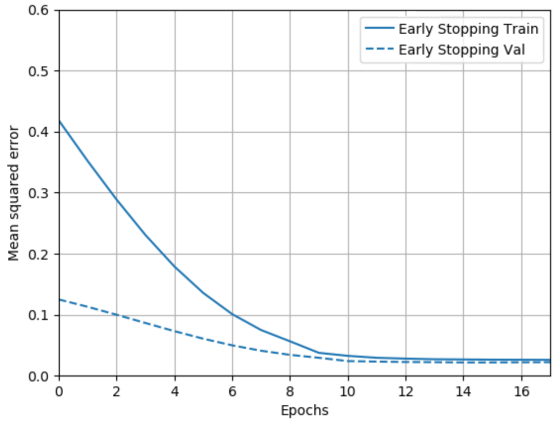

Figure SF.1b Caption: With the early stopping method

Figure SF.1b Alt text: Stop the training approach using the early stopping method when the model improvement on the validation dataset stops and the prediction error for the validation dataset is getting worse.

Figure SF.1. Impact of early stopping method on obviating the overfitting situation.

S.3. Data generation and parameter settings

Table ST.3. Parameters used to generate random instances

Notation	Generated by:	Unit
HP_i	Beta distribution; Beta ($\alpha = 2, \beta = 3$) scaled between [5-30]	cm
AP_i	Normal distribution; $\mathcal{N}(\mu = 500, \sigma = 150)$	cm^2
VP_i	Normal distribution; $\mathcal{N}(\mu = 1500, \sigma = 400)$	cm^3
D_i	Uniform distribution; $\mathcal{U}$ (50, 60×$\frac{I}{M}$)	hr
TP_i	Uniform distribution; $\mathcal{U}$ (3, 10)	\$
$\delta_m^{k\,k'}$	Uniform distribution; $\mathcal{U}$ (1, 2)	hr
δ_m^{0k}	Uniform distribution; $\mathcal{U}$ (0.5, 0.8×$\min_{\forall k'} \delta_m^{k'k}$)	hr
Γ_m^k	Uniform distribution; $\mathcal{U}$ (0.03, 0.05)	hr
Ω_m^k	Uniform distribution; $\mathcal{U}$ (0.5, 0.7)	hr
BA_m	Random choice from [800, 900, 1000, 1100, 1200]	cm^2
BH_m	Uniform distribution; $\mathcal{U}$ (25, 35)	cm

Table ST.4. Parameters of the proposed algorithms

Description	Value
Population size	[50 – 300]
Crossover probability	0.1
Mutation probability	0.95
Number of clusters	[1 – 4]
Training condition	Each 25% of maximum iteration for small- and medium-size problems and each 20% of maximum time for large size problems
Size of training data set	[200 – 1000]
Learning coefficient	0.1

S.4. Evaluation metrics

SPacing (SP): It concerns the relative distance between successive solutions in the Pareto front, which is calculated by Equation (1).

$$Sp = \sqrt{\frac{1}{n-1} \times \sum_{i=1}^{n} \left(\bar{d} - d_i\right)^2} \tag{1}$$

while

$$d_i = \min_j \left\{ \sum_{k=1}^{r} \left| obj_k^i - obj_k^j \right| \right\} \qquad i, j \in \{1,2,\ldots,n\}; i \neq j \tag{2}$$

$$\bar{d} = \sum_i \frac{d_i}{n} \tag{3}$$

where n and r represent the number of non-dominated solutions and objective functions, respectively. Accordingly, an algorithm with a lower *SP* has a better performance and indicating that the solutions are dispersed uniformly in the Pareto front (Wang et al. 2021).

Set Coverage (SC): It is applied to compare Pareto fronts resulted from various algorithms. This metric is calculated by Equation (4), in which $C(A,B)$ shows the percentage of solutions in front *B* that is dominated at least by a solution in front A. If $C(A,B) > C(B,A)$, A is much preferable. Moreover, it is worth mentioning that $C(A,B) \neq 1 - C(B,A)$ and $C(B,A)$ is calculated to determine the performance of algorithm *A*. Hence, a lower value is more desirable.

$$C(A,B) = \frac{|\{b \in B | a \in A : a \succ b\}|}{|B|} \tag{4}$$

Domination Degree (DD): DD is also utilized to compare different algorithms' Pareto fronts based on Equation (5).

$$Z(A,B) = \frac{z(A)}{z(A) + z(B)} \tag{5}$$

while

$$z(A) = \sum_{i=1}^{n} b_i \tag{6}$$

where n is the number of non-dominated solutions in algorithm A, and b_i is the number of solutions that dominate solution i of algorithm A. Accordingly, an efficient algorithm has a lower value of DD.

