Omrani, H., Z. Oveysi, **A. Emrouznejad**, and T. Teplova (2022). A Mixed Integer Network DEA with Shared Inputs and Undesirable Outputs for Performance Evaluation: Efficiency Measurement of Bank Branches. *Journal of the Operational Research Society,* https://doi.org/10.1080/01605682.2022.2064783.

A Mixed Integer Network DEA with Shared Inputs and Undesirable Outputs for Performance Evaluation: Efficiency Measurement of Bank Branches

Hashem Omrani[1*], Zeynab Oveysi [2], Ali Emrouznejad[3], and Tamara Teplova[4]

Abstract

Conventional DEA performs like a "black box" and provides no information about sub-processes. In some cases, such as banks, providing services is made up of interactive and interdependent processes. Also, in real world applications, inputs could be shared among these sub-processes. Moreover, due to the characteristics of some variables, such as number of employees, only integer values could be assigned to them. Hence, to address these shortcomings, in this study, a mixed integer network DEA (MI-NDEA) with shared inputs and undesirable outputs has been proposed to evaluate the efficiency of decision making units. The proposed model considers integer values for some of the input variables. Also, it assumes that some inputs are shared among different stages of the production process. To illustrate the capability of the model, the efficiency of "Internet banking", "profitability", "production" and "overall" performance of a set of bank branches have been evaluated and results are discussed. The results indicate that the mean of overall efficiency for all branches is high. However, some branches are not efficient enough in the "Production" stage or "Profitability" stage. To identify the source of inefficiency in such branches, projection values have been calculated and recommendations have been made for policy makers.

Keywords: Data Envelopment Analysis, Banking; Efficiency; Integer programming

[*] Corresponding Author, Tel: +98-4433554180, Fax: +98-4413554181, Email address: h.omrani@uut.ac.ir

[1] Faculty of Industrial Engineering, Urmia University of Technology, Urmia, Iran, h.omrani@uut.ac.ir

[2] Department of Industrial and Manufacturing Systems Engineering, University of Missouri, Columbia, MO 65211, USA, zofg8@mail.missouri.edu

[3] Surrey Business School, University of Surrey, Guildford, UK, a.emrouznejad@surrey.ac.uk

[4] National Research University Higher School of Economics, Russian Federation, toma@sani-k.ru

1. Introduction

Banking industry is one of the key drivers of each country's economy. An unstable and inefficient banking system may cause economic collapse in a country (Goyal et al., 2019). Thus, evaluating the performance of bank branches is essentially important to decision makers and has become one of the most important activities for bank managers (Strantza et al., 2003). There are several factors that have changed the methods of performance evaluation for banking systems over time. Major financial crises such as the 2007-08 global financial crisis and major structural changes such as technological advances, changes in market structures, and new regulatory processes have significantly changed the methods of evaluating the performance of financial institutions over the last 20 years (de Abreu et al., 2019). As a result, there is a lot of pressure on banks to enhance their performance and competitiveness. In fact, managers try to assess the performance of banks and identify the strengths and weaknesses of each in order to actively compete with their competitors (Omrani et al., 2022).

There have been many research studies dedicated to the performance assessment of banks and branches. For instance, Barros and Wanke (2014) evaluated the efficiency of Brazilian banks from 1998 to 2010 using a Bayesian dynamic frontier model. Their results showed that Brazilian banks improved their efficiency over time. Thaker et al. (2021) used DEA to evaluate the technical, cost, and profit efficiency of Indian banks. Then, they applied regression analysis to examine the impact of corporate governance bank characteristics and other characteristics on bank efficiency. Among all the different approaches, data envelopment analysis (DEA) has been widely used to evaluate the banks and branches performances.

DEA, first introduced by Charnes et al. (1978), is a nonparametric method which has been used for evaluating the efficiency of DMUs for many years (Emrouznejad et al., 2010). For example, Yeh (1996) used DEA together with financial ratios to identify inefficient banks. Their method also could provide bank regulators with some information about different financial dimensions that were linked to the financial operational decisions of banks. Lin et al. (2009) evaluated the operating performances of 117 branches of a certain bank in Taiwan using DEA approach. However, conventional DEA models have some significant shortcomings when it comes to the performance assessment of banks. First, they consider each bank as a "black box" that uses some inputs such as staff, supply cost, and computers to generate some outputs such as number of accounts and number of transactions (Omrani et al., 2022). In this case, DEA provides no information about the source of inefficiency for DMU managers to improve the efficiency of their DMUs

(Lewis and Sexton, 2004). However, in reality, the overall process of each bank is made up of some sub-processes that are linked and interacted with each other and the performance of each sub-process affects the overall performance.

To overcome the first shortcoming of conventional DEA models, many studies have focused on the network DEA. In a network DEA model, each DMU is assumed to have more than one stage. Initial inputs are consumed to generate intermediate measures. Intermediate measures flow between different stages to connect them and subsequently generate the final outputs (Cook et al., 2010). The network DEA model can identify the source of inefficiencies. Moreover, for the organizations with the same efficiency score, it can detect the *stage* that each organization is inefficient on (Sexton and Lewis, 2003). There are a lot of research studies that consider banking process as a two-stage DEA ((Fukuyama and Weber, 2010), (Wanke and Barros, 2014), (Wang et al., 2014), (Shi et al., 2021)). For instance, Wanke and Barros (2014) used a two-stage DEA model to measure the efficiency of Brazilian banking. In the first stage, they evaluated the cost efficiency while in the second stage they assessed the productive efficiency. Fukuyama et al. (2020) developed a two-stage DEA model to analyze the cost inefficiency of 25 Turkish banks from 2007 to 2016. The model consisted of fund-raising process as the first stage and revenue-generation process as the second one. Zha et al. (2016) used dynamic two-stage slack-based DEA to assess productivity and profitability of Chinese banks over the period 2008-2011. However, there are a few studies that consider three stages in their DEA network. For example, Boloori and Pourmahmoud (2016) used a modified slack-based network DEA (NDEA) to find branches efficiency scores as well as efficient targets for branches of Mehr-e-Eghtesad Bank in Tehran. They considered a network with three processes of deposit attraction, deposit allocation and banking services provision. Zhou et al. (2019) developed a multi-period network DEA model with three stages of capital organization, capital allocation, and profitability to evaluate the efficiency of banking system under uncertain environment. Mahmoudabadi and Emrouznejad (2019) used a SBM-NDEA model to evaluate the efficiency of banks. The overall efficiency was calculated as the weighted average of three stages: production, intermediation, and social welfare. In most of the reviewed papers, production and profitability are considered as the main two processes to evaluate the performance of banks with. In this study, we have added one more stage, Internet banking, to the DEA network.

With the advent of Internet banking, and due to its notable advantages for both customers and banks, Internet banking has become one of the profitable competitive area in the banking industry (Ho and Wu, 2009). It helps banks to reduce costs and increases profits. It also enables customers to do transactions faster and easier (Stoica et al., 2015). Therefore, increasing the efficiency of this process as well as other processes has become important to the banking industry (Ho and Wu, 2009). However, there are a few studies that use DEA method to assess the efficiency of banks with Internet banking included. Stoica et al. (2015) used a combined DEA and principal component analysis (PCA) to analyze the impact of Internet banking on the efficiency of 24 Romanian banks. Ho and Wu (2009) also used the similar technique for assessing the performance of online banking for 32 banks in Taiwan. Both studies used the conventional DEA methods to assess the effects of online banking on the performance of bank branches. In this study, we assume a network DEA with three stages of "Internet banking", "production", and "profitability".

In many real-world case studies, there are some inputs that are shared among different stages of a network. Assume a banking system which is modeled as a network with three stages of production, Internet banking, and profitability. In such network, inputs such as the employee and cost are used by all three stages. Therefore, it is better to model such network as a DEA model with shared inputs. There are a few studies that has included this assumption while evaluating the efficiency of banks ((Phung et al., 2020), (Chen et al., 2010), (Zhou et al., 2019)). Chen et al. (2010) developed some DEA models to assess the performance of DMUs with two-stage processes with shared input resources. To illustrate the application of their model, they used a banking example with two processes that shared fixed assets, employees, and IT budget inputs. Zhou et al. (2019) proposed a multi-period, multi-stage DEA model, in which employee salaries and fixed assets were treated as shared input resources for all three stages.

The other shortcoming of conventional DEA models is that all the inputs and outputs take real numbers. However, in some cases such as the efficiency assessment of banks, some inputs such as the number of employees or some outputs such as the number of transactions can only take integer values (Wu and Zhou, 2015). Assuming real numbers for such inputs or outputs may cause inaccuracy in efficiency analysis of banks. Moreover, to increase the efficiency of branches, bank managers would like to know about the projection values. Therefore, considering integer values for some variable is critical to evaluate DMUs. To deal with this issue, Wu and Zhou (2015) proposed a mixed-objective integer-valued DEA model that improved both inputs and outputs simultaneously. They applied their model to evaluate the performance of 42 university departments in IAUK, an example from Kuosmanen and Kazemi Matin (2009), to show the difference between their model and some existing models in the literature. The results showed that the efficiency scores and targets obtained from their model were more relevant and accurate. To the best of our knowledge, there is no research that has applied this integrality constraint for banking system in a network DEA. Therefore, this constraint is included in this study to have more reliable results.

Table 1 demonstrates more complete review of existing NDEA studies with shared resources, integer variables or undesirable outputs in banking system.

--- [Table 1 about here] --

According to the Table 1 and to the best of our knowledge, there is no paper that considered MI-NDEA model with three stages, shared inputs, input variables with integer values, and undesirable outputs, simultaneously.

In summary, the main contribution of this study is as follows:

- Considering a NDEA for assessing the efficiency score of “Internet banking”, “production”, and “profitability
- Developing a MI-NDEA model to assign integer values to some of the input variables
- Considering shared inputs for stages
- Considering both, desirable and undesirable outputs

The results from this study can help policy makers to evaluate the performance of different bank branches and if necessary, restructure the banking system in such a way that it is more efficient.

