Supplementary

S.I. Calculating Size of a Virtual Obstacle

The navigation space is an indoor environment, and has adequate features for detection and identification. Moreover, recent image procesing techniques are equipped with algorithms to predict size of a detected object with substantial accuracy.
With these assumptions, let us consider there are several closely spaced obsacles with no passage space in between (constraint boundaries will overlap each other). The collection of obstacles can be grouped together to form a larger obstacle. The centroid of the set of obstacles is computed by considering the centroid of each individual obstacle as a vertex of a planar polygon. The resultant point forms the centre of the virtual obstacle, (x_{obs}, y_{obs}). Let, the largest line segment joining the centroid of the set of obstacles and any of the centroids of the individial obstacles be denoted as r_1. Let, the line segment joinging the corresponding obstacle and its farthest vetrex be termed as r_2. Then, $r_1 + r_2$ forms the radius r_{obs} of the virtual obstacle. Any arbitrarily shaped obstacle can be treated like an irregular non-intersecting polygon. Hence,a similar argument can be applied to visible portions of the same. Figure 1 explains the mechanism pictorially. This method promotes a far more safe maneuver and is robust in avoiding local minima conditions compared to a standard wall-following strategy.

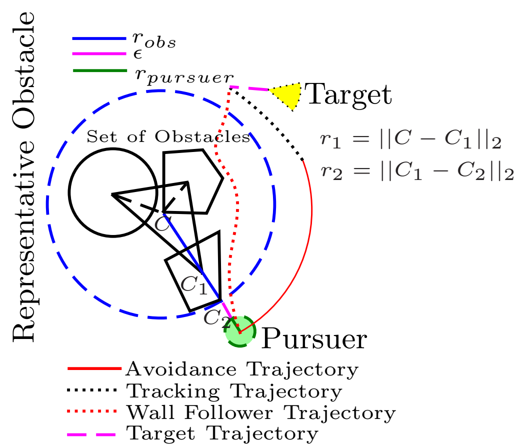

Figure 1. A virtual obstacle can be created from the closely located obstacles of arbitrary shapes, which allow no passage through them.

S.II. Derivations of the ODEs for Tracking and Collision Avoidance

Differentiating (4) (also, applicable to equation (13)) with respect to the control variables, $v(t)$ and $\theta(t)$ one can arrive at the following equations (S.1).

$$\begin{gathered} v(t) = \frac{1}{2w_v}\Big(\lambda_1(t)\cos(\theta(t)) + \lambda_2(t)\sin(\theta(t))\Big) \\ \Big(- \lambda_1 v(t)\sin(\theta(t)) + \lambda_2 v(t)\cos(\theta(t))\Big) = 0 \end{gathered} \tag{S.1}$$

Assuming the vehicle does not stop in the mission interval (persistent excitation condition), that is $v(t) > 0, \forall t \in [0, T_{int}]$, we can write the expression for heading to be $\theta(t) = \tan^{-1}\left(\frac{\lambda_2(t)}{\lambda_1(t)}\right)$. This further enables us to express the terms, $\cos(\theta(t))$ and $\sin(\theta(t))$ in terms of $\lambda_1(t)$ and $\lambda_2(t)$ as given in equation (S.2).

$$\begin{aligned}\sin(\theta(t)) &= \frac{\lambda_2(t)}{\sqrt{\lambda_1^2(t) + \lambda_2^2(t)}}\\ \cos(\theta(t)) &= \frac{\lambda_1(t)}{\sqrt{\lambda_1^2(t) + \lambda_2^2(t)}}\end{aligned} \tag{S.2}$$

Replacing the angles defined above, into the expressions for velocity in equation (S.1), we finally arrive at equation (5). Similarly the expressions for $v(t)$ and $\theta(t)$ can be derived for equation (14).

