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# Adaptive synchronization of unknown nonlinear networked systems with prescribed performance

Hashim A. Hashim\*, Sami El-Ferik, and Frank L. Lewis

**Abstract**—This paper proposes an adaptive tracking control with prescribed performance function for distributive cooperative control of highly nonlinear multi-agent systems. The use of such approach confines the tracking error within a large predefined set to a predefined smaller set. The key idea is to transform the constrained system into unconstrained one through the transformation of the output error. Agents' dynamics are assumed unknown, and the controller is developed for a strongly connected structured network. The proposed controller allows all agents to follow the trajectory of the leader node, while satisfying the necessary dynamic requirements. The proposed approach guarantees uniform ultimate boundedness for the transformed error as well as a bounded adaptive estimate of the unknown parameters and dynamics. Simulations include two examples to validate the robustness and smoothness of the proposed controller against highly nonlinear heterogeneous multi-agent system with uncertain time-variant parameters and external disturbances.

**Index Terms**—Prescribed performance, Transformed error, Multi-agents, Distributed adaptive control, Consensus, Transient, Steady-state error, Networked Systems, Distributed Adaptive Control, Robustness.

## I. INTRODUCTION

IN recent years, distributive cooperative control has gained popularity and attention among control researchers owing to its capabilities to mimic the social behavior of animals such as bees swarming, birds flocking, ants foraging, fish schooling and so forth. Indeed, the control scheme enables a group of agents to perform a task that can be daunting for an individual agent in a simpler and faster manner. Besides, the cooperation between agents allows information exchange, improving performance and increasing productivity through collaboration, which emulates the standard behavior in social groups. Cooperative control contributes to the betterments of many applications such as the control of autonomous mobile robot vehicles in energy and mineral explorations, space studies, surveillance and other areas. Agents are connected by a communication network and exchange useful information. In such case, they are considered as nodes. The group of agents may follow one or more real or virtual leaders. The network formed by all nodes creates a graph, which can be directed or undirected. Undirected graphs refer to no difference between nodes while directed graphs define the direction of the flow of information between the node and its neighbors.

In the literature, (Fax & Murray, 2004) and (Ren, Beard, & others, 2005) can be considered among the first pioneering studies addressing the consensus in multi-agent systems. Several other scholars contributed to the development in this field. For instance, the consensus of passive nonlinear systems has been addressed in (Chopra & Spong, 2006). Many research work such as (Olfati-Saber, Fax, & Murray, 2007)-(Zhou, Xia, Fu, & Li, 2015) investigated node consensus of cooperative tracking problem. Distributed tracking control for linear heterogeneous agents of MIMO systems with parameter uncertainties was established in (Y. Zhao, Duan, Wen, Li, & Chen, 2015). Cooperative tracking control for a single node has been studied in (Das & Lewis, 2010; Cao & Ren, 2012) and in the case of high-order dynamics in (Zhang & Lewis, 2012). The work in (Das & Lewis, 2010; Zhang & Lewis, 2012) developed a neuro-adaptive distributed control for heterogeneous agents with unknown nonlinear dynamics connected through a digraph. In (Das & Lewis, 2010), the authors considered nodes with first-order dynamic. Later on, in (Zhang & Lewis, 2012), high order systems have been addressed. In all previous studies, the input function in the node dynamics was assumed known.

On the other hand, cooperative tracking control problems of systems with unknown input function have been studied (Theodoridis, Boutalis, & Christodoulou, 2012) and (El-Ferik, Qureshi, & Lewis, 2014). In (Theodoridis et al., 2012), neuro-adaptive fuzzy was proposed to approximate unknown nonlinear dynamics and input functions. The centers of the output membership functions are determined based on off-line trials. A very fundamental assumption in all these studies is the one that considers the unknown nonlinear dynamics as well as the input function as linear in parameters (LIP) (see for instance (F. W. Lewis, Jagannathan, & Yesildirak, 1998) or (El-Ferik et al., 2014)). The goal is to guarantee the ultimate stability of the tracking error. Recently, there are several studies that were published addressing different issues related to adaptive control of multi-agent systems. These challenges include actuator fault (see for instance (Na, 2013), (Na, Chen, Ren, & Guo, 2014), (Tong, Wang, & Li, 2014), (Tong, Sui, & Li, 2015), (Y. Li & Tong, 2015) and (L. Zhao & Jia, 2016), switching network topology (Yang, Yue, & Dou, 2016), Predictor-based adaptive control (W. Wang, Wang, & Peng, 2016), etc. All these challenging practical issues could benefit from prescribed performance framework to guarantee performance. In particular, a practical implementation of neuro-adaptive prescribed performance control to compensate for friction using a turn table servo system has been reported in (Na et al., 2014). More implementation of such control

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approaches is really needed.

The distributed control of multi-agents attempts to tackle unknown nonlinearities, unmodeled dynamics, uncertainties, and disturbances. Estimation of the closed loop characteristics such as transient and steady state error is almost impossible to be represented analytically (Bechlioulis, Dimarogonas, & Kyriakopoulos, 2014). Alternatively, prescribed performance has been proposed as a means to seclude the error to an arbitrarily small set, where the convergence is constrained to a given range. The key idea in the approach is to transform the error from the restricted space to the unconstrained one. The following section gives necessary details about the method. At this stage, it is worth to mention that prescribed performance approach aims to satisfy the following objectives. The convergence error has to be less than the predefined value; the transformed error is bounded; the maximum overshoot is less than the prescribed constant; the system's controlled output is smooth; and the control signal is both bounded and smooth.

Developing a cooperative control approach for multi-agent systems with prescribed performance has many benefits. In this context, the specified performance ensures that the consensus output error starts within a large predefined set and then converges systematically into a predefined narrow set (Bechlioulis & Rovithakis, 2008). During transient and steady-state, the tracking error satisfies a known time-varying performance. Adaptive cooperative control with prescribed performance has then the ability to increase the robustness of the system's behavior and to reduce the control effort. The proper selection of the upper and lower bounds of the prescribed performance functions guarantees the convergence of error within predefined limits smoothly and systematically. In the literature, robust adaptive control with prescribed performance function for feedback linearizable systems has been designed in (Bechlioulis & Rovithakis, 2008). The design of neuro-adaptive controllers to handle unknown nonlinearities and disturbances has been considered in (Bechlioulis & Rovithakis, 2009)-(Yang, Ge, Wang, Li, & Hua, 2015) for different applications. The application of prescribed performance scheme with neural approximation included strict-feedback systems (Bechlioulis & Rovithakis, 2009), affine systems (J. Wang, Hovakimyan, & Cao, 2010), high order nonlinear systems (Bechlioulis & Rovithakis, 2014). Each of these studies considered different assumptions on the input matrix continuity. Further refinement of these results improved the neural network weights previously tuned using trial and errors to avoid neural nets in the controller design and redevelop adaptive control with prescribed performance based on fuzzy adaptive tuning in (Sun & Liu, 2014) and model reference adaptive control in (Mohamed, 2014). All previous studies considered a single autonomous system. Recently, (Bechlioulis et al., 2014) designed a control of a platoon with unknown nonlinear dynamics. Thus, the agents are set in a straight line, and each node sees only the one in front. This represents a particular network structure and a special formation for the nodes.

In this work, we propose a robust adaptive distributive control with prescribed performance for a group of nodes connected through a directed communication graph with known topology. The control law is fully distributed based on the fact that the control law of each agent respects the graph's topology and uses only the allowed local neighborhood information. Thus, the leader does not communicate with all the nodes. In our work, we consider a general network form characterized by its  $L$  and  $B$  matrices. The formation of the multi-agent systems can be anything including platooning. The synchronization error between nodes follows a prescribed performance to satisfy predefined characteristics imposed by the designer. Each node contains unknown nonlinear dynamics and time-varying uncertainties. The controller is developed to meet a predefined transient response and specific characteristics of the steady-state synchronization error for each node. The original form of the prescribed performance as originally proposed in (Bechlioulis & Rovithakis, 2008) is modified in this work to overcome the chattering in the control signal due to the interaction between nodes caused by the consensus algorithm the exchanging state information between nodes. The new approach guarantees stable dynamics with non-oscillatory, limited and smooth control signal.

The rest of the paper is organized as follows. Section II presents preliminaries of graph theory. Problem formulation and the derivation of the local error synchronization equation follow in Section III. Section IV contains the control law development as well as the stability proof of the connected graph. Simulations results are presented in Section V. Conclusion and future work are in Section VI.

**Notations:**The following symbols are used throughout the paper.

- $|\cdot|$  : absolute value of a real number;
- $\|\cdot\|$  : Euclidean norm of a vector;
- $\|\cdot\|_F$  : Frobenius norm of a matrix;
- $tr\{\cdot\}$  : trace of a matrix;
- $\sigma(\cdot)$  : set of singular values of a matrix, with the maximum singular value  $\bar{\lambda}$  and the minimum singular value  $\underline{\lambda}$ ;
- $P > 0$  : indicates that the matrix  $P$  is positive definite; ( $P \geq 0$ ) (positive semi-definite);
- $\mathcal{N}$  : set  $\{1, \dots, N\}$ ;
- $I_m$  : identity matrix of order  $m$ .
- $\underline{1}$  : unity vector  $[1, \dots, 1]^T \in \mathbb{R}^n$  where  $n$  is the required appropriate dimension.

## II. BASIC GRAPH THEORY

A graph is denoted by  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$  with a nonempty finite set of nodes (or vertices)  $\mathcal{V} = \{\mathcal{V}_1, \mathcal{V}_2, \dots, \mathcal{V}_n\}$ , and a set of edges (or arcs)  $E \subseteq \mathcal{V} \times \mathcal{V}$ .  $(\mathcal{V}_i, \mathcal{V}_j) \in E$  if there is an edge from node  $i$  to node  $j$ . The topology of a weighted graph is often described by the adjacency matrix  $A = [a_{ij}] \in \mathbb{R}^{N \times N}$  with weights  $a_{ij} > 0$  if  $(\mathcal{V}_j, \mathcal{V}_i) \in E$ ; otherwise  $a_{ij} = 0$ . Throughout the paper, the topology of the communication network is fixed, i.e.  $A$  is time-invariant, and the self-connectivity element  $a_{ii} = 0$ . A graph can be

directed or undirected. A directed graph is called digraph. The weight in-degree of a node  $i$  is defined as the sum of  $i$ -th row of  $A$ , i.e.,  $d_i = \sum_{j=1}^N a_{ij}$ . Define the diagonal in-degree matrix  $D = \text{diag}(d_1, \dots, d_N) \in \mathbb{R}^{N \times N}$  and the graph Laplacian matrix  $L = D - A$ . The set of neighbors of a node  $i$  is  $N_i = \{j | (\mathcal{V}_j \times \mathcal{V}_i) \in E\}$ . If node  $j$  is a neighbor of node  $i$ , then node  $i$  can get the information from node  $j$ , but not necessarily vice-versa. For an undirected graph, the neighborhood requires a mutual relation. A direct path from node  $i$  to node  $j$  is a sequence of successive edges in the form  $\{(\mathcal{V}_i, \mathcal{V}_k), (\mathcal{V}_k, \mathcal{V}_l), \dots, (\mathcal{V}_m, \mathcal{V}_j)\}$ . A digraph has a spanning tree if there is a node (called the root) having a possible direct path to every other node in the graph. A digraph is strongly connected if for any ordered pair of nodes  $[\mathcal{V}_i, \mathcal{V}_j]$  with  $i \neq j$ , there is a directed path from node  $i$  to node  $j$  (for more details, see (Ren & Beard, 2008) or (F. L. Lewis, Zhang, Hengster-Movric, & Das, 2013)).