The rest of the paper is organized as follows: Section 2 presents the proposed methodology. Section 3 presents an illustrative application of proposed model to 45 Agribank branches of West Azerbaijan Province of Iran. Section 4 summarizes the results. Managerial implications are presented in section 5. Finally, the conclusion of this study and potential future extensions are discussed in section 6.

2. Methodology

In this section, we discuss our proposed methodology to assess the efficiency of a network DEA with three stages of “Internet banking”, “production”, and “profitability”. In our model, we assume that some of the variables (e.g. number of employees, number of ATMs) can take only integer values and their projection values must be integer as well. It is also assumed that some of the input resources (e.g. cost, employees) are consumed by more than one stage.

2.1. Network DEA

DEA is a nonparametric method which has been used for evaluating the efficiency of DMUs such as banks for many years (Emrouznejad et al., 2010). In this study, we develop a network DEA model as illustrated in Fig. 1.

--- [Figure 1 about here] --

MI-NDEA with shared inputs

Fig.1 shows a network DEA with three stages of "Internet banking", "production", and "profitability" where some inputs are shared by more than one stage. Suppose there are *n* DMUs indicated by $DMU_j\,(j=1,2,...,n)$. Each DMU_j's whole process is divided into three sub-processes of Internet banking (stage 1), production (stage 2), and profitability (stage 3). Stage 1 has three types of inputs: (1) inputs that are only inputs to this stage (x_{ij}), (2) inputs that are shared between stage 1 and stage 2 (l_{pj}), and (3) inputs that are shared among all three stages (k_{mj}). Consuming x_{ij}, $(1-\beta_j)$ portion of l_{pj}, and α_{1j} portion of k_j, stage 1 generates z_{1bj} which are called intermediate measures and are assumed to be the inputs for stage three as well. Stage 2, on the other hand, has no input that is consumed by only this stage in our network. However, it uses β_j portion of l_{pj}, and α_{2j} portion of k_j to generate another set of intermediate measures, z_{2bj}, which are the inputs for stage 3 as well. Finally, stage 3 receives α_{3j} portion of k_j, outputs from stage 1, z_{1bj}, and outputs from stage 2, z_{2bj}, to generate the final desirable and undesirable outputs, y_{rj} and yb_{fj}.

2.2. Mathematical formulation

The sets, parameters, and decision variables of the model are listed below:

Sets.

I: set of Inputs
J: set of DMUs
P: set of shared input resources between stage 1 and stage 2
M: set of shared input resources among all three stages
R: set of desirable outputs

F : set of undesirable outputs

Data Parameters.

x_{ij} :	amount of input *i* consumed by DMU_j
k_{mj} :	amount of shared resource *m* consumed by all three stages of DMU_j
l_{pj} :	amount of shared resource *p* consumed by stage 1 and stage 2 of DMU_j
z_{1bj} :	amount of intermediate measure *b* generated by stage 1 of DMU_j
z_{2bj} :	amount of intermediate measure *b* generated by stage 2 of DMU_j
y_{rj} :	amount of desirable output *r* generated by DMU_j
yb_{fj} :	amount of undesirable output *f* generated by DMU_j
$R_i^x, R_p^l, R_m^k, R_r^y, R_f^b$:	ranges of input, shared resources, and outputs

Decision Variables.

ρ :	objective value
$\lambda_j, \mu_j, \gamma_j$:	intensity vectors for stage 1, stage 2, and stage 3, respectively
$\alpha_{1j}, \alpha_{2j}, \alpha_{3j}$:	portion of the shared resource k_{mj} goes into stages 1, 2, and 3 of DMU_j, respectively
β_j :	portion of the shared resource l_{pj} goes into stage 2
$S_i^x, S_p^{l_1}, S_p^{l_2}, S_m^{k_1}, S_m^{k_2}, S_m^{k_3}, S_r^y, S_f^{yb}$:	slacks related to input, shared resources, and outputs
$\theta_0, \theta_1, \theta_2, \theta_3$:	total efficiency, stage 1 efficiency, stage 2 efficiency, and stage 3 efficiency

Zhao et al. (2021) used the concepts of *natural disposability* and *managerial disposability* proposed by Sueyoshi and Goto (2012) to describe the adaptive strategies of bank to a regulation change on undesirable outputs. In natural disposability, a bank chooses to reduce the amount of its inputs to decrease undesirable outputs. However, in managerial disposability, a bank decides to increase the amount of its inputs to decrease undesirable outputs. To calculate the efficiency of banks under the assumptions of shared resources and adaptive strategies simultaneously, we use the sets of constraints proposed by Zhao et al. (2021) where it is assumed that the bank uses natural disposability strategy to reduce the undesirable outputs and increase the desirable outputs. The first set of constraints related to stage 1 are as follows:

$$\sum_{j=1}^{n}\lambda_j x_{ij} + S_i^x = x_{i0} \qquad i = 1,\ldots,I \tag{1}$$

$$\sum_{j=1}^{n}\lambda_j \left(1-\beta_j\right) l_{pj} + S_p^{l_1} = \left(1-\beta_0\right) l_{p0} \qquad p = 1,\ldots,P \tag{2}$$

$$\sum_{j=1}^{n}\lambda_j \alpha_{1j} k_{mj} + S_m^{k_1} = \alpha_{10} k_{m0} \qquad m = 1,\ldots,M \tag{3}$$

$$\sum_{j=1}^{n}\lambda_j z_{1bj} = z_{1b0} \qquad b = 1,\ldots,B \tag{4}$$

$$\sum_{j=1}^{n}\lambda_j = 1 \tag{5}$$

$$\lambda_j \geq 0 \qquad \forall \boldsymbol{j} \tag{6}$$

$$S_i^x, S_p^{l_1}, S_m^{k_1} \geq 0 \tag{7}$$

Where x_{i0}, l_{p0} , and k_{m0} are respectively, the i th input, p th shared resource (between stage 1 and stage 2), *m*th shared resource (among all three stages) for DMU_0 under evaluation. It is worth noting that the original form of constraints (1), (2), and (3) is in form of $\leq$ and adding the slack variables S_i^x , $S_p^{l_1}$, and $S_m^{k_1}$ has changed it to the equality form presented in the above equations. In a similar manner, the constraints for stage 2 are presented as follows:

$$\sum_{j=1}^{n}\mu_j \beta_j l_{pj} + S_p^{l_2} = \beta_0 l_{p0} \qquad p = 1,\ldots,P \tag{8}$$

$$\sum_{j=1}^{n}\mu_j \alpha_{2j} k_{mj} + S_m^{k_2} = \alpha_{20} k_{m0} \qquad m = 1,\ldots,M \tag{9}$$

$$\sum_{j=1}^{n}\mu_j z_{2tj} = z_{2t0} \qquad t = 1,\ldots,T \tag{10}$$

$$\sum_{j=1}^{n}\mu_j = 1 \tag{11}$$

$$\mu_j \geq 0 \tag{12}$$

$$S_p^{l_2}, S_m^{k_2} \geq 0 \tag{13}$$

The next set of constraint are related to stage 3:

$$\sum_{j=1}^{n} \gamma_j \alpha_{3j} k_{mj} + S_m^{k_3} = \alpha_{30} k_{m0} \qquad m = 1, \ldots, M \tag{14}$$

$$\sum_{j=1}^{n} \gamma_j z_{1bj} = z_{1b0} \qquad b = 1, \ldots, B \tag{15}$$

$$\sum_{j=1}^{n} \gamma_j z_{2tj} = z_{2t0} \qquad t = 1, \ldots, T \tag{16}$$

$$\sum_{j=1}^{n} \gamma_j y_{rj} - S_r^{y} = y_{r0} \qquad r = 1, \ldots, R \tag{17}$$

$$\sum_{j=1}^{n} \gamma_j yb_{fj} + S_f^{yb} = yb_{f0} \qquad f = 1, \ldots, F \tag{18}$$

$$\sum_{j=1}^{n} \gamma_j = 1 \tag{19}$$

$$\alpha_{1j} + \alpha_{2j} + \alpha_{3j} = 1 \tag{20}$$

$$\gamma_j \geq 0 \tag{21}$$

$$S_m^{k_3}, S_r^{y}, S_f^{yb} \geq 0 \tag{22}$$

Similar to constraints (1), (2), and (3), the original form of constraints (8), (9), and (14) is in form of $\leq$ and they all indicate that a bank reduces its inputs in all three stages. However, constraints (4), (10), (15), and (16) are in form of equality constraints and imply that the intermediate measures are nondiscretionary. In other words, it is assumed that intermediate measures are beyond the judgment of management.

To define the objective function, we use the Range Adjusted Measure (RAM). Beside RAM, there is a general network slacks-based inefficiency (NSBI) approach which proposed by Lozano (2016). NSBI have been applied in several applications and it can be easily implemented for any network of processes that some or all of them generate undesirable outputs. According to Sueyoshi et al. (2010), efficiency scores generated by DEA-RAM are larger than those of the radial DEA models. However, RAM have also some advantages such as easily considering both desirable and undesirable outputs in the unified analytical structure and it is translation invariant (Sueyoshi et al. 2010). For instance, due to the translation invariant, RAM can easily deal with the negative data (Sueyoshi and Sekitani, 2009). Therefore, to define the objective function, we use the RAM model which was first introduced by Cooper et al., (1999). The objective function is as follows:

$$\rho = \max \frac{1}{I+P+M+R+F}\left[\sum_{i=1}^{I}\left(\frac{S_i^x}{R_i^x}\right)+\sum_{p=1}^{P}\left(\frac{S_p^{l_1}+S_p^{l_2}}{R_p^l}\right)+\sum_{m=1}^{M}\left(\frac{S_m^{k_1}+S_m^{k_2}+S_m^{k_3}}{R_m^k}\right)+\sum_{r=1}^{R}\left(\frac{S_r^y}{R_r^y}\right)+\sum_{f=1}^{F}\left(\frac{S_f^{yb}}{R_f^b}\right)\right]$$

(23)

