## Appendix

### The Proof of Proposition 1

Recall that $u(t,e)$ is strictly increasing in $t$ but decreasing in $e$, Program (21) can be rewritten as

$$\max_{\tilde{x},\tilde{y},s_x,s_y} u\big(b+\alpha R(\tilde{x},\tilde{y})-\alpha R(-s_x,s_y),e\big)$$

$$= b+\alpha R(\tilde{x},\tilde{y})-\alpha R(-s_x,s_y)-v\left(e(s_x,s_y)\right)$$

$$= b+\alpha R(x,y)-v\left(e(s_x,s_y)\right)$$

$s.t.$ Constraints (20-1)- (20-3).

To maximize utility, the agent will pick $(s_x,s_y)$ to minimize $v\left(e(s_x,s_y)\right)$, i.e., the agent will select a local allocative efficient input-output combination $(\tilde{x},\tilde{y}) \in \arg\max_{(\tilde{x},\tilde{y})\in Z((x,y)|T)} R(\tilde{x},\tilde{y})$ as the underlying production mix. This completes the proof. QED.

### The Proof of Proposition 2

We prove this by contradiction. Assume $(\tilde{x},\tilde{y}) \notin \mathrm{Eff}\,T$, then by definition there exists an underlying production mix $(\tilde{x}',\tilde{y}')$ satisfying $(-\tilde{x}',\tilde{y}') \geq (-\tilde{x},\tilde{y}), (\tilde{x}',\tilde{y}') \neq (\tilde{x},\tilde{y})$ such that $(\tilde{x}',\tilde{y}') \in T$. By the definitions of profit function $R(x,y)$ and the set of underlying production mix $Z((x,y)|T)$, we induce that $R(\tilde{x}',\tilde{y}') > R(\tilde{x},\tilde{y})$ and $(\tilde{x}',\tilde{y}') \in Z((x,y)|T)$. With the help of **Proposition 1**, we have a contradiction such that $(\tilde{x},\tilde{y}) \notin L = \arg\max_{(x^*,y^*)\in Z((x,y)|T)} R(x^*,y^*)$ since $R(\tilde{x}',\tilde{y}') > R(\tilde{x},\tilde{y})$ and $(\tilde{x}',\tilde{y}') \in Z((x,y)|T)$. This completes the proof. QED.

### The Proof of Proposition 3

It is clear that $(\tilde{x},\tilde{y}) = (x-s_x, y+s_y) \in G$ holds by **Proposition 1**, combining $(s_x,s_y) \in \mathbb{R}_+^{m+h}$, we get $(x,y) = (\tilde{x}+s_x, \tilde{y}-s_y) \in G + \mathbb{R}_+^m \times (-\mathbb{R}_+^h)$. This completes the proof. QED.

### The Proof of Proposition 4

First, the agent is rational implies that the observed slack vector $s$ is the optimal solution of the utility maximization problem (23). We discuss the four cases separately in the following.

(i) In case with $v(e(s)) = g - \prod_{l=1}^{m+h} s_l^{\beta_l}$, to estimate the bargaining power of factor $\beta_l$, the utility maximization problem (23) can be rewritten as

$$\min_{s \in \mathbb{R}_+^{m+h}} \bar{U}(s) = \alpha \sum_{d=1}^{m+h} p_d s_d + g - \prod_{d=1}^{m+h} s_d^{\beta_d} - \Psi.$$

A necessary condition for the optimality of the observed slack vector $s$ in the above program is the first partial derivatives with respect to $s_l, l = 1,2, \dots, m+h$ are equal to zero, i.e.,

$$\frac{\partial \bar{U}(s)}{\partial s_l} = \alpha p_l - \frac{\beta_l \prod_{d=1}^{m+h} s_d^{\beta_d}}{s_l} = 0, l = 1,2, \dots, m+h.$$

The above formula can be reorganized as

$$\prod_{d=1}^{m+h} s_d^{\beta_d} = \frac{\alpha p_l s_l}{\beta_l}, l = 1,2, \dots, m+h.$$

For all $l = 1,2, \dots, m+h$ and $d = 1,2, \dots, m+h$, we thus have

$$\frac{\alpha p_l s_l}{\beta_l} = \frac{\alpha p_d s_d}{\beta_d}.$$

The above formula can be reorganized as

$$\frac{\beta_d}{\beta_l} = \frac{\alpha p_d s_d}{\alpha p_l s_l} = \frac{p_d s_d}{p_l s_l}.$$

Suppose $\beta_d = w p_d s_d$ and $\beta_l = w p_l s_l$ $(w > 0)$, we thus have $\sum_{d=1}^{m+h} w p_d s_d = 1$ because of $\sum_{d=1}^{m+h} \beta_d = 1$, from which we obtain

$$\beta_l = w p_l s_l = \frac{w p_l s_l}{\sum_{d=1}^{m+h} w p_d s_d} = \frac{p_l s_l}{\sum_{d=1}^{m+h} p_d s_d}.$$

