Appendix to

Optimal Contract Design in Sustainable Supply Chain: Interactive Impacts of Fairness Concern and Overconfidence

Appendix A: Proof of Section 4

For the sake of convenience, we denote $A = 4\beta - (\alpha - \delta)^2$, $B = 8\beta(2\lambda + 1) - (\lambda + 1)(\alpha - \delta)^2$, $C = 4\beta[2\lambda(2\lambda\phi + 2\phi + 1) + 1] - (\lambda + 1)^2(\alpha - \delta)^2$, and $D = 8\beta(2\lambda + 1)(1 - \eta) - (\lambda + 1)(\alpha - \delta)^2$. Under the assumption that $4\beta > \alpha(\alpha - \delta)$, we can derive that A, B, C, and D are all positive.

Proof of Lemma 1. By taking the first derivative of Π_{BC}^{SC} with respect to p and e, we can obtain

$$\frac{\partial \Pi_{BC}^{SC}}{\partial p} = \alpha e + \delta e + a - 2p,$$

$$\frac{\partial \Pi_{BC}^{SC}}{\partial e} = -\delta(\alpha e + a - p) + (-\delta e + p)\alpha - 2\beta e.$$

Therefore, the Hessian matrix is

$$H(p,e) = \begin{bmatrix} \frac{\partial^2 \Pi_{BC}^{SC}}{\partial p^2} & \frac{\partial^2 \Pi_{BC}^{SC}}{\partial p \partial e} \\ \frac{\partial^2 \Pi_{BC}^{SC}}{\partial e \partial p} & \frac{\partial^2 \Pi_{BC}^{SC}}{\partial e^2} \end{bmatrix} = \begin{bmatrix} -2 & \alpha + \delta \\ \alpha + \delta & -2\alpha\delta - 2\beta \end{bmatrix}.$$

The Hessian matrix of Π_{BC}^{SC} is a negative definite for all p and e because $H(p,e)$ satisfies the conditions that $-2 < 0$ and $4\beta - (\alpha - \delta)^2 > 0$. Let $\frac{\partial \Pi_{BC}^{SC}}{\partial p} = 0$ and $\frac{\partial \Pi_{BC}^{SC}}{\partial e} = 0$, we can get

$$e_{BC}^* = \frac{a(\alpha - \delta)}{4\beta - (\alpha - \delta)^2}, p_{BC}^* = \frac{a[(\alpha - \delta) + 2\beta]}{4\beta - (\alpha - \delta)^2}.$$

Proof of Corollary 1.

$\frac{\partial e_{BC}^*}{\partial \alpha} = \frac{a[(\alpha-\delta)^2+4\beta]}{A^2} > 0$, $\frac{\partial p_{BC}^*}{\partial \alpha} = \frac{a[\delta(\alpha-\delta)^2+4\alpha\beta]}{A^2} > 0$,
$\frac{\partial e_{BC}^*}{\partial \beta} = -\frac{4a(\alpha-\delta)}{A^2} < 0$, $\frac{\partial p_{BC}^*}{\partial \beta} = -\frac{2a(\alpha-\delta)(\alpha+\delta)}{A^2} < 0$,
$\frac{\partial e_{BC}^*}{\partial \delta} = -\frac{a[(\alpha-\delta)^2+4\beta]}{A^2} < 0$, $\frac{\partial p_{BC}^*}{\partial \delta} = -\frac{a[\alpha(\alpha-\delta)^2+4\delta\beta]}{A^2} < 0$.

Appendix B: Proof of Section 5

Proof of Lemma 2. Taking the second derivative of U_{WP}^R with respect to p, we have $\frac{\partial^2 U_{WP}^R}{\partial p^2} = -2 - 2\lambda < 0$. That is to say, U_{WP}^R is a concave function of p. Let $\frac{\partial U_{WP}^R}{\partial p} = 0$, we can obtain the retail price reaction $p(w,e) = \frac{\alpha e\lambda - \delta e\lambda + a\lambda + \alpha e + 2\lambda w + a + w}{2(\lambda+1)}$. Substituting $p(w,e)$ into U_{WP}^M, the Hessian matrix of U_{WP}^M is