Maximum Spread (MS): This metric is determined based on the maximum and minimum values of each objective function in the Pareto front as indicated by:

$$MS = \sqrt{\sum_{k=1}^{K} \left(\max_{i \in N} obj_k^i - \min_{i \in N} obj_k^i \right)} \tag{7}$$

where N is the number of non-dominated solutions, and K is the number of objective functions. An algorithm with greater MS is more satisfactory.

S.5. Procedure of dispatching rule 2

Figure SF.2 shows the procedure of dispatching rule 2. Accordingly, first, we assign parts to machines while the operators of the NSGA-II are used to improve this assignment during the solving procedure. Next, we form families in Step 2 so that the parts with the same material and machine make the same family. In Step 3, we form the jobs according to the family of parts so that each job cannot contain the parts with different families. Finally, we determine the sequence of jobs on each machine in Step 4.

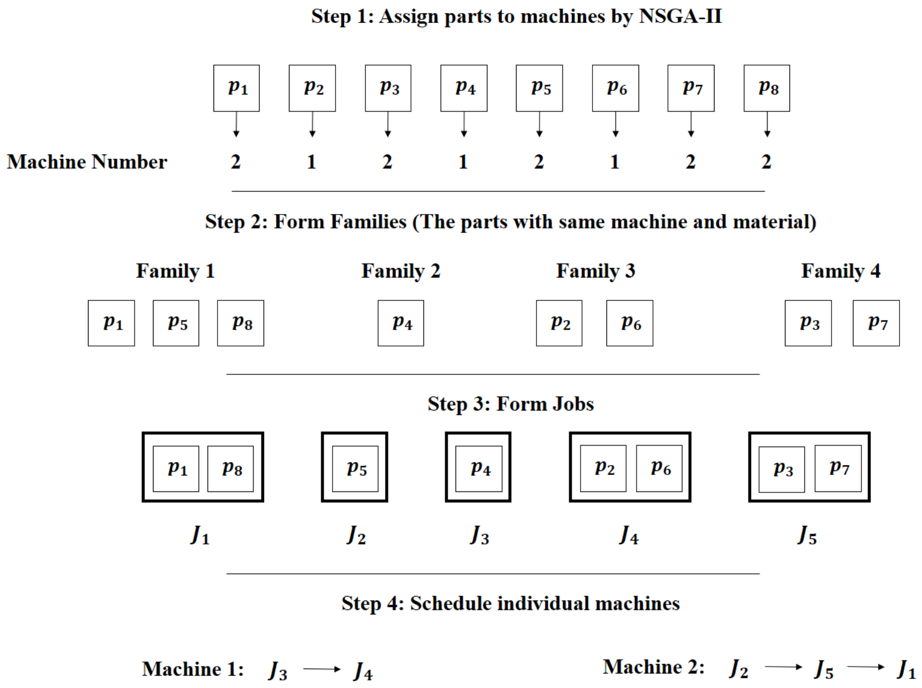

Figure SF.2 Caption: The procedure of dispatching rule 2.
Figure SF.2 Alt text: Schematic maner of the dispatching rule for parallel batching machine scheduling presented by Balasubramanian et al. (2004).

A simple heuristic approach is applied for batching and sequencing in NSGA-II_DR2, which can find acceptable solutions in a reasonable time (Devpura et al. 2001). In this heuristic, after processing a job at time t the L_i index is calculated for remained (unscheduled) parts as Equation (8).

$$L_i(t) = \left(\frac{TP_i}{pt_i}\right) exp\left(\frac{-\max\{D_i - pt_i - t, 0\}}{q\bar{p}}\right) \tag{8}$$

where $\bar{p}$ is the average processing time of the remaining unscheduled parts, q is a regulator parameter and PT_i is the processing time of part i, which is calculated based on Equation (9).

$$pt_i = \Gamma_m^k v_i + \Omega_m^k h_i \qquad k = MT_i \text{ and } m \text{ is the machine assigned to part } i \tag{9}$$

The parts in each family are ordered in a decreasing order of their L_i and a job of the first parts is formed for each family, respecting the capacity of the machine. Afterward, to schedule the created jobs at time t, first, the following index is calculated for each job j by Equation (10).

$$BL_j = \sum_{i \in BL_j} L_i(t) \tag{10}$$

The batch with the highest value of BL_j is scheduled on its assigned machine. This procedure continues until all batches are scheduled. After the completion of the scheduling phase, the objective functions are calculated, and then the NSGA-II operators are executed to generate new assignments of machines to parts.