Here $R_i^x, R_p^l, R_m^k, R_r^y$, and R_f^b are the ranges associated with inputs, shared resources, desirable and undesirable outputs, and they are calculated as follows:

$$\begin{aligned}
R_i^x &= \max_j\left(x_{ij}\right)-\min_j\left(x_{ij}\right)\\
R_m^k &= \max_j\left(k_{mj}\right)-\min_j\left(k_{mj}\right)\\
R_f^b &= \max_j\left(yb_{fj}\right)-\min_j\left(yb_{fj}\right)\\
R_p^l &= \max_j\left(l_{pj}\right)-\min_j\left(l_{pj}\right)\\
R_r^y &= \max_j\left(y_{rj}\right)-\min_j\left(y_{rj}\right)
\end{aligned} \tag{24}$$

The overall efficiency score (θ_0), stage 1 efficiency score (θ_1), stage 2 efficiency score (θ_2), and stage 3 efficiency score (θ_3) are calculated as follows:

$$\theta_0 = 1-\rho \tag{25}$$

$$\theta_1 = 1-\frac{1}{I+M+P}\left[\sum_{i=1}^{M}\left(\frac{S_i^x}{R_i^x}\right)+\sum_{p=1}^{P}\frac{S_p^{l_1}}{R_p^l}+\sum_{m=1}^{M}\frac{S_m^{k_1}}{R_m^k}\right] \tag{26}$$

$$\theta_2 = 1-\frac{1}{M+P}\left[\sum_{p=1}^{P}\frac{S_p^{l_2}}{R_p^l}+\sum_{m=1}^{M}\frac{S_m^{k_2}}{R_m^k}\right] \tag{27}$$

$$\theta_3 = 1-\frac{1}{M+R+F}\left[\sum_{m=1}^{M}\frac{S_m^{k_3}}{R_m^k}+\sum_{r=1}^{K}\frac{S_r^y}{R_r^y}+\sum_{f=1}^{F}\frac{S_f^{yb}}{R_f^b}\right] \tag{28}$$

According to the definition of objective function in equation (23) and efficiency scores in equations (25) – (28), the following theorem are concluded:

Definition DMU_o *is efficient if and only if* ρ *is equal to zero.*

Proof. The proof of theorem 1 is obvious based on equation (25). θ_0 is equal to one $\Leftrightarrow$ ρ is equal to zero.

Theorem DMU_o *is efficient if and only if all of its sub-processes are efficient.*

Proof. If θ_0 is equal to one $\Leftrightarrow$ ρ is equal to zero $\Leftrightarrow$ all the slack variable $(S_i^x, S_p^{l_1}, S_p^{l_2}, S_m^{k_1}, S_m^{k_2}, S_m^{k_3}, S_r^y, S_f^{yb})$

are equal to zero $\Leftrightarrow$ θ_1, θ_2, and θ_3 are equal to 1.

It is clear that since the numerators and denominators of (25) to (28) are analogous units, hence, the efficiency scores are unit invariant.

As we mentioned earlier, conventional DEA models assume inputs and outputs take real values. However, it is important to restrict some of variables (e.g. number of employees, number of ATM) to take integer values. In this study, we use the method proposed by Kuosmanen and Kazemi Matin (2009) to include the integrality restriction in our model. Therefore, constraints (1), (2), and (8) are replaced by the following constraints:

$$\begin{aligned} &\sum_{j=1}^{n} \lambda_j x_{ij} \le \bar{x}_{i0} && \forall i \\ &\bar{x}_{i0} + S_i^x = x_{i0} && \forall i \\ &\bar{x}_{i0} \quad \text{integer} && \forall i \end{aligned} \tag{29}$$

$$\begin{aligned} &\sum_{j=1}^{n} \lambda_j \left(1-\beta_j\right) l_{pj} \le \bar{l}_{p0} && \forall p \\ &\bar{l}_{p0} + S_p^{l_1} = \left(1-\beta_0\right) l_{p0} && \forall p \\ &\bar{l}_{p0} \quad \text{integer} && \forall p \end{aligned} \tag{30}$$

$$\begin{aligned} &\sum_{j=1}^{n} \mu_j \beta_j l_{pj} \le \bar{l}'_{p0} && \forall p \\ &\bar{l}'_{p0} + S_p^{l_2} = \beta_0 l_{p0} && \forall p \\ &\bar{l}'_{p0} \quad \text{integer} && \forall p \end{aligned} \tag{31}$$

Where $\bar{x}_{i0}$, and $\bar{l}_{p0}$ are the integer-valued reference points for inputs x_{i0} and l_{p0}, respectively.

Taking all the above discussion into consideration, the proposed mixed-integer non-linear model in this study is as follows:

$$\max \frac{1}{I+P+M+R+F}\left[\sum_{i=1}^{I}\left(\frac{S_i^x}{R_i^x}\right)+\sum_{p=1}^{P}\left(\frac{S_p^{l_1}+S_p^{l_2}}{R_p^l}\right)+\sum_{m=1}^{M}\left(\frac{S_m^{k_1}+S_m^{k_2}+S_m^{k_3}}{R_m^k}\right)+\sum_{r=1}^{R}\left(\frac{S_r^y}{R_r^y}\right)+\sum_{f=1}^{F}\left(\frac{S_f^{yb}}{R_f^b}\right)\right] \quad (32)$$

s.t.

$$\sum_{j=1}^{n}\lambda_j x_{ij} \le \bar{x}_{i0} \qquad \forall i \quad (33)$$

$$\bar{x}_{i0}+S_i^x = x_{i0} \qquad \forall i \quad (34)$$

$$\sum_{j=1}^{n}\lambda_j\left(1-\beta_j\right)l_{pj} \le \bar{l}_{p0} \qquad \forall p \quad (35)$$

$$\bar{l}_{p0}+S_p^{l_1}=\left(1-\beta_0\right)l_{p0} \qquad \forall p \quad (36)$$

$$\sum_{j=1}^{n}\lambda_j\alpha_{1j}k_{mj}+S_m^{k_1}=\alpha_{10}k_{m0} \qquad m=1,\dots,M \quad (37)$$

$$\sum_{j=1}^{n}\lambda_j z_{1bj}=z_{1b0} \qquad b=1,\dots,B \quad (38)$$

$$\sum_{j=1}^{n}\lambda_j=1 \quad (39)$$

$$\sum_{j=1}^{n}\mu_j\beta_j l_{pj} \le \bar{l}'_{p0} \qquad \forall p \quad (40)$$

$$\bar{l}'_{p0}+S_p^{l_2}=\beta_0 l_{p0} \qquad \forall p \quad (41)$$

$$\sum_{j=1}^{n}\mu_j\alpha_{2j}k_{mj}+S_m^{k_2}=\alpha_{20}k_{m0} \qquad m=1,\dots,M \quad (42)$$

$$\sum_{j=1}^{n}\mu_j z_{2tj}=z_{2t0} \qquad t=1,\dots,T \quad (43)$$

$$\sum_{j=1}^{n}\mu_j=1 \quad (44)$$

$$\sum_{j=1}^{n}\gamma_j\alpha_{3j}k_{mj}+S_m^{k3}=\alpha_{30}k_{m0} \qquad m=1,\dots,M \quad (45)$$

$$\sum_{j=1}^{n}\gamma_j z_{1bj}=z_{1b0} \qquad b=1,\dots,B \quad (46)$$

$$\sum_{j=1}^{n}\gamma_j z_{2tj} = z_{2t0} \qquad t=1,\ldots,T \tag{47}$$

$$\sum_{j=1}^{n}\gamma_j y_{rj} - S_r^{y} = y_{r0} \qquad r=1,\ldots,R \tag{48}$$

$$\sum_{j=1}^{n}\gamma_j yb_{fj} + S_f^{yb} = yb_{f0} \qquad f=1,\ldots,F \tag{49}$$

$$\sum_{j=1}^{n}\gamma_j = 1 \tag{50}$$

$$L_{1j}^{\alpha} \le \alpha_{1j} \le U_{1j}^{\alpha} \tag{51}$$

$$L_{2j}^{\alpha} \le \alpha_{2j} \le U_{2j}^{\alpha} \tag{52}$$

$$L_{3j}^{\alpha} \le \alpha_{3j} \le U_{3j}^{\alpha} \tag{53}$$

$$L_{j}^{\beta} \le \beta_{j} \le U_{j}^{\beta} \tag{54}$$

$$\alpha_{1j} + \alpha_{2j} + \alpha_{3j} = 1 \tag{55}$$

$$\overline{x}_{i0} \quad \text{integer} \qquad \forall i \tag{56}$$

$$\overline{l}_{p0}, \overline{l}'_{p0} \quad \text{integer} \qquad \forall p \tag{57}$$

$$\lambda_j, \mu_j, \gamma_j \ge 0 \qquad \forall j \tag{58}$$

$$S_i^{x}, S_p^{l_1}, S_p^{l_2}, S_m^{k_1}, S_m^{k_2}, S_m^{k_3}, S_r^{y}, S_f^{yb} \ge 0 \tag{59}$$

When we solve the above model for DMU_0 under evaluation, we get the optimal solution

$(\overline{x}_{i0}^{*}, \overline{l}_{p0}^{*}, \overline{l}'^{*}_{p0}, \lambda_j^{*}, \mu_j^{*}, \gamma_j^{*}, S_i^{x*}, S_p^{l_1*}, S_p^{l_2*}, S_m^{k_1*}, S_m^{k_2*}, S_m^{k_3*}, S_r^{y*}, S_f^{yb*}, \alpha_{1j}^{*}, \alpha_{2j}^{*}, \alpha_{3j}^{*}, \beta_j^{*})$. Then the projection

values for DMU_0 can be calculated as follows:

$$\hat{x}_{i0} = \overline{x}_{i0}^{*} \tag{60}$$

$$\hat{l}_{p0} = \overline{l}_{p0}^{*} + \overline{l}'^{*}_{p0} \tag{61}$$

$$\hat{k}_{m0} = \alpha_{10}^{*} k_{m0} - S_m^{k_1*} + \alpha_{20}^{*} k_{m0} - S_m^{k_2*} + \alpha_{30}^{*} k_{m0} - S_m^{k3*} \tag{62}$$

$$\hat{y}_{r0} = y_{r0} + S_r^{y*} \tag{63}$$

$$\hat{yb}_{f0} = yb_{f0} + S_f^{yb*} \qquad (64)$$

Where $\hat{x}_{i0}, \hat{l}_{p0}, \hat{k}_{m0}, \hat{y}_{r0}, \hat{yb}_{f0}$ are respectively projection values of input i, total shared input *p*, total shared input *m*, output *r* and output *f* for DMU_0 under evaluation. It is worth noting that $\bar{l}_{p0}^{*}$ and $\bar{l'}_{p0}^{*}$ are respectively the projection values of shared input *p* associated with stage 1 and stage 2 for DMU_0. Also, the projection values for shared input *m* associated with stage 1, stage 2, and stage 3 for DMU_0 are equal to $\alpha_{10}^{*}k_{m0} - S_m^{k_1*}$, $\alpha_{20}^{*}k_{m0} - S_m^{k_2*}$, $\alpha_{30}^{*}k_{m0} - S_m^{k3*}$.

