Noniterative Coordination in Multilevel Systems

Nonconvex Optimization and Its Applications

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Noniterative Coordination in Multilevel Systems

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Description and purpose of the work

The monograph discusses scientific research directed towards hierarchical system theory. It presents new theoretical results related to a new coordination strategy called "noniterative coordination". The theoretical background of this coordination is developed. Several practical implementations like traffic lights optimization, optimal data communications and others are given. The "noniterative coordination" extends the areas of multilevel methodology from traditional off-line applications like system design, planning, optimal problem solution, off-line resource allocation towards on-line processes like real time control, system management, on-line optimization and decision making. The main benefit of noniterative coordination is the reduced information transfer between hierarchical levels during evaluation of the global optimal control. This smaller size information transfer speeds up the management of the hierarchical system. The acceleration of the coordination results according to the following noniterative operational sequence:

a/ evaluation of the "suggestions" generated by the low level subsystems, subject to their local resources and goal functions;

b/ the coordinator modifies or confirms these "suggestions" in order to find the optimal solution of the global problem, solved by the full hierarchical system.

As the information transmitted between the levels is limited, noniterative coordination extends the multilevel approach for real time optimization processes . The monograph deals with two - level hierarchical systems and static optimization models. It can be used as a reference in lecture courses on optimal control for electrical and mechanical engineers. Moreover, it can be used for similar purposes by lectures in applied mathematics and informatics.

This research presents fundamentals and new directions of hierarchical multilevel theory and large scale systems. It can stimulate further investigations on modern control theory and practice.

The monograph is divided into four chapters and one appendix. The first chapter gives a general setting of the coordination in two - level hierarchical systems. It has been proved that the iterative nature of coordination is the general drawback which prevents the application of the multilevel theory for real time decision making, control and management processes. An extensive survey of hierarchical mathematical models used in system theory is given.

The second chapter presents the noniterative coordination methodology. Heuristic assumptions are explained which allow decreasing the coordination computations up to a single iteration. A formal description of the noniterative coordination is given.

The third chapter presents the linear-quadratic modeling of noniterative coordination. Its application to quadratic optimization problems allows to find explicit analytical solutions. These analytical solutions can be applied for real time and for closed loop control for non-stationary perturbed systems. The linear quadratic model of noniterative coordination has been implemented for traffic light control on street junctions; for optimal data transfer in simplex radio links; for optimal control of distributed systems.

The fourth chapter discusses mathematical models of noniterative coordination applying rational Pade functions. These models extend the feasible area of applications of noniterative coordination. Examples are given for non-linear optimization problem solutions.

The monographs uses matrix operations intensively. Several working rules for matrix differentiation of explicit and non-explicit scalar and matrix functions towards scalar and vector arguments are derived. Exercises and questions accompany the exposition which make the monograph suitable for a teaching tool.

The monograph has been written in very close communication between the authors: 1.1, 1.2, 1.3, 2.1, 3.5, 3.6, 3.7, 3.9, 3.12, 3.14, A1 – are written by K.Stoilova and 1.4, 1.5, 1.6, 2.2, 2.3, 3.1, 3.2, 3.3, 3.4, 3.8, 3.10, 3.11, 3.13, chapter IV, A2, A3 – by T.Stoilov.

Although this book is not a complete survey of the field of hierarchical system theory, the authors believe that readers will find it useful also in large scale systems, optimization and control theory.

PREFACE

Multilevel decision theory arises to resolve the contradiction between increasing requirements towards the process of design, synthesis, control and management of complex systems and the limitation of the power of technical, control, computer and other executive devices, which have to perform actions and to satisfy requirements in real time. This theory rises suggestions how to replace the centralised management of the system by hierarchical co-ordination of sub-processes. All sub-processes have lower dimensions, which support easier management and decision making. But the sub-processes are interconnected and they influence each other. Multilevel systems theory supports two main methodological tools: decomposition and co-ordination. Both have been developed and implemented in practical applications concerning design, control and management of complex systems.

In general, it is always beneficial to find the best or optimal solution in processes of system design, control and management. The real tendency towards the best (optimal) decision requires to present all activities in the form of a definition and then the solution of an appropriate optimization problem. Every optimization process needs the mathematical definition and solution of a well stated optimization problem. These problems belong to two classes: static optimization and dynamic optimization.

Static optimization problems are solved applying methods of mathematical programming: conditional and unconditional optimization. Dynamic optimization problems are solved by methods of variation calculus: Euler-Lagrange method; maximum principle; dynamical programming.

