

ON STRUCTURE AND ORGANIZATION: AN ORGANIZING PRINCIPLE

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ABSTRACT. We discuss the nature of structure and organization, and the process of making new Things. Hyperstructures are introduced as binding and organizing principles, and we show how they can transfer from one situation to another. A guiding example is the hyperstructure of higher order Brunnian rings and similarly structured many-body systems.

Keywords: Hyperstructure; organization; binding structure; Brunnian structure; many-body systems.

1. INTRODUCTION

In science and nature we study and utilize collections of objects by organizing them by relations and patterns in such a way that some structure emerges. Objects are bound together to form new objects. This process may be iterated in order to obtain higher order collections. Evolution works along these lines.

When things are being made or constructed it is via binding processes of some kind. This seems to be a very general and useful principle worthy of analyzing more closely. In other words we are asking for a general framework in which to study general many-body systems and their binding patterns as organizing principles.

2. EXAMPLES

Let us look at some examples of what we have in mind.

Example 1 (Links). A link is a disjoint union of embedded circles (or rings) in three dimensional space:

$$L: \coprod_{i=1}^n S_i^1 \rightarrow \mathbb{R}^3.$$

They may be linked in many ways. Linking is a kind of geometrical or topological binding as we see in the following examples.

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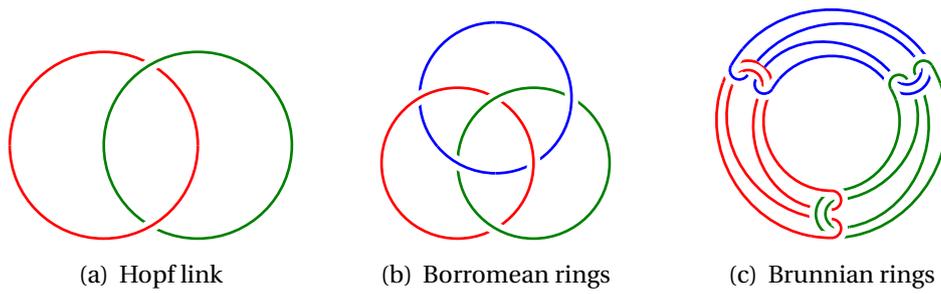


FIGURE 1

(Figures in colour are available at: <http://arxiv.org/pdf/1201.6228.pdf>)

In Baas (2013) the linking bonds have been extended to higher order links like:

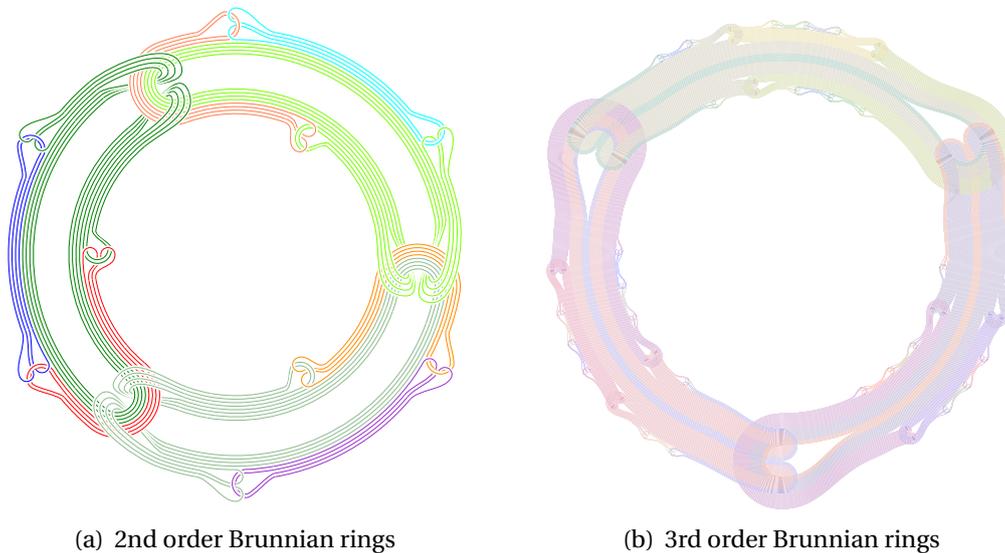


FIGURE 2

In order to iterate this process and study higher order links, it is preferable to study embeddings of tori:

$$L: \prod_{i=1}^n S_i^1 \times D^2 \rightarrow S^1 \times D^2$$

A second order Brunnian ring binds 9 circles (rings) together in a very subtle way, Figure 2(a). Higher order links (links of links of ...) provide a very good guiding example of what a general framework should cover. For more details, see Baas (2013).

In Baas (2009) and Baas and Seeman (2012) we discuss possible ways to synthesize such binding structures as molecules.

Example 2 (Many-body states). Efimov (Borromean, Brunnian) states in cold gases are bound states of three particles which are not bound two by two. Hence these states are analogous to Borromean and Brunnian rings. In Baas (2013) we have suggested that this analogy may be extended to higher order links and hence suggests higher order

versions of Efimov states. For example the second order Brunnian rings $2B(3,3)$, see Figure 2(a), suggest that there should exist bound states of 9 particles, bound 3 by 3 in a Brunnian sense, and that these clusters bound together again in a higher order sense as in:

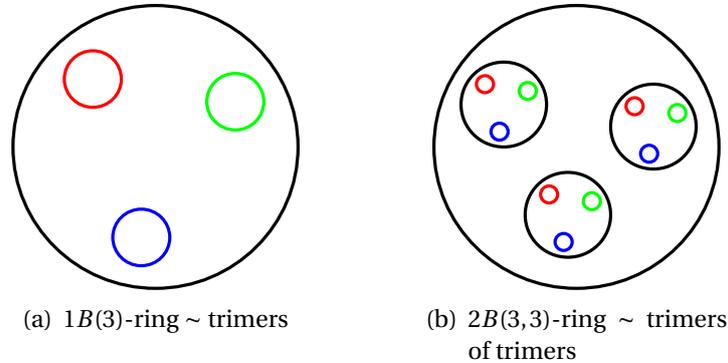


FIGURE 3

See also Baas (2013) for intermediate bound states between a trimer and a dimer and a trimer and a two-singleton. For a discussion of higher order Brunnian states, see Baas, Fedorov, Jensen, Riisager, Volosniev and Zinner (2012).

In general, clustering and higher order clustering of many-body systems represent a binding mechanism of particle systems — parametrizing the particles, in a way. One may ask for a general method to describe the binding of particles into higher clusters.

Example 3 (Clusters and decompositions). As pointed out in the previous example clusters of objects or data represent a binding mechanism between the objects in the cluster. The cluster of course depends on various defining criteria. Clusters of clusters... represent higher order versions. Similarly when we decompose a set or collection we may say that elements in the same part of a decomposition are bound together. In this case we get higher bonds as decompositions of decompositions...

Example 4 (Mathematical structures and organizations). We will give a few mathematical examples of how sets are organized into structures.

- a) Topological spaces. We organize the “points” of a set into open sets in such a way that they satisfy the axioms for a topology.
- b) Groups, Algebras, Vector spaces. The elements are organized by certain operations which satisfy the structure axioms.
- c) Manifolds. Organize the points into open sets and glue them together in a prescribed structured way. Gluing is an important example of a geometric binding mechanism.

In order to form higher order versions of these structures there may be many choices, but one way is through higher categories.

In a higher category of order n one is given objects (e.g. groups, topological spaces,...) which are organized by morphisms

$$X \xrightarrow{f} Y$$

between them.

Furthermore, there exist morphisms between morphisms — 2-morphisms:

$$\begin{array}{ccc}
 & f & \\
 X & \begin{array}{c} \curvearrowright \\ \Downarrow F \\ \curvearrowleft \end{array} & Y \\
 & g &
 \end{array}$$

and this continues up to n -morphisms between $(n - 1)$ -morphisms satisfying certain conditions.

Another type of higher order mathematical binding structure is a moduli space. These are spaces of structures — for example the space of all surfaces of a given genus.

3. HYPERSTRUCTURES

We will now introduce some new binding mechanisms of general collections of objects: physical, chemical, biological, sociological, abstract and mental.

This organization may bring to light some new and useful structure on the collections. We will discuss this in the following, extending the points of view of Baas (2013) — especially in the appendix.

The main concept we will use in order to do this is that of a *Hyperstructure* as introduced and studied in (Baas 1994a, Baas 1994b, Baas 1996, Baas 2006, Baas 2009).

Let us recall the basic construction from Baas (2006) and Baas (2009). We start with a set of objects X_0 — our basic units. To each subset (or families of elements)

$$S_0 \subset X_0$$

we assign a set of properties or states, $\Omega_0(S_0)$, so

$$\Omega_0: \mathcal{P}(X_0) \rightarrow \text{Sets}$$

where $\mathcal{P}(X) = \{A \mid A \subset X\}$ — the set of subsets — the power set, and Sets denotes a suitable set of sets. (In the language of category theory $\mathcal{P}(X_0)$ would be considered a category of subsets, Sets as some category of sets.) In our notation here we include properties and states of elements and subsets of S_0 in $\Omega_0(S_0)$.