3. Application to Agribank branches

In this section, the proposed MI-NDEA model is applied to evaluate “Internet banking”, “production”, “profitability”, and overall efficiency of 45 Agribank branches in West Azerbaijan Province. As the first official agricultural finance institution in Iran, Agribank was established in 1933 to support agricultural activities financially. Through its current 1914 branches nationwide, this bank offers a variety of banking services including deposit accounts for domestic and overseas clients, letters of credit, treasuries, currency exchange, and electronic banking services. To actively compete with their competitors, it is important to the policy makers to assess the performance of branches and identify the strengths and weaknesses of each.

To evaluate the performance of branches by mix integer NDEA, the first step is to define sub-processes and their corresponding inputs (both, sharable and non-sharable), intermediate measures, and outputs. Selection of sub-processes and their corresponding inputs, intermediate measures, and outputs in DEA applications depends on the purpose of research. To select relevant variables, this paper follows previous studies on assessing the performance of banks. We considered the most common variables used in the literature. For example, Wang et al. (2014) divided the whole operational process of a bank into two main sub-processes, production and profitability. They used fixed assets and labors as the inputs to the production process to generate bank deposits (intermediate measures). Then, the bank deposits were utilized by the profitability process to generate non-performing loans, interest incomes, and non-interest incomes. Omrani et al. (2019) used a bi-level multi-objective DEA to maximize profit and operational efficiencies, simultaneously. They used number of ATM, number of staff, total costs as the inputs for both efficiencies, value of deposits, value of loans, total profit, and total revenue as the outputs of profit efficiency, and value of deposits, value of loans, and number of cards as the outputs of operational efficiency. Chen et al. (2010) discussed that some of the inputs such as fixed assets and employees are associated with more than one stage and they should be considered as the shared inputs.

Due to the notable advantages of Internet banking for both customers and banks, there are also a few studies ((Stoica et al., 2015), (Ho and Wu, 2009)) that evaluate the performance of banks with respect to the Internet banking. In both research studies, authors assumed deposit, operation cost, employees, equipment as the inputs and revenue, and daily reach rate as the outputs.

In this study, the whole operation process of each bank is divided into three sub-processes of “Internet banking”, “production”, and “profitability”. In their Internet banking stage, the banks use ATMs (χ_{1j}), some portion of employees ($\left(1-\beta_j\right)l_{1j}$), and some portion of cost ($\alpha_{1j}k_j$) to generate credit cards (z_{11j}) and customers (z_{12j}) (intermediate measures). In the second stage, banks use the rest of employees ($\beta_j l_{1j}$), and some portion of cost ($\alpha_{2j}k_{1j}$) to generate loan (z_{21j}) and deposit (z_{22j}) (intermediate measures). Finally, in the last stage (profitability), banks use the generated intermediate measures by stage 1 and stage 2 (credit cards, customers, deposit, and loan) as well as some portion of cost ($\alpha_{3j}k_{1j}$) to generate profit (y_{1j}) as the desirable output and non-performing loan (yb_{1j}) as the undesirable output. In the proposed model, the portions are all decision variables with the upper and lower bounds set as $0.1 \le \alpha_{1j} \le 0.4$, $0.5 \le \alpha_{2j} \le 0.8$, $0.1 \le \alpha_{3j} \le 0.3$, $0.6 \le \beta_j \le 0.9$ for all *DMUs*. Table 2 presents the rest of data for 45 Agribank branches.

4. Results and Discussion

In the following subsections, the results of the proposed model are discussed and analyzed.

4.1. Efficiency Analysis

For evaluating each stage efficiency, the proposed MI-NDEA model has been applied and the results are presented in Table 3.

--- [Table 3 about here] --

According to the Table 3, B1, B39, and B44 are the most efficient branches among all branches with the overall efficiency score of 1. Followed by that, B45 has the second best performance by an overall efficiency score of 0.994. Branch 14 with the efficiency score of 0.817 is the worst branch among all branches. Also, B9 and B25 with the efficiency scores of 0.863 and 0.866 are the second and third worst ones. The mean efficiency of branches in overall, is 0.954. 28 branches have higher efficiency score than mean, which confirms that the overall efficiency of Agribank branches in the West Azerbaijan is acceptable.

Results indicates that 16 branches are efficient in Internet banking. B1, B3, B7, B12, B17, B20, B21, B22, B27, B31, B32, B38, B39, B40, B44, B45 with the efficiency score of 1 have the highest efficiency score among 45 branches. Followed by that, B25 has the second best performance in Internet banking with an efficiency scores of 0.999. Indeed, these branches could convert cost, ATM and employees to credit card and customer more efficiently than the other branches. On the other hand, B4 with the efficiency score of 0.895 has the weakest performance in Internet banking. B2 and B36 with the efficiency scores of 0.946 and 0.954 have the second and third worse performance, respectively. The mean of efficiency score for all branches is 0.988 which is higher than the mean of other stages as well as the overall mean. Also, the mean efficiency score of branches indicates that the average performance of branches in Internet banking is pretty high.

In production analysis, B1, B4, B24, B35, B39, B41, B42, B43, and B44 can completely use cost and employee to generate loan and deposit. In contrary, B17 with the efficiency score of 0.784 indicates the worse performance among 45 branches. Also, B3 with the efficiency score of 0.785 has the second weakest performance among branches. In comparison with the overall, Internet banking, and profitability, the production stage has the lowest mean of efficiency score with the mean of 0.951.

In profitability, 31 branches are efficient with the efficiency score of 1. Indeed, these 31 branches could increase profit and decrease non-performing loan by using customer, credit cards, loan and deposit. B14 with the efficiency score of 0.703 has the lowest efficiency score. The average efficiency scores of branches in profitability is higher than production and overall with the mean of 0.968.

Fig.2 shows the density plots for efficiency scores of all three stages and overall performance.

-- [Figure 2 about here] --

As can be seen from Fig.2, the peak of Internet banking and profitability density plots are closer to 1 compared to the ones for production and overall. In fact, it is expected to see such plots with 16 branches with the Internet banking efficiency score equal to one and 31 branches with the profitability efficiency score equal to 1. Moreover, the distribution of production density plot confirms that most of bank branches are not efficient enough in this stage, while the Internet banking density plot distribution shows that there is no branch with low efficiency score in this stage.

4.2. Projection values

In this section the source of inefficiency for the inefficient branches has been evaluated using projection values. Projection values are obtained using Eq. (60) to (64) and the values are presented in Table4.

--- [Table 4 about here] --

As it can be seen in Table 4, variables with the integer characteristics such as number of employees and ATMs get integer projection values as well which make the results more reliable and interpretable.

Moreover, the projection values associated with the inputs (See Table.4) are less than or equal to the original values of inputs (See Table. 2). In fact, adopting natural disposability strategy in our model, we expect an inefficient branch reduces the amount of some inputs to reduce undesirable outputs and increase the desirable outputs. The projection values presented in Table 4. confirms this issue. For example, the efficiency score of branch 4 (B4) for Internet banking, profitability, production, and overall is 0.895, 1, 1 and 0.937, respectively. It seems that the Internet banking section is not as efficient as the other sections in this branch. Projection values obtained for this branch (See Table.4), suggests reducing the number of employees in Internet banking section from 6 people ($\left(1-\beta_4\right)l_{14}$) to 2 and lowering the cost from 145.837 ($\alpha_{14}k_4$) to 43.873. After applying projection values for B4, the efficiency scores are changed to 0.943, 1, 1, 0.966 for Internet banking, profitability, production and overall, respectively. The improvement in the efficiency scores also implies that the projection values obtained from the model are valid.

5. Managerial implications

In this section, branches are divided into two groups, branches with high efficiency (group 1) and branches with lower efficiency (group 2). According to the features of each group, recommendations are made for Agribank managers. We choose a threshold of 97% to identify the most efficient branches. In other words, a branch is categorized as the most efficient branch if and only if it obtains an efficiency score of 97% or greater in all three stages as well as the overall performance. The threshold can be changed based on the managers opinion. Assuming the 97% threshold, 12 branches fall into group 1. In fact, these branches fall into area 1 of Fig. 3, Fig. 4, and Fig. 5. The common feature of all these branches is that, all have a profitability efficiency score of one which means they all fully utilize the intermediate measures including credit cards, customer, loan, and deposits as well as cost to generate the most profit possible and the least possible non-performing loan. These branches performance can be considered as benchmark and other branches can learn from group 1 branches to improve their efficiency. Although some branches in the second group have high Internet banking and production efficiency scores (e.g. B14), their low profitability performance results in a very low overall performance. One recommendation for policy makers in this case is to reduce the amount of non-performing loans. For example, for B14, the current non-performing loan value is 8955.5725 (See Table. 2). If B14 reduces its non-performing loan to 1282.427 (See Table 4.), this branch also falls into group 1 which is the most efficient ones.

For some other branches in the second group (e.g. B4, B24), Internet banking performance is the main cause of the low overall performance. Comparing the current number of ATMs, number of employees, and cost (See Table. 2) with their associated projection values (See Table. 4), we recommend mangers lowering the cost and number of employees assigned to the Internet banking section in these branches. For example, for B4, the current number of employees in the Internet banking is about 6 people ($\left(1-\beta_4\right)l_{14}$). However, the projection value associated with the number of employees in the Internet banking section for B4 is 2 (See Table. 4).