### III. PROBLEM FORMULATION

Consider the following nonlinear dynamics for the  $i$ th node

$$\dot{x}_i = A_{mi}x_i + B_{mi}u_i + f_i(x_i) + w_i \quad (1)$$

where the state node is  $x_i \in \mathbb{R}$ , the control signal node is  $u_i \in \mathbb{R}$  and the unknown disturbance for each node is  $w_i \in \mathbb{R}$ .  $A_{mi} \in \mathbb{R}$  and  $B_{mi} \in \mathbb{R}$  are known constants;  $f_i(x_i) \in \mathbb{R}$  is the unknown nonlinear part of the dynamics and assumed to be Lipschitz. From (1), the global dynamic can be written as

$$\dot{x} = A_m x + B_m u + f(x) + w \quad (2)$$

where  $x = [x_1, \dots, x_N]^T \in \mathbb{R}^N$ ,  $u = [u_1, \dots, u_N]^T \in \mathbb{R}^N$ ,  $f(x) = [f_1(x_1), \dots, f_N(x_N)]^T \in \mathbb{R}^N$ ,  $w = [w_1, \dots, w_N]^T \in \mathbb{R}^N$ ,  $A_m = \text{diag}\{A_{m1}, \dots, A_{mN}\}$ , and  $B_m = \text{diag}\{B_{m1}, \dots, B_{mN}\}$ .  $x_0$  is the leader's state and it represents the desired synchronization trajectory according to the following equation

$$\dot{x}_0 = A_m x_0 + f(x_0, t) \quad (3)$$

where  $x_0 \in \mathbb{R}$  is the leader state node,  $f(x_0, t) \in \mathbb{R}$  is the nonlinear part of the leader node's dynamic. The model presented in (2)-(3) is very similar to the one treated in (Das & Lewis, 2010), with the exception that the dynamic of the agent as described in (2) is more representative of real systems. The local synchronization error function for agent  $i$  can be described as in (X. Li, Wang, & Chen, 2004)-(Khoo, Xie, & Man, 2009).

$$e_i = \sum_{j \in N_i} a_{ij}(x_i - x_j) + b_i(x_i - x_0) \quad (4)$$

where  $a_{ij} \geq 0$  and  $a_{ii} = 0$ .  $a_{ij} > 0$  if agent  $i$  is directed to agent  $j$ ,  $b_i \geq 0$ . The network is such that  $b_i > 0$  for at least one agent  $i$ . Hence, equation (4) can be written in the global error form as

$$\begin{aligned} e &= -(L + B)(\underline{1}x_0 - x) = (L + B)(x - \underline{1}x_0) \\ &= (L + B)(\tilde{x}) \end{aligned} \quad (5)$$

where the global error is  $e = [e_1, \dots, e_N]^T \in \mathbb{R}^N$ , global state vector is  $\underline{1}x_0 = \underline{x}_0 \in \mathbb{R}^N$ , the Laplacian matrix is  $L \in \mathbb{R}^{N \times N}$ ,

$B \in \mathbb{R}^{N \times N}$  with  $B = \text{diag}\{b_i\}$  and  $\underline{1} = [1, \dots, 1]^T \in \mathbb{R}^N$ . Note that  $\tilde{x} = x - \underline{1} \cdot x_0$ , and  $\underline{f}(x_0, t) = \underline{1} \cdot f(x_0, t)$ . For more details, the proof of equation (5) is stated in (F. L. Lewis et al., 2013).

The derivative error dynamics of (5) is

$$\dot{e} = (L + B)(A_m \tilde{x} + f(x) + B_m u + w - \underline{f}(x_0, t)) \quad (6)$$

**Remark 1.** The communication graph is considered strongly connected. Thus, if  $b_i \neq 0$  for at least one  $i$ ,  $i = 1, \dots, N$  then  $(L + B)$  is an irreducible diagonally dominant  $M$ -matrix and hence nonsingular (Qu, 2009).

**Remark 2.** (see (F. L. Lewis et al., 2013)) If agent state is  $x_i \in \mathbb{R}^n$  and the leader state  $x_0 \in \mathbb{R}^n$  where  $n > 0$ , then  $e, x \in \mathbb{R}^{nN}$  and equation (5) will be

$$e = ((L + B) \otimes \mathbb{I}_N)(x - \underline{1}x_0) \quad (7)$$

where  $\otimes$  is the Kronecker product.

Also, one should note that  $B \neq 0$  for a strongly connected graph with

$$\|\beta\| \leq \|e\|/\lambda(L + B) \quad (8)$$

where  $\lambda(L + B)$  is the minimum singular value of  $(L + B)$  (F. L. Lewis et al., 2013).

A performance function  $\rho(t)$  is associated with the error component  $e(t)$  and is defined as a smooth function such as  $\rho(t) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$  is a positive decreasing function  $\lim_{t \rightarrow \infty} \rho(t) = \rho_\infty > 0$ . The prescribed performance function can be written as

$$\rho_i(t) = (\rho_{i0} - \rho_{i\infty}) \exp^{-\ell_i t} + \rho_{i\infty} \quad (9)$$

where  $\rho_{i0}, \rho_{i\infty}$  and  $\ell_i$  are appropriately defined positive constants. In order to overcome the difficulty caused through the synchronization algorithm and achieve the desired prescribed performance, the following time varying constraints are proposed:

$$-\delta_i \rho_i(t) < e_i(t) < \rho_i(t), \quad \text{if } e_i(t) > 0 \quad (10)$$

$$-\rho_i(t) < e_i(t) < \delta_i \rho_i(t), \quad \text{if } e_i(t) < 0 \quad (11)$$

for all  $t \geq 0$  and  $0 < \delta_i \leq 1$ , and  $i = 1, \dots, N$ .

**Remark 3.** The dynamic constraints (10) and (11) represent a modification of the ones in (Bechlioulis & Rovithakis, 2008), and (Mohamed, 2014). In these papers, the constraints are conditioned on  $e(0)$  as follows

$$-\delta \rho(t) < e(t) < \rho(t), \quad \text{if } e(0) > 0 \quad (12)$$

$$-\rho(t) < e(t) < \delta \rho(t), \quad \text{if } e(0) < 0 \quad (13)$$

Figure (1) shows the tracking error of controller with prescribed performance as it transits from a large to a smaller set in accordance with equations (10) and (11).

Due to the interaction between agents' dynamics, such constraints will lead to instability. Upon crossing this reference, the system becomes unstable under the original formulation (12) and (13). However, the switching based on  $e_i(t)$  provides the necessary control to keep the system stable.

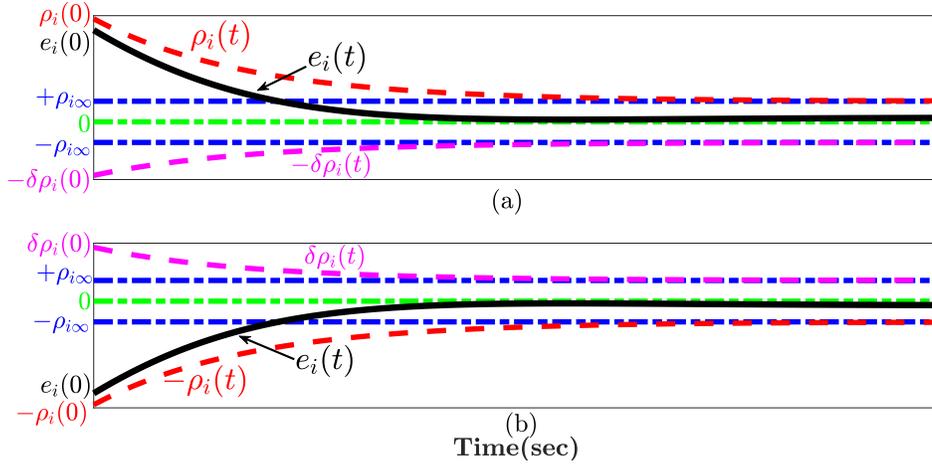


Fig. 1. Graphical representation of tracking error with prescribed performance (a) Prescribed performance of (12); (b) Prescribed performance of (13).

**Remark 4.** In the control with prescribed performance framework as presented as in (Bechlioulis & Rovithakis, 2008; Mohamed, 2014), the knowledge of the sign of  $e_i(0)$  is sufficient to maintain the same robust controller for all  $t > 0$  and satisfy the performance constraints (no switching occurs after  $t = 0$ ). However, in the case of multi-agent systems, the synchronization error (7) creates a coupling between the different states of each agent. The interactions created may force the synchronization error to violate the desired performance constraints and exit the compact set if keep the same controller based solely on the sign  $e_i(0)$ ,  $i = 1, \dots, N$ . Switching is rather needed at any time  $t$  to keep the error within the compact sets.

One should notice from Fig. (1) that the tracking error in the case of multi-agent systems may exceed the lower (or upper) bound (in green color). Upon crossing this constraint, the system becomes unstable under the control based on  $e_i(0)$ . However, the switching based on  $e_i(t)$  provides the necessary control effort to keep the system stable.