$$H(w,e) = \begin{bmatrix} \frac{\partial^2 U_{WP}^M}{\partial w^2} & \frac{\partial^2 U_{WP}^M}{\partial w \partial e} \\ \frac{\partial^2 U_{WP}^M}{\partial e \partial w} & \frac{\partial^2 U_{WP}^M}{\partial e^2} \end{bmatrix} = \begin{bmatrix} -\frac{2\lambda+1}{1+\lambda} & \frac{(\alpha+3\delta)\lambda+\alpha+\delta}{2+2\lambda} \\ \frac{(\alpha+3\delta)\lambda+\alpha+\delta}{2+2\lambda} & -\frac{\alpha\delta\lambda+\delta^2\lambda+\alpha\delta+2\beta\lambda+2\beta}{1+\lambda} \end{bmatrix}.$$

Under the assumption that $4\beta > \alpha(\alpha - \delta)$, we can find the above Hessian matrix is a negative definite for all w and e. Let $\frac{\partial U_{WP}^M}{\partial w} = 0$ and $\frac{\partial U_{WP}^M}{\partial e} = 0$, we can get $w_{WP}^* = \frac{a[4\beta-\delta(\alpha-\delta)](\lambda+1)}{8\beta(2\lambda+1)-(\lambda+1)(\alpha-\delta)^2}$

and $e^*_{WP} = \frac{a(\alpha-\delta)(\lambda+1)}{8\beta(2\lambda+1)-(\lambda+1)(\alpha-\delta)^2}$. Substituting w^*_{WP} and e^*_{WP} into $p(w,e)$, we obtain $p^*_{WP} = \frac{a[6\beta(2\lambda+1)+\delta(\alpha-\delta)(\lambda+1)]}{8\beta(2\lambda+1)-(\lambda+1)(\alpha-\delta)^2}$.

Proof of Corollary 2.

$$\frac{\partial w^*_{WP}}{\partial \lambda} = -\frac{8a\beta[\delta(\alpha-\delta)+4\beta]}{B^2} < 0,$$
$$\frac{\partial e^*_{WP}}{\partial \lambda} = -\frac{8a\beta(\alpha-\delta)}{B^2} < 0,$$
$$\frac{\partial p^*_{WP}}{\partial \lambda} = -\frac{2a\beta(3\alpha+\delta)(\alpha-\delta)}{B^2} < 0.$$

Proof of Corollary 3. The result presented in Corollary 3 can be verified immediately based on the results shown in Lemma 1 and Lemma 2.

Proof of Lemma 3. Similar to the proof process of Lemma 2, we use the backward induction to obtain the following optimal decision variables.

$$e^*_{RS} = \frac{(\alpha-\delta)(\lambda+1)^2 a}{4\beta[2\lambda(2\lambda\phi+2\phi+1)+1]-(\lambda+1)^2(\alpha-\delta)^2},$$
$$w^*_{RS} = \frac{[(\alpha-\delta)(\lambda+1)^2\delta\phi+4\beta(4\lambda\phi^2+4\phi^2+(\phi-\lambda)^2)]a}{4\beta[2\lambda(2\lambda\phi+2\phi+1)+1]-(\lambda+1)^2(\alpha-\delta)^2},$$
$$p^*_{RS} = \frac{[(\alpha-\delta)(\lambda+1)^2\delta+2\beta((\lambda+1)(8\lambda\phi+1)+2(\phi-\lambda^2))]a}{4\beta[2\lambda(2\lambda\phi+2\phi+1)+1]-(\lambda+1)^2(\alpha-\delta)^2}.$$

Proof of Proposition 1. The result presented in Proposition 1 can be verified immediately based on the results shown in Lemma 3.

