GENERAL ARTICLES

The evaluation of mobile health Apps: A psychological perception-based probabilistic linguistic belief thermodynamic multiple attribute decision making method(supplementary materials)

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ARTICLE HISTORY
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Appendix A.

Example 1. *Let a LTS be* $S = \{s_{-2} = very\ dissatisfied, s_{-1} = dissatisfied, s_0 = general, s_1 = satisfied, s_2 = very\ satisfied\}$*. Suppose that two decision makers* e_1 *and* e_2 *evaluate the alternative* x *with the PLTSs* $\{s_{-2}(0.6), s_{-1}(0.4)\}$ *and* $\{s_{-1}(0.8), s_0(0.2)\}$*. The risk aversion parameters* θ *of the two decision makers are 2.25 and 0.5, respectively. We integrate the decision makers' opinions by (a) with* α *and* β *are 0.88. Thus, the aggregated PLTS is* $\{s_{-1.42}(0.48), s_{-0.77}(0.12), s_{-0.46}(0.32), s_{-0.09}(0.08)\}$*.*

Example 2. *(Continuation of Example 1) For the aggregated PLTS* $\{s_{-1.42}(0.48), s_{-0.77}(0.12), s_{-0.46}(0.32), s_{-0.09}$ $(0.08)\}$*, we adjust it by using (3)-(6). In this case,* $s_{-1.42}(0.48)$ *is adjusted to* $\{s_{-2}(0.2), s_{-1}(0.28)\}$*.* $s_{-0.77}(0.12)$ *is adjusted to* $\{s_{-1}(0.09).s_0(0.03)\}$*.* $s_{-0.46}(0.32)$ *is adjusted to* $\{s_{-1}(0.15), s_0(0.17)\}$*.* $s_{-0.09}(0.08)$ *is adjusted to* $\{s_{-1}(0.01), s_0(0.07)\}$*. Thus, we obtain* $L'(p) = \{s_{-2}(0.2), s_{-1}(0.53), s_0(0.27)\}$*.*

Example 3. *Suppose that the evaluation outcome of the alternative* x *of two decision makers* e_1 *and* e_2 *is the PLTS* $\{s_{-2}(0.2), s_{-1}(0.3), s_0(0.1), s_1(0.3), s_2(0.1)\}$*. We assume that the risk aversion parameters* θ *of the two decision makers are 0.5 and 2.25. Meanwhile,* α *and* β *are 0.88. With the aid of the transformation function pg, the psychological perception values of the decision makers* e_1 *and* e_2 *are* $\{0.0(0.2), 0.15(0.3), 0.33(0.1), 0.70(0.3), 1.0(0.1)\}$ *and* $\{0.0(0.2), 0.32(0.3), 0.69(0.1), 0.86(0.3), 1.0(0.1)\}$*, respectively. In order to explain the influence of risk attitudes, we also transform the PLTS with the transformation function g of Definition 2 of our formal paper. The corresponding comparison results are displayed in Fig. 1 of Appendix A. For the results of Fig. 1 of Appendix A, there are differences between the transformation results of the functions g and pg. Based on the*

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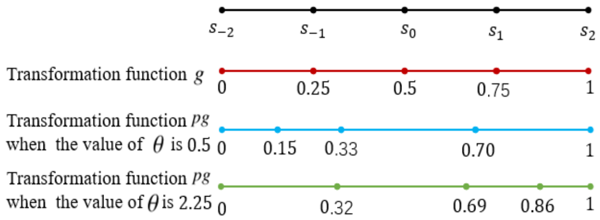

Figure A1. The comparison results of the transformation functions g and pg with different risk attitudes

Table A1. The calculation results of PLTSs by $(a)-(e)$ and $(i)-(v)$

(a)
$\{s_{-2}(0.07), s_{-1}(0.14), s_0(0.15), s_1(0.30), s_2(0.34)\}$
(b)
$\{s_{-2}(0.63), s_{-1}(0.06), s_0(0.05), s_1(0.15), s_2(0.11)\}$
(c)
$\{s_{-2}(0.54), s_{-1}(0.25), s_0(0.14), s_1(0.05), s_2(0.02)\}$
(d)
$\{s_{-2}(0.20), s_{-1}(0.08), s_0(0.27), s_1(0.14), s_2(0.31)\}$
(e)
$\{s_{-2}(0.48), s_{-1}(0.12), s_0(0.18), s_1(0.12), s_2(0.10)\}$
(i)
$\{s_{-2}(0.04), s_{-1}(0.14), s_0(0.14), s_1(0.35), s_2(0.33)\}$
(ii)
$\{s_{-2}(0.43), s_{-1}(0.08), s_0(0.10), s_1(0.19), s_2(0.20)\}$
(iii)
$\{s_{-2}(0.50), s_{-1}(0.29), s_0(0.12), s_1(0.08), s_2(0.01)\}$
(iv)
$\{s_{-2}(0.20), s_{-1}(0.07), s_0(0.22), s_1(0.18), s_2(0.33)\}$
(v)
$\{s_{-2}(0.42), s_{-1}(0.18), s_0(0.22), s_1(0.08), s_2(0.10)\}$

transformation results of the function g, the transformation results of pg have smaller values when the value of θ is 0.5 that the decision maker is the risk appetite and have greater values when the value of θ is 2.25 that the decision maker is the risk aversion. In addition, in order to validate the influence of risk attitudes for the operations of PLTSs, we calculate the above two PLTSs with $(a)-(e)$ of Definition 7 and $(i)-(v)$ of Section II. The calculation results are displayed in Table I.

Example 4. *Let the PHFE be* $\{0(0.2), 0.25(0.3), 0.5(0.1), 0.75(0.3), 1(0.1)\}$. *At present, we need to transform the PHFE to a PLTS by the function pg^{-1} when the risk aversion parameter θ of two decision makers are* 0.5 *and* 2.25, *respectively. α and β are 0.88. When θ is 0.5, the PLTS is* $\{s_{-2}(0.2), s_{-0.41}(0.3), s_{0.41}(0.1), s_{1.17}(0.3), s_2(0.1)\}$. *When θ is 2.25, the transformed PLTS is* $\{s_{-2}(0.2), s_{-1.2}(0.3), s_{-0.47}(0.1), s_{0.3}(0.3), s_2(0.1)\}$. *For comparison, we also transform the PHFE by the transformation function g^{-1} of Definition 2. The corresponding comparison results are shown in Fig. 2 of Appendix B.*

In Fig. 2 of Appendix A, based on the transformation results of the function g^{-1},

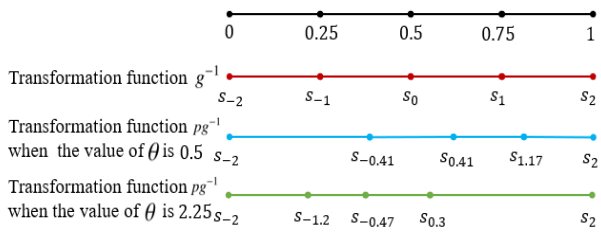

Figure A2. The comparison results of transformation functions g^{-1} and pg^{-1} with different risk attitudes

the transformation results of pg^{-1} have greater linguistic terms when the value of θ is 0.5 and acquire smaller linguistic terms when the value of θ is 2.25. Decision makers with the risk appetite tend to give alternatives better evaluations and decision makers with risk aversion tend to give alternatives worse evaluations.