Then we want to assign a set of bonds, relations, relationships or interactions of each subset S_0 — depending on properties and states. Here we will just call them bonds. Let us define

$$\Gamma_0 = \{(S_0, \omega_0) \mid S_0 \in \mathcal{P}(X_0), \omega_0 \in \Omega_0(S_0)\}$$

$$B_0: \Gamma_0 \rightarrow \text{Sets.}$$

In our previous notion of hyperstructures the set X_0 represents the systems or agents (S_i), Ω_0 the observables (Obs), B_0 the interactions (Int) and a specific choice of $b_0 \in B_0(S_0, \omega_0)$ represents the resultant “bond” system giving rise to the next level of objects — called R in previous papers, like S_i , Obs, Int, see (Baas 1994a, Baas 1994b, Baas 1996, Baas and Helvik 2005).

We will often implicitly assume in the following that given a bond we know what it binds. We may require that the set of all bonds of (S_0, ω_0) — $B_0(S_0, \omega_0)$ satisfies the following condition:

$$(*) \quad \left\{ \begin{array}{l} \text{For all } S_0, S'_0, \omega_0, \omega'_0, S_0 \neq S'_0 \implies B_0(S_0, \omega_0) \cap B_0(S'_0, \omega'_0) = \emptyset \\ \text{In some cases we may impose the stronger condition: } B_0 \text{ injective.} \end{array} \right.$$

(In other situations this is too strong a condition. For example if we want to consider colimits as bonds, then the ∂_i in the following are not well-defined. The (*) condition

ensures that the bonds “know” what they bind.)

Example 5 (Geometric example). S_0 = a finite number of manifolds, ω_0 = property of being smooth, put $B_0(S_0, \omega_0)$ = the set of all smooth manifolds with boundary equal (isomorphic) to the disjoint union of the manifolds in S_0 . See Figure 4.

We now just formalize in a general setting the procedure we described in Baas (2009, Section 5).

Let us form the next level and define:

$$X_1 = \{b_0 \mid b_0 \in B_0(S_0, \omega_0), S_0 \in \mathcal{P}(X_0), \omega_0 \in \Omega_0(S_0)\},$$

by definition the image set of B_0 , and

$$\begin{array}{c} X_1 \\ \downarrow \partial_0 \\ \mathcal{P}(X_0) \end{array}$$

given by $\partial_0(b_0) = S_0$.

If B_0 is injective we have a factorization:

$$\begin{array}{ccc} X_1 & & \\ \downarrow \partial_0 & \searrow \partial'_0 & \\ & & \Gamma_0 \\ & \swarrow \text{projection} & \\ \mathcal{P}(X_0) & & \end{array}$$

Depending on the actual situation we may consider ∂_0 and ∂'_0 as boundary maps.

X_1 represents the bonds of collections of elements or interactions in a dynamical context. But the bonds come along with the collection they bind just as morphisms in mathematics come along with sources and targets. Similarly at this level we introduce properties and state spaces and sets of bonds as follows:

$$\Omega_1 : \mathcal{P}(X_1) \rightarrow \text{Sets}$$

(Ω_1 then represents the emergent properties as in Baas (1994a))

$$\Gamma_1 = \{(S_1, \omega_1) \mid S_1 \in \mathcal{P}(X_1), \omega_1 \in \Omega_1(S_1)\}$$

$$B_1 : \Gamma_1 \rightarrow \text{Sets.}$$

B_1 satisfying the corresponding (*) condition.

Then we form the next level set:

$$X_2 = \{b_1 \mid b_1 \in B_1(S_1, \omega_1), S_1 \in \mathcal{P}(X_1), \omega_1 \in \Omega_1(S_1)\}$$

and

$$\begin{array}{c} X_2 \\ \downarrow \partial_1 \\ \mathcal{P}(X_1) \end{array}$$

$\partial_1(b_1) = S_1$.

We now iterate this procedure up to a general level N :

$$\Omega_{N-1}: \mathcal{P}(X_{N-1}) \rightarrow \text{Sets}$$

$$B_{N-1}: \Gamma_{N-1} \rightarrow \text{Sets}$$

B_{N-1} satisfying the corresponding (*) condition

$$X_N = \{b_{N-1} \mid b_{N-1} \in B_{N-1}(S_{N-1}, \omega_{N-1}), S_{N-1} \in \mathcal{P}(X_{N-1}), \omega_{N-1} \in \Omega_{N-1}(S_{N-1})\}$$

This is not a recursive procedure since at each level new assignments take place. The higher order bonds extend the notion of higher morphisms in higher categories.

Let us write

$$\mathcal{X} = \{X_0, \dots, X_N\}$$

$$\Omega = \{\Omega_0, \dots, \Omega_{N-1}\}$$

$$\mathcal{B} = \{B_0, \dots, B_{N-1}\}$$

$$\partial = \{\partial_0, \dots, \partial_{N-1}\}.$$

where

$$\begin{array}{c} X_{i+1} \\ \downarrow \partial_i \\ \mathcal{P}(X_i) \end{array}$$

The ∂_i 's generalize the source and target maps in the category theoretical setting, and we think of them as generalized boundary maps. An Observer mechanism is implicit in the Ω_i 's. Sometimes one may also want to require maps $I_i: X_i \rightarrow X_{i+1}$ or $I_i: \mathcal{P}(X_i) \rightarrow X_{i+1}$ — generalizing the identity — such that $\partial_i \circ I_i = \text{id}$. As for ∂_0 one may also for ∂_i consider ∂'_i .

Further mathematical properties to be satisfied will be discussed elsewhere, for example composition of bonds. We will then also discuss how to associate a topological space to a hyperstructure — a generalized Nerve construction. Bonds may also have internal structures like topological spaces, manifolds, algebras, vector spaces, wave functions, fields, etc.

The intuition behind this is:

- X_0 = objects like atoms, molecules, manifolds, genes, organisms
- X_1 = bonds ~ relations, aggregates, clusters, interactions, processes, ...
- X_2 = bonds of bonds ~ relations of relations, aggregates of aggregates (possibly overlapping), clusters of clusters, interactions of interactions, processes of processes, ... etc.

Definition 1. The system $\mathcal{H} = (\mathcal{X}, \Omega, \mathcal{B}, \partial)$ where the elements are related as described, we call *a hyperstructure of order N* .

Sometimes one may want to organize a set of agents for a specific purpose. One way to do this is to put a hyperstructure on it organizing the agents to fulfill a given goal. This applies to both concrete physical and abstract situations.

An example of this is the procedure whereby we organize molecules (or abstract topological bonds) into rings, 2-rings, ..., n -rings representing new topological structures Baas and Seeman (2012).

In many cases it is natural to view the bonds as geometric or topological spaces. For example if a surface F has three circles as boundary components, see Figure 4, $\partial F = S_1 \cup S_2 \cup S_3$, we may say that F is a geometric bond of the circles. Clearly there may be many. This is in analogy with chemical bonds.

Furthermore, if we have a manifold B such that its boundary is

$$\partial B = A_1 \cup \dots \cup A_n \cup \tilde{A},$$

see Figure 5, we may say that B is a bond of A_1, \dots, A_n . Even more general is the following situation: given a topological space of some kind Y and let Z_1, \dots, Z_n be subspaces of Y . Then we say that Y is a bond of Z_1, \dots, Z_n , see Figure 6, thinking of Z_1, \dots, Z_n as “the boundary” of Y .

In a hyperstructure of higher order we may let Y represent a top level bond, then the Z_i 's will represent bonds of other spaces:

$$Y = B_0(Z_1, \dots, Z_n)$$

$$Z_i = B_{1,i}(W_{1,i}, \dots, W_{n,i})$$

etc., see Figure 7.

This slightly extends the pictures of Baas (2006) and represent what we could call a *geometric hyperstructure*. This concept relates to topological quantum field theory and will be studied in a separate paper, see Section 8 for some further remarks.

Hyperstructures and higher order bonds may be viewed as a huge extension of cobordisms (manifolds with boundary) and chemical bonds and interlockings.

Furthermore, the whole scheme of thinking may be applied to interactions and ways of connecting and interlocking people, groups of people, social and economic systems and organizations, organisms, genes, data, etc. For some of these aspects, see Baas (2006).

By describing all such types of systems by means of hyperstructures one may create entirely new structures which may be useful both in old and new contexts.