Proof of Corollary 4.

$$\frac{\partial e^*_{RS}}{\partial \phi} = -\frac{4a\beta(\alpha-\delta)(\lambda+1)^2(2\lambda+1)^2}{C^2} < 0,$$
$$\frac{\partial p^*_{RS}}{\partial \phi} = \frac{-4a\beta(\lambda+1)(2\lambda+1)^2[\alpha(\alpha-\delta)(\lambda+1)-2\beta(2\lambda+1)]}{C^2} > 0.$$

Proof of Lemma 4. Similar to the proof process of Lemma 2, we use the backward induction to obtain the following optimal decision variables.

$$e^*_{CS} = \frac{(\alpha-\delta)(\lambda+1)a}{8\beta(2\lambda+1)(1-\eta)-(\alpha-\delta)^2(\lambda+1)},$$
$$w^*_{CS} = \frac{(\lambda+1)[\delta(\alpha-\delta)+4\beta(1-\eta)]a}{8\beta(2\lambda+1)(1-\eta)-(\alpha-\delta)^2(\lambda+1)},$$
$$p^*_{CS} = \frac{[\delta(\alpha-\delta)(\lambda+1)+6\beta(2\lambda+1)(1-\eta)]a}{8\beta(2\lambda+1)(1-\eta)-(\alpha-\delta)^2(\lambda+1)}.$$

Proof of Corollary 5.

$$\frac{\partial e^*_{CS}}{\partial \lambda} = -\frac{8a\beta(\alpha-\delta)(1-\eta)}{D^2} < 0,$$
$$\frac{\partial w^*_{CS}}{\partial \lambda} = -\frac{4\beta(1-\eta)^2+\alpha\delta(1-\eta)+\delta(\alpha-\delta)}{D^2} < 0,$$
$$\frac{\partial p^*_{CS}}{\partial \lambda} = -\frac{2a\beta(3\alpha-\delta)(\alpha+\delta)(1-\eta)}{D^2} < 0.$$

Proof of Proposition 2. Compare the optimal carbon reduction effort under the four game models, we can find that

$$e^*_{CS} > e^*_{WP} > e^*_{RS}.$$

Moreover, we have $e^*_{BC} - e^*_{CS} = \frac{4a\beta(\alpha-\delta)[3\lambda+1-2(2\lambda+1)\eta]}{AD}$. Therefore, if the cost-sharing rate η is relatively high (i.e., $\frac{1}{2} < \eta < \frac{3\lambda+1}{4\lambda+2}$), then $e^*_{BC} > e^*_{CS}$. If the cost-sharing rate η is relatively low (i.e., $0 < \eta < \frac{1}{2}$), then $e^*_{CS} > e^*_{BC}$.

Proof of Proposition 3. Compare the optimal retail prices under the three game models (i.e., BC, WP, and CS contracts), we can find that $p^*_{CS} > p^*_{WP}$ and $p^*_{CS} > p^*_{BC}$. Moreover, we have $p^*_{BC} - p^*_{WP} = \frac{2a\beta(5\alpha^2\lambda-4\alpha\delta\lambda-\delta^2\lambda+2\alpha^2-2\alpha\delta-8\beta\lambda-4\beta)}{AB}$. Therefore, when the investment cost coefficient is relatively high (i.e., $\beta > \frac{(\alpha-\delta)(5\alpha+\delta)}{8}$) and the fairness concern intensity is relatively low (i.e., $\lambda < \frac{2[\alpha(\alpha-\delta)-2\beta]}{8\beta-(\alpha-\delta)(5\alpha+\delta)}$), then $p^*_{WP} > p^*_{BC}$. When the investment cost coefficient is relatively low (i.e., $\beta < \frac{(\alpha-\delta)(5\alpha+\delta)}{8}$) and the fairness concern intensity is relatively high (i.e., $\lambda > \frac{2[\alpha(\alpha-\delta)-2\beta]}{8\beta-(\alpha-\delta)(5\alpha+\delta)}$), then $p^*_{BC} > p^*_{WP}$. Therefore, Proposition 3 is verified.

Proof of Corollary 6. Notice that

$\Pi^{M*}_{WP} - \Pi^{M*}_{CS} = \frac{a^2\beta[16\beta(1+\lambda)(1+2\lambda)(1-\eta)-(1+\lambda)^2(\alpha-\delta)^2(2-\eta)]}{BD} < 0$,

$\Pi^{M*}_{CS} - \Pi^{M*}_{RS} = \frac{a^2\beta[(1+\lambda)^3(\alpha-\delta)^2(2-\eta)-4(1+\lambda)(1+2\lambda)(1-\eta)(2\phi+2\lambda+\phi+3)\beta]}{CD} < 0$.