For some other branches (e.g. B3, B7, B10, B11, B17), low efficiency score in the production stage caused them to fall in the second group. Similar to the Internet banking stage and with comparing the current and projection values for the number of employees, and cost, we recommend mangers lowering the cost and number of employees assigned to the production section in these branches. For example, for B3, the current number of employees in the production section is about 14 people ($\beta_3 l_{13}$). However, the projection value associated with the number of employees in the production section for B3 is 9 (See Table. 4).

Fig. 3, Fig. 4, Fig. 5 plot the stages efficiency two by two and highlight the branches that are highly efficient in one or two stages but not efficient enough in another stage.

--[Figure 3 about here] ---

--[Figure 4 about here] ---

--[Figure 5 about here] ---

6. Summary and Conclusion

Due to the importance of role of banks in economy of countries, performance evaluation of banks especially bank branches are substantially important for bank managers and decision-makers. In this paper, 45 Agribank branches of West Azerbaijan Province of Iran are evaluated. For this purpose, a novel MI-NDEA with shared inputs and undesirable outputs are proposed. The proposed model could consider integer values for some variables. Moreover, input data could be shared among different stages. Using the proposed model, Internet banking, profitability, production, and overall efficiency scores of all branches have been evaluated. The results indicate that the mean of overall efficiency for most branches is acceptable for Agribank managers. However, some branches are not efficient enough in the “production” stage or “profitability” stage. Production stage has the least efficiency score among all three stages. There are only around 20% of branches that are 100% efficient in this stage. However, around 69% of bank branches are 100% efficient in the profitability stage, and around 36% of branches are 100% efficient in the Internet banking stage. Despite of the high percentage of bank branches with efficiency score of 1 in the profitability stage, this stage is the next stage with the lowest mean of efficiency score. To identify the source of inefficiencies, projection values are calculated and based on that some recommendations are made for policy makers. Moreover, based on the efficiency scores of branches in each stage, we divided branches into two subgroups, subgroup 1 and subgroup 2. The first subgroup are the most efficient branches and the second subgroup are the rest of branches. Recommendation for improving the branches in the second subgroup are made in this paper. One limitation of this study is to consider lower and upper bounds for portion of shared inputs used by different stages. Various portions could lead to different solutions. So, one direction for the future study, could be applying uncertainty for portion of shared inputs used by different stages which could make the results more reliable. Also, applying uncertain data parameters has value added for future studies. Interested researchers can also incorporate preferences of DMs into the evaluations to increase the discrimination power and reduce weight flexibility of network DEA.

Acknowledgement

The article was prepared within the framework of the Basic Research Program at HSE University. The authors would like to thank *Professor Said Salhi*, The Editor-in-Chief of *Journal of the Operational Research Society*, associate editor and two anonymous reviewers for their insightful comments and suggestions. As results this paper has been improved substantially.

References

Barros, C.P., & Wanke, P. (2014). Banking efficiency in Brazil. *Journal of International Financial Markets, Institutions and Money*, 28, 54-65.

Boloori, F., & Pourmahmoud, J. (2016). A modified SBM-NDEA approach for the efficiency measurement in bank branches. *Operational Research*, 16(2), 301-326.

Charnes, A., Cooper, W.W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research*, 2, 429-44.

Chen, Y., Du, J., Sherman, H.D., & Zhu, J. (2010). DEA model with shared resources and efficiency decomposition. *European Journal of Operational Research*, 207(1), 339-349.

Cook, W.D., & Hababou, M. (2001). Sales performance measurement in bank branches. *Omega*, 29, 299-307.

Cook, W.D., Hababou, M., & Tuenter, H.J.H. (2000). Multicomponent efficiency measurement and shared inputs in data envelopment analysis: an application to sales and service performance in bank branches. *Journal of Productivity Analysis*, 14, 209-224.

Cook, W.D., Zhu, J., Bi, G., & Yang, F. (2010). Network DEA: Additive efficiency decomposition. *European Journal of Operational Research*, 207(2), 1122-1129.

Cooper, W.W., Park, K.S., & Pastor, J.T. (1999). RAM: a range adjusted measure of inefficiency for use with additive models, and relations to other models and measures in DEA. *Journal of Productivity Analysis*, 11, 5-42.

de Abreu, E.S., Kimura, H., & Sobreiro, V.A. (2019). What is going on with studies on banking efficiency? *Research in International Business and Finance*, 47, 195-219.

Dia, M., Golmohammadi, A., & Takouda, P.M. (2020). Relative Efficiency of Canadian Banks: A Three-Stage Network Bootstrap DEA. *Journal of Risk and Financial Management*, 13(4), 1-25.

Emrouznejad, A., Anouze, A.L., & Thanassoulis, E. (2010). A semi-oriented radial measure for measuring the efficiency of decision making units with negative data, using DEA. *European Journal of Operational Research*, 200, 297-304.

Fukuyama, H., Matousek, R., & Tzeremes, N.G. (2020). A Nerlovian cost inefficiency two-stage DEA model for modeling banks' production process: Evidence from the Turkish banking system. *Omega*, 95, 102198.

Fukuyama, H., Matousek, R., & Tzeremes, N.G. (2021). Minimum distance efficiency measure in bank production: A directional slack inefficiency approach. *Journal of the operational research society*, 1-13, DOI: 10.1080/01605682.2021.1943020

Fukuyama, H., & Weber, W.L. (2010). A slacks-based inefficiency measure for a two-stage system with bad outputs. *Omega*, 38, 398-409.

Goyal, J., Singh, M., Singh, R, & Aggarwal, A. (2019). Efficiency and technology gaps in Indian banking sector: Application of meta-frontier directional distance function DEA approach. *The Journal of Finance and Data Science*, 5(3), 156-72.

Ho, C-T.B., & Wu, D.D. (2009). Online banking performance evaluation using data envelopment analysis and principal component analysis. *Computers & operations research*, 36(6), 1835-1842.

Jahanshahloo, G.R., Amirteimoori, A.R. & Kordrostami, S. (2004). Multi-component performance, progress and regress measurement and shared inputs and outputs in DEA for panel data: an application in commercial bank branches. *Applied Mathematics and Computation*, 151, 1-16.

Kuosmanen, T., & Kazemi Matin, R. (2009). Theory of integer-valued data envelopment analysis. *European Journal of Operational Research*, 192, 658-67.

Lewis, H.F., & Sexton, T.R. (2004). Network DEA: efficiency analysis of organizations with complex internal structure. *Computers & operations research*, 31(9), 1365-1410.

Lin, T.T., Lee, C-C., & Chiu, T-F. (2009). Application of DEA in analyzing a bank's operating performance. *Expert systems with applications*, 36, 8883-8891.

Lozano, S. (2016). Slacks-based inefficiency approach for general networks with bad outputs: An application to the banking sector. *Omega*, 60, 73-84

Mahmoudabadi, M.Z., & Emrouznejad, A. (2019). Comprehensive performance evaluation of banking branches: A three-stage slacks-based measure (SBM) data envelopment analysis. *International Review of Economics & Finance*, 64, 359-376.

Omrani, H., Alizadeh, A., Emrouznejad, A., & Oveysi, Z. (2022). A novel best-worst-method two-stage data envelopment analysis model considering decision makers' preferences: An application in bank branches evaluation. *International Journal of Finance & Economics*, 1–18.

Omrani, H., Mohammadi, S. & Emrouznejad, A. (2019). A bi-level multi-objective data envelopment analysis model for estimating profit and operational efficiency of bank branches. *RAIRO-operations research*, *53*(5), 1633-1648.

Phung, M-T., Cheng, C-P., Guo, C., & Kao, C-Y. (2020). Mixed Network DEA with Shared Resources: A Case of Measuring Performance for Banking Industry. *Operations Research Perspectives*, 7, 100173.

Sexton, T.R., & Lewis, H.F. (2003). Two-stage DEA: An application to major league baseball. *Journal of Productivity Analysis*, 19, 227-249.

Shi, X., Emrouznejad, A., & Yu, W. (2021). Overall efficiency of operational process with undesirable outputs containing both series and parallel processes: A SBM network DEA model. *Expert systems with applications*, 178, 115062.

Stoica, O., Mehdian, S., & Sargu, A. (2015). The impact of internet banking on the performance of Romanian banks: DEA and PCA approach. *Procedia Economics and Finance*, 20, 610-622.

Strantza, C, Damaskos, X., Doumpos, M., & Zopounidis, C. (2003). A multicriteria methodology for developing a performance measurement model for bank branches. *Operational Research*, 3(3), 307-323.

Sueyoshi, T., & Goto, M. (2012). Data envelopment analysis for environmental assessment: Comparison between public and private ownership in petroleum industry. *European Journal of Operational Research*, 216(3), 668-678.

Sueyoshi, T., Goto, M., & Ueno, T. (2010). Performance analysis of US coal-fired power plants by measuring three DEA efficiencies. *Energy Policy*, 38, 1675-1688.

Sueyoshi, T., & Sekitani, K. (2009). An occurrence of multiple projections in DEA-based measurement of technical efficiency: Theoretical comparison among DEA Models from desirable properties. *European Journal of Operational Research*, 196, 764-794.

Thaker, K, Charles, V., Pant, A., & Gherman, T. (2021). A DEA and random forest regression approach to studying bank efficiency and corporate governance. *Journal of the operational research society*: 1-28, DOI: 10.1080/01605682.2021.1907239

Wang, K., Huang, W., Wu, J., & Liu, Y-N. (2014). Efficiency measures of the Chinese commercial banking system using an additive two-stage DEA. *Omega*, 44, 5-20.

Wanke, P, & Barros, C. (2014). Two-stage DEA: An application to major Brazilian banks. *Expert systems with applications*, 41(5), 2337-2344.

Wu, J., & Zhou, Z. (2015). A mixed-objective integer DEA model. *Annals of Operations Research*, 228, 81-95.