Equation (5) can be used to replace the control variables in equation (2) to yield the time derivatives of $x(t)$ and $y(t)$ in equations (6) (Also, equation (14) can be used to replace the control variables in equation (2) to obtain the time derivatives of the states in (15)). Therefore, we obtain equation (S.3).

$$\begin{aligned}\dot{x}(t) &= \frac{\lambda_1(t)}{2w_v}\\ \dot{y}(t) &= \frac{\lambda_2(t)}{2w_v}\end{aligned} \tag{S.3}$$

Equation (6) and (15) represent the derivatives of the Hamiltonians defined in equation (4) and (13) respectively, with respect to the states and costates. Equations (6) and (15) have been derived according to the relation (S.4), where $\mathcal{H}$ may represent the Hamiltonian constructed for tracking or collision avoidance.

$$\begin{aligned}\frac{\partial\mathcal{H}}{\partial\lambda_1(t)} &= \dot{x}(t)\\ \frac{\partial\mathcal{H}}{\partial\lambda_2(t)} &= \dot{y}(t)\\ -\frac{\partial\mathcal{H}}{\partial x(t)} &= \dot{\lambda}_1(t)\\ -\frac{\partial\mathcal{H}}{\partial y(t)} &= \dot{\lambda}_2(t)\end{aligned} \tag{S.4}$$

S.III. Existence-Uniqueness of Tracking Trajectory

Existence and uniqueness of the tracking trajectory can be proved by showing that the second order system corresponding to equation (6) are Lipschitz continuous functions for given state boundary values. Let us define a new set of variables, $\tilde{x}(t)$, $\tilde{y}(t)$, $\tilde{\lambda}_1(t)$, $\tilde{\lambda}_2(t)$, such that

$$\begin{aligned}\tilde{x}(t) &= x(t) - x_{tar}(t), \quad \tilde{\lambda}_1(t) = \lambda_1(t) - 2w_v v_{tar}\cos(\theta_{tar})\\ \tilde{y}(t) &= y(t) - y_{tar}(t), \quad \tilde{\lambda}_2(t) = \lambda_2(t) - 2w_v v_{tar}\sin(\theta_{tar})\end{aligned} \tag{S.5}$$

Equations in (6) are thereby linearly transformed into equation (S.6).

$$\begin{aligned}\dot{\tilde{x}}(t)&=\frac{\tilde{\lambda}_1(t)}{2w_v},\ \dot{\tilde{\lambda}}_1(t)=2w_d(\tilde{x}(t))\\ \dot{\tilde{y}}(t)&=\frac{\tilde{\lambda}_2(t)}{2w_v},\ \dot{\tilde{\lambda}}_2(t)=2w_d(\tilde{y}(t))\end{aligned}\tag{S.6}$$

The transformed equations in (S.6) can be re-written as second order differential equations (S.7) in $\tilde{x}$ and $\tilde{y}$, where, $f(\tilde{x}) = -(\frac{w_d}{w_v})\tilde{x}(t)$ and $f(\tilde{y}) = -(\frac{w_d}{w_v})\tilde{y}(t)$.

$$\ddot{\tilde{x}}(t)-\left(\frac{w_d}{w_v}\right)\tilde{x}(t)=0,\quad \ddot{\tilde{y}}(t)-\left(\frac{w_d}{w_v}\right)\tilde{y}(t)=0\tag{S.7}$$

(S.7) presents a second order boundary value problem with known $\tilde{x}$ and $\tilde{y}$ at both the boundaries. Suppose, $(\tilde{x}_1, \tilde{y}_1)$ and $(\tilde{x}_2, \tilde{y}_2)$ are two tuples satisfying equation (S.7) and $w_o = -(\frac{w_d}{w_v})$. Triangle inequality can be applied to write equation (S.8).

$$\begin{aligned}|f(\tilde{x}_1)-f(\tilde{x}_2)|&=|(w_o)(\tilde{x}_1-\tilde{x}_2)|\leq|w_o|\cdot|\tilde{x}_1-\tilde{x}_2|\\ |f(\tilde{y}_1)-f(\tilde{y}_2)|&=|(w_o)(\tilde{y}_1-\tilde{y}_2)|\leq|w_o|\cdot|\tilde{y}_1-\tilde{y}_2|\end{aligned}\tag{S.8}$$

$f(\tilde{x})$ and $f(\tilde{y})$ are assumed to be continuous bounded functions in time over $[0, T_{int}]$. To prove that $f(\tilde{x})$ and $f(\tilde{y})$ are Lipschitzian, non-negative constants, K_x and K_y can always be chosen, such that, $|w_o|\leq K_x$ and $|w_o|\leq K_y$. A positive real $T_{int}<\frac{\pi}{max\{K_x,K_y\}}$ assures existence of exactly one solution [31].