Multilevel theory makes use and develops decomposition approaches applied for solving both mathematical programming and variation problems. Such decomposition techniques allow the original complex optimization problem to be reduced to a set of low order optimization subproblems. Then the solution of the complex problem is found as a vector of the subproblem solutions. To find the aggregated solution, the subproblems are influenced, called coordination by means to give the appropriate subproblem solutions. As a result, instead of direct solution of a high order and complex optimization problem, multilevel theory manages and coordinates the solutions of low order optimization subproblems, which will give solutions equal to the initial global optimization problem. Such methodology, consisting of decomposition to subproblems and coordination between them, leads to the model of hierarchical systems. This is the classical model in multilevel theory, and it is applied sequentially for systems with multiple hierarchical levels. The basic investigations in the multilevel theory are done for two - level control system, see Fig. 1:

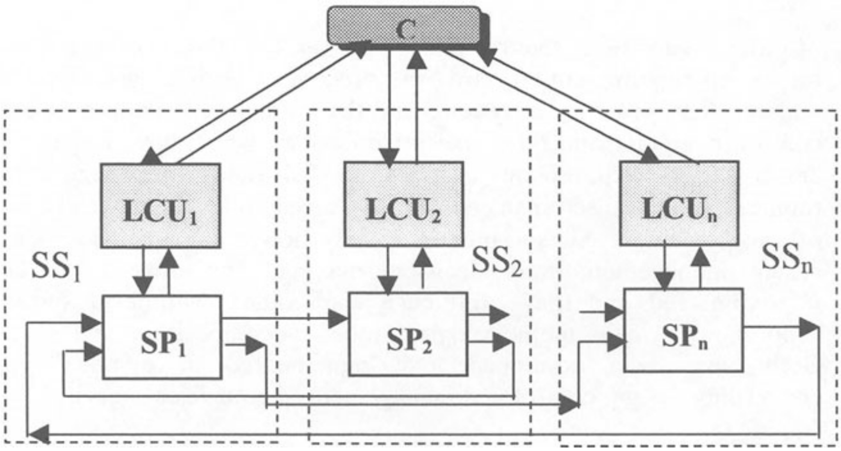

Fig.1. Two-level hierarchical system

The two-level hierarchical system is made up of N interconnected subsystems SS_i , $i=1,..,N$. Each subsystem SS_i consists the couple of a local control unit LCU_i and a subprocess SP_i . The LCU_i influences and manages only the subprocess SP_i . The subprocesses are connected to each other. The management of the subsystem is modelled by the solutions of appropriate optimisation subproblems. LCU_i evaluates its control independently, neglecting the interconnection links between the SP_i , $i=1,..,N$. This independence of the LCU_i management and the real connections between the SP_i leads to a contradiction between the subsystems. These contradictions are expressed in non-satisfaction of balances and equilibrium of material, energy, finance, information flows between the subsystems. To overcome these errors, a new second level control unit, named coordinator, is introduced in the total control scheme. The coordination task is to influence over the subsystems and their control decisions by means to achieve equilibrium in the subsystem connections. This influence is expressed as modifications of the local goals of the subsystem management and/or by appropriate allocation of the common system resources. The optimal coordination will smooth the contradictions between the subsystems and the interconnection flows will conform to the subsystem management.

In a hierarchical system with two levels three general optimisation problems are defined and solved: a global initial optimisation problem; local subsystem optimisation subproblems; a coordination problem.

The global optimisation problem is the one originally defined which has to be solved by the multilevel hierarchical system. Nor the coordinator or the local subsystems can solve it.

The local optimisation subproblems are the mathematical description of subsystem management. They differ from the global problem and they always have lower dimensions then the original one.

The coordination problem is solved by the coordinator. It differs both from the global one and the local subproblems.

These three problems constitute the functional structure of the hierarchical system. Respectively the management of the hierarchical system consists of a sequence of coordination and subproblem solutions and transfer of these solutions between the hierarchical levels, see Fig.2. In the terminology of [Mesarovich at al.,1973] this research deals with multilevel hierarchy which concerns decision making problem solution, system control and management.

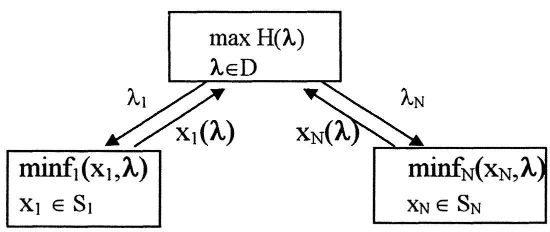

Fig.2. Subproblems in a hierarchical system (an illustrative example)

Multilevel systems operate in an iterative manner. Multiple evaluations of the coordination vector λ are performed and respective calculations of the local controls $x_i(\lambda)$, $i=1,N$ are done. Iterative information transfer takes place: λ from the coordinator to the subsystems and the corresponding solutions $x_i(\lambda)$ from the subsystems to the coordinator. In the end of this iterative computational process the optimal solution of the initial global optimization problem is found. This manner of computation and subproblem solutions determine the iterative character of hierarchical system management.

The iterative methodology of decomposition and coordination has been applied in various practical hierarchical systems. Respectively, appropriate mathematical models for multilevel management have been derived. The common feature in these models is the equality which is supposed to hold between the multilevel system management and the sequence of optimisation problem solution. To prove this equality some examples are given below which examines the hierarchical management of the complex system. Then a survey of a range of mathematical models used in hierarchical system theory is presented.