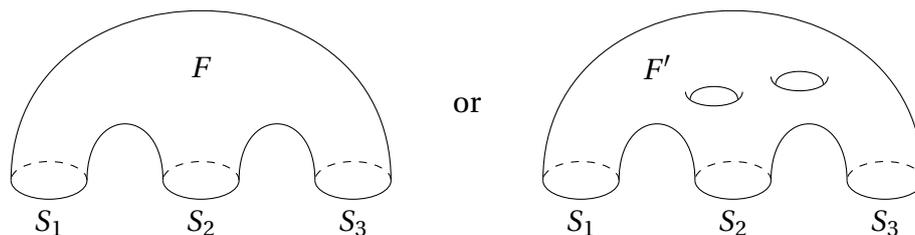


FIGURE 4

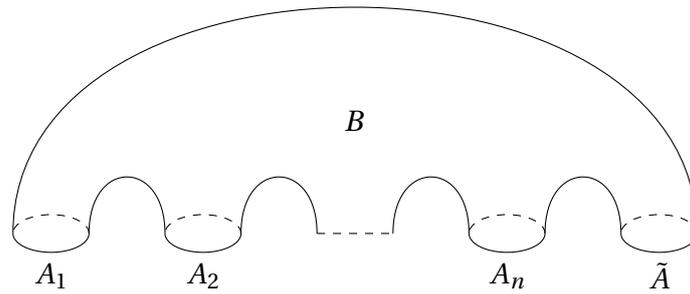


FIGURE 5

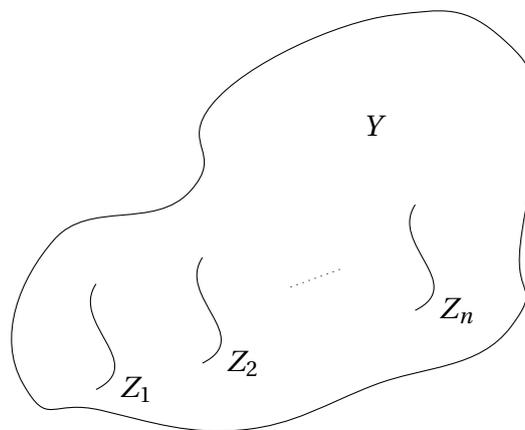


FIGURE 6

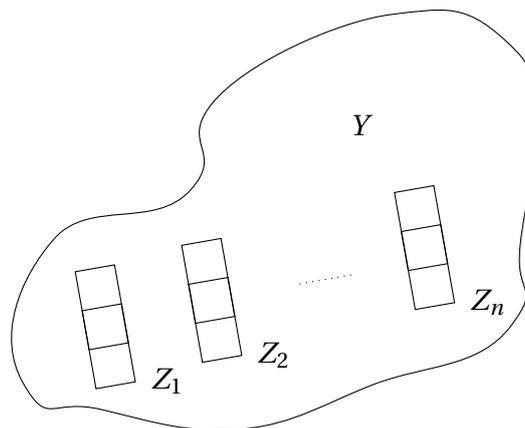


FIGURE 7

We could also call a hyperstructure a binding structure since it really binds the elements of a collection. To make the notion simpler we will suppress the states in the following, and we will express the associated binding mechanisms of collections as simple as possible. At the end of Baas (2006) we offer a categorical version which is more restrictive.

Further examples of hyperstructures:

- a) *Higher order links* as in Example 1. Here the starting set X_0 is a collection of rings, the observed state is circularity and the bonds are the links. Then one observes “circularity” (or embedding in a torus) of the Brunnian links and continues the process by forming rings of rings ... as described in Baas (2013).
- b) *Hierarchical clustering* and multilevel decompositions are typical examples of hyperstructures. The bonds are given by level-wise membership as in Example 2. We should point out that hyperstructures encompass and are far more sophisticated and subtle than hierarchies.
- c) *Higher categories* are examples of hyperstructures as well where the higher morphisms are interpreted as higher bonds.
- d) *Compositions*. Often collections of objects and data may be organized into a composition of mappings

$$S_1 \xleftarrow{\varphi_1} S_2 \xleftarrow{\varphi_2} \dots \xleftarrow{\varphi_{n-1}} S_n$$

where we may think of $s_i \in S_i$ as a bond of the elements in $\varphi_i^{-1}(s_i)$, and $s_{i+1} \in \varphi^{-1}(s_i)$ is again a bond of the elements in $\varphi_{i+1}^{-1}(s_{i+1})$, etc. See Baas (2006) and references therein. Similarly one may say that a subset (or space if we have more structure) $Z_i \subset S_i$ is a bond of subsets in $\varphi_i^{-1}(Z_i)$. Composition models of hyperstructures are particularly interesting when the S_i 's and mappings have more structure, for example being smooth manifolds and smooth mappings. In that case there is an interesting stability theory, see Baas (2006) and references therein.

- e) *Higher Order Cobordisms*. In geometry and topology we consider kinds of generalized surfaces in arbitrary dimensions called manifolds. These may be smooth and have various additional structures. Amongst manifolds there is a very important notion of cobordism, and we will illustrate how cobordisms of manifolds with boundaries and corners are important as bonds.

Two manifolds A_1 and A_2 (with or without boundary) of dimension n are cobordant if and only if there exists an $(n + 1)$ dimensional manifold B such that

$$\partial B = (A_1 \cup A_2) \cup \hat{A}$$

∂ stands for boundary and \cup means glued together along the common boundary

$$\partial(A_1 \cup A_2) = \partial \hat{A},$$

see Figure 8.

In this paper we are interested in studies of structures etc. So let us see what cobordisms of cobordisms or more generally bonds of bonds mean in this geometric setting.

Let B_1 be a bond between A_1 and A_2 and let B_2 be a bond between \tilde{A}_1 and \tilde{A}_2 . Then C is bond (cobordism) between B_1 and B_2 if and only if

$$\partial C = (B_1 \cup B_2) \cup \hat{B}$$

where \cup means glued together along common boundary: $\partial(B_1 \cup B_2) = \partial \hat{B}$, C is of dimension $(n + 2)$ and \hat{B} of dimension $(n + 1)$, see Figure 9.

Furthermore, a third order bond between C_1 and C_2 will be given by an $(n+3)$ manifold D such that

$$\partial D = (C_1 \cup C_2) \cup \hat{C}$$

etc., see Figure 10.

In this formal description we have just considered two “components” in the boundary, hence a cobordism is then considered as a bond between two parts like a morphism in a category. But clearly the geometry extends to any finite number of components, hence we consider a cobordism as the prototype of a geometric bond between several objects:

$$B(A_1, \dots, A_n) \text{ if and only if } \partial B = (A_1 \cup \dots \cup A_n) \cup \hat{A}.$$

Mathematically this requires that we study manifolds with decomposed boundaries, whose boundary components again are decomposed, etc. (as introduced and studied in Baas (1973)) or manifolds with higher order corners (corners of corners etc.), see Figure 11.

Hyperstructures seem like the correct mathematical structure to describe this situation.