S.6. Numerical results

Table ST.5. Performance of the proposed algorithms based on the evaluation metrics in small-sized problems

Sample	Methods	SC	DD	SP	MS	Time (s)
8-2-2	NSGAII-DR1	0.29	0.12	47.63	21.19	5.2
	NSGAII-DR2	1.00	0.68	31.82	13.50	5.6
	NSGAII-RLS	0.29	0.18	28.86	20.80	13.1
	SPEA2-LLS	0.00	0.00	65.38	21.19	17.2
	NSGAII-LLS	0.00	0.00	22.32	21.19	15.5
	AUGMECON	0.00	0.00	38.56	23.12	378.1
9-2-2	NSGAII-DR1	0.85	0.40	92.77	45.29	5.8
	NSGAII-DR2	1.00	0.43	136.14	31.91	6.1
	NSGAII-RLS	0.50	0.14	49.84	37.44	15.2
	SPEA2-LLS	0.17	0.006	49.10	31.29	20.0
	NSGAII-LLS	0.00	0.00	130.94	36.81	18.3
	AUGMECON	0.00	0.00	275.46	37.82	412.9
10-2-2	NSGAII-DR1	0.57	0.10	104.47	29.06	7.3
	NSGAII-DR2	1.00	0.71	103.08	35.60	7.5
	NSGAII-RLS	0.33	0.05	108.90	31.66	16.8
	SPEA2-LLS	0.25	0.021	43.20	31.66	23.4
	NSGAII-LLS	0.25	0.09	110.42	31.66	21.2
	AUGMECON	0.00	0.00	201.85	31.66	655.7
11-2-2	NSGAII-DR1	0.14	0.06	643.45	52.56	7.7
	NSGAII-DR2	0.13	0.31	662.56	54.20	8.0
	NSGAII-RLS	0.33	0.62	143.28	51.59	17.2
	SPEA2-LLS	0.00	0.00	195.79	52.56	25.5
	NSGAII-LLS	0.00	0.00	643.45	52.56	23.1
	AUGMECON	0.00	0.00	643.45	52.56	869.2
12-2-2	NSGAII-DR1	1.00	0.17	140.19	43.37	6.6
	NSGAII-DR2	1.00	0.49	155.26	37.92	8.2
	NSGAII-RLS	1.00	0.16	53.43	43.30	22.4
	SPEA2-LLS	0.63	0.07	94.43	39.90	28.6
	NSGAII-LLS	0.50	0.09	63.72	36.40	26.8
	AUGMECON	0.00	0.00	93.00	41.84	3600

Table ST.6. Performance of the proposed Algorithms based on the evaluation metrics in medium-sized problems