Appendix B.

B.1. The proof of Proposition 1

Proof. Based on the prospect theory, we consider the linguistic term $s_t(t < gen)$ as the loss and the linguistic term $s_t(t >= gen)$ as the gain. In the light of (1), we obtain the transformation function of linguistic term s_t of $L(P)$, represented as follows:

$$v(s_t) = \begin{cases} (t - gen)^{\alpha}, & t \in [gen, \tau^{+}] \\ -\theta(gen - t)^{\beta}, & t \in [\tau^{-}, gen) \end{cases}.$$

Because $\gamma^l \in [0, 1]$ in the PHFE $h_\gamma(p)$, we further give the normalization of $v(s_t)$ as follows:

$$\begin{aligned} \gamma^l = \tilde{v}(s_t) &= \begin{cases} \frac{(t-gen)^{\alpha}-v_{\min}}{v_{\max}-v_{\min}}, & t \in [gen, \tau^{+}] \\ \frac{-\theta(gen-t)^{\beta}-v_{\min}}{v_{\max}-v_{\min}}, & t \in [\tau^{-}, gen) \end{cases} \\ &= \begin{cases} \frac{(t-gen)^{\alpha}+\theta(gen-\tau^{-})^{\beta}}{(\tau^{+}-gen)^{\alpha}+\theta(gen-\tau^{-})^{\beta}}, & t \in [gen, \tau^{+}] \\ \frac{\theta((gen-\tau^{-})^{\beta}-(gen-t)^{\beta})}{(\tau^{+}-gen)^{\alpha}+\theta(gen-\tau^{-})^{\beta}}, & t \in [\tau^{-}, gen) \end{cases}, \end{aligned}$$

where $v_{\min} = -\theta(gen - \tau^{-})^{\beta}$ and $v_{\max} = (\tau^{+} - gen)^{\alpha}$. Then, we obtain the transformation function pg of the PLTS $L(P)$ as follows:

$$\begin{aligned} pg(L(P)) = &\left\{[\frac{(t - gen)^{\alpha} + \theta(gen - \tau^{-})^{\beta}}{(\tau^{+} - gen)^{\alpha} + \theta(gen - \tau^{-})^{\beta}}](p^t) | t \in [gen, \tau^{+}]\right\} \\ &\cup \left\{[\frac{\theta((gen - \tau^{-})^{\beta} - (gen - t)^{\beta})}{(\tau^{+} - gen)^{\alpha} + \theta(gen - \tau^{-})^{\beta}}](p^t) | t \in [\tau^{-}, gen)\right\}. \end{aligned}$$

Thus, the statement of Proposition 1 holds. □

B.2. The proof of Proposition 2

Proof. Based on $\tilde{v}(s_t)$ in the proof of Proposition 1, we obtain the inverse function $\tilde{v}(\gamma^l)^{-1}$, represented as follows:

$$\begin{aligned} s_t &= \tilde{v}(\gamma^l)^{-1} \\ &= \begin{cases} s_{-((1-\gamma^l)(gen-\tau^-)^\beta - \frac{\gamma^l}{\theta}(\tau^+-gen)^\alpha)^{\frac{1}{\beta}}+gen}, \gamma^l \in [0, \frac{\theta(gen-\tau^-)^\beta}{(\tau^+-gen)^\alpha+\theta(gen-\tau^-)^\beta}) \\ s_{(\gamma^l(\tau^+-gen)^\alpha+\theta(\gamma^l-1)(gen-\tau^-)^\beta)^{\frac{1}{\alpha}}+gen}, \gamma^l \in [\frac{\theta(gen-\tau^-)^\beta}{(\tau^+-gen)^\alpha+\theta(gen-\tau^-)^\beta}, 1] \end{cases}. \end{aligned}$$

Through the calculation of $\tilde{v}(\gamma^l)^{-1}$, we obtain the transformation function pg^{-1} of the PHFE $h_\gamma(p)$, displayed as follows:

$$\begin{aligned} & pg^{-1}(h_\gamma(p)) \\ & = \Big\{ s_{(\gamma^l(\tau^+-gen)^\alpha+\theta(\gamma^l-1)(gen-\tau^-)^\beta)^{\frac{1}{\alpha}}+gen}(p^l) \\ & |\gamma^l \in [\frac{\theta(gen-\tau^-)^\beta}{(\tau^+-gen)^\alpha+\theta(gen-\tau^-)^\beta}, 1]\Big\} \\ & \cup \Big\{ s_{-((1-\gamma^l)(gen-\tau^-)^\beta - \frac{\gamma^l}{\theta}(\tau^+-gen)^\alpha)^{\frac{1}{\beta}}+gen}(p^l) \\ & |\gamma^l \in [0, \frac{\theta(gen-\tau^-)^\beta}{(\tau^+-gen)^\alpha+\theta(gen-\tau^-)^\beta})\Big\}. \end{aligned}$$

Therefore, the proof of Proposition 2 is completed. □

B.3. The proof of Proposition 3

Proof. According to the idea of prospect theory, we consider the linguistic term $s_t(t < 0)$ as the loss and the linguistic term $s_t(t >= 0)$ as the gain. In the light of (1), we obtain the transformation function of linguistic term s_t of $L(P)$, represented as follows:

$$v(s_t) = \begin{cases} t^\alpha, & t \in [0, \tau] \\ -\theta(-t)^\beta, & t \in [-\tau, 0) \end{cases}.$$

Due to that $\gamma^l \in [0, 1]$ in the PHFE $h_\gamma(p)$, we normalize $v(s_t)$ as follows:

$$\begin{aligned} \gamma^l = \tilde{v}(s_t) &= \begin{cases} \frac{t^\alpha - v_{\min}}{v_{\max}-v_{\min}}, & t \in [0, \tau] \\ \frac{-\theta(-t)^\beta - v_{\min}}{v_{\max}-v_{\min}}, & t \in [-\tau, 0) \end{cases} \\ &= \begin{cases} \frac{t^\alpha+\theta\tau^\beta}{\tau^\alpha+\theta\tau^\beta}, & t \in [0, \tau] \\ \frac{\theta(\tau^\beta-(-t)^\beta)}{\tau^\alpha+\theta\tau^\beta}, & t \in [-\tau, 0) \end{cases}, \end{aligned}$$

where $v_{\min} = -\theta\tau^\beta$ and $v_{\max} = \tau^\alpha$. Based on $\tilde{v}(s_t)$, we obtain the transformation function pg of PLE $L(P)$ as follows:

$$pg(L(P)) = \Big\{[\frac{t^\alpha+\theta\tau^\beta}{\tau^\alpha+\theta\tau^\beta}](p^t)|t \in [0, \tau]\Big\} \cup \Big\{[\frac{\theta(\tau^\beta-(-t)^\beta)}{\tau^\alpha+\theta\tau^\beta}](p^t)|t \in [-\tau, 0)\Big\}.$$