Therefore, $\Pi^{M*}_{WP} < \Pi^{M*}_{CS} < \Pi^{M*}_{RS}$.

Appendix C: Equilibrium results of Section 6.

The equilibrium results under the benchmark centralized contract (BC contract) without fairness concern are shown as follows.

$e^*_{BC} = -a(\alpha k + \alpha - \delta)/(\alpha^2k^2 + 2\alpha^2k - 2\alpha\delta k + \alpha^2 - 2\alpha\delta + \delta^2 - 4\beta)$.

$p^*_{BC} = -a(\alpha\delta k + \alpha\delta - \delta^2 + 2\beta)/(\alpha^2k^2 + 2\alpha^2k - 2\alpha\delta k + \alpha^2 - 2\alpha\delta + \delta^2 - 4\beta)$.

The equilibrium results under the wholesale price contract (WP contract) with considering fairness concern are shown as follows.

$e^*_{WP} = -(\alpha k\lambda + \alpha k + \alpha\lambda - \delta\lambda + \alpha - \delta)a/(\alpha^2k^2\lambda + \alpha^2k^2 + 2\alpha^2k\lambda - 2\alpha\delta k\lambda + 2\alpha^2 + \alpha^2\lambda - 2\alpha\delta k - 2\alpha\delta\lambda + \delta^2\lambda + \alpha^2 - 2\alpha\delta - 16\beta\lambda + \delta^2 - 8\beta)$.

$w^*_{WP} = -(\alpha\delta k\lambda + \alpha\delta k + \alpha\delta\lambda - \delta^2\lambda + \alpha\delta + 4\beta\lambda - \delta^2 + 4\beta)a/(\alpha^2k^2\lambda + \alpha^2k^2 + 2\alpha^2k\lambda - 2\alpha\delta k\lambda + 2\alpha^2k + \alpha^2\lambda - 2\alpha\delta k - 2\alpha\delta\lambda + \delta^2\lambda + \alpha^2 - 2\alpha\delta - 16\beta\lambda + \delta^2 - 8\beta)$.

$p^*_{WP} = -(\alpha\delta k\lambda + \alpha\delta k + \alpha\delta\lambda - \delta^2\lambda + \alpha\delta + 12\beta\lambda - \delta^2 + 6\beta)a/(\alpha^2k^2\lambda + \alpha^2k^2 + 2\alpha^2k\lambda - 2\alpha\delta k\lambda + 2\alpha^2k + \alpha^2\lambda - 2\alpha\delta k - 2\alpha\delta\lambda + \delta^2\lambda + \alpha^2 - 2\alpha\delta - 16\beta\lambda + \delta^2 - 8\beta)$.

The equilibrium results under the revenue-sharing contract (RS contract) with considering fairness concern are shown as follows.

$e^*_{RS} = -(\alpha k\lambda^2 + 2\alpha k\lambda + \alpha\lambda^2 - \delta\lambda^2 + \alpha k + 2\alpha\lambda - 2\delta\lambda + \alpha - \delta)a/(\alpha^2k^2\lambda^2 + 2\alpha^2k^2\lambda + 2\alpha^2k\lambda^2 - 2\alpha\delta k\lambda^2 + \alpha^2k^2 + 4\alpha^2k\lambda + \alpha^2\lambda^2 - 4\alpha\delta k\lambda - 2\alpha\delta\lambda^2 - 16\beta\lambda^2\phi + \delta^2\lambda^2 + 2\alpha^2k + 2\alpha^2\lambda - 2\alpha\delta k - 4\alpha\delta\lambda - 16\beta\lambda\phi + 2\delta^2\lambda + \alpha^2 - 2\alpha\delta - 8\beta\lambda - 4\beta\phi + \delta^2 - 4\beta)$.