S.IV. Existence-Uniqueness of Avoidance Trajectory

It can be shown by Lipschitz continuity, that one and only one solution exists. For a split boundary value problem, existence and uniqueness of the solution are simultaneously valid in the domain $[t_3, t_3 + \Delta]$, only if the boundary values are zero. Linear transformations are applied to $x(t)$ and $y(t)$ and new variables are defined as $\tilde{x}(t)$, $\tilde{y}(t)$.

Let, $\tilde{x}(t)=x(t)-l_x(t)$ and $\tilde{y}(t)=y(t)-l_y(t)$, such that

$$\begin{aligned}\tilde{x}(t_3)&=0,\quad \tilde{y}(t_3)=0\\ \dot{\tilde{x}}(t_3+\Delta)&=0,\quad \dot{\tilde{y}}(t_3+\Delta)=0\end{aligned}\tag{S.9}$$

The linear functions of time are defined as in (S.10) such that (S.11) holds.

$$\begin{aligned}l_x(t)&=x(t_3)-(t_3-t)\frac{\lambda_1(t_3+\Delta)}{2w_v}\\ l_y(t)&=y(t_3)-(t_3-t)\frac{\lambda_2(t_3+\Delta)}{2w_v}\end{aligned}\tag{S.10}$$

$$\begin{aligned}l_x(t_3)&=x(t_3),\quad \dot{l}_x(t_3+\Delta)=\dot{x}(t_3+\Delta)\\ l_y(t_3)&=y(t_3),\quad \dot{l}_y(t_3+\Delta)=\dot{y}(t_3+\Delta)\end{aligned}\tag{S.11}$$

$\dot{l}_x(t)$ and $\dot{l}_y(t)$ are constants and so, $\ddot{\tilde{x}}(t)$=$\ddot{x}(t)$ and $\ddot{\tilde{y}}(t)$=$\ddot{y}(t)$. This leads to a pair of second order ordinary differential equations (S.12) in $\tilde{x}(t)$ and $\tilde{y}(t)$, where, p=$\frac{\lambda_3}{w_v}$ (λ_3 is a constant).

$$\begin{aligned}\ddot{\tilde{x}}(t)+p(\tilde{x}(t)+l_x(t)-x_{obs})=0\\ \ddot{\tilde{y}}(t)+p(\tilde{y}(t)+l_y(t)-y_{obs})=0\end{aligned} \tag{S.12}$$

Defining $f(t,\tilde{x})$=$p(\tilde{x}(t)+l_x(t)-x_{obs})$ and $f(t,\tilde{y})$=$p(\tilde{y}(t)+l_y(t)-y_{obs})$, we arbitrarily choose $(\tilde{x}_1,\tilde{y}_1)$ and $(\tilde{x}_2,\tilde{y}_2)$ to be two tuples satisfying equation (S.12) at any $t\in[t_3,t_3+\Delta]$. Applying triangle inequality gives (S.13).

$$\begin{aligned}|f(t,\tilde{x}_1)-f(t,\tilde{x}_2)|&=|p(\tilde{x}_1(t)+l_x(t)-x_{obs})-p(\tilde{x}_2(t)+l_x(t)-x_{obs})|\\ &\leq|p|\cdot|\tilde{x}_1(t)-\tilde{x}_2(t)\|\\ |f(t,\tilde{y}_1)-f(t,\tilde{y}_2)|&=|p(\tilde{y}_1(t)+l_y(t)-y_{obs})-p(\tilde{y}_2(t)+l_y(t)-y_{obs})|\\ &\leq|p|\cdot|\tilde{y}_1(t)-\tilde{y}_2(t)|\end{aligned} \tag{S.13}$$

Hence, $f(t,\tilde{x})$ and $f(t,\tilde{y})$ are Lipschitzian as, it is always possible to choose two non-negative constants, K_x and K_y, such that, $|p|\leq K_x$ and $|p|\leq K_y$. Assuming $f(t,\tilde{x})$ and $f(t,\tilde{y})$ are continuous in $t\in[t_3,t_3+\Delta]$, both existence and uniqueness of the transformed problem is guaranteed [31]. Since, $l_x(t)$ and $l_y(t)$ are also Lipschitz continuous, the original problem will also have one and only one solution.