Thus, the statement of Proposition 3 holds. □

B.4. *The proof of Proposition 4*

Proof. Based on $\tilde{v}(s_t)$ in the proof of Proposition 3, we obtain the inverse function $\tilde{v}(\gamma^l)^{-1}$, displayed as follows:

$$s_t = \tilde{v}(\gamma^l)^{-1} = \begin{cases} s_{-((1-\gamma^l)\tau^\beta - \frac{\gamma^l}{\theta}\tau^\alpha)^{\frac{1}{\beta}}}, & \gamma^l \in [0, \frac{\theta\tau^\beta}{\tau^\alpha+\theta\tau^\beta}) \\ s_{(\gamma^l\tau^\alpha+\theta(\gamma^l-1)\tau^\beta)^{\frac{1}{\alpha}}}, & \gamma^l \in [\frac{\theta\tau^\beta}{\tau^\alpha+\theta\tau^\beta}, 1] \end{cases}.$$

Then, we obtain the transformation function pg^{-1} of the PHFE $h_\gamma(p)$, displayed as follows:

$$\begin{aligned} pg^{-1}(h_\gamma(p)) = &\Big\{ s_{(\gamma^l\tau^\alpha+\theta(\gamma^l-1)\tau^\beta)^{\frac{1}{\alpha}}}(p^l) | \gamma^l \in [\frac{\theta\tau^\beta}{\tau^\alpha+\theta\tau^\beta}, 1] \Big\} \\ &\cup \Big\{ s_{-((1-\gamma^l)\tau^\beta - \frac{\gamma^l}{\theta}\tau^\alpha)^{\frac{1}{\beta}}}(p^l) | \gamma^l \in [0, \frac{\theta\tau^\beta}{\tau^\alpha+\theta\tau^\beta}) \Big\}. \end{aligned}$$

Therefore, the proof of Proposition 4 is completed. □

B.5. *The proof of Property 1*

Proof. When $t \in [\tau^-, gen)$, the derivative of the transformation function pg with respect to θ is shown as follows:

$$\begin{aligned} \frac{dpg}{d\theta} &= \frac{d\left(\frac{\theta((gen-\tau^-)^\beta-(gen-t)^\beta)}{(\tau^+-gen)^\alpha+\theta(gen-\tau^-)^\beta}\right)}{d\theta} \\ &= \frac{((gen-\tau^-)^\beta-(gen-t)^\beta)((\tau^+-gen)^\alpha+\theta(gen-\tau^-)^\beta)}{\left((\tau^+-gen)^\alpha+\theta(gen-\tau^-)^\beta\right)^2} \\ &- \frac{\theta((gen-\tau^-)^\beta-(gen-t)^\beta)(gen-\tau^-)^\beta}{\left((\tau^+-gen)^\alpha+\theta(gen-\tau^-)^\beta\right)^2} \\ &= \frac{(\tau^+-gen)^\alpha((gen-\tau^-)^\beta-(gen-t)^\beta)}{\left((\tau^+-gen)^\alpha+\theta(gen-\tau^-)^\beta\right)^2}. \end{aligned}$$

Due to that $t \in [\tau^-, gen)$, we have: $\frac{dpg}{d\theta} = \frac{(\tau^+-gen)^\alpha((gen-\tau^-)^\beta-(gen-t)^\beta)}{\left((\tau^+-gen)^\alpha+\theta(gen-\tau^-)^\beta\right)^2} \geq 0$.

When $t \in [gen, \tau^+]$, the derivative of the transformation function pg with respect to θ is displayed as follows:

$$\begin{aligned} \frac{dpg}{d\theta} &= \frac{d\left(\frac{(t-gen)^\alpha+\theta(gen-\tau^-)^\beta}{(\tau^+-gen)^\alpha+\theta(gen-\tau^-)^\beta}\right)}{d\theta} \\ &= \frac{(gen-\tau^-)^\beta((\tau^+-gen)^\alpha+\theta(gen-\tau^-)^\beta)}{\left((\tau^+-gen)^\alpha+\theta(gen-\tau^-)^\beta\right)^2} \\ &- \frac{((t-gen)^\alpha+\theta(gen-\tau^-)^\beta)(gen-\tau^-)^\beta}{\left((\tau^+-gen)^\alpha+\theta(gen-\tau^-)^\beta\right)^2} \\ &= \frac{(gen-\tau^-)^\beta((\tau^+-gen)^\alpha-(t-gen)^\alpha)}{\left((\tau^+-gen)^\alpha+\theta(gen-\tau^-)^\beta\right)^2}. \end{aligned}$$

Owing to $t \in [gen, \tau^+]$, $\frac{dpg}{d\theta} = \frac{(gen-\tau^-)^\beta((\tau^+-gen)^\alpha-(t-gen)^\alpha)}{\left((\tau^+-gen)^\alpha+\theta(gen-\tau^-)^\beta\right)^2} \geq 0$. Therefore, the proof of Property 1 is completed. □

B.6. The proof of Property 2

Proof. When $\gamma^l \in [0, \frac{\theta(gen-\tau^-)^\beta}{(\tau^+-gen)^\alpha+\theta(gen-\tau^-)^\beta})$, the derivative of the transformation function pg^{-1} with respect to θ is shown as follows:

$$\begin{aligned}
&\frac{dpg^{-1}}{d\theta} \\
&= \frac{d(-((1-\gamma^l)(gen-\tau^-)^\beta - \frac{\gamma^l}{\theta}(\tau^+-gen)^\alpha)^{\frac{1}{\beta}} + gen)}{d\theta} \\
&= -\frac{1}{\beta}((1-\gamma^l)(gen-\tau^-)^\beta - \frac{\gamma^l}{\theta}(\tau^+-gen)^\alpha)^{\frac{1}{\beta}-1} * \frac{\gamma^l(\tau^+-gen)^\alpha}{\theta^2} \\
&= -\frac{1}{\beta}((gen-\tau^-)^\beta - \gamma^l(\frac{(\tau^+-gen)^\alpha+\theta(gen-\tau^-)^\beta}{\theta}))^{\frac{1}{\beta}-1} * \frac{\gamma^l(\tau^+-gen)^\alpha}{\theta^2}.
\end{aligned}$$