$w^*_{RS} = -(\alpha\delta k\lambda^2\phi + 2\alpha\delta k\lambda\phi + \alpha\delta\lambda^2\phi + 16\beta\lambda^2\phi^2 - \delta^2\lambda^2\phi + \alpha\delta k\phi + 2\alpha\delta\lambda\phi - 16\beta\lambda^2\phi + 16\beta\lambda\phi^2 - 2\delta^2\lambda\phi + \alpha\delta\phi + 4\beta\lambda^2 - 8\beta\lambda\phi + 4\beta\phi^2 - \delta^2\phi)a/(\alpha^2k^2\lambda^2 + 2\alpha^2k^2\lambda + 2\alpha^2k\lambda^2 - 2\alpha\delta k\lambda^2 + alpha^2k^2 + 4\alpha^2k\lambda + \alpha^2\lambda^2 - 4\alpha\delta k\lambda - 2\alpha\delta\lambda^2 - 16\beta\lambda^2\phi + \delta^2\lambda^2 + 2\alpha^2k + 2\alpha^2\lambda - 2\alpha\delta k - 4\alpha\delta\lambda - 16\beta\lambda\phi + 2\delta^2\lambda + \alpha^2 - 2\alpha\delta - 8\beta\lambda - 4\beta\phi + \delta^2 - 4\beta)$.

$p^*_{RS} = -(\alpha\delta k\lambda^2 + 2\alpha\delta k\lambda + \alpha\delta\lambda^2 + 16\beta\lambda^2\phi - \delta^2\lambda^2 + \alpha\delta k + 2\alpha\delta\lambda - 4\beta\lambda^2 + 16\beta\lambda\phi - 2\delta^2\lambda + \alpha\delta + 2\beta\lambda + 4\beta\phi - \delta^2 + 2\beta)a/(\alpha^2k^2\lambda^2 + 2\alpha^2k.^2\lambda + 2\alpha^2k\lambda^2 - 2\alpha\delta k\lambda^2 + \alpha^2k^2 + 4\alpha^2k\lambda + \alpha^2\lambda^2 - 4\alpha\delta k\lambda -$

$2\alpha\delta\lambda^2-16\beta\lambda^2\phi+\delta^2\lambda^2+2\alpha^2k+2\alpha^2\lambda-2\alpha\delta k-4\alpha\delta\lambda-16\beta\lambda\phi+2\delta^2\lambda+\alpha^2-2\alpha\delta-8\beta\lambda-4\beta\phi+\delta^2-4\beta)$.

The equilibrium results under the cost-sharing contract (CS contract) with considering fairness concern are shown as follows.

$e^*_{CS} = -(\alpha k\lambda + \alpha k + \alpha\lambda - \delta\lambda + \alpha - \delta)a/(\alpha^2k^2\lambda + \alpha^2k^2 + 2\alpha^2k\lambda - 2\alpha\delta k\lambda + 2\alpha^2k + \alpha^2\lambda - 2\alpha\delta k - 2\alpha\delta\lambda + 16\beta\eta\lambda + \delta^2\lambda + \alpha^2 - 2\alpha\delta + 8\beta\eta - 16\beta\lambda + \delta^2 - 8\beta)$.

$w^*_{CS} = -(\alpha\delta k\lambda + \alpha\delta k + \alpha\delta\lambda - 4\beta\eta\lambda - \delta^2\lambda + \alpha\delta - 4\beta\eta + 4\beta\lambda - \delta^2 + 4\beta)a/(\alpha^2k^2\lambda + \alpha^2k^2 + 2\alpha^2k\lambda - 2\alpha\delta k\lambda + 2\alpha^2k + \alpha^2\lambda - 2\alpha\delta k - 2\alpha\delta\lambda + 16\beta\eta\lambda + \delta^2\lambda + \alpha^2 - 2\alpha\delta + 8\beta\eta - 16\beta\lambda + \delta^2 - 8\beta)$.

$p^*_{CS} = -(\alpha\delta k\lambda + \alpha\delta k + \alpha\delta\lambda - 12\beta\eta\lambda - \delta^2\lambda + \alpha\delta - 6\beta\eta + 12\beta\lambda - \delta^2 + 6\beta)a/(\alpha^2k^2\lambda + \alpha^2k^2 + 2\alpha^2k\lambda - 2\alpha\delta k\lambda + 2\alpha^2k + \alpha^2\lambda - 2\alpha\delta k - 2\alpha\delta\lambda + 16\beta\eta\lambda + \delta^2\lambda + \alpha^2 - 2\alpha\delta + 8\beta\eta - 16\beta\lambda + \delta^2 - 8\beta)$.