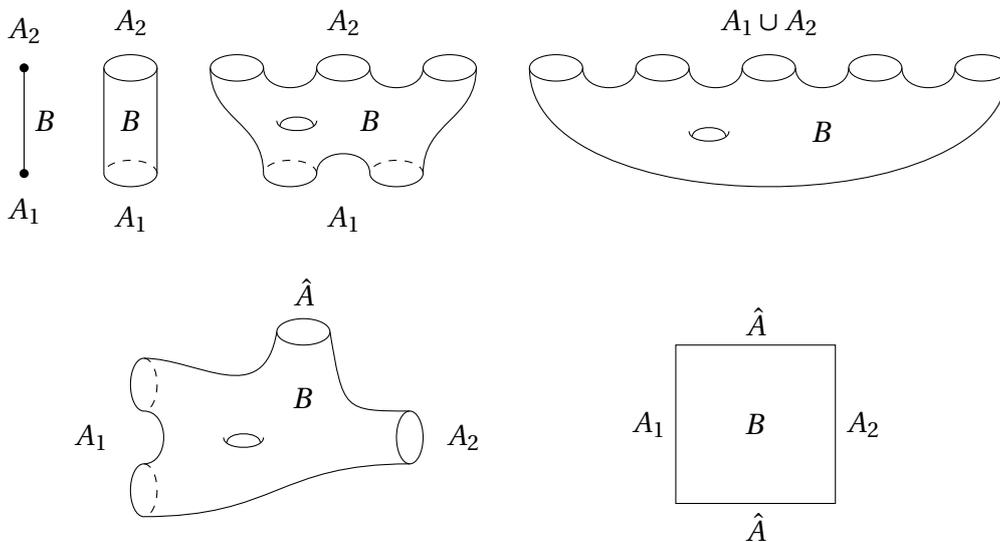


FIGURE 8

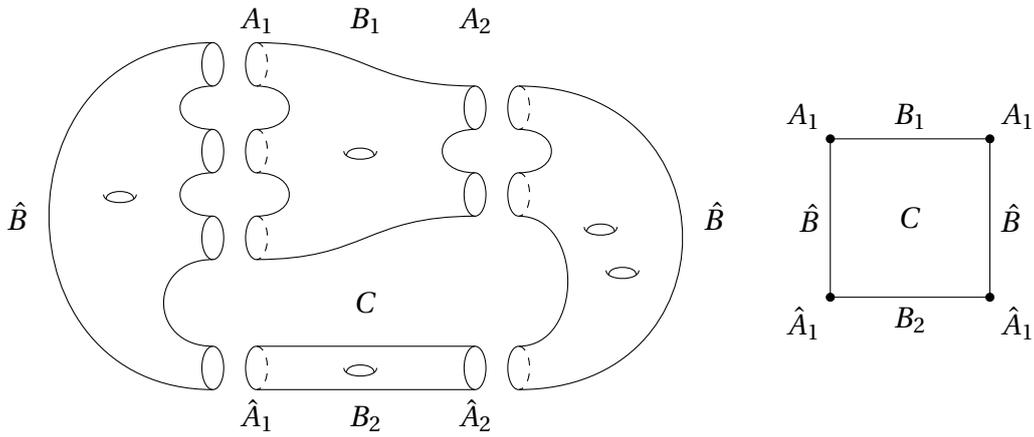


FIGURE 9

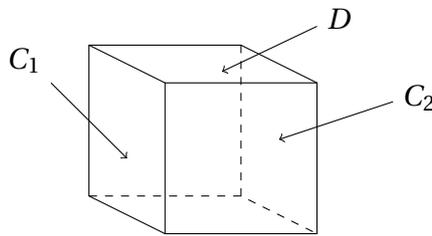


FIGURE 10: D is schematically represented by a cube

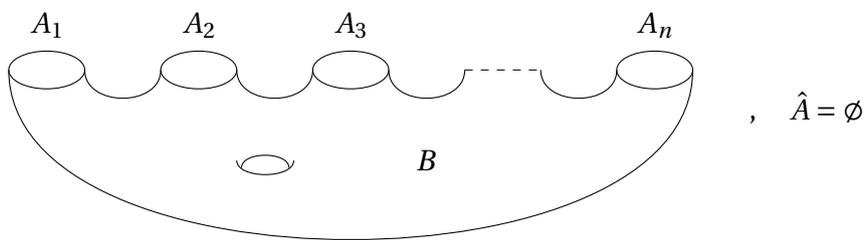


FIGURE 11

In the framework we have introduced the geometric examples in the figures correspond to:

$$X_0 = \{\text{the set of circles in some high dimensional space}\}$$

no states, $\Omega_0 = \emptyset$.

$S_0 \in \mathcal{P}(X_0)$ means that S_0 is a disjoint union of circles. $b_0 \in B_0(S_0)$ is then given by a surface having the circles of S_0 as its boundary

$$X_1 = \{\text{the set of surfaces with boundary equal the union of circles}\}$$

$S_1 \in \mathcal{P}(X_1)$ is a disjoint union of such surfaces.

$B_1(S_1)$ is then given by a 3 dimensional manifold having the surfaces of S_1 as parts of its boundary, but possibly glued together along common boundaries

with additional parts — the \hat{B} 's. For more details on hyperstructured glueing and decomposition processes, see Baas (1973) and Baas, Cohen and Ramírez (2006).

In this way it goes on up to a desired dimension. If in addition we add states in the form of letting the Ω_i 's take vector spaces (Hilbert spaces, or some other algebraic structure) as values we enter the situation of topological quantum field theory which we will not pursue here, see Section 8.

- f) *Limits*. In category theory we form limits and colimits of a collection of objects — more precisely, given a functor

$$F: I \rightarrow \mathcal{C}$$

we form the colimit:

$$\operatorname{colim} F = \operatorname{colim} c_i, \quad F(i) = c_i.$$

The colimit binds the collection or pattern of objects c_i into one simple object reflecting the complexity of the pattern. In this sense it is a bond in the hyperstructure framework if we drop the condition giving rise to the “boundary” maps ∂_i .

If we require the ∂_i 's to exist, then the bond knows which objects it binds. In the colimit this is not the case. Hence we consider hyperstructures with and without ∂_i 's.

Colimits may also be iterated. For example we may consider situations where each c_i already is a colimit of other colimits etc. Expressed in a different way we consider a multivariable functor

$$F: I_1 \times I_2 \times \cdots \times I_k \rightarrow \mathcal{C}$$

and form the iterated colimit

$$\mathcal{H}: \operatorname{colim}_{I_k} \cdots \operatorname{colim}_{I_2} \operatorname{colim}_{I_1} F.$$

This is clearly an iterated binding structure in the hyperstructure framework which we will discuss in the next section. The colimits over the various indexing categories represent the bonds of the various levels, see (Ehresmann and Vanbremeersch 1996, Baas, Ehresmann and Vanbremeersch 2004) for a general context. For a categorical discussion of hyperstructure, see Appendix B in Baas (2006).

4. A METAPHOR

Let us illustrate using a metaphor what we mean by putting a hyperstructure on an already existing structure, system or situation.

Suppose we are given a society or organization of agents, and we want to act upon it in the manner of wielding political power, governing a society or nation. A possible procedure is: create a kind of “political party” organization. A structural design of the organization is needed, rules of action (“ideology”) and incentives (“goals”). The fundamental task is to create an organization — a “party” — starting with “convinced” individuals, then suitable groups of individuals, groups of groups,...

Basically this is putting a hyperstructure on the society of agents which may act as an ideological amplifier from individuals to the society. This can be done independently

of an existing societal organization that one wants to act upon. In such a hyperstructure the bonds may depend on a goal (ideology) and incentives like solving common problems, infrastructure, healthcare, poverty, etc. In physics it could be minimizing or releasing energy to obtain stability. All such factors will play a role in the build up of the hyperstructure in the form of choosing bonds, states, etc. such that they support the goals or “ideology”.

Having established a hyperstructure, then let it act by the “ideology” in the sense that it should be instructible — like a superdemocracy. Hence it may be instructed to maintain, replace or improve the existing structure of society or change it to achieve certain goals. This is what political parties and other organizations do.

A hyperstructure on a society (or space) will facilitate the achievement of desired dynamics for agents or other objects through the bonds which may act dynamically — like fusion of old bonds to new bonds.

If the hyperstructure is given as

$$\mathcal{H} = \{B_0, B_1, \dots, B_n\},$$

then one may initiate a dynamics at the lowest level

$$B_0 \rightarrow B'_0$$

which may require relatively few resources or little energy. Then these changes of actions and states will propagate through the higher bonds

$$\begin{aligned} B_1 &\rightarrow B'_1 \\ &\vdots \\ B_n &\rightarrow B'_n \end{aligned}$$

leading to a major desired change at the top level, depending on the nature of \mathcal{H} . This is how a political organization works. These aspects are elaborated in Section 8.

The degree of detail of the hyperstructure will depend on the situation and information available — like how rich mathematically the hyperstructure can be. One may also ask:

What is the sociology of a space? $\mathcal{H}(X)$ represents the “organization of X ” or “the society of X ”. What can be obtained by a political structure on X depends on X and $\mathcal{H}_{\text{political}}(X)$.

This shows through the metaphor how general the idea is and how it may usefully apply in many situations. Another interesting idea is how to use this metaphor in the study of the genome as a society of genes. One would like to put on a hyperstructure whose ideology should be to maintain the structure (homeostasis), avoiding and discarding unwanted growth like cancer.

How could one possibly create and represent such a “genomic political party”? Possibly by an organized collection (hyperstructure) of drugs (or external fields) that acts on bonds of genes. The protein P53 may already have such a role at a high level in an existing hyperstructure.

5. BINDING STRUCTURES

We will now discuss the main issue in this paper, namely how to organize a collection of objects (possibly with an existing structure) into a new structure.

Let us assume that we are given a basic collection of objects

$$Z = \{z_i\}_{i \in I}$$

I finite, countable or uncountable. Z may be the elements of a set or a space. Let us also assume that we are given a hyperstructure of order n in the sense of the previous section.

$$\mathcal{H}: \quad \{X_0, X_1, \dots, X_n\} \\ \{B_0, B_1, \dots, B_{n-1}\}$$

In order to simplify the notation we do not write the Ω 's and ∂ 's.

How to put an \mathcal{H} -structure on Z ?

This means describing how to bind the objects in Z together into new higher order objects following the pattern given by the bond structure in \mathcal{H} . The idea is as follows.

We represent the collection Z as a collection of elements in X_0 — the basic set on which \mathcal{H} is built.

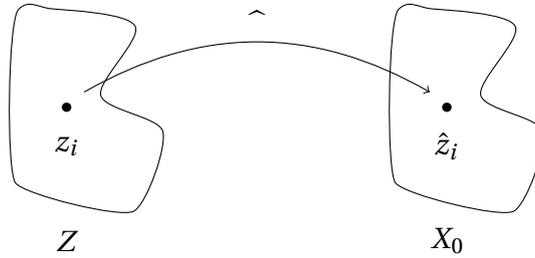


FIGURE 12

Hence we get a new collection of objects (or elements) in X_0 —

$$\hat{Z} = \{\hat{z}_i\}_{i \in I} \subset X_0.$$

This is similar to the correspondence or analogy in Examples 1 and 2 where particles are represented as rings. More on this later.

On X_0 we have bonds B_0 and can apply these to the new collection \hat{Z} in X_0 . Therefore we put a bond structure on Z as follows.