To infer the lower and upper bounds of the underlying maximal effort level, the individual rationality constraint can be rewritten as

$$\Psi - \alpha \sum_{l=1}^{m+h} p_l s_l - g + \prod_{l=1}^{m+h} s_l^{\beta_l} \geq Q,$$

By the definition of the cost of effort, we have

$$g - \prod_{l=1}^{m+h} s_l^{\beta_l} \geq 0.$$

Combining the optimal estimates of bargaining power of factors, we thus obtain

$$g \leq \Psi + \prod_{l=1}^{m+h} s_l^{\frac{p_l s_l}{\sum_{d=1}^{m+h} p_d s_d}} - \alpha \sum_{l=1}^{m+h} p_l s_l - Q = g^{ub},$$

$$g \geq \prod_{l=1}^{m+h} s_l^{\frac{p_l s_l}{\sum_{d=1}^{m+h} p_d s_d}} = g^{lb}.$$

Subsequently, the inference about the cost of effort is

$$v(e(s)) = g - \prod_{l=1}^{m+h} s_l^{\frac{p_l s_l}{\sum_{d=1}^{m+h} p_d s_d}}, g \in \left[\prod_{l=1}^{m+h} s_l^{\frac{p_l s_l}{\sum_{d=1}^{m+h} p_d s_d}}, \Psi + \prod_{l=1}^{m+h} s_l^{\frac{p_l s_l}{\sum_{d=1}^{m+h} p_d s_d}} - \alpha \sum_{l=1}^{m+h} p_l s_l - Q\right].$$

This completes the first part of the proposition.

(ii) In case with $v(e(s)) = g - \min\{\beta_1 s_1, \beta_2 s_2, \dots, \beta_{m+h} s_{m+h}\}$, to estimate the relative importance of factor $\beta_l$, the utility maximization problem (23) can be rewritten as

$$\min_{s \in \mathbb{R}_+^{m+h}} \bar{U}(s) = \alpha \sum_{d=1}^{m+h} p_d s_d + g - \min\{\beta_1 s_1, \beta_2 s_2, \dots, \beta_{m+h} s_{m+h}\} - \Psi.$$

A necessary condition for the optimality of the observed slack vector $s$ in the above program is $\beta_1 s_1 = \beta_2 s_2 = \cdots = \beta_{m+h} s_{m+h}$, we record this result as a corollary.

**Corollary 1**. In case with $v(e(s)) = g - \min\{\beta_1 s_1, \beta_2 s_2, \dots, \beta_{m+h} s_{m+h}\}$, the optimal estimates of relative importance of the factors $\beta_l, l = 1,2, \dots, m+h$ satisfy $\beta_1 s_1 = \beta_2 s_2 = \cdots = \beta_{m+h} s_{m+h}$.

For ease of exposition, let $\tau = \beta_1 s_1 = \beta_2 s_2 = \cdots = \beta_{m+h} s_{m+h}$, the optimal value of Program (23) is

$$\alpha \sum_{d=1}^{m+h} p_d s_d + g - \tau - \Psi.$$

Suppose $\beta_1 s_1 = \beta_2 s_2 = \cdots = \beta_{m+h} s_{m+h}$ does not always hold, there exists at least one input or output (denoted by $l$) satisfying $\beta_l s_l < \tau$ since $\sum_{d=1}^{m+h} \beta_d = 1$. Then the corresponding value of Program (23) is

$$\alpha \sum_{d=1}^{m+h} p_d s_d + g - \min_{l \in \{l | \beta_l s_l < \tau\}} \beta_l s_l - \Psi.$$

Note that

$$\alpha \sum_{d=1}^{m+h} p_d s_d + g - \min_{l \in \{l | \beta_l s_l < \tau\}} \beta_l s_l - \Psi > \alpha \sum_{d=1}^{m+h} p_d s_d + g - \tau - \Psi,$$

which implies any deviation from $\beta_1 s_1 = \beta_2 s_2 = \cdots = \beta_{m+h} s_{m+h}$ leads to the increase of the value of Program (23). To minimize the value of Program (23), therefore, $\beta_1 s_1 = \beta_2 s_2 = \cdots = \beta_{m+h} s_{m+h}$ is optimal. This completes the proof of **Corollary 1.**

With the help of **Corollary 1**, let us combine $\tau = \beta_1 s_1 = \beta_2 s_2 = \cdots = \beta_{m+h} s_{m+h}$ and