In order to transform the constrained error of the nonlinear system (10) and (11) to an unconstrained one, a transformed error  $\epsilon_i$  is defined as

$$\epsilon_i = \psi \left( \frac{e_i(t)}{\rho_i(t)} \right) \quad (14)$$

or equivalently,

$$e_i(t) = \rho_i(t) S(\epsilon_i) \quad (15)$$

where  $\epsilon_i$ ,  $S_i(\cdot)$  and  $\psi_i^{-1}(\cdot)$  are all smooth functions,  $i = 1, 2, \dots, N$ .  $S(\cdot) = \psi^{-1}(\cdot)$  and  $S(\cdot)$  satisfy the following properties:

**Property 1.** 1)  $S_i(\epsilon_i)$  is smooth and strictly increasing.

- 2)  $-\underline{\delta}_i < S(\epsilon_i) < \bar{\delta}_i$ , if  $e_i(t) \geq 0$   
 $-\bar{\delta}_i < S(\epsilon_i) < \underline{\delta}_i$ , if  $e_i(t) < 0$

3)

$$\left. \begin{aligned} \lim_{\epsilon_i \rightarrow -\infty} S(\epsilon_i) &= -\underline{\delta}_i \\ \lim_{\epsilon_i \rightarrow +\infty} S(\epsilon_i) &= \bar{\delta}_i \end{aligned} \right\} \quad \text{if } e_i(t) \geq 0$$

$$\left. \begin{aligned} \lim_{\epsilon_i \rightarrow -\infty} S(\epsilon_i) &= -\bar{\delta}_i \\ \lim_{\epsilon_i \rightarrow +\infty} S(\epsilon_i) &= \underline{\delta}_i \end{aligned} \right\} \quad \text{if } e_i(t) < 0$$

where

$$S(\epsilon_i) = \begin{cases} \frac{\bar{\delta}_i \exp^{\epsilon_i} - \underline{\delta}_i \exp^{-\epsilon_i}}{\exp^{\epsilon_i} + \exp^{-\epsilon_i}}, & \bar{\delta}_i > \underline{\delta}_i \text{ if } e_i(t) \geq 0 \\ \frac{\underline{\delta}_i \exp^{\epsilon_i} - \bar{\delta}_i \exp^{-\epsilon_i}}{\exp^{\epsilon_i} + \exp^{-\epsilon_i}}, & \underline{\delta}_i > \bar{\delta}_i \text{ if } e_i(t) < 0 \end{cases} \quad (16)$$

Now, consider the general form of the smooth function

$$S(\epsilon_i) = \frac{\bar{\delta}_i \exp^{\epsilon_i} - \underline{\delta}_i \exp^{-\epsilon_i}}{\exp^{\epsilon_i} + \exp^{-\epsilon_i}} \quad (17)$$

and the transformed error

$$\begin{aligned} \epsilon_i &= S^{-1} \left( \frac{e_i(t)}{\rho_i(t)} \right) \\ &= \frac{1}{2} \begin{cases} \ln \frac{\bar{\delta}_i + e_i(t)/\rho_i(t)}{\bar{\delta}_i - e_i(t)/\rho_i(t)}, & \text{with } \bar{\delta}_i > \underline{\delta}_i \text{ if } e_i(t) \geq 0 \\ \ln \frac{\underline{\delta}_i + e_i(t)/\rho_i(t)}{\underline{\delta}_i - e_i(t)/\rho_i(t)}, & \text{with } \underline{\delta}_i > \bar{\delta}_i \text{ if } e_i(t) < 0 \end{cases} \end{aligned} \quad (18)$$

**Remark 5.** In the previous set of equations,  $\underline{\delta}_i$  and  $\bar{\delta}_i$  exchange values depending on the sign of  $e_i(t)$ . One should note that the highest value of both involves subtracting the absolute value of  $e_i(t)/\rho_i(t)$  and the lowest includes the addition of the absolute value of  $e_i(t)/\rho_i(t)$ .

Using this remark and the fact that  $\rho_i(t) > 0$ , equation (18) can be rewritten as

$$\epsilon_i = \frac{1}{2} \begin{cases} \ln \frac{\bar{\delta}_i + |e_i(t)|/\rho_i(t)}{\bar{\delta}_i - |e_i(t)|/\rho_i(t)}, & \text{if } e_i(t) \geq 0 \\ -\ln \frac{\underline{\delta}_i + |e_i(t)|/\rho_i(t)}{\underline{\delta}_i - |e_i(t)|/\rho_i(t)}, & \text{if } e_i(t) < 0 \end{cases} \quad (19)$$

with  $\bar{\delta}_i > \underline{\delta}_i$

Thus, the transformed error can be expressed in more compact form as follows:

$$\epsilon_i = \frac{1}{2} \text{sign}(e_i(t)/\rho_i(t)) \cdot \ln \left( \frac{\bar{\delta}_i + |e_i(t)|/\rho_i(t)}{\bar{\delta}_i - |e_i(t)|/\rho_i(t)} \right), \quad (20)$$

with  $\bar{\delta}_i > \underline{\delta}_i$

And to attenuate the effect of chattering, the following form of the transformed error will be considered

$$\epsilon_i = \frac{1}{2\sqrt{\pi}} \text{erf} \left( \frac{\xi e_i(t)}{\rho_i(t)} \right) \cdot \ln \left( \frac{\bar{\delta}_i + |e_i(t)|/\rho_i(t)}{\bar{\delta}_i - |e_i(t)|/\rho_i(t)} \right), \quad (21)$$

with  $\bar{\delta}_i > \underline{\delta}_i$

where  $\text{erf}(\xi e/\rho) = \frac{2}{\sqrt{\pi}} \int_0^{\xi e} e^{-a^2} da$ .  $\xi > 0$  is a design parameter.

**Remark 6.** The primary role of  $\xi$  is to make the  $\text{erf}(\xi e)$  as close as possible to  $\text{sign}(e)$ . Ideally,  $\xi$  is selected to be as big as possible. For instance,  $|\text{erf}(\xi e)| \approx 1$  when  $|e| > \Delta = \frac{2}{\xi}$ . Therefore, if  $\xi = 200$  then  $|\text{erf}(e)| \approx 1$  when  $|e| > 0.01$ . However, while the derivative is smooth the more one selects a big  $\xi$  the more there is a risk of chattering.

For simplification, let  $x(t) = x$ ,  $e(t) = e$ ,  $\epsilon(t) = \epsilon$  and  $\rho(t) = \rho$ . After algebraic manipulations, the derivative of transformed error when  $|e|/\rho \geq \Delta/\xi$  can be approximated by:

$$\dot{\epsilon}_i = \frac{1}{2\rho_i} \left( \frac{1}{\bar{\delta}_i + |e_i|/\rho_i} + \frac{1}{\bar{\delta}_i - |e_i|/\rho_i} \right) \left( \dot{e}_i - \frac{e_i \dot{\rho}_i}{\rho_i} \right) \quad (22)$$

**Remark 7.** As mentioned earlier, the selection of the high gain  $\xi$  can make the absolute value of the error function converge to 1 for a small value of the ratio  $|e(t)|/\rho = \Delta/\rho$ . In our analysis, we will use (22) to show that the control will generate a UUB error dynamic that will converge to a ball around zero with a radius that can be made as small as desired depending on the selection of  $\xi$ . Thus, the error may not converge to zero.

Let

$$r_i = \frac{1}{2\rho_i} \left( \frac{1}{\bar{\delta}_i + |e_i|/\rho_i} + \frac{1}{\bar{\delta}_i - |e_i|/\rho_i} \right) \quad (23)$$

From (6) and (22), the global synchronization of the transformed error can be obtained as

$$\dot{\epsilon} = R(L+B)(A_m x + f(x) + B_m u + w - \underline{f}(x_0, t)) - R\dot{\Upsilon}\Upsilon^{-1}e(t) \quad (24)$$

where the control at the level of each node is of the form  $u_i = -c\epsilon_i + \nu$ ; the value of  $\nu$  represents the part of the control action necessary to tackle the uncertainties and takes into account the estimation errors in the adaptation rule (see (35));  $\epsilon = [\epsilon_1, \dots, \epsilon_N]^T \in \mathbb{R}^N$ ,  $\Upsilon = \text{diag}\{\rho_i(t)\}$  and  $\dot{\Upsilon} = \text{diag}\{\dot{\rho}_i(t)\}$ ,  $i = 1, \dots, N$ ;  $R$  is such that  $R = \text{diag}[r_1(t), \dots, r_N(t)]$  with  $R > 0$  and  $\dot{R} < 0$ ;  $\dot{\Upsilon}\Upsilon^{-1} < 0$  with  $\lim_{t \rightarrow \infty} \dot{\Upsilon}\Upsilon^{-1} = 0$ . Before proceeding further, the following definitions are needed (see (Das & Lewis, 2010)).

**Definition 1.** The global error  $e(t) \in \mathbb{R}^N$  is uniformly ultimately bounded (UUB) if there exists a compact set  $\Omega \subset \mathbb{R}^N$  so that  $\forall e(t_0) \in \Omega$  there exists a bound  $B$  and a time  $t_f(B, e(t_0))$ , both independent of  $t_0 \geq 0$ , such that  $\|e(t)\| \leq B$  so that  $\forall t > t_0 + t_f$ .

**Definition 2.** The control node trajectory  $x_0(t)$  given by (1) is cooperative UUB with respect to solutions of node dynamics (3) if there exists a compact set  $\Omega \subset \mathbb{R}^N$  so that  $\forall x_i(t_0) - x_0(t_0) \in \Omega$ , there exist a bound  $B$  and a time  $t_f(B, x(t_0) - x_0(t_0))$ , both independent of  $t_0 \geq 0$ , such that  $\|x(t_0) - x_0(t_0)\| \leq B$ ,  $\forall i, \forall t > t_0 + t_f$ .

#### IV. ADAPTIVE PROJECTION APPROXIMATION

Using linear parametrization of nonlinear systems (for more details see A.8 in (Hovakimyan & Cao, 2010)) The agent  $i$ 's nonlinear dynamics in (1) can be written as

$$\dot{x}_i = A_{mi}x_i + B_{mi}u_i + \theta_i \|x_i\|_\infty + \sigma_i(x_i, t) \quad (25)$$

with  $\theta_i \in \mathbb{R}$  is an unknown but bounded time varying parameter and  $\sigma_i \in \mathbb{R}$  is the part that includes all unknown nonlinearities and external disturbances,  $w$ .  $\Theta_i$  and  $\Delta_i$  are known compact sets where  $\theta_i \in \Theta_i$  and  $\sigma_i \in \Delta_i$ . In the remaining of the paper, the following assumptions will be considered.

**Assumption 1.** (Hovakimyan & Cao, 2010)

- 1) Leader's states are bounded by  $\|x_0\| \leq x_0$ .
- 2) Leader's nonlinear dynamic is unknown and bounded such as  $\|f_0(x_0, t)\| \leq F_M$ .
- 3) Uniform boundedness of the unknown parameters:  $\|\theta(t)\| \leq \theta_M$  and  $\|\sigma(t)\| \leq \sigma_M$  for all  $t > 0$
- 4) Uniform boundedness of the rate of variation of parameters:  $\theta(t)$  and  $\sigma(t)$  are continuously differentiable with uniformly bounded derivatives.  $\|\dot{\theta}(t)\| \leq d_\theta < \infty$  and  $\|\dot{\sigma}(t)\| \leq d_\sigma < \infty$  for all  $t \geq 0$ .

One should note that the values of the estimation bounds are not necessary known.

**Assumption 2.** Matrix  $A_{mi}, B_{mi}$  are known and  $B_{mi}^{-1}$  exists.

Let  $\hat{\theta}$  and  $\hat{\sigma}$  be the approximation of  $\theta$  and  $\sigma$  respectively. Then,

$$\tilde{\theta}_i = \theta_i - \hat{\theta}_i \quad (26)$$

$$\tilde{\sigma}_i = \sigma_i - \hat{\sigma}_i \quad (27)$$

**Remark 8.** The communication graph is considered strongly connected. Thus, if  $b_i \neq 0$  for at least one  $i$ ,  $i = 1, \dots, N$  then  $(L+B)$  is an irreducible diagonally dominant M-matrix and hence nonsingular (Das & Lewis, 2010). The control signal of local agent  $i$  can be given by

**Lemma 1.** (see (F. L. Lewis et al., 2013) for more details.) Let  $L$  be an irreducible matrix and  $B \neq 0$  such as  $(L+B)$  is nonsingular, then we can define

$$q = [q_1, \dots, q_N]^T = (L+B)^{-1} \cdot \mathbf{1} \quad (28)$$

$$P = \text{diag}\{p_i\} = \text{diag}\{1/q_i\} \quad (29)$$

Then,  $P > 0$  and the matrix  $Q$  defined as

$$Q = P(L+B) + (L+B)^T P = P \left[ S(L+B) + (L+B)^T S \right] P \quad (30)$$

is also positive definite with  $S = P^{-1}$

The gist of the idea is that  $Q = S(L+B) + (L+B)^\top S$  is diagonally strictly dominant, and since it is a symmetric M-matrix, then it is positive definite. Based on this lemma, the following Proposition holds

**Proposition 1.** *Let  $R$  a positive definite diagonal matrix, and  $L$ ,  $B$ ,  $P$  and  $S$  as defined in Lemma 1, then the matrix  $Q$  defined as*

$$Q = PR(L+B) + (L+B)^\top RP \quad (31)$$

is positive definite.