Definition 2. $B_0(Z) = B_0(\hat{Z})$

This means that we pull back the bonds from the hyperstructure \mathcal{H} on X_0 to Z . Since $\hat{Z} \subset X_0$ we get an induced hyperstructure on \hat{Z} . This means that

$$B_1(\hat{Z}) \\ B_2(\hat{Z}) \\ \vdots \\ B_{n-1}(\hat{Z})$$

with the $\Omega_i, \partial_i, \Gamma_i$ coming along.

Remark. If we already have a good hyperstructure on Z , we just keep it via the identity representation ($Z = X_0$) and use it in the binding process we will describe.

With a hyperstructure on the given collection Z we can introduce new higher order clusters and patterns of interactions.

Definition 3. An \mathcal{H} -binding (or clustering) structure on Z — denoted by $\mathcal{H}(Z)$ — is given as follows:

Let $S_0 \subset Z$ and $b_0 \in B_0(S_0)$, $z \in Z$ is an element of a b_0 -cluster ($\text{Cl}(b_0)$) if $z \in S_0$.

Furthermore, let $S_1 \subset B_0(Z)$ and $b_1 \in B_1(S_1)$, then $b_0 \in B_0(Z)$ is an element of a b_1 -cluster ($\text{Cl}(b_1)$) if $b_0 \in S_1$.

If $z \in \text{Cl}(b_0)$, and $b_0 \in \text{Cl}(b_1)$, then we have a second order clustering and write $z \in \text{Cl}(b_0, b_1)$. In the same way we proceed to n -th order clustering by requiring

$$z \in \text{Cl}(b_0, b_1, \dots, b_{n-1})$$

in an obvious extension of the notation.

This describes the general \mathcal{H} -binding (or clustering) principle. The same principle applies to the extended representation picture:

$$\begin{array}{ccc} Z & \xrightarrow{R_0} & X_0 \xrightarrow{I_0} Z \\ & & \vdots \\ Z & \xrightarrow{R_n} & X_n \xrightarrow{I_n} Z \end{array}$$

interpreted in the natural way: R giving a binding structure and I inducing a “parametrization” or decomposition by taking inverse images. The figure indicates that we may represent or induce at any level, but most of the time we use level 0. One may construct a decomposition of Z via a hyperstructure \mathcal{H} , by starting with the top bonds B_n (reverse the direction of the binding process). Furthermore, one may then bind the lowest level (smallest) elements of the decomposition (B_0 -bonds) to a new hyperstructure $\hat{\mathcal{H}}$. The situation may also be extended to the R 's and I 's being of relational character.

The \mathcal{H} -binding principle that we have described is in a way also a Transfer Principle of Organization — showing how to transfer structure and organization from one universe to another (this is more general than functors between categories). For example one may use it to transfer deep geometrical bonds to other interacting systems, like particle systems as described in Baas (2013) and Baas (2010).

The idea may be easier to grasp in the case that the hyperstructure is given by a composition:

$$\mathcal{H}: S_1 \xleftarrow{\varphi_1} S_2 \xleftarrow{\varphi_2} \dots \xleftarrow{\varphi_{n-1}} S_n.$$

In this case a given collection Z should be represented in S_n . For simplicity let us consider the identity representation $Z = S_n$. The elements of S_{n-1} represent B_0 , hence $z \in \text{Cl}(s_{n-1})$ if $z \in \varphi_{n-1}^{-1}(s_{n-1})$.

Similarly

$$s_{n-1} \in \text{Cl}(s_{n-2}) \quad \text{if} \quad s_{n-1} \in \varphi_{n-2}^{-1}(s_{n-2})$$

and 2nd order clusters are formed $\text{Cl}(s_{n-1}, s_{n-2})$. Hence $z \in \text{Cl}(s_{n-1}, s_{n-2}, \dots, s_1)$ gives an n -th order clustering scheme of Z .

This discussion shows what we mean by putting a hyperstructure on a collection of objects. In a way the hyperstructure acts as a parametrization of the new objects formed by higher bonds. An important point here is the choice of representations, and how to choose them in an interesting and relevant way. Z may be any collection of elements, subsets, subspaces, elements of a decomposition, etc. $\mathcal{H}(Z)$ organizes Z into a “society” of objects.

Putting a hyperstructure on a “situation” is not meant to be restricted to a set or a space, but could be a non-mathematical “situation”, a category of some kind or in

general an already existing hyperstructure (or families of such). The transfer of binding structures may be considered as a kind of generalized “functoriality”.

In fact one may think of a given situation S in representing level 0, and then always look for a higher order associated situation in the form of a hyperstructure — $\mathcal{H}(S)$, which may give a better understanding of the given situation (or object). For example:

$$S: \text{a set } X_0 \rightsquigarrow \mathcal{H}(X_0)$$

$$S: \text{a category } \mathcal{C}_0 \rightsquigarrow \mathcal{H}(\mathcal{C}_0)$$

This idea is a bit similar to the idea of associating complexes and resolutions to groups, modules, algebras, categories, etc. — derived objects (or situations).

Sometimes, a representation is given in a natural way by the nature of the object. For example molecules being represented by their own geometric form. On the other hand one may choose more abstract representations embedding the collection in a universe rich with structures and possibilities for interesting bindings and interactions.

In this way we put a hyperstructure on a situation by somehow inducing one from a known one. If a good model does not exist, one may have to create a new suitable hyperstructure for further use. In either case the hyperstructure enables us to form organized Abstract Matter in the sense of Baas (2006) in order to handle a collection of objects or a situation and achieve certain goals. Binding structures represent the *Principle of how we make Things*.

As pointed out in Baas (2013) one may study the collection in a selected universe and then ask whether the “abstract” binding structure may be realized in the original universe. Hence we may use an \mathcal{H} -structure to synthesize new bond states.

This is exactly the situation we study in Baas (2013) where Z is a collection of particles in a cold gas. We represent the particles by rings as in Examples 1 and 2. The hyperstructure of higher order links — $\mathcal{H}(\text{Rings})$ — pulls back to a hyperstructure of the particle collection — $\mathcal{H}(\text{Particles})$. It is a verified fact, see Baas (2013) and references therein, that to Brunnian (or Borromean) rings there corresponds quantum mechanical states — Efimov states with the same binding patterns.

On this basis it is tempting to suggest that there is a similar correspondence between higher order links and states corresponding to the higher order clusters of bound particles — higher order Brunnian states. This is the main suggestion of Baas (2013).

For example the binding pattern of a second order Brunnian ring $2B(3, 3)$ suggests a bond or particle state of 3 particles bound to trimers and 3 trimers bound to a single state. This is the key idea in our guiding examples of the general setting of binding structures we have introduced.

We may summarize our discussion in the following figure:

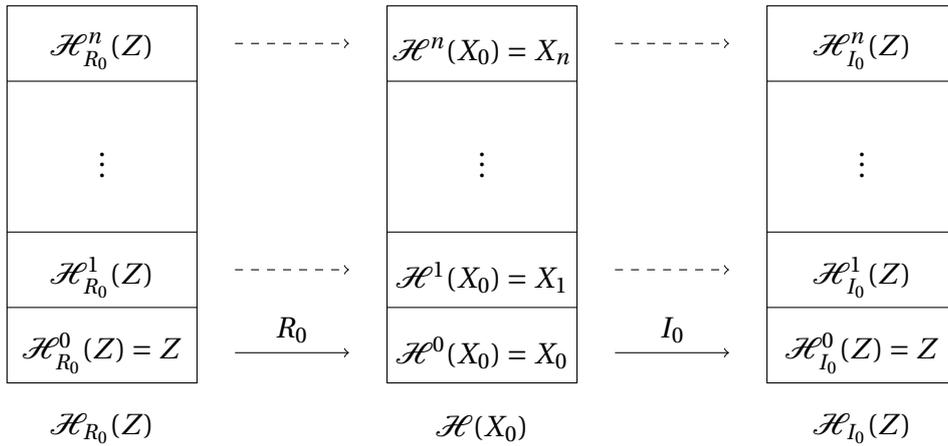


FIGURE 13

illustrating the two basic principles:

- (I) The Hyperstructure Principle — which is an organizing principle and a guiding principle for structural architecture and engineering.
- (II) The Transfer Principle — which is a way to transfer a hyperstructure.