Because $((gen-\tau^-)^\beta - \gamma^l(\frac{(\tau^+-gen)^\alpha+\theta(gen-\tau^-)^\beta}{\theta}))^{\frac{1}{\beta}-1} > ((gen-\tau^-)^\beta - \frac{\theta(gen-\tau^-)^\beta}{(\tau^+-gen)^\alpha+\theta(gen-\tau^-)^\beta}(\frac{(\tau^+-gen)^\alpha+\theta(gen-\tau^-)^\beta}{\theta}))^{\frac{1}{\beta}-1}$ $= 0$ and $\frac{\gamma^l(\tau^+-gen)^\alpha}{\theta^2} \geq 0$, we can obtain: $\frac{dpg^{-1}}{d\theta} \leq 0$. When $\gamma^l \in [\frac{\theta(gen-\tau^-)^\beta}{(\tau^+-gen)^\alpha+\theta(gen-\tau^-)^\beta}, 1]$, the derivative of the transformation function pg^{-1} with respect to θ is displayed as follows:

$$\begin{aligned}
&\frac{dpg^{-1}}{d\theta} \\
&= \frac{d((\gamma^l(\tau^+-gen)^\alpha + \theta(\gamma^l-1)(gen-\tau^-)^\beta)^{\frac{1}{\alpha}} + gen)}{d\theta} \\
&= \frac{1}{\alpha}(\gamma^l(\tau^+-gen)^\alpha + \theta(\gamma^l-1)(gen-\tau^-)^\beta)^{\frac{1}{\alpha}-1}(\gamma^l-1)(gen-\tau^-)^\beta \\
&= \frac{1}{\alpha}(\gamma^l((\tau^+-gen)^\alpha + \theta(gen-\tau^-)^\beta) - \theta(gen-\tau^-)^\beta)^{\frac{1}{\alpha}-1}(\gamma^l-1)(gen-\tau^-)^\beta.
\end{aligned}$$

Due to that $(\gamma^l((\tau^+-gen)^\alpha + \theta(gen-\tau^-)^\beta) - \theta(gen-\tau^-)^\beta)^{\frac{1}{\alpha}-1} \geq (\frac{\theta(gen-\tau^-)^\beta}{(\tau^+-gen)^\alpha+\theta(gen-\tau^-)^\beta}((\tau^+-gen)^\alpha + \theta(gen-\tau^-)^\beta) - \theta(gen-\tau^-)^\beta)^{\frac{1}{\alpha}-1} = 0$ and $(\gamma^l-1)(gen-\tau^-)^\beta \leq 0$, we have: $\frac{dpg^{-1}}{d\theta} \leq 0$. Therefore, the proof of Property 2 is completed. □

Appendix C.

$$LM^1 = \begin{array}{cc} & \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} & \left[\begin{array}{cccc} \{s_0(0.1), s_1(0.5), s_2(0.1)\} & \{s_1(0.7), s_2(0.1)\} & \{s_{-2}(0.6), s_{-1}(0.1)\} & \{s_{-2}(0.2), s_{-1}(0.7)\} \\ \{s_{-2}(0.1), s_0(0.6)\} & \{s_{-2}(0.7), s_0(0.1)\} & \{s_{-1}(0.6), s_0(0.1), s_1(0.2)\} & \{s_{-1}(0.1), s_0(0.1), s_1(0.6)\} \\ \{s_1(0.8), s_2(0.1)\} & \{s_{-2}(0.3), s_{-1}(0.6)\} & \{s_1(0.2), s_2(0.5)\} & \{s_{-1}(0.5), s_0(0.3)\} \\ \{s_{-2}(0.1), s_{-1}(0.7), s_0(0.1)\} & \{s_1(0.1), s_2(0.8)\} & \{s_0(0.1), s_1(0.8)\} & \{s_{-2}(0.8), s_{-1}(0.1)\} \\ \{s_1(0.6), s_2(0.1)\} & \{s_1(0.8)\} & \{s_{-2}(0.5), s_{-1}(0.1), s_0(0.2)\} & \{s_{-2}(0.6), s_{-1}(0.1), s_0(0.1)\} \\ \{s_0(0.2), s_1(0.3), s_2(0.4)\} & \{s_{-1}(0.3), s_1(0.5)\} & \{s_{-2}(0.5), s_{-1}(0.4)\} & \{s_{-2}(0.3), s_{-1}(0.4), s_0(0.1)\} \end{array}\right] \end{array},$$

$$LM^2 = \begin{array}{cc} & \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} & \left[\begin{array}{cccc} \{s_1(0.4), s_2(0.1)\} & \{s_0(0.4), s_1(0.1)\} & \{s_{-1}(0.1), s_0(0.1), s_1(0.3)\} & \{s_{-1}(0.2), s_0(0.1), s_1(0.2)\} \\ \{s_{-1}(0.2), s_0(0.2), s_1(0.1)\} & \{s_0(0.2), s_1(0.3)\} & \{s_0(0.1), s_1(0.3)\} & \{s_{-1}(0.4), s_0(0.1)\} \\ \{s_0(0.1), s_1(0.3), s_2(0.2)\} & \{s_{-1}(0.1), s_0(0.3), s_1(0.1)\} & \{s_{-1}(0.2), s_0(0.3)\} & \{s_0(0.2), s_1(0.2)\} \\ \{s_0(0.1), s_1(0.3), s_2(0.2)\} & \{s_0(0.3), s_1(0.2)\} & \{s_1(0.3), s_2(0.2)\} & \{s_{-2}(0.3), s_{-1}(0.1), s_0(0.2)\} \\ \{s_1(0.1), s_2(0.4)\} & \{s_0(0.1), s_1(0.4), s_2(0.1)\} & \{s_0(0.1), s_1(0.2), s_2(0.3)\} & \{s_{-1}(0.1), s_0(0.3)\} \\ \{s_{-2}(0.4), s_{-1}(0.1)\} & \{s_{-1}(0.4), s_0(0.1), s_1(0.1)\} & \{s_{-2}(0.1), s_{-1}(0.3)\} & \{s_1(0.3), s_2(0.1)\} \end{array}\right] \end{array},$$

$$LM^3 = \begin{array}{cc} & \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} & \left[\begin{array}{cccc} \{s_{-2}(0.5), s_{-1}(0.3), s_0(0.1)\} & \{s_{-1}(0.1), s_0(0.1), s_1(0.6)\} & \{s_{-1}(0.2), s_1(0.4), s_2(0.1)\} & \{s_{-2}(0.4), s_{-1}(0.5)\} \\ \{s_1(0.4), s_2(0.4)\} & \{s_{-1}(0.2), s_0(0.4), s_1(0.2)\} & \{s_{-2}(0.4), s_{-1}(0.5)\} & \{s_0(0.2), s_1(0.6)\} \\ \{s_0(0.1), s_1(0.4), s_2(0.4)\} & \{s_0(0.1), s_1(0.3), s_2(0.4)\} & \{s_{-2}(0.4), s_{-1}(0.4)\} & \{s_{-2}(0.5), s_{-1}(0.4)\} \\ \{s_1(0.5), s_2(0.4)\} & \{s_0(0.1), s_1(0.3), s_2(0.4)\} & \{s_{-2}(0.8), s_{-1}(0.1)\} & \{s_0(0.5), s_1(0.3), s_2(0.1)\} \\ \{s_{-1}(0.1), s_0(0.2), s_1(0.6)\} & \{s_{-2}(0.6), s_0(0.3)\} & \{s_0(0.3), s_1(0.5)\} & \{s_{-2}(0.3), s_{-1}(0.4)\} \\ \{s_{-2}(0.2), s_{-1}(0.2), s_0(0.4)\} & \{s_0(0.6), s_1(0.3)\} & \{s_{-1}(0.4), s_0(0.4)\} & \{s_{-2}(0.4), s_{-1}(0.5)\} \end{array}\right] \end{array},$$