**Proof:**

Since  $(L+B)$  is a nonsingular M-matrix and  $R > 0$  is diagonal, then  $R(L+B)$  is a non-singular M-Matrix.

$$(L+B)q = \mathbf{1} > 0 \quad (32)$$

Let  $S = \text{diag}\{q_i\}$  then

$$R(L+B)S\mathbf{1} = R(L+B)q = R\mathbf{1} > 0 \quad (33)$$

which means strict diagonal dominance of  $R(L+B)S$ .

$$\begin{aligned} Q &= PR(L+B) + (L+B)^\top RP \\ &= P \left[ R(L+B)S + S(L+B)^\top R \right] P \end{aligned} \quad (34)$$

$R(L+B)S + S(L+B)^\top R$  is symmetric and strictly diagonally dominant. Therefore,  $Q$  is positive definite.

The control signal of local nodes is given by

$$u_i = B_{mi}^{-1} \left( -c\epsilon_i - A_{mi} \tilde{x}_i - \hat{\theta}_i \|x_i\|_\infty - \hat{\sigma}_i \right) \quad (35)$$

where the control gain  $c > 0$  and the overall control signal

$$u = B_m^{-1} \left( -c\epsilon - A_m \tilde{x} - \hat{\theta} \|x\|_\infty - \hat{\sigma} \right) \quad (36)$$

with  $\|x\|_\infty = [\|x_1\|_\infty, \dots, \|x_N\|_\infty]^\top$ . Let the adaptive estimates of  $\theta$  and  $\sigma$  updated according to

$$\dot{\hat{\theta}}_i = (\Gamma_i x_i \epsilon_i^\top p_i r_i (d_i + b_i))^\top - k \Gamma_i \hat{\theta}_i \quad (37)$$

$$\dot{\hat{\sigma}}_i = (\Gamma_i \epsilon_i^\top p_i r_i (d_i + b_i))^\top - k \Gamma_i \hat{\sigma}_i \quad (38)$$

with  $\Gamma_i \in \mathbb{R}^+$  and  $k > 0$ .  $c$  and  $k$  are scalar design parameters.

**Theorem 1.** *Consider the strong connected digraph of the network in (1) with adaptive estimates in (37) and (38) satisfying Assumptions (1) and under the control law, then the distributed multi-agent system is UUB stable if the tuning gain  $k$  and  $c$  satisfy the following conditions*

$$k = \frac{c\lambda(Q)}{2} \quad (39)$$

and

$$c\lambda(Q) > \frac{1}{2}(x_m + 1)\bar{\lambda}(P)\bar{\lambda}(A) \quad (40)$$

with  $P$  and  $Q$  are defined in Lemma 1.

**Proof:** using (36), equation (6) becomes

$$\dot{\epsilon} = (L+B)(-c\epsilon + \tilde{\theta} \|x\|_\infty + \tilde{\sigma} - \underline{f}(x_0, t)) \quad (41)$$

Consider the following Lyapunov candidate function

$$V = \frac{1}{2}\epsilon^\top P\epsilon + \frac{1}{2}\tilde{\theta}^\top \Gamma^{-1}\tilde{\theta} + \frac{1}{2}\tilde{\sigma}^\top \Gamma^{-1}\tilde{\sigma} \quad (42)$$

with  $P > 0$  as defined in (29),  $\gamma \in \mathbb{R}^+$  was mentioned in (37) and  $\Gamma = \text{diag}\{\gamma_i\}$ . The derivative of (42) is

$$\dot{V} = \epsilon^\top P\dot{\epsilon} + \tilde{\theta}^\top \Gamma^{-1}\dot{\tilde{\theta}} + \tilde{\sigma}^\top \Gamma^{-1}\dot{\tilde{\sigma}} \quad (43)$$

Let  $P_1 = PR = RP$  and  $Q = P_1(L+B) + (L+B)^\top P_1$ . Using equations (36)-(39) and (41) to replace  $\dot{\epsilon}$ ,  $\dot{\tilde{\theta}}$  and  $\dot{\tilde{\sigma}}$  respectively, one can write

$$\begin{aligned} \dot{V} &= -\frac{1}{2}c\epsilon^\top Q\epsilon - \epsilon^\top P_1(L+B)\underline{f}(x_0, t) - k\tilde{\theta}^\top \tilde{\theta} - k\tilde{\sigma}^\top \tilde{\sigma} \\ &\quad + k\tilde{\sigma}^\top \sigma + k\tilde{\theta}^\top \theta + \epsilon^\top P_1 A \tilde{\theta} \|x\|_\infty + \epsilon^\top P_1 A \tilde{\sigma} \\ &\quad - \epsilon^\top P_1 \dot{\Upsilon} \Upsilon^{-1} e(t) + \tilde{\theta}^\top \Gamma^{-1} \dot{\tilde{\theta}} + \tilde{\sigma}^\top \Gamma^{-1} \dot{\tilde{\sigma}} \end{aligned} \quad (44)$$

On the other hand,

$$e(t) = \Upsilon S(\epsilon)$$

Therefore

$$\begin{aligned} \dot{V} &= -\frac{1}{2}c\epsilon^\top Q\epsilon - \epsilon^\top P_1(L+B)\underline{f}(x_0, t) - k\tilde{\theta}^\top \tilde{\theta} - k\tilde{\sigma}^\top \tilde{\sigma} \\ &\quad + k\tilde{\sigma}^\top \sigma + k\tilde{\theta}^\top \theta + \epsilon^\top P_1 A \tilde{\theta} \|x\|_\infty + \epsilon^\top P_1 A \tilde{\sigma} \\ &\quad - \epsilon^\top P \dot{\Upsilon} S(\epsilon) + \tilde{\theta}^\top \Gamma^{-1} \dot{\tilde{\theta}} + \tilde{\sigma}^\top \Gamma^{-1} \dot{\tilde{\sigma}} \end{aligned} \quad (45)$$

one should note that  $\Lambda(t) = -P\dot{\Upsilon}$  is a positive definite diagonal matrix for  $\forall t$  and  $t \xrightarrow{\text{lim}} \infty, \Lambda(t) = 0$

$$\begin{aligned} \dot{V} &= -\frac{1}{2}c\epsilon^\top Q\epsilon - \epsilon^\top P_1(L+B)\underline{f}(x_0, t) - k\tilde{\theta}^\top \tilde{\theta} - k\tilde{\sigma}^\top \tilde{\sigma} \\ &\quad + k\tilde{\sigma}^\top \sigma + k\tilde{\theta}^\top \theta + \epsilon^\top P_1 A \tilde{\theta} \|x\|_\infty + \epsilon^\top P_1 A \tilde{\sigma} + \epsilon^\top \Lambda \bar{\delta} \\ &\quad + \tilde{\theta}^\top \Gamma^{-1} \dot{\tilde{\theta}} + \tilde{\sigma}^\top \Gamma^{-1} \dot{\tilde{\sigma}} \end{aligned} \quad (46)$$

$$\bar{\delta} = \max\{\bar{\delta}_1, \dots, \bar{\delta}_N\}.$$

$$\begin{aligned} \dot{V} &\leq -\frac{1}{2}c\lambda(Q)\|\epsilon\|^2 + \bar{\lambda}(P_1)\bar{\lambda}(L+B)F_M\|\epsilon\| - k\|\tilde{\theta}\|^2 \\ &\quad - k\|\tilde{\sigma}\|^2 + \bar{\lambda}(P_1)\bar{\lambda}(A)x_M\|\tilde{\theta}\|\|\epsilon\| + \bar{\lambda}(P_1)\bar{\lambda}(A)\|\epsilon\|\|\tilde{\sigma}\| \\ &\quad + k\|\tilde{\sigma}\|\sigma_M + k\|\tilde{\theta}\|\theta_M + \bar{\delta}\bar{\lambda}(A)\|\epsilon\| + \bar{\lambda}(\Gamma^{-1})d_\theta\|\tilde{\theta}\| \\ &\quad + \bar{\lambda}(\Gamma^{-1})d_\sigma\|\tilde{\sigma}\| \end{aligned} \quad (47)$$

Define

$$\begin{aligned} z &= \left[ \|\epsilon\| \quad \|\tilde{\theta}\| \quad \|\tilde{\sigma}\| \right]^\top \\ H &= \begin{bmatrix} \frac{1}{2}c\lambda(Q) & -\frac{1}{2}\bar{\lambda}(P_1)\bar{\lambda}(A)x_M & -\frac{1}{2}\bar{\lambda}(P_1)\bar{\lambda}(A) \\ -\frac{1}{2}\bar{\lambda}(P_1)\bar{\lambda}(A)x_M & k & 0 \\ -\frac{1}{2}\bar{\lambda}(P_1)\bar{\lambda}(A) & 0 & k \end{bmatrix} \\ h &= \begin{bmatrix} \bar{\lambda}(P_1)(\bar{\lambda}(L+B)F_M + \bar{\delta}\bar{\lambda}(A)) \\ k\theta_M + \bar{\lambda}(\Gamma^{-1})d_\theta \\ k\sigma_M + \bar{\lambda}(\Gamma^{-1})d_\sigma \end{bmatrix} \end{aligned}$$

then

$$\dot{V} \leq -z^\top H z + h^\top z \quad (48)$$

then equation (48) can be written as

$$\dot{V} \leq -z^\top H z + h^\top z \quad (49)$$

and  $\dot{V} \leq 0$  if and only if  $H$  is positive definite and

$$\|z\| > \frac{\|h\|}{\underline{\lambda}(H)} \quad (50)$$

The Lyapunov candidate in (42) can be described by

$$\begin{aligned} & \frac{1}{2} \underline{\lambda}(P) \|\epsilon\|^2 + \frac{\underline{\lambda}(\Gamma^{-1})}{2} \|\tilde{\theta}\|^2 + \frac{\underline{\lambda}(\Gamma^{-1})}{2} \|\tilde{\sigma}\|^2 \\ & \leq V \leq \\ & \frac{1}{2} \bar{\lambda}(P) \|\epsilon\|^2 + \frac{\bar{\lambda}(\Gamma^{-1})}{2} \|\tilde{\theta}\|^2 + \frac{\bar{\lambda}(\Gamma^{-1})}{2} \|\tilde{\sigma}\|^2 \end{aligned} \quad (51)$$

or

$$\begin{aligned} & \frac{1}{2} z^\top \begin{bmatrix} \underline{\lambda}(P) & 0 & 0 \\ 0 & \underline{\lambda}(\Gamma^{-1}) & 0 \\ 0 & 0 & \underline{\lambda}(\Gamma^{-1}) \end{bmatrix} z \leq V \leq \\ & \frac{1}{2} z^\top \begin{bmatrix} \bar{\lambda}(P) & 0 & 0 \\ 0 & \bar{\lambda}(\Gamma^{-1}) & 0 \\ 0 & 0 & \bar{\lambda}(\Gamma^{-1}) \end{bmatrix} z \end{aligned} \quad (52)$$