$\sum_{d=1}^{m+h} \beta_d = 1$, we get $\sum_{d=1}^{m+h} \frac{\tau}{s_d} = 1$, i.e.,

$$\tau = \frac{1}{\sum_{d=1}^{m+h} \frac{1}{s_d}} = \beta_l s_l,$$

from which we obtain

$$\beta_l = \frac{1}{s_l \left( \sum_{d=1}^{m+h} \frac{1}{s_d} \right)}, l = 1,2, \dots, m+h.$$

To infer the lower and upper bounds of the underlying maximal effort level, the individual rationality constraint can be rewritten as

$$\Psi - \alpha \sum_{l=1}^{m+h} p_l s_l - g + \min\{\beta_1 s_1, \beta_2 s_2, \dots, \beta_{m+h} s_{m+h}\} \geq Q.$$

By the definition of the cost of effort, we have

$$g - \min\{\beta_1 s_1, \beta_2 s_2, \dots, \beta_{m+h} s_{m+h}\} \geq 0.$$

Combining the optimal estimates of relative importance of factors, we thus obtain

$$g \leq \Psi + \frac{1}{\sum_{l=1}^{m+h} \frac{1}{s_l}} - \alpha \sum_{l=1}^{m+h} p_l s_l - Q = g^{ub},$$

$$g \geq \frac{1}{\sum_{l=1}^{m+h} \frac{1}{s_l}} = g^{lb}.$$

Subsequently, the inference about the cost of effort is

$$v\big(e(s)\big) = g - \frac{1}{\sum_{d=1}^{m+h} \frac{1}{s_d}}, g \in \left[ \frac{1}{\sum_{l=1}^{m+h} \frac{1}{s_l}}, \Psi + \frac{1}{\sum_{l=1}^{m+h} \frac{1}{s_l}} - \alpha \sum_{l=1}^{m+h} p_l s_l - Q \right].$$

This completes the second part of the proposition.

(iii) In case with $v\big(e(s)\big) = \left( g - \prod_{l=1}^{m+h} s_l^{\beta_l} \right)^2$, to estimate the bargaining power of factor $\beta_l$, the utility maximization problem (23) can be rewritten as

$$\min_{s \in \mathbb{R}_+^{m+h}} \bar{U}(s) = \alpha \sum_{d=1}^{m+h} p_d s_d + \left( g - \prod_{d=1}^{m+h} s_d^{\beta_d} \right)^2 - \Psi.$$

A necessary condition for the optimality of the observed slack vector $s$ in the above program is the first partial derivatives with respect to $s_l, l = 1,2, \dots, m+h$ are equal to zero, i.e.,

$$\frac{\partial \bar{U}(s)}{\partial s_l} = \alpha p_l + 2\left(g - \prod_{d=1}^{m+h} s_d^{\beta_d}\right)\left(-\frac{\beta_l \prod_{d=1}^{m+h} s_d^{\beta_d}}{s_l}\right) = 0, l = 1,2, \dots, m+h.$$

The above formula can be reorganized as

$$2\left(g - \prod_{d=1}^{m+h} s_d^{\beta_d}\right)\prod_{d=1}^{m+h} s_d^{\beta_d} = \frac{\alpha p_l s_l}{\beta_l}, l = 1,2, \dots, m+h.$$

For all $l = 1,2, \dots, m+h$ and $d = 1,2, \dots, m+h$, we thus have

$$\frac{\alpha p_l s_l}{\beta_l} = \frac{\alpha p_d s_d}{\beta_d}.$$

The above formula can be reorganized as

$$\frac{\beta_d}{\beta_l} = \frac{\alpha p_d s_d}{\alpha p_l s_l} = \frac{p_d s_d}{p_l s_l}.$$

Suppose $\beta_d = \gamma p_d s_d$ and $\beta_l = \gamma p_l s_l$ ($\gamma > 0$), we thus have $\sum_{d=1}^{m+h} \gamma p_d s_d = 1$ because of $\sum_{d=1}^{m+h} \beta_d = 1$, from which we obtain

$$\beta_l = \gamma p_l s_l = \frac{\gamma p_l s_l}{\sum_{d=1}^{m+h} \gamma p_d s_d} = \frac{p_l s_l}{\sum_{d=1}^{m+h} p_d s_d}.$$

To infer the lower and upper bounds of the underlying maximal effort level, the individual rationality constraint can be rewritten as

$$\Psi - \alpha \sum_{l=1}^{m+h} p_l s_l - \left(g - \prod_{l=1}^{m+h} s_l^{\beta_l}\right)^2 \geq Q.$$