$$LM^4 = \begin{array}{cc} & \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} & \begin{bmatrix} \{s_{-2}(0.2), s_{-1}(0.3), s_0(0.3)\} & \{s_{-1}(0.4), s_0(0.5)\} & \{s_1(0.4), s_2(0.3)\} & \{s_0(0.5), s_1(0.4)\} \\ \{s_0(0.2), s_1(0.3), s_2(0.4)\} & \{s_{-1}(0.3), s_0(0.3)\} & \{s_{-1}(0.4), s_0(0.5)\} & \{s_{-1}(0.3), s_0(0.5)\} \\ \{s_{-1}(0.5), s_0(0.3), s_1(0.1)\} & \{s_{-2}(0.4), s_{-1}(0.5)\} & \{s_{-1}(0.2), s_0(0.1), s_1(0.5)\} & \{s_{-2}(0.2), s_{-1}(0.4), s_0(0.3)\} \\ \{s_1(0.4), s_2(0.3)\} & \{s_0(0.2), s_1(0.3), s_2(0.4)\} & \{s_{-1}(0.4), s_0(0.2), s_1(0.2)\} & \{s_0(0.2), s_1(0.5), s_2(0.1)\} \\ \{s_{-1}(0.4), s_0(0.3)\} & \{s_{-2}(0.3), s_{-1}(0.1), s_0(0.3)\} & \{s_{-1}(0.5), s_0(0.3)\} & \{s_{-2}(0.2), s_{-1}(0.4), s_0(0.2)\} \\ \{s_{-2}(0.4), s_{-1}(0.3)\} & \{s_0(0.2), s_1(0.4), s_2(0.2)\} & \{s_0(0.4), s_1(0.3), s_2(0.2)\} & \{s_{-1}(0.3), s_0(0.4), s_1(0.2)\} \end{bmatrix} \end{array},$$

$$LM^5 = \begin{array}{cc} & \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} & \begin{bmatrix} \{s_1(0.6), s_2(0.3)\} & \{s_1(0.5), s_2(0.4)\} & \{s_1(0.5), s_2(0.3)\} & \{s_0(0.4), s_1(0.5)\} \\ \{s_0(0.5), s_1(0.4)\} & \{s_0(0.4), s_1(0.4), s_2(0.1)\} & \{s_{-1}(0.4), s_0(0.2), s_1(0.2)\} & \{s_{-2}(0.2), s_{-1}(0.4), s_0(0.1)\} \\ \{s_0(0.5), s_1(0.3), s_2(0.1)\} & \{s_0(0.3), s_1(0.2), s_2(0.3)\} & \{s_{-1}(0.2), s_0(0.6), s_1(0.1)\} & \{s_{-1}(0.2), s_0(0.7)\} \\ \{s_{-1}(0.5), s_0(0.4)\} & \{s_{-2}(0.3), s_{-1}(0.5)\} & \{s_{-1}(0.1), s_0(0.3), s_1(0.4)\} & \{s_1(0.3), s_2(0.6)\} \\ \{s_{-2}(0.3), s_{-1}(0.1), s_{-0}(0.5)\} & \{s_0(0.4), s_1(0.4)\} & \{s_{-2}(0.5), s_{-1}(0.2), s_0(0.1)\} & \{s_0(0.7), s_1(0.2)\} \\ \{s_{-1}(0.3), s_0(0.2), s_1(0.3)\} & \{s_0(0.3), s_1(0.3), s_2(0.3)\} & \{s_1(0.4), s_2(0.4)\} & \{s_{-1}(0.3), s_0(0.4), s_1(0.1)\} \end{bmatrix} \end{array}.$$

$$R = \begin{array}{cc} & \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{array}{c} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{array} & \begin{bmatrix} 1 & 2 & 4 & 3 \\ 3 & 2 & 4 & 1 \\ 4 & 3 & 2 & 1 \\ 1 & 4 & 2 & 3 \\ 4 & 2 & 3 & 1 \end{bmatrix} \end{array}.$$

$$\bar{E}^1 = \begin{array}{cc} & \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} & \begin{bmatrix} \{s_{-2}(0.02), s_{-1}(0.51), s_0(0.07), s_2(0.10)\} & \{s_{-2}(0.16), s_{-1}(0.54), s_2(0.10)\} & \{s_{-2}(0.69), s_{-1}(0.01)\} & \{s_{-2}(0.50), s_{-1}(0.40)\} \\ \{s_{-2}(0.24), s_{-1}(0.46)\} & \{s_{-2}(0.75), s_{-1}(0.05)\} & \{s_{-2}(0.68), s_{-1}(0.22)\} & \{s_{-2}(0.04), s_{-1}(0.18), s_0(0.58)\} \\ \{s_{-1}(0.69), s_0(0.11), s_2(0.10)\} & \{s_{-2}(0.80), s_{-1}(0.10)\} & \{s_{-2}(0.09), s_{-1}(0.11), s_2(0.50)\} & \{s_{-2}(0.21), s_{-1}(0.46), s_0(0.13)\} \\ \{s_{-2}(0.63), s_{-1}(0.27)\} & \{s_{-2}(0.02), s_{-1}(0.08), s_2(0.80)\} & \{s_{-2}(0.40), s_{-1}(0.50)\} & \{s_{-2}(0.84), s_{-1}(0.06)\} \\ \{s_{-1}(0.52), s_0(0.08), s_2(0.10)\} & \{s_{-2}(0.19), s_{-1}(0.61)\} & \{s_{-2}(0.72), s_{-1}(0.08)\} & \{s_{-2}(0.64), s_{-1}(0.12), s_0(0.04)\} \\ \{s_{-2}(0.05), s_{-1}(0.41), s_0(0.04), s_2(0.40)\} & \{s_{-2}(0.37), s_{-1}(0.43)\} & \{s_{-2}(0.85), s_{-1}(0.05)\} & \{s_{-2}(0.47), s_{-1}(0.29), s_0(0.04)\} \end{bmatrix} \end{array},$$