Yang, C., & Liu, H-M. (2012). Managerial efficiency in Taiwan bank branches: A network DEA. *Economic Modelling*, 29(2), 450-461.

Yeh, Q-J. (1996). The application of data envelopment analysis in conjunction with financial ratios for bank performance evaluation. *Journal of the operational research society*, 47(8), 980-988.

Zha, Y., Liang, N., Wu, M., & Bian, Y. (2016). Efficiency evaluation of banks in China: A dynamic two-stage slacks-based measure approach. *Omega*, 60, 60-72.

Zhao, L., Zhu, Q., & Zhang, L. (2021). Regulation adaptive strategy and bank efficiency: A network slacks-based measure with shared resources. *European Journal of Operational Research*, 295(1), 348-362.

Zhou, X., Xu, Z., Chai, J., Yao, L., Wang, S., & Lev, B. (2019). Efficiency evaluation for banking systems under uncertainty: A multi-period three-stage DEA model. *Omega*, 85, 68-82.

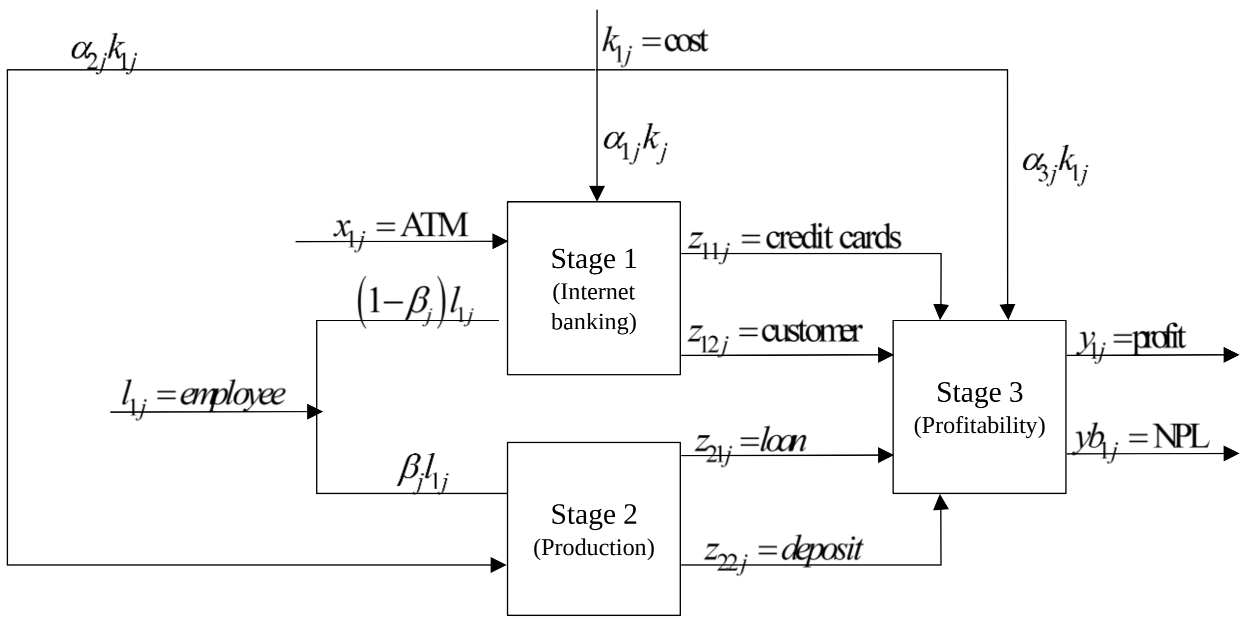

Figure 1. Three stage network process with shared inputs

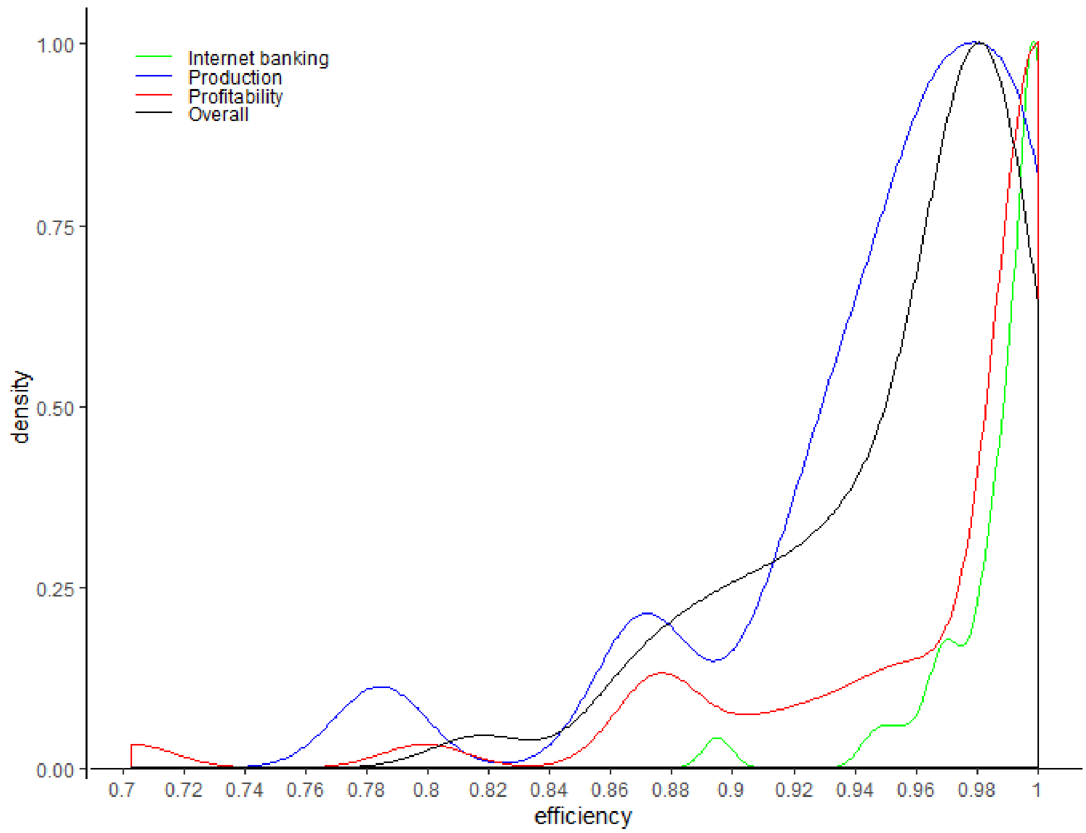

Figure 2. Density plots of stages and overall efficiencies of Agribank branches

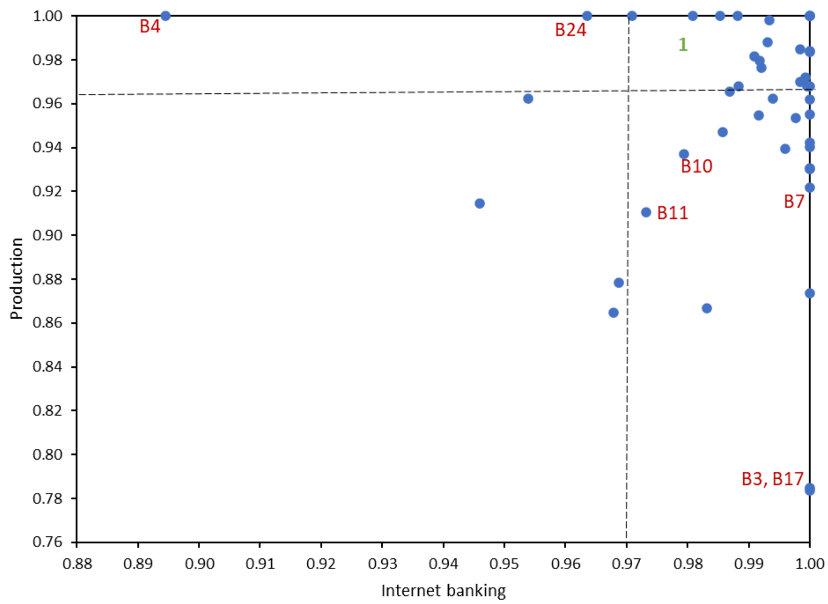

Figure 3. Internet banking efficiency vs. Production efficiency

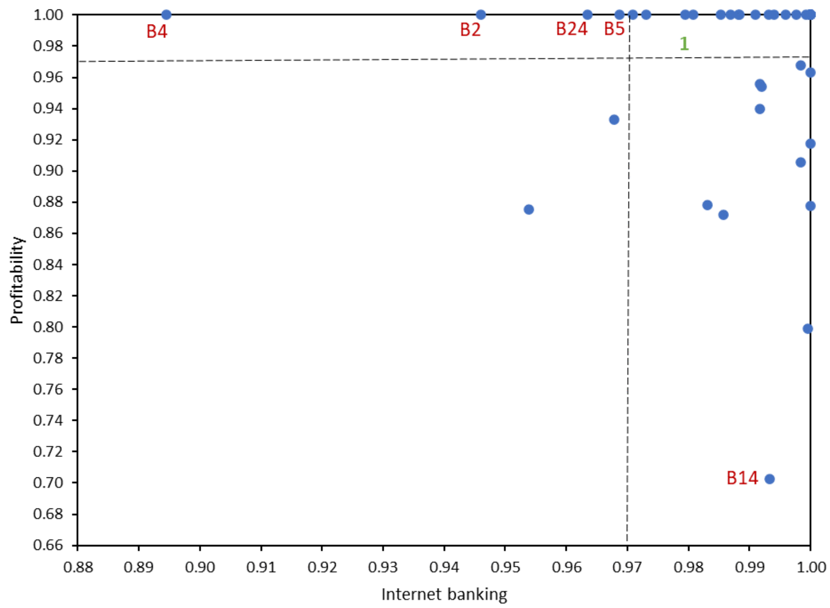

Figure 4. Internet banking efficiency vs. Profitability efficiency

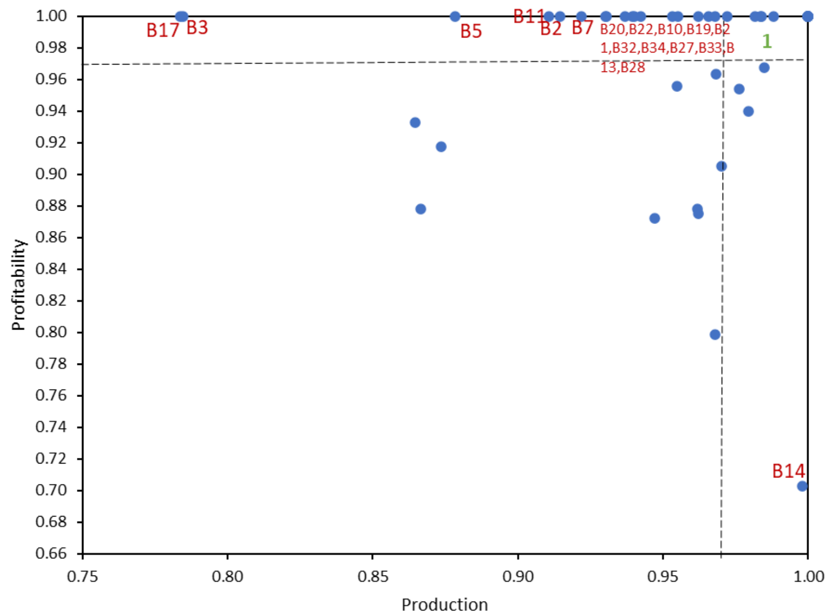

Figure 5. Production efficiency vs. Profitability efficiency

Table 1: Literature review of DEA studies on banking industry

Author/year	Method	Network structure	# DMUs/ Observations	Inputs	Intermediate variables	Outputs	Key findings	Shared input	Integer variable	Undesirable Output
(Cook, Hababou, and Tuenter 2000)	DEA	-	20	# service staff, # sales staff, # support staff, # other staff	-	# counter level deposits, # transfers between accounts, # retirement savings plan openings, # mortgage accounts opened	Extended usual DEA to determine the best resource split that optimizes the aggregate efficiency score	√	-	-
(Cook and Hababou 2001)	Goal programming version of the additive DEA	-	20	# full time equivalent service staff, # full time equivalent sales staff, # full time equivalent support staff, # full time equivalent "other" staff		# menu account transactions, # Visa cash advances, # commercial deposit transactions, # RSP account openings, # mortgages transacted, # variable rate consumer loans transacted	Developed a goal programming version of the additive DEA that allows to have multicomponent measures.	√	-	-
(Jahanshahloo, Amirteimoori, and Kordrostami 2004)	DEA	-	39	-	-	-	Characterized each DMU and its component in a panel in efficiency terms and determined progress and regress for each component	√	-	-
(Lin, Lee, and Chiu 2009)	DEA	-	117	# staff, interest expense, deposit operating amount, current deposit operating amount	-	loan operating amount, interest revenue, operating revenue, and earning	Used DEA to evaluate operating performance of branches of a bank in Taiwan. There were many inefficient branches. The average overall technical efficiency of branches was low probably due to lower loan-to-deposit ratio which caused excessive input waste.	-	-	-
(Fukuyama and Weber 2010)	Slacks- based inefficiency	Two-stage (deposit generation,	869	labour, capital, equity	raised funds	loans, investments, other business	Developed a SBM-NDEA for a system with undesirable outputs and used their	-	-	√

		loans and securities investments production)				activities, nonperforming loans	method to evaluate the performance of Japanese banks			
(Wanke and Barros 2014)	Centralized two-stage DEA	Two-stage (cost efficiency, productive efficiency	40	# branches, # employees	administrative expenses, personnel expenses	equity, permanent assets	Used a centralized two-stage DEA model to optimize cost efficiency stage and productive efficiency stage, simultaneously for Brazilian banks.	-	-	-
(Wang et al. 2014)	Additive two-stage DEA	Two-stage (Deposit producing, profit earning)	16	fixed assets, labours	bank deposits	non-interest incomes, interest incomes, non-performing loans	Utilized two-stage DEA model to assess the operational performance of the Chinese banks. Deposit producing and profit earning were two stages considered in this study. The inefficiency of the Chinese banking system was mainly because of the inefficiency of the deposit producing sub-process. The disposal of non-performing loans and the join-equity reform of the state-owned commercial banks improved the efficiency.	-	-	√
(Boloori and Pourmahmoud 2016)	SBM-NDEA	Three-stage (deposit attraction process, deposit allocation process, banking services)	73	personnel cost, operation cost, interest cost	deposits	interest income, fee income, fund transfer income	Introduced a modified SBM-NDEA model to evaluate the efficiency of bank branches. The modifications composed of some changes in the overall efficiency measure and contribution of intermediate factors in the constraints based on their categorization.	√	-	√
(Zhou et al. 2019)	Multi-period three-stage DEA	Three-stage (capital organization, capital allocation, profitability)	16	employees' salaries, fixed asserts, interest payments	deposits, due from banks, total loans	net interest incomes, non-performing loans	Developed a multi-period three-stage DEA model to evaluate the efficiency of banking system under uncertain environment. Triangular type-2 fuzzy numbers were considered for non-performing loans.	√	-	√