Appendix D: Proof of Section 6

Proof of Proposition 4. Based on the equilibrium results shown in Appendix C, we can obtain that $e^*_{CS} > e^*_{RS}$, $e^*_{CS} > e^*_{BC}$, $e^*_{RS} > e^*_{WP}$, and $e^*_{BC} > e^*_{WP}$ are always hold. Moreover, on the one hand, when the fairness concern intensity $\lambda < \frac{2\phi+\sqrt{\phi}}{1-4\phi}$, $e^*_{RS} > e^*_{BC}$. On the other hand, when the fairness concern intensity $\frac{2\phi+\sqrt{\phi}}{1-4\phi} < \lambda < 1$, $e^*_{RS} < e^*_{BC}$.

Therefore, when the fairness concern intensity λ is relatively low (i.e., $0 < \lambda < \frac{2\phi+\sqrt{\phi}}{1-4\phi}$), then $e^*_{CS} > e^*_{RS} > e^*_{BC} > e^*_{WP}$.

When the fairness concern intensity λ is relatively high (i.e., $\frac{2\phi+\sqrt{\phi}}{1-4\phi} < \lambda < 1$), then $e^*_{CS} > e^*_{BC} > e^*_{RS} > e^*_{WP}$.

Proof of Proposition 5. According to the equilibrium results shown in Appendix C, we can obtain that $p^*_{CS} > p^*_{BC}$, $p^*_{BC} > p^*_{RS}$, and $p^*_{BC} > p^*_{WP}$ are always hold. Moreover, on the one hand, when the fairness concern intensity meets $0 < \lambda < \frac{2\beta-(k+1)^2\alpha^2-\delta(k+1)\alpha}{(k+1)^2\alpha^2-\delta(k+1)\alpha-4\beta}$, $p^*_{RS} > p^*_{WP}$. On the other hand, when the fairness concern intensity meets $\frac{2\beta-(k+1)^2\alpha^2-\delta(k+1)\alpha}{(k+1)^2\alpha^2-\delta(k+1)\alpha-4\beta} < \lambda < 1$, $p^*_{WP} > p^*_{RS}$.

Therefore, when the fairness concern intensity λ is relatively low (i.e., $0 < \lambda < \frac{2\beta-(k+1)^2\alpha^2-\delta(k+1)\alpha}{(k+1)^2\alpha^2-\delta(k+1)\alpha-4\beta}$), then $p^*_{CS} > p^*_{BC} > p^*_{RS} > p^*_{WP}$.

When the fairness concern intensity λ is relatively high (i.e., $\frac{2\beta-(k+1)^2\alpha^2-\delta(k+1)\alpha}{(k+1)^2\alpha^2-\delta(k+1)\alpha-4\beta} < \lambda < 1$), then $p^*_{CS} > p^*_{BC} > p^*_{WP} > p^*_{RS}$.

Proof of Proposition 6. Based on the equilibrium results shown in Appendix C, we can obtain that $\frac{\partial e^*_{WP}}{\partial k} > 0$, $\frac{\partial e^*_{RS}}{\partial k} > 0$, $\frac{\partial e^*_{CS}}{\partial k} > 0$, $\frac{\partial p^*_{WP}}{\partial k} > 0$, $\frac{\partial p^*_{RS}}{\partial k} > 0$, and $\frac{\partial p^*_{CS}}{\partial k} > 0$ accordingly.

Proof of Proposition 7. Substitute the equilibrium results shown in Appendix C into the utility functions of the manufacturer and the retailer which are summarized in Table 5, we can obtain the optimal profits for the supply chain members under the three decentralized contracts. Then we can verify that $\frac{\partial \Pi^M_{WP}}{\partial k} > 0$, $\frac{\partial \Pi^M_{RS}}{\partial k} > 0$, $\frac{\partial \Pi^M_{CS}}{\partial k} > 0$, $\frac{\partial \Pi^R_{WP}}{\partial k} > 0$, $\frac{\partial \Pi^R_{RS}}{\partial k} > 0$, and $\frac{\partial \Pi^R_{CS}}{\partial k} > 0$.