Let

$$\begin{aligned} \Pi_{min} &= \begin{bmatrix} \underline{\lambda}(P) & 0 & 0 \\ 0 & \underline{\lambda}(\Gamma^{-1}) & 0 \\ 0 & 0 & \underline{\lambda}(\Gamma^{-1}) \end{bmatrix} \\ \Pi_{max} &= \begin{bmatrix} \bar{\lambda}(P) & 0 & 0 \\ 0 & \bar{\lambda}(\Gamma^{-1}) & 0 \\ 0 & 0 & \bar{\lambda}(\Gamma^{-1}) \end{bmatrix} \end{aligned}$$

(52) is equivalent to

$$\frac{1}{2} \underline{\lambda}(\Pi_{min}) \|z\|^2 \leq V \leq \frac{1}{2} \bar{\lambda}(\Pi_{max}) \|z\|^2 \quad (53)$$

then

$$V > \frac{1}{2} \underline{\lambda}(\Pi_{min}) \frac{\|h\|^2}{\underline{\lambda}(H)^2} \quad (54)$$

Defining  $c = \frac{2}{\underline{\lambda}(Q)}$ ,  $\gamma_1 = \frac{1}{2} \bar{\lambda}(P_1) \bar{\lambda}(A) x_M$ ,  $\gamma_2 = \frac{1}{2} \bar{\lambda}(P_1) \bar{\lambda}(A)$  and substitute (48), we have

$$H = \begin{bmatrix} c & -\gamma_1 & -\gamma_2 \\ -\gamma_1 & k & 0 \\ -\gamma_2 & 0 & k \end{bmatrix}$$

where  $H$  is positive definite matrix. If we select  $k = \frac{1}{2} c \underline{\lambda}(Q)$  and  $c \underline{\lambda}(Q) > \frac{1}{2} (x_M + 1) \bar{\lambda}(P) \bar{\lambda}(A)$ , then, we will have

$$\underline{\lambda}(H) = \frac{c \underline{\lambda}(Q) - \frac{1}{2} (x_M + 1) \bar{\lambda}(P) \bar{\lambda}(A)}{2}$$

And from 50, taking  $\|\cdot\|_1$  of  $h$ , define

$$s_1 = \bar{\lambda}(P_1) (\bar{\lambda}(L + B) F_M + \bar{\delta} \bar{\lambda}(A))$$

As such

$$\|z\| > \frac{s_1 \bar{\lambda}(\dot{\rho}) + k \theta_M + \bar{\lambda}(\Gamma^{-1}) d_\theta + k \sigma_M + \bar{\lambda}(\Gamma^{-1}) d_\sigma}{\underline{\lambda}(H)} \quad (55)$$

which implies

$$\|\epsilon\| > \frac{s_1 + k \theta_M + \bar{\lambda}(\Gamma^{-1}) d_\theta + k \sigma_M + \bar{\lambda}(\Gamma^{-1}) d_\sigma}{\underline{\lambda}(H)} \quad (56)$$

$$\|\tilde{\theta}\| > \frac{s_1 + k \theta_M + \bar{\lambda}(\Gamma^{-1}) d_\theta + k \sigma_M + \bar{\lambda}(\Gamma^{-1}) d_\sigma}{\underline{\lambda}(H)} \quad (57)$$

$$\|\tilde{\sigma}\| > \frac{s_1 + k \theta_M + \bar{\lambda}(\Gamma^{-1}) d_\theta + k \sigma_M + \bar{\lambda}(\Gamma^{-1}) d_\sigma}{\underline{\lambda}(H)} \quad (58)$$

Also from (53), we have

$$\|z\| \leq \sqrt{\frac{2V}{\underline{\lambda}(S)}}, \quad \|z\| \geq \sqrt{\frac{2V}{\bar{\lambda}(S)}} \quad (59)$$

Then, equation (49) can be written as

$$\dot{V} \leq -H_\alpha V + h_\alpha \sqrt{V} \quad (60)$$

with  $H_\alpha = \frac{2\underline{\lambda}(H)}{\underline{\lambda}(\Pi)}$  and  $h_\alpha = \frac{\sqrt{2}\|h\|}{\sqrt{\underline{\lambda}(\Pi)}}$  which equivalent to

$$\frac{2d\sqrt{V}}{dt} \leq -H_\alpha \sqrt{V} + h_\alpha \quad (61)$$

$$\sqrt{V} \leq \exp^{-H_\alpha t/2} \left( \sqrt{V(0)} - \frac{h_\alpha}{H_\alpha} \right) + \frac{h_\alpha}{H_\alpha} \quad (62)$$

That can be written as

$$\sqrt{V} \leq \sqrt{V(0)} \leq \sqrt{V(0)} + \frac{h_\alpha}{H_\alpha} \quad (63)$$

Finally, the algorithm of nonlinear single node dynamics such as equation (1) can be summarized briefly as 1.

- 1) Define the system known parameters  $A_{mi}, B_{mi}$ .
- 2) Define the control design parameters such as  $\Gamma_i, p_i, d_i, b_i, k$  and  $c$ .
- 3) Evaluate local error synchronization from equation (4).
- 4) Evaluate the prescribed performance function from equation (9).
- 5) Evaluate  $r_i$  from equation (23).
- 6) Evaluate transformed error from equation (21).
- 7) Evaluate control signal from equation (35).
- 8) Evaluate adaptive estimates from equations (37) and (38).
- 9) Go to step 3).

**Remark 9.** If we have  $x_i \in \mathbb{R}^n, n > 1$ , then  $u_i \in \mathbb{R}^n, f_i(x_i) \in \mathbb{R}^n, \theta_i \in \mathbb{R}^n, \sigma_i \in \mathbb{R}^n, r_i = \text{diag}\{r_{i1}, \dots, r_{in}\} \in \mathbb{R}^{n \times n}$ , then the problem can be extended easily and the estimated weight will be written as

$$\dot{\hat{\theta}}_i = (\Gamma_i \|x_i\|_\infty \epsilon_i^T (p_i \mathbb{I}_n) r_i ((d_i + b_i) \mathbb{I}_n))^\top - k \Gamma_i \hat{\theta}_i \quad (64)$$

$$\dot{\hat{\sigma}}_i = (\Gamma_i \epsilon_i^T (p_i \mathbb{I}_n) r_i ((d_i + b_i) \mathbb{I}_n))^\top - k \Gamma_i \hat{\sigma}_i \quad (65)$$

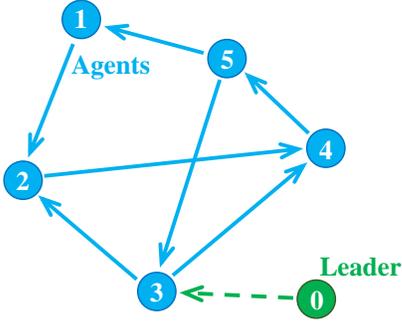


Fig. 2. Strongly connected graph of one leader and five agents.

## V. EXAMPLES AND SIMULATIONS

**Example 1:** Consider the digraph composed of five nodes strongly connected and having a single leader connected to node 3. The pinning gains between connected nodes are assumed equal to 1 as in Fig. 2. The nonlinear dynamics of the different agents are as follows

$$\begin{aligned}\dot{x}_1 &= x_1^3 + u_1 + a_1 \cos(t) \\ \dot{x}_2 &= x_2^2 + u_2 + a_2 \cos(t) \\ \dot{x}_3 &= x_3^4 + u_3 + a_3 \cos(t) \\ \dot{x}_4 &= x_4 + u_4 + a_4 \cos(t) \\ \dot{x}_5 &= x_5^5 + u_5 + a_5 \cos(t)\end{aligned}$$

$a_i, i = 1, \dots, 5$  are bounded randomly generated constant amplitudes. The leader dynamics was selected  $\dot{x}_0 = f_0(x_0, t) = 0$  with desired consensus value equal to 2. Nonlinearities and disturbances are assumed to be unknown in all nodes. The control parameters of the system are  $\rho_\infty = 0.05 \times \mathbf{1}_{5 \times 5}$ ,  $\rho_0 = 7 \times \mathbf{1}_{1 \times 5}$ ,  $l = 7 \times \mathbf{1}_{1 \times 5}$ ,  $\Gamma = 150 \mathbb{I}_{5 \times 5}$ ,  $\bar{\delta} = 7$ ,  $\underline{\delta} = 1$ ,  $c = 100$ ,  $k = 0.8$ ,  $\alpha = 20$  and  $x_0 = 2$ ,  $x(0) = [0.8230, -0.9001, -2.5351, -1.4567, -0.7553]^\top$ .

Figures 3 and 4 show the output performance, control signal and transformed error respectively for the proposed control algorithm using (18) for the transformed error. Fig. 3 shows the severe chattering in the control effort. Although the oscillation of synchronization error values satisfy the prescribed performance conditions, the switching in error signs caused switching in transformed errors as shown in Fig. 4 which consequently causes chattering in the control signal as clearly revealed in Fig. 3.

The proposed control with the new prescribed performance function as in (21) Fig. 5 and 6 show the output performance, control signal and transformed error respectively. A significant improvement in the control effort and transformed error can be clearly observed.