$$\bar{E}^2 = \begin{array}{cc} & \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} & \begin{bmatrix} \{s_{-1}(0.34), s_0(0.06), s_2(0.10)\} & \{s_{-2}(0.23), s_{-1}(0.27)\} & \{s_{-2}(0.28), s_{-1}(0.22)\} & \{s_{-2}(0.08), s_{-1}(0.20), s_0(0.22)\} \\ \{s_{-2}(0.19), s_{-1}(0.29), s_0(0.01)\} & \{s_{-2}(0.17), s_{-1}(0.33)\} & \{s_{-2}(0.19), s_{-1}(0.21)\} & \{s_{-2}(0.17), s_{-1}(0.29), s_0(0.04)\} \\ \{s_{-2}(0.02), s_{-1}(0.33), s_0(0.04), s_2(0.20)\} & \{s_{-2}(0.26), s_{-1}(0.24)\} & \{s_{-2}(0.37), s_{-1}(0.13)\} & \{s_{-1}(0.14), s_0(0.26)\} \\ \{s_{-2}(0.02), s_{-1}(0.33), s_0(0.04), s_2(0.20)\} & \{s_{-2}(0.20), s_{-1}(0.30)\} & \{s_{-2}(0.13), s_{-1}(0.17), s_2(0.20)\} & \{s_{-2}(0.34), s_{-1}(0.17), s_0(0.08)\} \\ \{s_{-1}(0.09), s_0(0.01), s_2(0.40)\} & \{s_{-2}(0.14), s_{-1}(0.36), s_1(0.10)\} & \{s_{-2}(0.15), s_{-1}(0.15), s_2(0.30)\} & \{s_{-2}(0.04), s_{-1}(0.23), s_0(0.13)\} \\ \{s_{-2}(0.47), s_{-1}(0.03)\} & \{s_{-2}(0.41), s_{-1}(0.19)\} & \{s_{-2}(0.36), s_{-1}(0.04)\} & \{s_{-1}(0.03), s_0(0.27), s_2(0.10)\} \end{bmatrix} \end{array},$$

$$\bar{E}^3 = \begin{array}{cc} & \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} & \begin{bmatrix} \{s_{-2}(0.74), s_{-1}(0.16)\} & \{s_{-2}(0.27), s_{-1}(0.53)\} & \{s_{-2}(0.35), s_{-1}(0.25), s_2(0.10)\} & \{s_{-2}(0.61), s_{-1}(0.29)\} \\ \{s_{-1}(0.34), s_0(0.06), s_2(0.40)\} & \{s_{-2}(0.42), s_{-1}(0.38)\} & \{s_{-2}(0.84), s_{-1}(0.06)\} & \{s_{-1}(0.18), s_0(0.62)\} \\ \{s_{-2}(0.02), s_{-1}(0.42), s_0(0.06), s_2(0.40)\} & \{s_{-2}(0.12), s_{-1}(0.28), s_2(0.40)\} & \{s_{-2}(0.75), s_{-1}(0.05)\} & \{s_{-2}(0.67), s_{-1}(0.23)\} \\ \{s_{-1}(0.43), s_0(0.07), s_2(0.40)\} & \{s_{-2}(0.12), s_{-1}(0.28), s_2(0.40)\} & \{s_{-2}(0.89), s_{-1}(0.01)\} & \{s_{-1}(0.32), s_0(0.48), s_2(0.10)\} \\ \{s_{-2}(0.12), s_{-1}(0.69), s_0(0.08)\} & \{s_{-2}(0.75), s_{-1}(0.15)\} & \{s_{-2}(0.40), s_{-1}(0.40)\} & \{s_{-2}(0.47), s_{-1}(0.23)\} \\ \{s_{-2}(0.44), s_{-1}(0.36)\} & \{s_{-2}(0.14), s_{-1}(0.46), s_2(0.30)\} & \{s_{-2}(0.61), s_{-1}(0.19)\} & \{s_{-2}(0.61), s_{-1}(0.29)\} \end{bmatrix} \end{array},$$

$$\bar{E}^4 = \begin{array}{cc} & \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} & \begin{bmatrix} \{s_{-2}(0.49), s_{-1}(0.31)\} & \{s_{-2}(0.59), s_{-1}(0.31)\} & \{s_{-2}(0.17), s_{-1}(0.23), s_2(0.30)\} & \{s_{-1}(0.33), s_0(0.57)\} \\ \{s_{-2}(0.05), s_{-1}(0.41), s_0(0.04), s_2(0.40)\} & \{s_{-2}(0.47), s_{-1}(0.43)\} & \{s_{-2}(0.67), s_{-1}(0.23)\} & \{s_{-2}(0.13), s_{-1}(0.46), s_0(0.21)\} \\ \{s_{-2}(0.44), s_{-1}(0.45), s_0(0.01)\} & \{s_{-2}(0.82), s_{-1}(0.08)\} & \{s_{-2}(0.45), s_{-1}(0.35)\} & \{s_{-2}(0.37), s_{-1}(0.41), s_0(0.13)\} \\ \{s_{-1}(0.34), s_0(0.06), s_2(0.30)\} & \{s_{-2}(0.17), s_{-1}(0.33), s_2(0.40)\} & \{s_{-2}(0.56), s_{-1}(0.24)\} & \{s_{-1}(0.17), s_0(0.53), s_2(0.10)\} \\ \{s_{-2}(0.36), s_{-1}(0.34)\} & \{s_{-2}(0.54), s_{-1}(0.16)\} & \{s_{-2}(0.63), s_{-1}(0.17)\} & \{s_{-2}(0.37), s_{-1}(0.35), s_0(0.08)\} \\ \{s_{-2}(0.62), s_{-1}(0.08)\} & \{s_{-2}(0.19), s_{-1}(0.41), s_2(0.20)\} & \{s_{-2}(0.38), s_{-1}(0.32), s_2(0.20)\} & \{s_{-2}(0.13), s_{-1}(0.43), s_0(0.35)\} \end{bmatrix} \end{array},$$

$$\bar{E}^5 = \begin{array}{cc} & \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} & \begin{bmatrix} \{s_{-1}(0.52), s_0(0.08), s_2(0.30)\} & \{s_{-2}(0.12), s_{-1}(0.38), s_2(0.40)\} & \{s_{-2}(0.21), s_{-1}(0.29), s_2(0.30)\} & \{s_{-1}(0.29), s_0(0.61)\} \\ \{s_{-2}(0.12), s_{-1}(0.72), s_0(0.06)\} & \{s_{-2}(0.30), s_{-1}(0.50), s_2(0.10)\} & \{s_{-2}(0.56), s_{-1}(0.24)\} & \{s_{-2}(0.37), s_{-1}(0.29), s_0(0.04)\} \\ \{s_{-2}(0.12), s_{-1}(0.64), s_0(0.04), s_2(0.10)\} & \{s_{-2}(0.20), s_{-1}(0.30), s_2(0.30)\} & \{s_{-2}(0.60), s_{-1}(0.30)\} & \{s_{-2}(0.08), s_{-1}(0.52), s_0(0.29)\} \\ \{s_{-2}(0.46), s_{-1}(0.44)\} & \{s_{-2}(0.72), s_{-1}(0.08)\} & \{s_{-2}(0.45), s_{-1}(0.35)\} & \{s_{-1}(0.03), s_0(0.27), s_2(0.60)\} \\ \{s_{-2}(0.49), s_{-1}(0.41)\} & \{s_{-2}(0.30), s_{-1}(0.50)\} & \{s_{-2}(0.74), s_{-1}(0.06)\} & \{s_{-1}(0.43), s_0(0.47)\} \\ \{s_{-2}(0.27), s_{-1}(0.49), s_0(0.04)\} & \{s_{-2}(0.22), s_{-1}(0.38), s_2(0.30)\} & \{s_{-2}(0.17), s_{-1}(0.23), s_2(0.40)\} & \{s_{-2}(0.13), s_{-1}(0.42), s_0(0.26)\} \end{bmatrix} \end{array}.$$