For the situation in Example 2 the figure looks like:

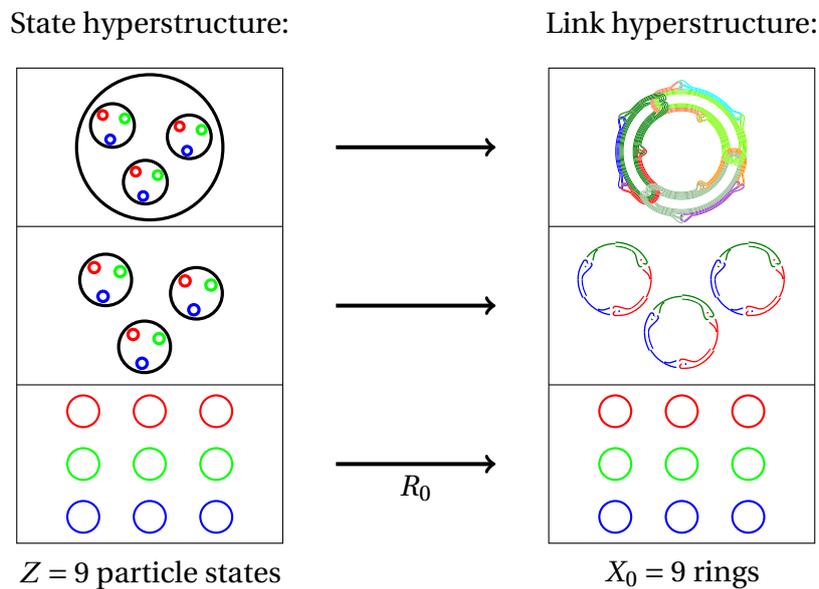


FIGURE 14

The binding structures and diagrams in Figure 13 described here may be considered as extensions of pasting diagrams in higher categories. A sheaf type formulation has been given in Appendix B in Baas (2006). Using the Principles described here one may

induce an action on a totality Z , by acting on individual elements and letting the action propagate through $\mathcal{H}(Z)$ to the top level as in political processes and in social and business organizations. In this way many small actions may lead to major global actions and change of state. Hence \mathcal{H} acts as an amplifier.

6. ANALYSIS AND SYNTHESIS

How do we synthesize new objects or structures from old ones? A common procedure both in nature and science is to bind objects together to form new objects with new properties, then use these properties in forming new bonds and new higher order objects. This is precisely what a hyperstructure does!

Let Z be a collection of objects. By putting a hyperstructure on Z — $\mathcal{H}(Z)$ we have a binding scheme of new higher order synthetic objects. If $\mathcal{H}(Z)$ is pulled back from a given structure, the problem may be to tune the environment of Z in such a way that the binding pattern of $\mathcal{H}(Z)$ may be realized.

On the other hand Z may be considered as a global object that we want to analyze by decomposing it into smaller and smaller pieces. By putting a hyperstructure on Z — $\mathcal{H}(Z)$ — we have seen in the previous section how it gives rise to a higher order clustering decomposition of Z .

It is interesting to notice that if we take the smallest pieces in the decomposition (lowest level elements) as our basic set $Z(\text{Dec})$, we may put a new hyperstructure $\widehat{\mathcal{H}}$ on it and recapture Z as the top level of $\widehat{\mathcal{H}}(Z(\text{Dec}))$.

This shows that hyperstructures are useful in the *synthesis* of new collections of objects from given ones and in the *analysis* of them as well. It is very useful to put a hyperstructure on a collection of objects in order to manipulate the collection towards certain goals. We will discuss various applications in the following.

7. APPLICATIONS OF THE BINDING STRUCTURE

Many interactions in science and nature may be described and handled as organized and structured collections of objects — in certain contexts called many-body systems. In the following we would like to point out that hyperstructures of bindings may be interesting and useful in many areas.

- a) *Physics.* \mathcal{H} -structures of many body systems (particles) may — as we have already discussed — give rise to new and exotic states of matter (\mathcal{H} -states), see Examples 1 and 2.
- b) *Chemistry.* Hyperstructures like higher order links are interesting models for the synthesis of new molecules and materials — like higher order Brunnian rings, see Baas (2009) and Baas and Seeman (2012).

More generally we may consider a hyperstructure \mathcal{H} where the bonds are spaces like manifolds or CW-complexes built up of cells.

Let us think of a collection of molecules each represented by a point in space. Then they may form



FIGURE 15

This representation increases the dimension by one. Similarly:

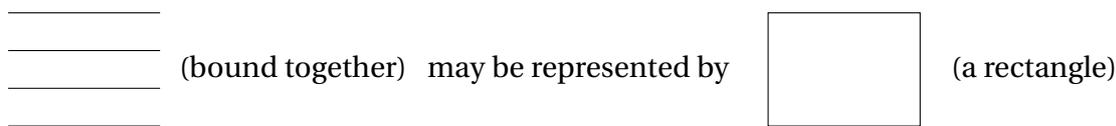


FIGURE 16

Collection of rectangles forming new chains:

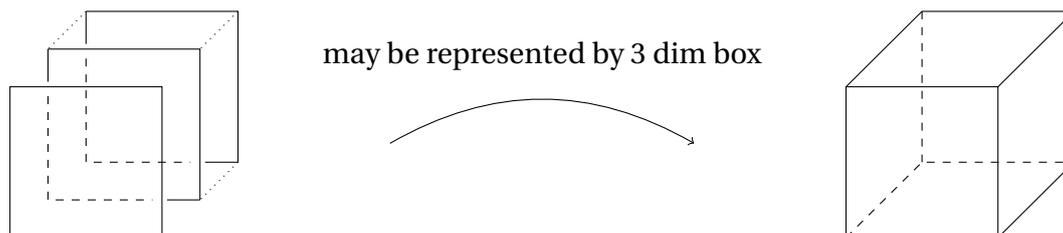


FIGURE 17

Then we form chains of 3 dimensional boxes again to be represented by a 4 dimensional box, etc. We may also introduce holes and we may continue up to a desired dimension.

Then we may glue the molecular cells together following the topological patterns (for example homotopy type) of the bond spaces B_n in each dimension.

In this way we get organized molecules in three dimensions with the structure induced from a higher dimensional binding structure in \mathcal{H} . Clearly many other similar representations are possible. This is a very useful and important principle.

We could also have molecules representing the bonds as in the following figures:

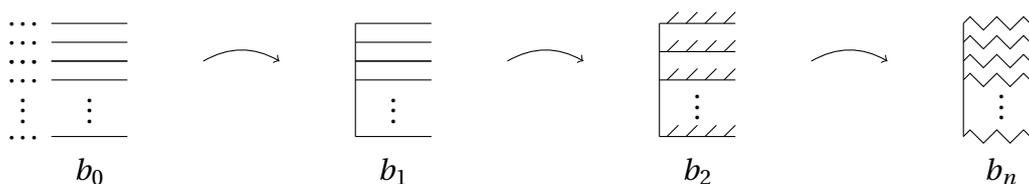


FIGURE 18

describing the process:

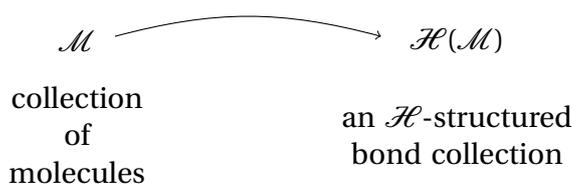


FIGURE 19

- c) *Social and economic systems.* We may here consider populations of individuals, social or economic units. Then it may be useful to consider them as many body systems in the physical examples and introduce higher order binding structures, see examples in Baas (2013) and Section 4.

One may for example discuss Brunnian investments of n agents and continuation to higher order which may be interesting in certain contexts.

- d) *Biology.* Here we may consider collections of genes, cells, pathways, neurons, etc. as many body systems and bind them together in new ways according to a given \mathcal{H} -structure. For example within tissue engineering one may make \mathcal{H} -type tissues for various purposes.
- e) *Logic.* We may introduce \mathcal{H} -type bindings of logical types and data-structures. New “laws of thought” are possible based on a logic of \mathcal{H} -type bindings as “deductions” and states/observations in \mathcal{H} representing the semantics.
- f) *Networks.* In Baas (2009) we argued that in many situations networks are inadequate and should be replaced by hyperstructures. Pairwise binding or interactions would then be replaced by \mathcal{H} -bindings. Look at the Brunnian hyperstructure of links as in the introduction.

- g) *Brain systems.* Extend natural and artificial neural networks to \mathcal{H} -structures of neurons as follows.

Let Z be a collection of real or abstract neurons. Then the $\mathcal{H}(Z)$ binding structure represent new interaction patterns “parametrized” by \mathcal{H} , possibly representing new types of higher order cognitive functions and properties. See also Baas (1996).

- h) *Correlations.* One may think of correlations as relations and bindings of variables. An interesting possibility would be to extend pair correlations to \mathcal{H} -type correlations of higher clusters. They could possibly have Brunnian properties as follows:

$$\text{Corr}(X, Y) = \text{Corr}(Y, Z) = \text{Corr}(X, Z) = 0, \quad \text{but} \quad \widehat{\text{Corr}}(X, Y, Z) \neq 0.$$

This is in analogy with cup products and Massey products in the study of Brunnian links. To detect higher order Brunnian linking one introduces higher order Massey products. In the correlation language this would mean putting

$$\hat{A} = \widehat{\text{Corr}}(X_1, Y_1, Z_1) = 0, \quad \hat{B} = \widehat{\text{Corr}}(X_2, Y_2, Z_2) = 0, \quad \hat{C} = \widehat{\text{Corr}}(X_3, Y_3, Z_3) = 0$$

and finding a \hat{C} such that $\hat{C}(\hat{A}, \hat{B}, \hat{C}) \neq 0$ represents a second order correlation.