By the definition of the cost of effort, we have

$$\left(g - \prod_{l=1}^{m+h} s_l^{\beta_l}\right)^2 \geq 0.$$

Combining the optimal estimates of bargaining power of factors, we thus obtain

$$g \leq \sqrt{\Psi - \alpha \sum_{l=1}^{m+h} p_l s_l - Q} + \prod_{l=1}^{m+h} s_l^{\frac{p_l s_l}{\sum_{d=1}^{m+h} p_d s_d}} = g^{ub},$$

$$g \geq \prod_{l=1}^{m+h} s_l^{\frac{p_l s_l}{\sum_{d=1}^{m+h} p_d s_d}} = g^{lb}.$$

Subsequently, the inference about the cost of effort is

$$v\big(e(s)\big) = \left(g - \prod_{l=1}^{m+h} s_l^{\frac{p_l s_l}{\sum_{d=1}^{m+h} p_d s_d}}\right)^2, g \in \left[\prod_{l=1}^{m+h} s_l^{\frac{p_l s_l}{\sum_{d=1}^{m+h} p_d s_d}}, \sqrt{\Psi - \alpha \sum_{l=1}^{m+h} p_l s_l - Q} + \prod_{l=1}^{m+h} s_l^{\frac{p_l s_l}{\sum_{d=1}^{m+h} p_d s_d}}\right].$$

This completes the third part of the proposition.

(iv) In case with $v(e(s)) = (g - \min\{\beta_1 s_1, \beta_2 s_2, \dots, \beta_{m+h} s_{m+h}\})^2$, the utility maximization problem (23) can be rewritten as

$$\min_{s\in\mathbb{R}_+^{m+h}} \bar{U}(s) = \alpha \sum_{d=1}^{m+h} p_d s_d + (g - \min\{\beta_1 s_1, \beta_2 s_2, \dots, \beta_{m+h} s_{m+h}\})^2 - \Psi.$$

Similar to the proof of part (ii) of the proposition above, a necessary condition for the optimality of the observed slack vector $s$ in the above program is $\beta_1 s_1 = \beta_2 s_2 = \dots = \beta_{m+h} s_{m+h}$, from which we have

$$\beta_l = \frac{1}{s_l\left(\sum_{d=1}^{m+h}\frac{1}{s_d}\right)}, l = 1,2, \dots, m+h.$$

To infer the lower and upper bounds of the underlying maximal effort level, the individual rationality constraint can be rewritten as

$$\Psi - \alpha \sum_{l=1}^{m+h} p_l s_l - (g - \min\{\beta_1 s_1, \beta_2 s_2, \dots, \beta_{m+h} s_{m+h}\})^2 \geq Q.$$

By the definition of the cost of effort, we have

$$(g - \min\{\beta_1 s_1, \beta_2 s_2, \dots, \beta_{m+h} s_{m+h}\})^2 \geq 0 \geq 0.$$

Combining the optimal estimates of relative importance of factors, we thus obtain

$$g \leq \sqrt{\Psi - \alpha \sum_{l=1}^{m+h} p_l s_l - Q} + \frac{1}{\sum_{l=1}^{m+h}\frac{1}{s_l}} = g^{ub},$$

$$g \geq \frac{1}{\sum_{l=1}^{m+h}\frac{1}{s_l}} = g^{lb}.$$

Subsequently, the inference about the cost of effort is

$$(e(s)) = \left(g - \frac{1}{\sum_{l=1}^{m+h}\frac{1}{s_l}}\right)^2, g \in \left[\frac{1}{\sum_{l=1}^{m+h}\frac{1}{s_l}}, \sqrt{\Psi - \alpha \sum_{l=1}^{m+h} p_l s_l - Q} + \frac{1}{\sum_{l=1}^{m+h}\frac{1}{s_l}}\right].$$

This completes the final part of the proposition. QED.

**The Nonparametric Frontier Technology**

In line with the production theory, the nonparametric frontier technology $T$ satisfies the following standard axioms (Banker et al., 1984; Podinovski et al., 2018; Kerstens et al., 2019).

**Axiom 1.** *Inclusion of Observations*. $(x_j, y_j) \in T$ for any $j, j = 1,2, \dots, n$.

**Axiom 2.** *Convexity*. $T$ is convex.

**Axiom 3.** *Free Disposability*. If $(x, y) \in T$, then for any $x' \geq x, 0 \leq y' \leq y$, $(x', y') \in T$.

**Axiom 4.** *Minimum Extrapolation*. $T$ is the smallest set.

With the help of the classical axioms above, an algebraic representation of convex nonparametric frontier technology under variable return to scale can be expressed as

$$T = \left\{(x, y) \in \mathbb{R}_{+}^{m+h} \middle| \exists \lambda_j \in \mathbb{R}_0 : x \geq \sum_{j=1}^{n} \lambda_j x_j , y \leq \sum_{j=1}^{n} \lambda_j y_j , \sum_{j=1}^{n} \lambda_j = 1\right\}$$