$$Q^1 = \begin{array}{cc} & \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} & \begin{bmatrix} 0.9168 & 0.8763 & 0.8735 & 0.8598 \\ 0.8943 & 0.8614 & 0.8735 & 0.8957 \\ 0.8594 & 0.8264 & 0.9224 & 0.9015 \\ 0.8318 & 0.9217 & 0.8362 & 0.8249 \\ 0.8923 & 0.8477 & 0.8918 & 0.8998 \\ 0.9025 & 0.8486 & 0.8400 & 0.9020 \end{bmatrix} \end{array},$$

$$Q^2 = \begin{array}{cc} & \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} & \begin{bmatrix} 0.9312 & 0.9311 & 0.9283 & 0.9316 \\ 0.9260 & 0.9388 & 0.9433 & 0.9347 \\ 0.9302 & 0.9264 & 0.9239 & 0.9477 \\ 0.9303 & 0.9171 & 0.9310 & 0.9056 \\ 0.9404 & 0.9247 & 0.9281 & 0.9482 \\ 0.9210 & 0.9121 & 0.9324 & 0.9375 \end{bmatrix} \end{array},$$

$$Q^3 = \begin{array}{cc} & \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} & \begin{bmatrix} 0.8674 & 0.8736 & 0.9092 & 0.8651 \\ 0.9207 & 0.8859 & 0.8423 & 0.8912 \\ 0.9303 & 0.9327 & 0.8555 & 0.8485 \\ 0.9139 & 0.9385 & 0.8292 & 0.8710 \\ 0.8598 & 0.8656 & 0.8847 & 0.8988 \\ 0.8804 & 0.8861 & 0.9023 & 0.8593 \end{bmatrix} \end{array},$$

$$Q^4 = \begin{array}{cc} & \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} & \begin{bmatrix} 0.8783 & 0.8613 & 0.9098 & 0.8934 \\ 0.9110 & 0.8824 & 0.8805 & 0.9077 \\ 0.8866 & 0.8496 & 0.8819 & 0.9232 \\ 0.9306 & 0.9192 & 0.8793 & 0.8955 \\ 0.9029 & 0.9085 & 0.8962 & 0.9143 \\ 0.8710 & 0.9302 & 0.9098 & 0.8943 \end{bmatrix} \end{array},$$

$$Q^5 = \begin{array}{cc} & \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} & \begin{bmatrix} 0.8868 & 0.9119 & 0.9081 & 0.8614 \\ 0.8765 & 0.8921 & 0.9025 & 0.8959 \\ 0.8977 & 0.9409 & 0.8665 & 0.8763 \\ 0.8497 & 0.8326 & 0.8759 & 0.9008 \\ 0.8855 & 0.9009 & 0.8698 & 0.8853 \\ 0.8825 & 0.9245 & 0.9059 & 0.8787 \end{bmatrix} \end{array}.$$

$$\bar{E}X^1 = \begin{array}{cc} & \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} & \begin{bmatrix} \{s_{-2}(0.06), s_{-1}(0.48), s_0(0.07), s_2(0.10)\} & \{s_{-2}(0.22), s_{-1}(0.48), s_2(0.10)\} & \{s_{-2}(0.69), s_{-1}(0.01)\} & \{s_{-2}(0.54), s_{-1}(0.36)\} \\ \{s_{-2}(0.28), s_{-1}(0.42)\} & \{s_{-2}(0.76), s_{-1}(0.04)\} & \{s_{-2}(0.70), s_{-1}(0.20)\} & \{s_{-2}(0.06), s_{-1}(0.21), s_0(0.53)\} \\ \{s_{-2}(0.08), s_{-1}(0.62), s_0(0.10), s_2(0.10)\} & \{s_{-2}(0.81), s_{-1}(0.09)\} & \{s_{-2}(0.09), s_{-1}(0.11), s_2(0.50)\} & \{s_{-2}(0.25), s_{-1}(0.44), s_0(0.12)\} \\ \{s_{-2}(0.67), s_{-1}(0.23)\} & \{s_{-2}(0.03), s_{-1}(0.07), s_2(0.80)\} & \{s_{-2}(0.47), s_{-1}(0.43)\} & \{s_{-2}(0.85), s_{-1}(0.05)\} \\ \{s_{-2}(0.04), s_{-1}(0.48), s_0(0.08), s_2(0.10)\} & \{s_{-2}(0.26), s_{-1}(0.54)\} & \{s_{-2}(0.72), s_{-1}(0.08)\} & \{s_{-2}(0.65), s_{-1}(0.11), s_0(0.04)\} \\ \{s_{-2}(0.08), s_{-1}(0.38), s_0(0.04), s_2(0.40)\} & \{s_{-2}(0.42), s_{-1}(0.38)\} & \{s_{-2}(0.86), s_{-1}(0.04)\} & \{s_{-2}(0.49), s_{-1}(0.27), s_0(0.04)\} \end{bmatrix} \end{array}.$$

$$\bar{E}X^2 = \begin{array}{cc} & \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} & \begin{bmatrix} \{s_{-2}(0.02), s_{-1}(0.33), s_0(0.05), s_2(0.10)\} & \{s_{-2}(0.24), s_{-1}(0.26)\} & \{s_{-2}(0.29), s_{-1}(0.21)\} & \{s_{-2}(0.10), s_{-1}(0.20), s_0(0.21)\} \\ \{s_{-2}(0.21), s_{-1}(0.28), s_0(0.01)\} & \{s_{-2}(0.19), s_{-1}(0.31)\} & \{s_{-2}(0.20), s_{-1}(0.20)\} & \{s_{-2}(0.18), s_{-1}(0.28), s_0(0.04)\} \\ \{s_{-2}(0.04), s_{-1}(0.32), s_0(0.04), s_2(0.20)\} & \{s_{-2}(0.27), s_{-1}(0.23)\} & \{s_{-2}(0.38), s_{-1}(0.12)\} & \{s_{-2}(0.01), s_{-1}(0.14), s_0(0.25)\} \\ \{s_{-2}(0.04), s_{-1}(0.32), s_0(0.04), s_2(0.20)\} & \{s_{-2}(0.22), s_{-1}(0.28)\} & \{s_{-2}(0.14), s_{-1}(0.16), s_2(0.20)\} & \{s_{-2}(0.36), s_{-1}(0.17), s_0(0.08)\} \\ \{s_{-1}(0.08), s_0(0.01), s_2(0.40)\} & \{s_{-2}(0.17), s_{-1}(0.33), s_2(0.10)\} & \{s_{-2}(0.16), s_{-1}(0.14), s_2(0.30)\} & \{s_{-2}(0.05), s_{-1}(0.23), s_0(0.12)\} \\ \{s_{-2}(0.47), s_{-1}(0.03)\} & \{s_{-2}(0.42), s_{-1}(0.18)\} & \{s_{-2}(0.37), s_{-1}(0.03)\} & \{s_{-1}(0.04), s_0(0.26), s_2(0.10)\} \end{bmatrix} \end{array}.$$