**Example 2: (MIMO case)** Consider the same problem as in Fig. 2 with 3 inputs and 3 outputs nonlinear systems. The nonlinear dynamics of the graph are now

$$\begin{aligned}\dot{x}_j &= Ax_j + Bu_j + \theta_j x_j + f_j(x_j) + D_j(t) \\ y_j &= Cx_j\end{aligned}$$

where  $x_j \in \mathbb{R}^{3 \times 1}$  is the state vector,  $u_j \in \mathbb{R}^{3 \times 1}$  is the input vector,  $y_j \in \mathbb{R}^{3 \times 1}$  is the output vector.  $A, B$  and  $C \in \mathbb{R}^{3 \times 3}$

are known constant matrices.

$$A = \begin{bmatrix} -20 & 22 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & -3 \end{bmatrix}, \quad B = \mathbb{I}_{3 \times 3}, \quad C = \mathbb{I}_{3 \times 3},$$

$f_j(x_j) \in \mathbb{R}^{3 \times 1}$  is the system nonlinear vector,  $D_j(t) \in \mathbb{R}^{3 \times 1}$  is the system disturbance vector,  $\theta_j \in \mathbb{R}^{3 \times 1}$ . Each of  $f_j(x_j), D_j(t), \theta_j$  are assumed to be completely unknown.

$$f_j(x_j) = \begin{bmatrix} a_{1,j} x_{3,j} x_{1,j} + 0.2 \sin(x_{1,j} a_{1,j}) \\ -a_{2,j} x_{1,j} x_{3,j} - 0.2 a_{2,j} \cos(a_{2,j} x_{3,j} t) x_{1,j} \\ a_{3,j} x_{1,j} x_{2,j} \end{bmatrix},$$

$$D_j(t) = \begin{bmatrix} 1 + b_{1,j} \sin(b_{1,j} t) \\ 1.2 \cos(b_{2,j} t) \\ \sin(0.5 b_{3,j} t) + \cos(b_{3,j} t) - 1 \end{bmatrix},$$

$$\theta_j = [\theta_j^1 \quad \theta_j^2 \quad \theta_j^3],$$

$$\theta_j^1 = \begin{bmatrix} 3c_{1,j} \sin(0.5t) \\ 0.9 \sin(0.2c_{2,j} t) \\ 0.5 \sin(0.13c_{3,j} t) \end{bmatrix},$$

$$\theta_j^2 = \begin{bmatrix} 2c_{1,j} \sin(0.4c_{1,j} t) \cos(0.3t) \\ 2.5 \sin(0.3c_{2,j} t) + 0.3 \cos(t) \\ 0.6c_{3,j} \cos(0.15t) \end{bmatrix},$$

$$\theta_j^3 = \begin{bmatrix} 0.7 \sin(0.2c_{1,j} t) \\ 1.0 \sin(0.1c_{2,j} t) \\ 1.5 \cos(0.7c_{3,j} t) + 1.6c_{3,j} \sin(0.3t) \end{bmatrix},$$

$a, b, c$  are matrices that were selected with different input elements to introduce heterogeneity into the system and therefore different control efforts have to be implemented.

$$a = \begin{bmatrix} 1.5 & 0.5 & 0.7 & 1.3 & 0.7 \\ 0.5 & 1.4 & 0.1 & 1.3 & 2.4 \\ 2.8 & 1.4 & 0.6 & 0.7 & 0.6 \end{bmatrix},$$

$$b = \begin{bmatrix} 0.5 & 1.5 & 1.1 & 1.6 & 0.3 \\ 0.7 & 1.2 & 1.3 & 0.5 & 0.3 \\ 1.1 & 1.4 & 1.6 & 0.6 & 1.0 \end{bmatrix},$$

$$c = \begin{bmatrix} 1.5 & 2.5 & 0.5 & 1.7 & 0.7 \\ 0.5 & 1.7 & 1.1 & 0.3 & 0.4 \\ 0.8 & 0.4 & 2.2 & 0.9 & 1.4 \end{bmatrix},$$

The leader's dynamics is selected such that  $x_0 = [3\cos(0.7t), 2\cos(0.8t), 1.5\cos(t)]^\top$ . The other parameters of the problem are defined as  $\rho_\infty = 0.05 \times \mathbf{1}_{3 \times 5}$ ,  $\rho_0 = 7 \times \mathbf{1}_{3 \times 5}$ ,  $l = 7 \times \mathbf{1}_{3 \times 5}$ ,  $\Gamma = 150 \mathbb{I}_{5 \times 5}$ ,  $\bar{\delta} = 7$ ,  $\underline{\delta} = 1$ ,  $c = 100$ ,  $k = 0.8$ ,  $\alpha = 50$ . Initial conditions of  $x(0) = [1.6399, 1.6639, -2.1864, 0.1160, -2.7805, -2.2175, -0.1489, 2.2989, -1.3038, 0.5571, -0.5959, 1.6760, -2.4743, 0.0488, 0.8288]$ . The robustness of the proposed controller against time variant uncertainties in parameters, time-variant disturbances and high nonlinearities are tested in this example considering the formula in (21). Fig. 7 shows the output performance of the proposed controller for the MIMO case. The control input in the connected graph is shown in Fig. 8. Errors and transformed errors for the three outputs are depicted in Fig. 9, 10 and 11. Fig. 12 shows the phase plane plot starting from different initial conditions and the synchronization to the desired trajectory. The results demonstrate the performance of the proposed robust controller.

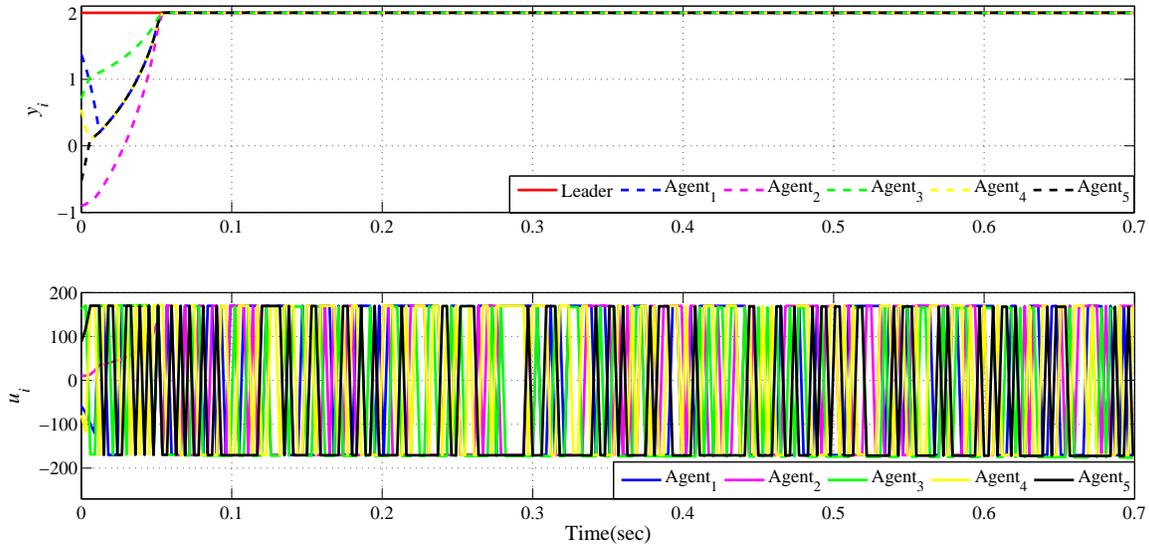


Fig. 3. Output performance and control signal using (18) for the transformed error.

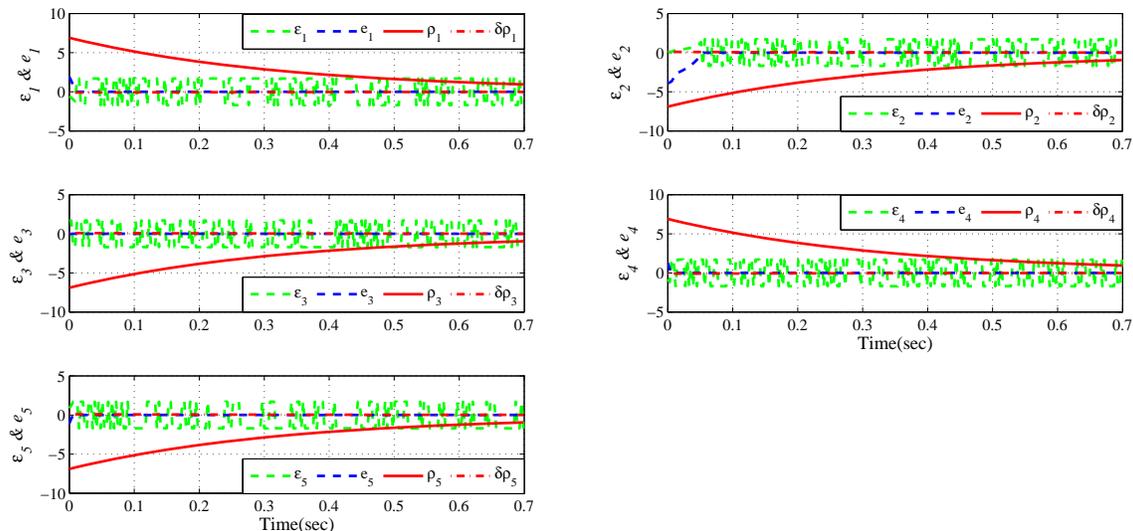


Fig. 4. Error and Transformed Error using (18) for the transformed error.

## VI. CONCLUSION

In this paper, a distributed adaptive tracking control of nonlinear uncertain multi-agent systems with prescribed performance is proposed. Under such controller, the tracking error is confined from within a predefined large set to a smaller set according to a given performance. Agents' dynamics were assumed unknown. The control law is fully distributed based on the fact that the control of each agent respects the strongly connected graph's topology and includes only the allowed local neighborhood information. The proposed approach guarantees uniform ultimate boundedness for the transformed error. Simulations include two examples to validate the robustness and smoothness of the proposed controller against highly

nonlinear heterogeneous multi-agent system with time-variant uncertain parameters and external disturbances. In future work, control of multi-agents with networks that are weakly connected or have variable topology will be studied. Under such controllers  $L$  and  $B$  could time varying and an additional but practical challenges. Systems subjects to actuator failure, saturation or hysteresis will benefit from prescribed performance control and represent interesting areas of further development. Implementation of such control approach on a real system is an area needing research effort.

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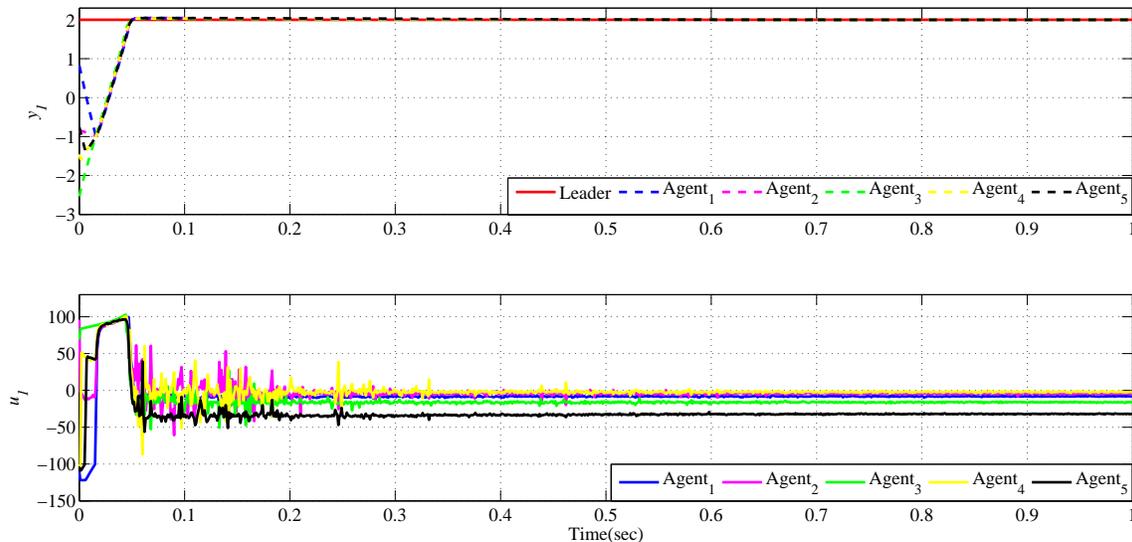


Fig. 5. Output performance and control signal with the new prescribed performance function as in (21).