Sample	Methods	SP	MS	SC	DD	Time (s)
15-2-2	NSGAII-DR1	0.62	0.11	236.1	47.8	9.4
	NSGAII-DR2	0.91	0.66	43.0	33.2	9.9
	NSGAII-RLS	0.50	0.09	143.1	63.3	32.1
	SPEA2-LLS	0.55	0.08	94.1	53.0	38.3
	NSGAII-LLS	0.37	0.04	145.2	63.3	36.7
15-3-2	NSGAII-DR1	1.00	0.28	166.5	52.0	10.1
	NSGAII-DR2	0.90	0.49	42.7	24.0	11.0
	NSGAII-RLS	0.85	0.16	677.2	56.3	29.1
	SPEA2-LLS	0.41	0.03	139.7	57.0	35.2
	NSGAII-LLS	0.33	0.01	170.2	57.0	33.1
20-2-3	NSGAII-DR1	1.00	0.14	234.5	76.4	14.1
	NSGAII-DR2	1.00	0.22	198.3	47.2	15.4
	NSGAII-RLS	0.88	0.49	991.2	91.9	40.5
	SPEA2-LLS	0.85	0.12	75.7	54.9	51.6
	NSGAII-LLS	0.22	0.10	243.1	54.8	46.2
25-3-3	NSGAII-DR1	1.00	0.36	239.1	70.3	23.4
	NSGAII-DR2	1.00	0.64	977.8	101.7	30.7
	NSGAII-RLS	0.3	0.004	348.3	56.8	41.5
	SPEA2-LLS	0.36	0.005	378.7	53.1	53.0
	NSGAII-LLS	0.33	0.005	1519.0	72.4	48.3
30-3-3	NSGAII-DR1	0.92	0.27	213.2	64.9	29.8
	NSGAII-DR2	1.00	0.30	88.5	36.6	41.9
	NSGAII-RLS	1.00	0.38	106.1	42.7	45.8
	SPEA2-LLS	0.57	0.03	267.3	43.7	55.6
	NSGAII-LLS	0.18	0.004	69.7	41.8	51.7
35-3-3	NSGAII-DR1	0.54	0.02	111.5	50.5	39.6
	NSGAII-DR2	0.42	0.04	167.9	59.0	68.3
	NSGAII-RLS	0.94	0.25	411.6	67.9	65.3
	SPEA2-LLS	0.66	0.09	137.6	67.5	78.2
	NSGAII-LLS	0.94	0.59	273.2	71.9	74.6
40-3-3	NSGAII-DR1	1.00	0.13	319.6	48.7	43.5
	NSGAII-DR2	1.00	0.54	116.8	41.4	87.2
	NSGAII-RLS	1.00	0.17	93.6	55.3	68.1
	SPEA2-LLS	0.41	0.14	123.9	65.6	82.8
	NSGAII-LLS	0.00	0.00	56.4	40.8	76.4
45-3-4	NSGAII-DR1	1.00	0.43	231.6	58.1	54.0
	NSGAII-DR2	0.83	0.09	109.4	30.0	98.1
	NSGAII-RLS	1.00	0.43	1451.5	102.4	83.8
	SPEA2-LLS	0.44	0.02	580.6	99.9	97.0
	NSGAII-LLS	0.20	0.01	257.6	59.3	91.2
50-3-4	NSGAII-DR1	1.00	0.17	195.9	54.6	76.6
	NSGAII-DR2	1.00	0.13	122.7	48.2	130.1
	NSGAII-RLS	1.00	0.25	365.7	53.2	88.0
	SPEA2-LLS	0.21	0.04	265.4	78.1	108.5
	NSGAII-LLS	0.56	0.38	37.6	45.9	101.4
50-4-4	NSGAII-DR1	1.00	0.71	251.5	87.3	65.6
	NSGAII-DR2	1.00	0.12	79.9	48.2	194.5
	NSGAII-RLS	1.00	0.11	205.3	53.7	114.7
	SPEA2-LLS	0.77	0.03	700.4	64.7	139.8
	NSGAII-LLS	0.21	0.009	81.9	55.2	130.5
60-4-4	NSGAII-DR1	0.50	0.01	54.1	44.3	105.7
	NSGAII-DR2	1.00	0.32	1740.1	105.9	289.7
	NSGAII-RLS	1.00	0.52	70.6	50.5	189.4
	SPEA2-LLS	0.38	0.11	416.6	74.0	216.1
	NSGAII-LLS	0.50	0.02	453.5	49.9	207.2

70-5-4	NSGAII-DR1	1.00	0.25	134.4	48.2	155.5
	NSGAII-DR2	1.00	0.32	53.9	26.5	475.7
	NSGAII-RLS	0.91	0.22	152.4	48.1	282.9
	SPEA2-LLS	0.72	0.18	202.0	81.3	324.3
	NSGAII-LLS	0.00	0.00	86.5	45.3	310.5
80-5-4	NSGAII-DR1	1.00	0.63	79.8	39.2	230.6
	NSGAII-DR2	0.76	0.10	128.1	45.1	861.7
	NSGAII-RLS	0.66	0.34	89.6	48.5	420.1
	SPEA2-LLS	0.68	0.26	64.5	62.3	475.0
	NSGAII-LLS	0.00	0.00	30.2	31.3	458.6
90-5-4	NSGAII-DR1	1.00	0.28	1148.0	71.2	292.7
	NSGAII-DR2	1.00	0.53	452.1	96.1	1421.2
	NSGAII-RLS	0.82	0.38	340.8	48.2	653.2
	SPEA2-LLS	0.57	0.17	516.9	45.4	722.3
	NSGAII-LLS	0.10	0.002	107.7	71.0	698.1

Table ST.7. Performance of the proposed algorithms based on the evaluation metrics in large-sized problems