$$
\bar{E}X^3 = \begin{array}{cc}
 & \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\
\begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} &
\left[\begin{array}{cccc}
\{s_{-2}(0.76), s_{-1}(0.14)\} & \{s_{-2}(0.33), s_{-1}(0.47)\} & \{s_{-2}(0.36), s_{-1}(0.24), s_2(0.10)\} & \{s_{-2}(0.64), s_{-1}(0.26)\} \\
\{s_{-2}(0.02), s_{-1}(0.32), s_0(0.05), s_2(0.40)\} & \{s_{-2}(0.45), s_{-1}(0.35)\} & \{s_{-2}(0.85), s_{-1}(0.05)\} & \{s_{-2}(0.02), s_{-1}(0.22), s_0(0.57)\} \\
\{s_{-2}(0.05), s_{-1}(0.40), s_0(0.05), s_2(0.40)\} & \{s_{-2}(0.14), s_{-1}(0.26), s_2(0.40)\} & \{s_{-2}(0.76), s_{-1}(0.04)\} & \{s_{-2}(0.70), s_{-1}(0.20)\} \\
\{s_{-2}(0.03), s_{-1}(0.40), s_0(0.07), s_2(0.40)\} & \{s_{-2}(0.13), s_{-1}(0.27), s_2(0.40)\} & \{s_{-2}(0.89), s_{-1}(0.01)\} & \{s_{-2}(0.03), s_{-1}(0.34), s_0(0.43), s_2(0.10)\} \\
\{s_{-2}(0.20), s_{-1}(0.62), s_0(0.08)\} & \{s_{-2}(0.77), s_{-1}(0.13)\} & \{s_{-2}(0.44), s_{-1}(0.36)\} & \{s_{-2}(0.49), s_{-1}(0.21)\} \\
\{s_{-2}(0.48), s_{-1}(0.32)\} & \{s_{-2}(0.18), s_{-1}(0.42), s_2(0.30)\} & \{s_{-2}(0.62), s_{-1}(0.18)\} & \{s_{-2}(0.64), s_{-1}(0.26)\}
\end{array}\right]
\end{array}.
$$

$$
\bar{E}X^4 = \begin{array}{cc}
 & \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\
\begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} &
\left[\begin{array}{cccc}
\{s_{-2}(0.52), s_{-1}(0.28)\} & \{s_{-2}(0.62), s_{-1}(0.28)\} & \{s_{-2}(0.19), s_{-1}(0.21), s_2(0.30)\} & \{s_{-2}(0.03), s_{-1}(0.35), s_0(0.52)\} \\
\{s_{-2}(0.08), s_{-1}(0.38), s_0(0.04), s_2(0.40)\} & \{s_{-2}(0.51), s_{-1}(0.39)\} & \{s_{-2}(0.69), s_{-1}(0.21)\} & \{s_{-2}(0.16), s_{-1}(0.44), s_0(0.19)\} \\
\{s_{-2}(0.48), s_{-1}(0.41), s_0(0.01)\} & \{s_{-2}(0.83), s_{-1}(0.07)\} & \{s_{-2}(0.49), s_{-1}(0.31)\} & \{s_{-2}(0.39), s_{-1}(0.39), s_0(0.12)\} \\
\{s_{-2}(0.02), s_{-1}(0.33), s_0(0.05), s_2(0.30)\} & \{s_{-2}(0.19), s_{-1}(0.31), s_2(0.40)\} & \{s_{-2}(0.59), s_{-1}(0.21)\} & \{s_{-2}(0.01), s_{-1}(0.20), s_0(0.49), s_2(0.10)\} \\
\{s_{-2}(0.39), s_{-1}(0.31)\} & \{s_{-2}(0.55), s_{-1}(0.15)\} & \{s_{-2}(0.64), s_{-1}(0.16)\} & \{s_{-2}(0.39), s_{-1}(0.33), s_0(0.08)\} \\
\{s_{-2}(0.63), s_{-1}(0.07)\} & \{s_{-2}(0.22), s_{-1}(0.38), s_2(0.20)\} & \{s_{-2}(0.41), s_{-1}(0.29), s_2(0.20)\} & \{s_{-2}(0.16), s_{-1}(0.42), s_0(0.32)\}
\end{array}\right]
\end{array}.
$$

$$
\bar{E}X^5 = \begin{array}{cc}
 & \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\
\begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} &
\left[\begin{array}{cccc}
\{s_{-2}(0.05), s_{-1}(0.48), s_0(0.08), s_2(0.30)\} & \{s_{-2}(0.14), s_{-1}(0.36), s_2(0.40)\} & \{s_{-2}(0.23), s_{-1}(0.27), s_2(0.30)\} & \{s_{-2}(0.03), s_{-1}(0.32), s_0(0.54)\} \\
\{s_{-2}(0.19), s_{-1}(0.66), s_0(0.05)\} & \{s_{-2}(0.34), s_{-1}(0.46), s_2(0.10)\} & \{s_{-2}(0.58), s_{-1}(0.22)\} & \{s_{-2}(0.39), s_{-1}(0.27), s_0(0.04)\} \\
\{s_{-2}(0.17), s_{-1}(0.59), s_0(0.04), s_2(0.10)\} & \{s_{-2}(0.21), s_{-1}(0.29), s_2(0.30)\} & \{s_{-2}(0.63), s_{-1}(0.27)\} & \{s_{-2}(0.14), s_{-1}(0.50), s_0(0.26)\} \\
\{s_{-2}(0.51), s_{-1}(0.39)\} & \{s_{-2}(0.73), s_{-1}(0.07)\} & \{s_{-2}(0.49), s_{-1}(0.31)\} & \{s_{-1}(0.05), s_0(0.25), s_2(0.60)\} \\
\{s_{-2}(0.53), s_{-1}(0.37)\} & \{s_{-2}(0.34), s_{-1}(0.46)\} & \{s_{-2}(0.75), s_{-1}(0.05)\} & \{s_{-2}(0.04), s_{-1}(0.43), s_0(0.43)\} \\
\{s_{-2}(0.31), s_{-1}(0.45), s_0(0.04)\} & \{s_{-2}(0.25), s_{-1}(0.35), s_2(0.30)\} & \{s_{-2}(0.19), s_{-1}(0.21), s_2(0.40)\} & \{s_{-2}(0.17), s_{-1}(0.40), s_0(0.23)\}
\end{array}\right]
\end{array}.
$$

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