(Mahmoudabadi and Emrouznejad 2019)	SBM-NDEA	Three-stage (production, intermediation, social welfare)	37	employees, fixed assets, non-operating costs, interest expenses	bank deposits, bank facilities	employment, # transactions, # accounts, interest income, non-interest income	Used a SBM-NDEA model to evaluate the efficiency of banks. The overall efficiency was calculated as the weighted average of three stages: production, intermediation, and social welfare.	-	-	-

(Dia, Golmohammadi, and Takouda 2020)	NDEA	Three-stage (production, investment, revenue generation)	6	total assets, # employees, other operating costs	deposits, loans, securities, impaired loans	interest incomes, non-interest incomes	Proposed a NDEA with bootstrapping to assess the performance of six Canadian banks over 2000-2017	-	-	-
(Shi, Emrouznejad, and Yu 2021)	SBM-NDEA	Two-stage (deposit producing, deposit utilizing)	16	fixed assets, labour	disposable deposit, deposit reserve	non-interest incomes, interest incomes, non-performing loans	Developed a SBM-NDEA to evaluate the operational performance of DMUs with undesirable outputs and series and parallel processes. Evaluated Chinese commercial banks during 2012–2016 and showed that the overall inefficiency was mainly caused by the profit generating process and estimated the required adjustment of variables for an inefficient bank.	-	-	√
(Fukuyama, Matousek, and Tzeremes 2021)	minimum distance DEA	-	721	total # employees, banks' physical capital, banks' deposits levels	-	net loans, total securities	Proposed a minimum distance DEA model to be able to set more realistic benchmark targets for the evaluation of bank's efficiencies. The method was based on the idea of Koopmans strong efficiency.	-	√	-
This study	MI-NDEA	Three- stage (production, Internet banking, profitability)	45	ATMs, employees, cost	credit cards, customers, loan, deposit	profit, non-performing loans	Proposed MI-NDEA with shared inputs and undesirable outputs to evaluate the efficiency of DMUs	√	√	√

Table 2. Data

Branch	Number of ATM (x_1)	Number of FTE employees (l_1)	Total Cost (*10^6 Rials) (k_1)	Issued credit-cards (z_{11})	Customers (z_{12})	Value of loans (*10^9 Rials) (z_{21})	Value of deposits (*10^9 Rials) (z_{22})	Profit (*10^9 Rials) (y_1)	Non-performing loans (*10^6 Rials) (yb_1)
1	12	26	803.6903	8641	240	456.9545	3,078.2227	216.3477	2104.1726
2	2	13	456.2391	3954	305	265.0543	1,686.1282	164.0475	5666.9421
3	2	16	444.7804	4111	225	368.7397	1,602.5204	133.1462	1216.0753
4	4	15	364.5931	4412	182	357.2202	1,550.5406	130.8336	8688.4892
5	3	9	250.6122	3392	252	237.8434	829.3599	81.0172	721.8233
6	3	8	218.1744	2380	248	160.4613	510.8911	47.3679	1744.5523
7	1	7	302.6559	1715	95	170.4509	1,138.1223	112.3815	6737.2447
8	1	4	141.4904	1067	35	68.5860	547.8821	55.4326	2449.1622
9	1	8	321.8510	1720	179	103.6143	754.2955	64.8720	5428.1248
10	1	6	159.9544	1505	53	81.5783	512.8198	49.2663	2084.6524
11	1	6	170.8670	1111	251	67.1668	269.9968	27.8552	896.6692
12	1	7	177.6070	1924	157	128.5899	269.5623	32.9509	2483.8294
13	1	4	158.7300	801	35	141.4374	292.2960	23.3413	2127.2801
14	1	3	124.9480	589	96	62.3319	357.0245	36.9815	8955.5725
15	1	6	152.2141	1009	80	110.4535	389.3656	36.5903	3508.2122
16	1	5	166.2720	1112	73	105.4613	538.2798	49.4907	1665.0244
17	1	15	521.7200	2277	2098	345.5890	1,655.2093	142.5752	5923.0252
18	1	4	144.1345	960	107	86.9623	416.0810	42.4832	2170.2642
19	1	4	135.6508	774	50	49.4155	222.4854	19.8094	1091.8398
20	1	4	152.9222	908	30	70.5395	232.8019	30.9178	1746.9221
21	1	4	138.9811	1338	693	103.7880	229.9153	25.9783	6650.2362
22	2	4	170.4549	1199	28	99.2206	366.9763	33.2611	1282.8917
23	1	4	149.3540	800	52	134.1878	268.2196	14.9820	1948.7911
24	1	5	142.8128	1171	122	313.8654	445.2498	34.5878	762.7641
25	1	4	116.0409	853	62	55.6811	302.8277	31.3022	6357.2690
26	1	6	175.3735	1218	99	209.1332	759.6675	78.0035	7816.1549
27	2	4	143.6919	926	21	118.8133	240.9129	24.1849	4125.9035
28	1	4	113.9082	485	62	61.8093	178.1847	14.5364	176.1522
29	1	4	164.3434	877	89	131.3822	205.4017	27.5747	2843.2224
30	1	4	129.2903	866	114	115.0234	422.0802	39.6994	1028.5187
31	1	5	109.2209	1354	127	110.0016	271.6193	26.7246	974.9953
32	1	5	131.3778	1483	48	122.9265	152.5336	12.4166	118.9133
33	1	4	118.5861	959	73	46.0458	186.8068	19.7564	1185.0888
34	1	4	113.5400	1108	316	55.3028	108.8218	10.9988	1902.0738
35	1	5	107.9984	472	36	27.9253	587.9349	61.1722	5664.3141
36	2	5	122.5742	1077	69	76.9666	256.8362	22.4325	3291.5658
37	1	3	111.2424	437	39	60.1895	274.1113	27.9989	166.8611
38	2	3	87.4687	500	24	57.0945	154.1113	15.6073	955.8050
39	1	3	115.0203	422	27	19.9501	277.6825	31.0410	4076.6317
40	1	4	112.5790	817	253	51.5218	152.7133	17.4613	3732.2479
41	1	3	89.7252	695	82	29.9634	100.4059	10.2145	1936.1228
42	1	3	102.1829	805	50	63.9573	34.4926	35.3593	1151.9454
43	1	4	132.1742	619	166	28.5537	47.7151	4.3503	14.7169
44	1	4	104.5030	1459	171	63.6045	12.7278	13.8202	8.3323
45	1	3	90.0268	1030	34	57.3079	80.8977	7.8625	412.3640