- i) *Mathematics.* As already indicated in some of the examples collections of mathematical objects may also be bound together in new ways modelled or parametrized by a hyperstructure, for example collections of spaces like manifolds and cell-complexes along with gluing bonds.

Assume that we have a collection of spaces $\mathcal{L} = \{L_i\}$ organized in a well-defined hyperstructure $\mathcal{H}(\mathcal{L})$. If we have another collection of spaces or objects $\mathcal{X} = \{X_i\}$ we may then induce an \mathcal{H} -structure on $\mathcal{X} — \mathcal{H}(\mathcal{X})$ and use it to study the collection \mathcal{X} .

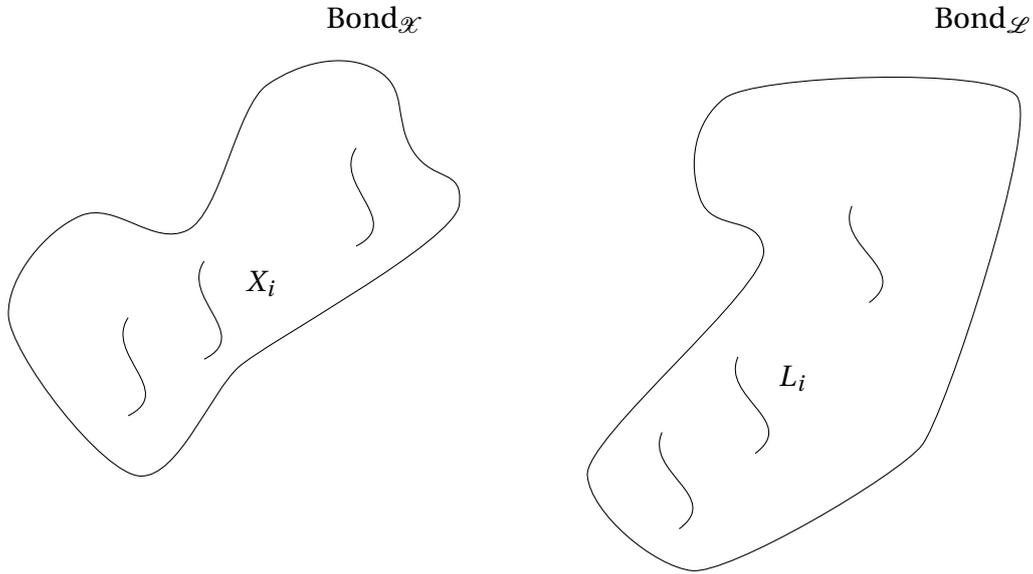


FIGURE 20

Interesting properties for $\mathcal{H}(\mathcal{L})$ may be asked for $\mathcal{H}(\mathcal{X})$ like Brunnian properties in a categorical setting, see Baas (2013).

We may take a family of spaces, for example simplicial complexes:

$$\mathcal{V} = \{V_i\}$$

and represent them in a family of manifolds

$$\mathcal{M} = \{M_i\}$$

on which there is a hyperstructure $\mathcal{H}(\mathcal{M})$ with given bonds such that

$$V_i \rightarrow M_i$$

and

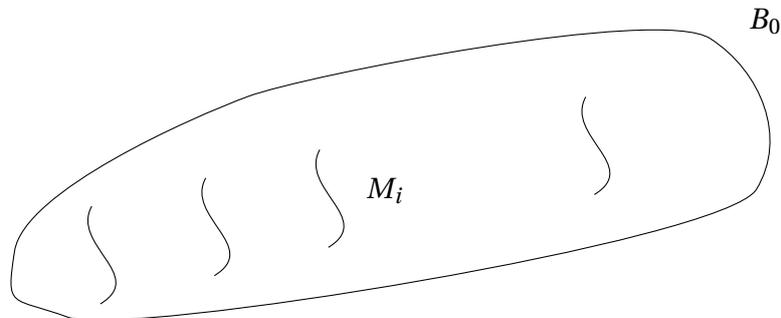


FIGURE 21

B_0 being a bond for $\{M_i\}$, for example by $M_i \subset B_0$. Then we may say that the $\{V_i\}$ binds by a pullback bond \hat{B}_0 . Similar for higher bonds. In this way one may introduce geometric structures for each level which otherwise may have been difficult. In the sense that hyperstructures extend the various notions of higher categories, one may also introduce \mathcal{H} -type bundles and stacks with transition and gluing morphisms replaced by appropriate bonds. Then one may hope to extend bundle notions like connections, curvature and holonomy in suitable contexts.

Let us follow up the examples and discussion of hyperstructures in Section 3.

How to produce hyperstructures?

We have seen that compositions of maps

$$S_1 \leftarrow S_2 \leftarrow \cdots \leftarrow S_n$$

naturally lead to hyperstructures on S_n . This may also be extended to a situation of compositions of functors and sub-categories.

Geometric bonds are basically constructed by binding families of spaces or sub-spaces of an ambient space:

$$\{A, \{A(\omega)\}\}, \quad \omega = (i_1, i_2, \dots, i_n), \quad i_j \in I_j$$

where

$$A \supset A(i_1) \supset A(i_1, i_2) \supset \cdots \supset A(\omega).$$

are successive bonds. Here the $A(\omega)$'s may be general spaces, manifolds, etc.

Generalized link and knot theory may be viewed as the study of embeddings of topological spaces in other topological spaces, but hyperstructures encompass much more. Still for geometric hyperstructures one may consider using and extending the mathematical theory of links and knots (quantum versions, quandles, etc., see Nelson (2011)) to the study of geometric hyperstructures.

Manifolds with singularities as introduced in Baas (1973) are also represented by such bond systems $A = \{A(\omega)\}$. So is also the Brunnian link hyperstructure

$$B = \{B(\omega)\} = \{B(n_1, n_2, \dots, n_k)\}.$$

This applies to structures in general — for example algebraic, topological, geometric, logical and categorical — presented as follows

$$\mathcal{C} = \{\mathcal{C}(\omega)\},$$

where structure $\mathcal{C}(\omega)$ binds the structures $\mathcal{C}(\omega, i)$ for example as substructures — like in higher order links and many-body systems.

This may be viewed as a kind of many-body problem of general structures, and represent a simple organizing principle for them.

All this shows that there is a plethora of possible applications of hyperstructured binding both in abstract and natural systems. Hyperstructures apply to all kinds of universes: mathematical, physical, chemical, biological, economic and social. Furthermore, the Transfer Principle makes it possible to connect them. Detailed applications will be the subject of future papers. The main point in this paper has been to illustrate the transfer of higher order binding structure as given in a hyperstructure. Ultimately one may also consider bindings coming from hyperstructures of hyperstructures as in the case of higher order links.

All the examples here and in Section 3 may be used to put a hyperstructure on sets, spaces, structures and situations by the methods described in Section 5. This may be useful to obtain actions like geometrical and physical fusion of objects in various situations similar to the “political/sociological” metaphor.

After all we “make things” through a hyperstructure principle as in modern engineering. This may be so since it is the way nature works through evolution, and after all we are ourselves products of such a process.

8. MULTILEVEL STATE SYSTEMS

We have here discussed the organization of many-body systems or general systems of collections of objects. The systems may be finite, infinite or even uncountable. We have advocated Hyperstructures as the guiding organizational principle. In this section we will discuss in more detail possible organization of the states of the system through level connections. We use the terminology in Section 3.

When we put a hyperstructure on a situation in order to obtain a certain goal or action often a dynamics is required on \mathcal{H} :

$$D: \mathcal{H} \rightarrow \mathcal{H}$$

which essentially changes the states.

In order to do this it is advantageous to be able to have as rich structures as possible as states: sets, spaces (manifolds), algebras, (higher) vector spaces, (higher) categories, etc.

For this reason and in order to cover as many interactions as possible we introduce the following extension.

Instead of letting the states (Ω_i) in \mathcal{H} take values in Sets we extend this to a family of prescribed *hyperstructures of states*:

$$\mathcal{S} = \{\mathcal{S}_0, \mathcal{S}_1, \dots, \mathcal{S}_n\}$$

where \mathcal{S}_i is a hyperstructure such that:

$$\begin{array}{l} \Omega_0 \text{ takes values in } \mathcal{S}_n \\ \vdots \\ \Omega_i \text{ takes values in } \mathcal{S}_{n-i} \\ \vdots \\ \Omega_n \text{ takes values in } \mathcal{S}_0. \end{array}$$

We want these level state structures to be connected in some way. Therefore we require that \mathcal{S} is organized into a hyperstructure itself with $\mathcal{S}_0, \dots, \mathcal{S}_n$ as the levels — or actually sets of bonds of states, with \mathcal{S}_0 being the top level (dual to \mathcal{H} itself). We furthermore assume that we have level connecting assignments or boundary maps δ_i which we will for short write as:

$$\mathcal{S}_0 \overset{\delta_1}{\leftarrow} \mathcal{S}_1 \overset{\delta_2}{\leftarrow} \dots \overset{\delta_n}{\leftarrow} \mathcal{S}_n.$$

Often the order of the hyperstructure will decrease moving from the bottom to the top — “integrating away complexity”. The δ_i ’s may be of assignment (functional) or relational type.