Sample	**Methods**	**SP**	**MS**	**SC**	**DD**	**Time (s)**
100-5-5	NSGAII-DR1	1.00	0.22	349.0	79.5	487.6
	NSGAII-DR2	1.00	0.27	89.7	48.2	>1800
	NSGAII-RLS	1.00	0.36	67.4	48.6	1082.6
	SPEA2-LLS	1.00	0.13	139.3	44.0	1196.3
	NSGAII-LLS	0.00	0.00	46.7	39.8	1112.7
110-5-5	NSGAII-DR1	1.00	0.42	489.50	80.10	512.2
	NSGAII-DR2	1.00	0.38	146.20	72.40	>1800
	NSGAII-RLS	1.00	0.16	275.60	45.11	1247.1
	SPEA2-LLS	1.00	0.03	393.50	46.47	1561.9
	NSGAII-LLS	0.00	0.00	83.20	54.49	1487.3
120-5-5	NSGAII-DR1	1.00	0.16	1221.7	97.1	662.7
	NSGAII-DR2	1.00	0.71	426.5	103.5	>1800
	NSGAII-RLS	1.00	0.12	124.1	66.4	1432.5
	SPEA2-LLS	0.04	0.00	493.4	89.7	1706.9
	NSGAII-LLS	0.00	0.00	356.0	66.5	1621.3
130-5-5	NSGAII-DR1	1.00	0.49	1150.13	160.29	732.6
	NSGAII-DR2	0.80	0.04	1686.20	115.78	>1800
	NSGAII-RLS	1.00	0.38	2660.60	135.52	1618.1
	SPEA2-LLS	0.72	0.08	3226.40	128.02	>1800
	NSGAII-LLS	0.12	0.00	1878.60	113.37	>1800
140-6-5	NSGAII-DR1	1.00	0.15	1097.0	107.0	1019.5
	NSGAII-DR2	1.00	0.39	466.2	95.4	>1800
	NSGAII-RLS	1.00	0.37	215.9	74.8	>1800
	SPEA2-LLS	0.00	0.00	839.4	93.9	>1800
	NSGAII-LLS	1.00	0.07	307.3	85.9	>1800
150-6-5	NSGAII-DR1	1.00	0.36	2112.16	105.19	1256.2
	NSGAII-DR2	1.00	0.16	1482.03	86.41	>1800
	NSGAII-RLS	1.00	0.27	1217.37	117.74	>1800
	SPEA2-LLS	1.00	0.19	1349.02	116.33	>1800
	NSGAII-LLS	0.00	0.00	1099.25	95.43	>1800
160-6-6	NSGAII-DR1	1.00	0.32	421.2	55.2	1480.2
	NSGAII-DR2	1.00	0.21	277.4	67.1	>1800
	NSGAII-RLS	0.75	0.29	360.7	56.8	>1800
	SPEA2-LLS	0.42	0.10	286.4	43.6	>1800
	NSGAII-LLS	0.26	0.05	122.8	47.9	>1800
170-6-6	NSGAII-DR1	1.00	0.33	87.83	138.98	>1800
	NSGAII-DR2	1.00	0.36	588.66	131.05	>1800
	NSGAII-RLS	1.00	0.20	2476.77	82.95	>1800
	SPEA2-LLS	0.00	0.00	1091.41	76.87	>1800

	NSGAII-LLS	1.00	0.10	1037.67	103.26	>1800
180-6-6	NSGAII-DR1	0.66	0.22	241.8	79.2	>1800
	NSGAII-DR2	0.90	0.25	144.2	63.6	>1800
	NSGAII-RLS	0.85	0.35	341.5	68.3	>1800
	SPEA2-LLS	0.43	0.08	176.4	81.6	>1800
	NSGAII-LLS	0.31	0.07	254.5	102.6	>1800
190-6-6	NSGAII-DR1	1.00	0.37	984.98	147.17	>1800
	NSGAII-DR2	1.00	0.33	2092.31	98.47	>1800
	NSGAII-RLS	1.00	0.26	3181.20	167.24	>1800
	SPEA2-LLS	0.83	0.03	1438.49	101.41	>1800
	NSGAII-LLS	0.00	0.00	2780.00	94.38	>1800
200-7-6	NSGAII-DR1	1.00	0.59	440.7	120.6	>1800
	NSGAII-DR2	1.00	0.10	501.9	128.2	>1800
	NSGAII-RLS	1.00	0.28	1634.1	136.5	>1800
	SPEA2-LLS	0.00	0.00	223.1	61.1	>1800
	NSGAII-LLS	0.37	0.008	578.7	132.2	>1800

References:

Devpura, Amit, John W Fowler, MW Carlyle, and I Perez. 2001. Minimizing total weighted tardiness on single batch process machine with incompatible job families. Paper presented at the Operations Research Proceedings. Springer, pp. 366-371.

Wang, Jianjiang, Xuejun Hu, Erik Demeulemeester, and Yan Zhao. 2021. "A bi-objective robust resource allocation model for the RCPSP considering resource transfer costs." *International Journal of Production Research* 59 (2), pp. 367-87.