Table 3. Stage and overall efficiencies of Agribank branches

Branch	Internet banking	Production	Profitability	Overall
1	1	1	1	1
2	0.946	0.915	1	0.933
3	1	0.785	1	0.914
4	0.895	1	1	0.937
5	0.969	0.879	1	0.933
6	0.968	0.865	0.933	0.886
7	1	0.922	1	0.969
8	0.999	0.972	1	0.988
9	0.983	0.867	0.878	0.863
10	0.979	0.937	1	0.962
11	0.973	0.911	1	0.948
12	1	0.874	0.918	0.900
13	0.987	0.966	1	0.978
14	0.993	0.998	0.703	0.817
15	0.986	0.947	0.872	0.893
16	0.992	0.955	0.956	0.950
17	1	0.784	1	0.913
18	0.992	0.980	0.940	0.951
19	0.996	0.939	1	0.973
20	1	0.930	1	0.972
21	1	0.940	1	0.976
22	1	0.931	1	0.972
23	0.992	0.976	0.954	0.958
24	0.963	1	1	0.978
25	0.9995	0.968	0.799	0.866
26	0.991	0.982	1	0.987
27	1	0.955	1	0.982
28	0.988	0.968	1	0.980
29	0.998	0.970	0.905	0.930
30	0.998	0.985	0.968	0.974
31	1	0.968	0.963	0.965
32	1	0.942	1	0.977
33	0.994	0.962	1	0.981
34	0.998	0.953	1	0.980
35	0.971	1	1	0.983
36	0.954	0.962	0.875	0.882
37	0.993	0.988	1	0.991
38	1	0.984	1	0.993
39	1	1	1	1
40	1	0.962	0.878	0.911
41	0.988	1	1	0.993
42	0.985	1	1	0.991
43	0.981	1	1	0.988
44	1	1	1	1
45	1	0.984	1	0.994
Mean	0.988	0.951	0.968	0.954

Table 4. Projection values

Branch	Number of ATM $(\hat{x}_{10})$	Stage 1 employees $(\tilde{l}^{*}_{10})$	Stage 2 employees $(\tilde{l}'^{*}_{10})$	Total number of employees $(\hat{l}_{10})$	Stage 1 cost $\alpha^{*}_{10}k_{10} - s_1^{k_1*}$	Stage 2 cost $\alpha^{*}_{20}k_{10} - s_1^{k_2*}$	Stage 3 cost $\alpha^{*}_{30}k_{10} - s_1^{k_3*}$	Total cost $(\hat{k}_{10})$	Profit $(\hat{y}_{10})$	Non-performing loan $(y\hat{b}_{10})$
1	12	3	23	26	321.476	401.845	80.369	803.690	216.348	2104.173
2	2	2	8	10	45.291	220.369	45.668	311.328	164.047	5666.942
3	2	2	9	11	44.478	203.296	44.478	292.252	133.146	1216.075
4	4	2	9	11	43.873	182.297	36.459	262.629	130.834	8688.489
5	2	2	4	6	34.035	108.709	25.061	167.805	81.017	721.823
6	2	1	3	4	17.730	61.782	20.012	99.523	51.961	516.475
7	1	1	6	7	30.266	130.381	30.266	190.912	112.381	6737.245
8	1	1	3	4	16.956	68.514	14.559	100.028	55.433	2449.162
9	1	1	4	5	22.819	87.906	24.466	135.191	65.059	2818.278
10	1	1	3	4	15.420	56.235	15.995	87.650	49.266	2084.652
11	1	1	2	3	10.605	50.928	17.087	78.620	27.855	896.669
12	1	1	3	4	17.761	54.409	14.948	87.117	32.973	308.001
13	1	1	3	4	9.776	55.754	15.874	81.404	23.341	2127.280
14	1	1	2	3	10.605	59.853	12.552	83.010	36.982	1282.427
15	1	1	3	4	12.333	57.943	15.343	85.619	37.532	352.173
16	1	1	3	4	10.689	58.997	17.703	87.388	49.515	816.394
17	1	2	10	12	52.172	200.985	52.172	305.329	142.575	5923.025
18	1	1	3	4	10.095	53.995	15.669	79.760	42.687	777.149
19	1	1	2	3	9.470	48.184	13.565	71.219	19.809	1091.840
20	1	1	2	3	15.398	53.111	15.387	83.896	30.918	1746.922
21	1	1	2	3	13.898	56.774	13.898	84.570	25.978	6650.236
22	2	1	2	3	17.577	64.471	20.194	102.242	33.261	1282.892
23	1	1	3	4	11.659	54.735	12.748	79.141	24.020	961.923
24	1	1	3	4	9.774	71.406	14.281	95.462	34.588	762.764
25	1	1	2	3	11.493	54.770	12.464	78.727	31.640	1221.919
26	1	1	4	5	21.350	115.381	24.250	160.981	78.004	7816.155
27	2	1	3	4	14.369	50.872	14.369	79.611	24.185	4125.903
28	1	1	2	3	13.083	48.894	12.146	74.124	14.536	176.152
29	1	1	3	4	16.840	59.600	13.587	90.027	27.889	650.592
30	1	1	3	4	9.379	56.146	14.946	80.471	39.699	454.580
31	1	1	3	4	10.922	51.088	14.043	76.053	30.306	224.228
32	1	1	3	4	13.138	53.606	13.138	79.881	12.417	118.913
33	1	1	2	3	18.385	52.183	12.097	82.664	19.756	1185.089
34	1	1	2	3	13.842	47.666	11.392	72.900	10.999	1902.074
35	1	1	3	4	11.644	54.026	10.821	76.491	61.172	5664.314
36	1	1	2	3	9.299	50.784	12.740	72.822	25.344	241.895
37	1	1	2	3	12.210	55.955	11.209	79.374	27.999	166.861
38	2	1	2	3	8.747	46.600	8.747	64.094	15.607	955.805
39	1	1	2	3	25.785	66.768	22.467	115.020	31.041	4076.632
40	1	1	2	3	11.258	44.151	14.532	69.940	17.461	694.333
41	1	1	2	3	10.441	44.864	8.974	64.278	10.214	1936.123
42	1	1	2	3	9.367	51.091	10.218	70.677	35.359	1151.945
43	1	1	3	4	11.644	66.087	13.218	90.949	4.350	14.717
44	1	1	3	4	41.801	52.252	10.450	104.503	13.820	8.332
45	1	1	2	3	9.033	49.038	9.035	67.107	7.863	412.364

Table 5. Portion of shared input resources among stages

Branch	α_1	α_2	α_3	β
1	0.400	0.500	0.100	0.885
2	0.258	0.642	0.100	0.739
3	0.100	0.800	0.100	0.875
4	0.400	0.500	0.100	0.600
5	0.145	0.755	0.100	0.778
6	0.100	0.600	0.300	0.875
7	0.100	0.800	0.100	0.857
8	0.131	0.766	0.103	0.750
9	0.102	0.658	0.239	0.770
10	0.100	0.800	0.100	0.600
11	0.400	0.500	0.100	0.833
12	0.100	0.800	0.100	0.857
13	0.239	0.661	0.100	0.750
14	0.200	0.500	0.300	0.667
15	0.140	0.611	0.249	0.717
16	0.111	0.617	0.271	0.734
17	0.100	0.800	0.100	0.867
18	0.193	0.578	0.229	0.750
19	0.105	0.795	0.100	0.719
20	0.101	0.799	0.101	0.750
21	0.100	0.800	0.100	0.750
22	0.103	0.778	0.118	0.750
23	0.193	0.593	0.215	0.750
24	0.400	0.500	0.100	0.600
25	0.106	0.602	0.292	0.749
26	0.142	0.720	0.138	0.748
27	0.100	0.800	0.100	0.750
28	0.254	0.640	0.107	0.676
29	0.124	0.623	0.253	0.750
30	0.100	0.600	0.300	0.750
31	0.100	0.600	0.300	0.800
32	0.100	0.800	0.100	0.800
33	0.174	0.724	0.102	0.664
34	0.125	0.775	0.100	0.712
35	0.400	0.500	0.100	0.600
36	0.100	0.600	0.300	0.600
37	0.244	0.656	0.101	0.667
38	0.100	0.800	0.100	0.667
39	0.224	0.580	0.195	0.667
40	0.100	0.600	0.300	0.750
41	0.400	0.500	0.100	0.667
42	0.400	0.500	0.100	0.667
43	0.400	0.500	0.100	0.750
44	0.400	0.500	0.100	0.750
45	0.100	0.799	0.100	0.667