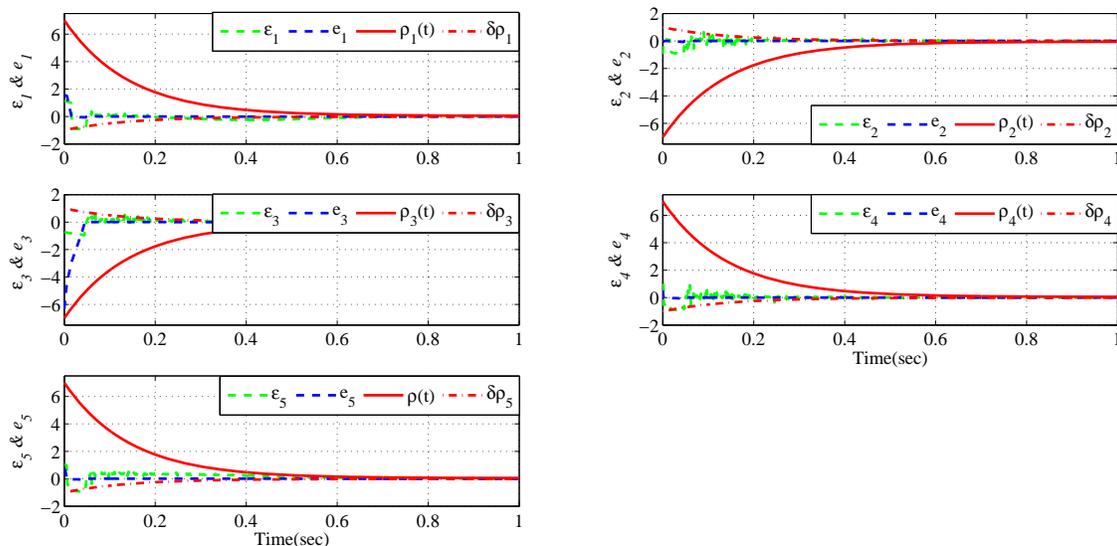


Fig. 6. Error and transformed error with the new prescribed performance function as in (21).

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#### REFERENCES

- Bechlioulis, C. P., Dimarogonas, D. V., & Kyriakopoulos, K. J. (2014). Robust control of large vehicular platoons with prescribed transient and steady state performance. In *Decision and control (cdc), 2014 IEEE 53rd annual conference on* (pp. 3689–3694).
- Bechlioulis, C. P., & Rovithakis, G. A. (2008). Robust adaptive control of feedback linearizable MIMO nonlinear systems with prescribed performance. *Automatic Control, IEEE Transactions on*, 53(9), 2090–2099.
- Bechlioulis, C. P., & Rovithakis, G. A. (2009). Adaptive control with guaranteed transient and steady state tracking error bounds for strict feedback systems. *Automatica*, 45(2), 532–538.
- Bechlioulis, C. P., & Rovithakis, G. A. (2014). A low-complexity global approximation-free control scheme with prescribed performance for unknown pure feedback systems. *Automatica*, 50(4), 1217–1226.
- Cao, Y., & Ren, W. (2012). Distributed coordinated tracking with reduced interaction via a variable structure approach. *Automatic Control, IEEE Transactions on*, 57(1), 33–48.
- Chopra, N., & Spong, M. W. (2006). Passivity-based control of multi-agent systems. In *Advances in robot control* (pp. 107–134). Springer.

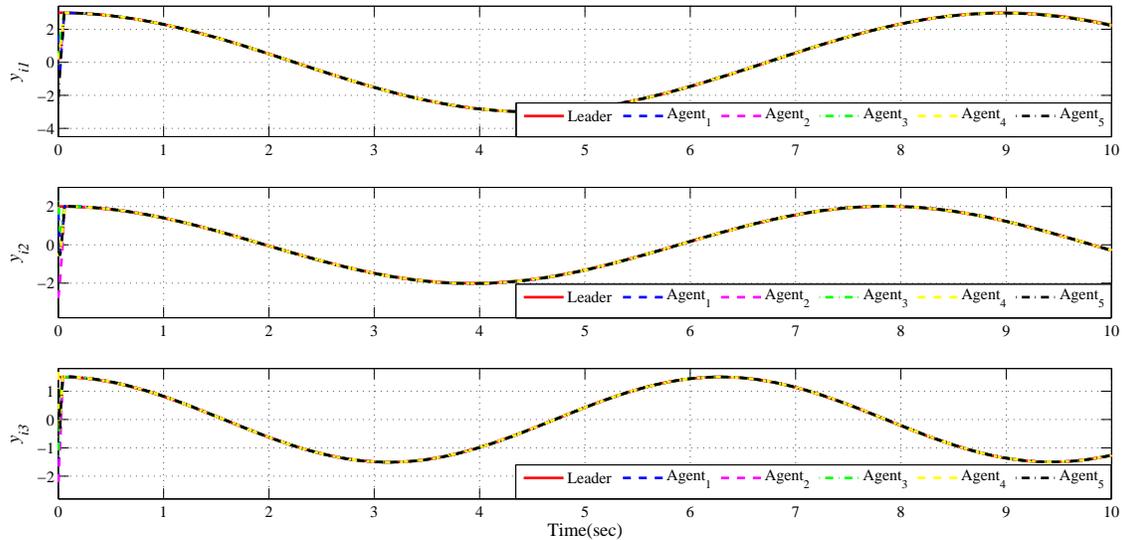


Fig. 7. Output performance in the MIMO case using (21).

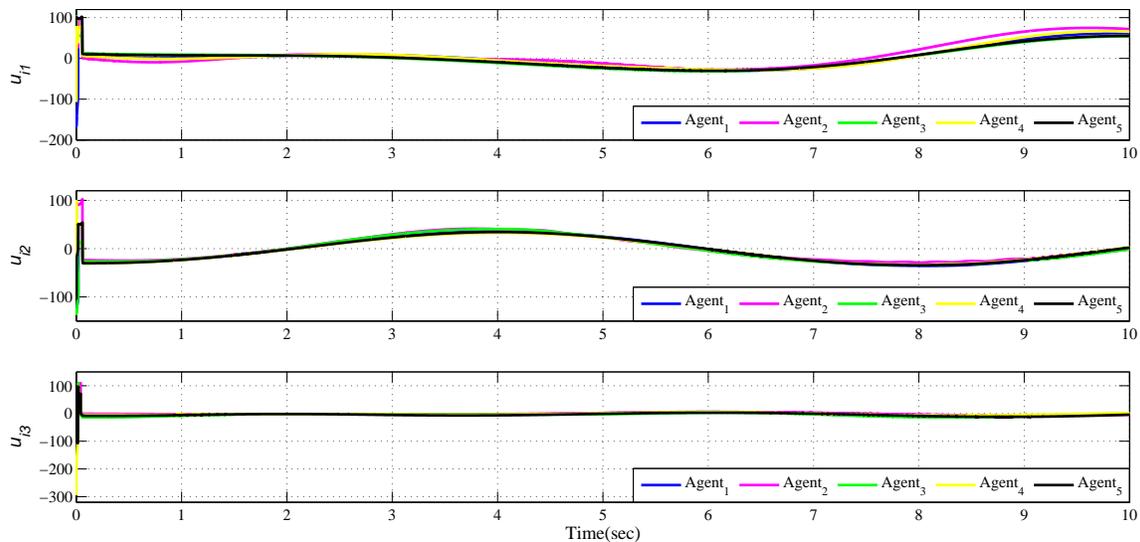


Fig. 8. Control Signal in the MIMO case using (21).

Das, A., & Lewis, F. L. (2010). Distributed adaptive control for synchronization of unknown nonlinear networked systems. *Automatica*, 46(12), 2014–2021.

El-Ferik, S., Qureshi, A., & Lewis, F. L. (2014). Neuro-adaptive cooperative tracking control of unknown higher-order affine nonlinear systems. *Automatica*, 50(3), 798–808.

Fax, J. A., & Murray, R. M. (2004). Information flow and cooperative control of vehicle formations. *Automatic Control, IEEE Transactions on*, 49(9), 1465–1476.

Hovakimyan, N., & Cao, C. (2010). *L1 adaptive control theory: Guaranteed robustness with fast adaptation* (Vol. 21). Siam.

Khoo, S., Xie, L., & Man, Z. (2009). Robust finite-time consensus tracking algorithm for multirobot systems. *Mechatronics, IEEE/ASME Transactions on*, 14(2), 219–228.

Lewis, F. L., Zhang, H., Hengster-Movric, K., & Das, A. (2013). *Cooperative control of multi-agent systems: optimal and adaptive design approaches*. Springer Science & Business Media.

Lewis, F. W., Jagannathan, S., & Yesildirak, A. (1998). *Neural network control of robot manipulators and non-linear systems*. CRC Press.

Li, X., Wang, X., & Chen, G. (2004). Pinning a complex dynamical network to its equilibrium. *Circuits and Sys-*

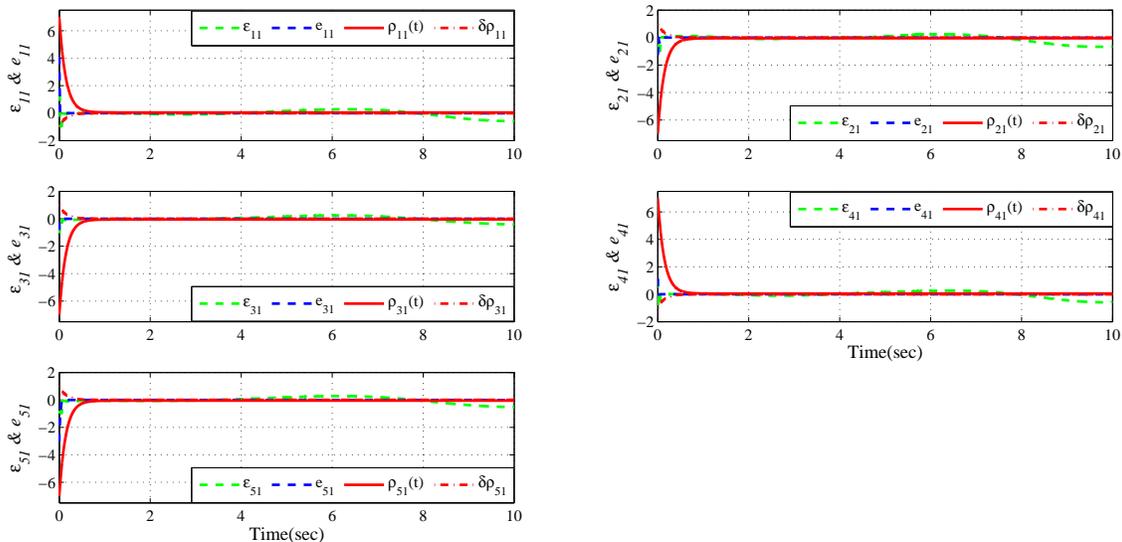


Fig. 9. Error and Transformed Error in the MIMO case using (21) for  $x_{1,j}$  where  $j = 1, \dots, 5$ .