This shows how to form hyperstructures of hyperstructures. In such cases one may actually use existing hyperstructures to form bonds and states in new hyperstructures.

The assignment of a state to a bond (or collection of bonds) is a kind of representation:

$$\Omega: \mathcal{H} \rightsquigarrow \mathcal{S} \quad \text{or} \quad \Omega_i: B_i \rightsquigarrow \mathcal{S}_{n-i}$$

such that if

$$\Omega_i(b_i) = \omega_i \quad \text{and} \quad \partial_{i-1} b_i = \{b_{i-1}(\omega_{i-1})\},$$

then

$$\delta_{n-i+1} \Omega_{i-1} \partial_{i-1}(b_i) = \Omega_i(b_i)$$

in simplified notation ($\{-\}$ meaning a family or subset of objects). This is a balancing equation of bonds and states at the various levels.

Let us illustrate this by an example.

Example 6. This is basically a version of Example d) in Section 3 and studied in Baas (2006) in connection with genomic structure. \mathcal{H} is given by sets

$$G_0 < G_1 < G_2 < \cdots < G_n$$

meaning that there exist maps

$$g_i: G_i \rightarrow \mathcal{P}(G_{i+1}).$$

To each G_i we assign a state space S_i — possibly a manifold.

Then the state hyperstructure reduces to the composition of (smooth) mappings:

$$S_0 \leftarrow S_1 \leftarrow \cdots \leftarrow S_n.$$

In order to influence global states from local actions it is a reasonable and general procedure to put a hyperstructure on the systems with a multilevel state structure given by another hyperstructure \mathcal{S} with level relations as described:

$$\mathcal{S}_0 \leftarrow \mathcal{S}_1 \leftarrow \cdots \leftarrow \mathcal{S}_n.$$

The idea is then to act on \mathcal{S}_n by introducing a suitable dynamics and let the actions propagate through the hyperstructure to the global level. This is similar to social systems and may be called the “Democratic Method of Action”. In other situations one may want to move from high to low level states.

It is especially useful if the level relations are functional assignments:

$$\mathcal{S}_0 \leftarrow \mathcal{S}_1 \leftarrow \cdots \leftarrow \mathcal{S}_n.$$

For example if the \mathcal{S}_i 's are categories of some kind, the arrows would represent functors and if the \mathcal{S}_i 's are spaces, the arrows represent mappings.

Let us specify the mappings

$$\partial_{i-1}: X_i \rightarrow \mathcal{P}(X_{i-1})$$

by

$$\partial_{i-1} b_i = \{b_{i-1}(\omega_{i-1})\},$$

where $\{-\}$ just means a family or subsets of objects, and requiring assignments (mappings):

$$\prod_{\substack{\text{m} \\ \mathcal{S}_{n-i+1}}} \Omega_{i-1}(\{b_{i-1}\}) \xrightarrow{\delta_{n-i+1}} \prod_{\substack{\text{m} \\ \mathcal{S}_{n-i}}} \Omega_i(b_i)$$

The $\hat{\delta}$'s are then state level connectors. If the \mathcal{S}_i 's have a tensor product we will often require

$$\bigotimes \{\Omega_{i-1}(b_{i-1})\} \xrightarrow{\hat{\delta}_{n-i+1}} \Omega_i(b_i)$$

More schematically this may be described as:

$$\{b(i_n)\} \xrightarrow{\hat{\delta}} \{b(i_{n-1}, i_n)\} \xrightarrow{\hat{\delta}} \cdots \xrightarrow{\hat{\delta}} \{b(i_0, \dots, i_n)\}$$

for bonds, and for states:

$$\Omega_n(\{b(i_n)\}) \xleftarrow{\hat{\delta}} \Omega_{n-1}(\{b(i_{n-1}, i_n)\}) \xleftarrow{\hat{\delta}} \cdots \xleftarrow{\hat{\delta}} \Omega_0(\{b(i_0, \dots, i_n)\}).$$

This is an extension of the framework for extended Topological Quantum Field Theories (see Lawrence (1996)) where one considers manifolds with general boundaries and corners

$$M = \{M(\omega)\} \quad \text{like in Section 7 i)}$$

bound by generalized cobordisms.

At the state level one assigns to these manifolds higher order algebraic structures such as a version k -vector spaces ($k - \mathcal{V}$) or k -algebras as follows (Lawrence 1996):

$$\begin{array}{ccc} M(\omega) & \longrightarrow & Z(\omega) \in k - \mathcal{V} \\ (\text{codim} = k) & & (\text{order} = k) \end{array}$$

where levels of states are connected via geometrically induced pairings:

$$k - \mathcal{V} \otimes k - \mathcal{V} \rightarrow (k - 1) - \mathcal{V}$$

for all k . $0 - \mathcal{V}$ being the scalars \mathbb{C} in the case of complex vector spaces.

In this sense we can control and regulate the global state from the lowest level which is clearly a desirable thing in systems of all kinds. This is useful in the following situation. Let A be a desired action or task which may be “un-managable” in a given system or context. Furthermore, let $M = \{m_\alpha\}$ be a collection of “managable” actions in the system.

Put $X_0 = M$ and design a hyperstructure \mathcal{H} (like a society or factory) where then A appears as $X_n = B_n$ — the top level bond of the hyperstructure \mathcal{H} . Hence \mathcal{H} will act as a propagator from {managable actions} to {desired actions} by dynamically regulating the states of \mathcal{H} as described. This procedure applies to general systems and in the sense of Waddington (1977) one may say that hyperstructures are “Tools for Thought” and creation of novelty.

In general systems one may often want to change the global state in a desired way and it may be difficult since it would require large resources (“high energy”). But via a hyperstructure it may be possible introducing managable local actions and change of states which may require small resources (“low energy”), in other words small actions are being organized into large actions via \mathcal{H} . This is similar to changing the action of a society or organization by influencing individuals. If one wants to join two opposing societies (or nations) into one, it may take less resources to act on individuals to obtain the global effect. The photosynthesis works along the same lines — collecting, organizing and amplifying energy.

It seems like an interesting idea to suggest the use of hyperstructures in order to facilitate fusion of various types of systems, for example “particles” in biology, chemistry and physics. Even nuclear fusion may profit from this perspective. The hyperstructure in question may be introduced on the system itself or surrounding space and forces.

9. ORGANIZING PRINCIPLES

Binding structures and hyperstructures as we have described them are basically organizing principles of collections of objects. They apply to all kinds of collections and general systems, and as organizing principles they may be particularly interesting in physical matter (condensed matter) of atoms, molecules, etc.

R. Laughlin has advocated the importance of organizing principles in condensed matter physics in understanding for example superconductivity, the quantum Hall effect, phonons, topological insulators etc. See Laughlin and Pines (2000). He suggests that the very precise measurements made of important physical constants in these situations come from underlying organizing principles of matter.

Our binding- and hyperstructures are organizing principles that when introduced to physical matter should lead to new emergent properties. It is like in our Example 1. If we are given a random tangle of links, a new and non-trivial geometric order emerges when we put a higher order Brunnian structure on the collection of links.

In one way it is analogous to logical systems, where organized statements are more likely to be decidable than random ones. Similarly in biology: language, memory, spatial recognition, etc. are related to similar organizing principles.

Entangled states are studied in quantum mechanics and higher order versions are suggested Zeilinger, Horne and Greenberg (1992). Greenberger–Horne–Zeilinger (GHZ) states are analogous to Borromean and Brunnian rings, see Aravind (1997). From our previous discussion we are naturally led to suggest higher order entangled states organized by a hyperstructure \mathcal{H} using the transfer principle. Could such a process lead to collections of particles forming global/macroscopic quantum states? Could also an \mathcal{H} -structure act as a kind of (geometric) protectorate of a desired quantum state from thermodynamic disturbances (in high temperature superconductivity for example)?

The binding principle may be applied in two ways — in particular in condensed matter physics.

- I. Putting a binding- or hyperstructure on a collection of objects. Then collections of bound structures will appear, and they may have interesting emergent properties. For example with respect to precise measurements of involved constants of nature.
- II. Putting a binding- or hyperstructure on the ambient space (space-time) of a collection for example using various fields etc. This will introduce a structure on the collection and may result in bindings and fusion of particles and objects, or splitting (fission), stabilizing them into new patterns with new emergent properties.

In other words space, fields and reactors may all be organized by binding principles. We suggest that putting a binding- or hyperstructure on a collection, situation or system is a very fundamental and useful organizing principle.

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Related links of interest:

- New Scientist¹
- MIT Tech Review²

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¹<http://www.newscientist.com/article/mg20927942.300-make-way-for-mathematical-matter.htm>

²<http://www.technologyreview.com/view/422055/topologist-predicts-new-form-of-matter/>