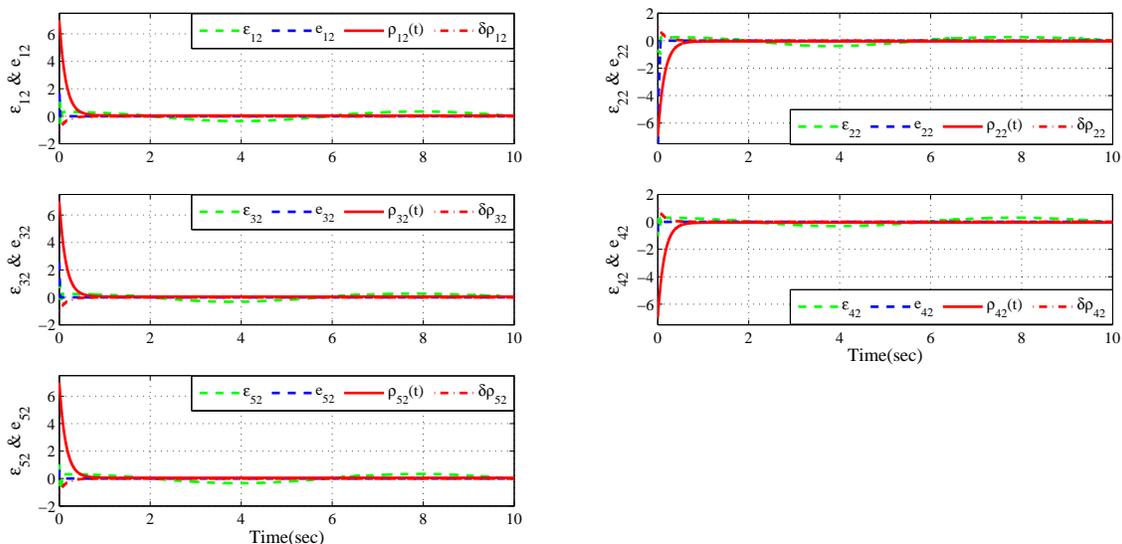


Fig. 10. Error and Transformed Error in the MIMO case using (21) for  $x_{2,j}$  where  $j = 1, \dots, 5$ .

tems I: Regular Papers, *IEEE Transactions on*, 51(10), 2074–2087.

Li, Y., & Tong, S. (2015). Prescribed performance adaptive fuzzy output-feedback dynamic surface control for nonlinear large-scale systems with time delays. *Information Sciences*, 292, 125–142.

Mohamed, H. A. H. (2014). *Improved robust adaptive control of high-order nonlinear systems with guaranteed performance* (M.Sc). King Fahd University Of Petroleum & Minerals.

Na, J. (2013). Adaptive prescribed performance control of nonlinear systems with unknown dead zone. *International Journal of Adaptive Control and Signal*

*Processing*, 27(5), 426–446.

Na, J., Chen, Q., Ren, X., & Guo, Y. (2014). Adaptive prescribed performance motion control of servo mechanisms with friction compensation. *Industrial Electronics, IEEE Transactions on*, 61(1), 486–494.

Olfati-Saber, R., Fax, J. A., & Murray, R. M. (2007). Consensus and cooperation in networked multi-agent systems. *Proceedings of the IEEE*, 95(1), 215–233.

Qu, Z. (2009). *Cooperative control of dynamical systems: applications to autonomous vehicles*. Springer Science & Business Media.

Ren, W., & Beard, R. W. (2008). *Distributed consensus in multi-vehicle cooperative control*. Springer.

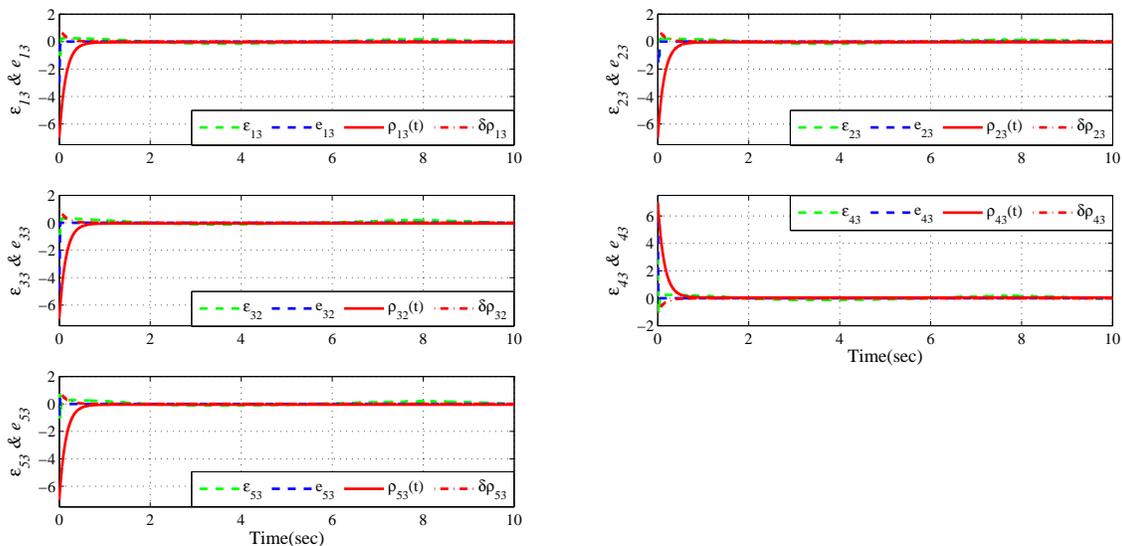


Fig. 11. Error and Transformed Error in the MIMO case using (21) for  $x_{3,j}$  where  $j = 1, \dots, 5$

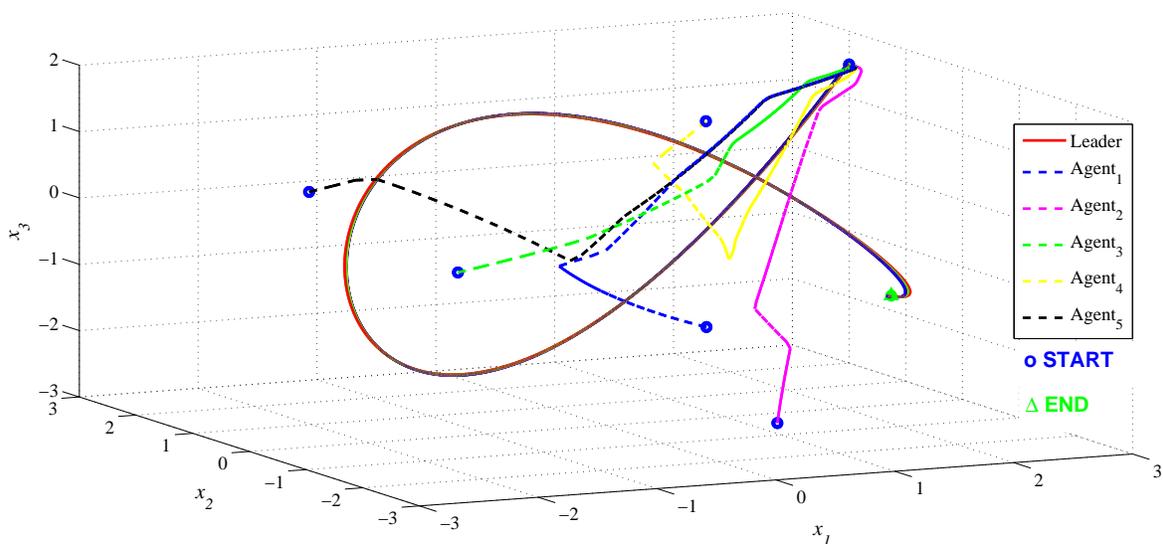


Fig. 12. Phase Plan for motion synchronization in the MIMO system case under different initial conditions.

Ren, W., Beard, R. W., & others. (2005). Consensus seeking in multiagent systems under dynamically changing interaction topologies. *IEEE Transactions on automatic control*, 50(5), 655–661.

Sun, Y., & Liu, H. (2014). Fuzzy Adaptive Prescribed Performance Control for MIMO Uncertain Chaotic Systems in Nonstrict Feedback Form. *Discrete Dynamics in Nature and Society*, 2014.

Theodoridis, D. C., Boutalis, Y. S., & Christodoulou, M. A. (2012). Direct adaptive neuro-fuzzy trajectory tracking of uncertain nonlinear systems. *International Journal of Adaptive Control and Signal Processing*, 26(7), 660–688.

Tong, S., Sui, S., & Li, Y. (2015). Fuzzy adaptive output feedback control of mimo nonlinear systems with partial tracking errors constrained. *Fuzzy Systems, IEEE Transactions on*, 23(4), 729–742.

Tong, S., Wang, T., & Li, Y. (2014). Fuzzy adaptive actuator failure compensation control of uncertain stochastic nonlinear systems with unmodeled dynamics. *Fuzzy Systems, IEEE Transactions on*, 22(3), 563–574.

Wang, J., Hovakimyan, N., & Cao, C. (2010). Verifiable adaptive flight control: unmanned combat aerial vehicle and aerial refueling. *Journal of guidance, control, and dynamics*, 33(1), 75–87.

Wang, W., Wang, D., & Peng, Z. (2016). Predictor-based adap-

- tive dynamic surface control for consensus of uncertain nonlinear systems in strict-feedback form. *International Journal of Adaptive Control and Signal Processing*.
- Yang, Y., Ge, C., Wang, H., Li, X., & Hua, C. (2015). Adaptive neural network based prescribed performance control for teleoperation system under input saturation. *Journal of the Franklin Institute*, 352(5), 1850–1866.
- Yang, Y., Yue, D., & Dou, C. (2016). Distributed adaptive output consensus control of a class of heterogeneous multi-agent systems under switching directed topologies. *Information Sciences*, 345, 294–312.
- Zhang, H., & Lewis, F. L. (2012). Adaptive cooperative tracking control of higher-order nonlinear systems with unknown dynamics. *Automatica*, 48(7), 1432–1439.
- Zhao, L., & Jia, Y. (2016). Neural network-based adaptive consensus tracking control for multi-agent systems under actuator faults. *International Journal of Systems Science*, 47(8), 1931–1942.
- Zhao, Y., Duan, Z. S., Wen, G. H., Li, Z. K., & Chen, G. R. (2015). Fully distributed tracking control for non-identical multi-agent systems with matching uncertainty. *International Journal of Adaptive Control and Signal Processing*, 29(8), 1024–1037. Retrieved from <http://dx.doi.org/10.1002/acs.2520> doi: 10.1002/acs.2520
- Zhou, N., Xia, Y., Fu, M., & Li, Y. (2015). Distributed cooperative control design for finite-time attitude synchronisation of rigid spacecraft. *IET Control Theory & Applications*.

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