Supplementary Materials to "Robust multivariate functional discriminant coordinates"

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In this supporting material, the results of the simulation studies considered in Section 3 of the paper are presented in Figures 6-45. Moreover, Figures 1-5 and 46 depict the generated and real functional data investigated in the paper.

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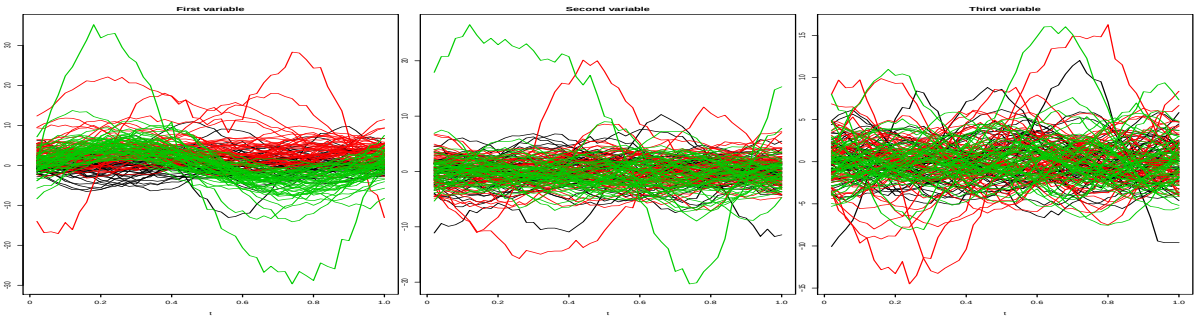

Figure 1: Realizations of three functional variables for Setup D1 with 40% multivariate t_3-distribution contamination and $L = 3$ groups, $n_i = 50$, $i = 1, 2, 3$.

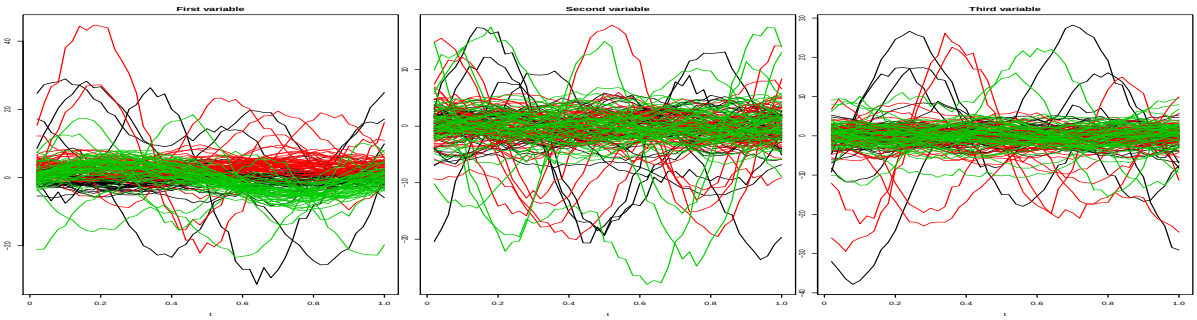

Figure 2: Realizations of three functional variables for Setup D1 with 10% scale contamination and $L = 3$ groups, $n_i = 50$, $i = 1, 2, 3$, $\kappa = 25$.

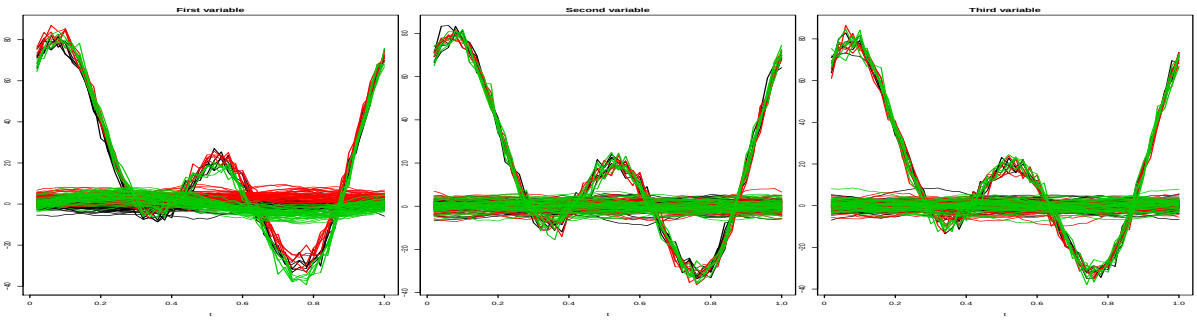

Figure 3: Realizations of three functional variables for Setup D1 with 10% one-direction shift location contamination and $L = 3$ groups, $n_i = 50$, $i = 1, 2, 3$, $v = 10$.

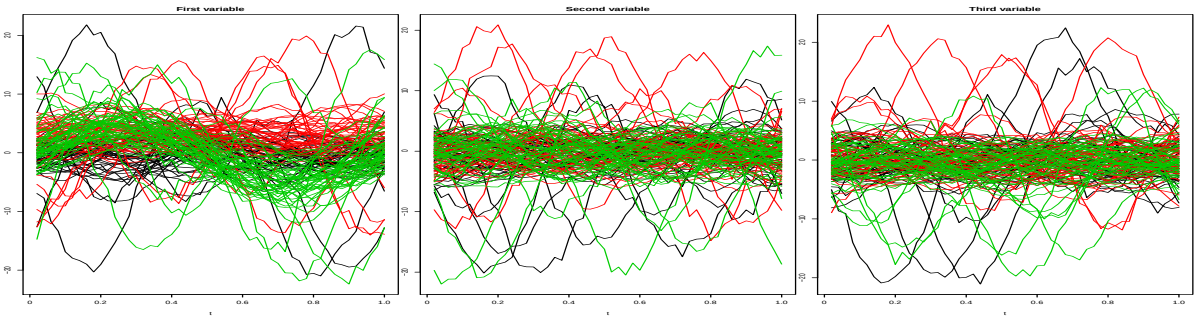

Figure 4: Realizations of three functional variables for Setup D1 with 10% radial location contamination and $L = 3$ groups, $n_i = 50$, $i = 1, 2, 3$, $v = 10$.

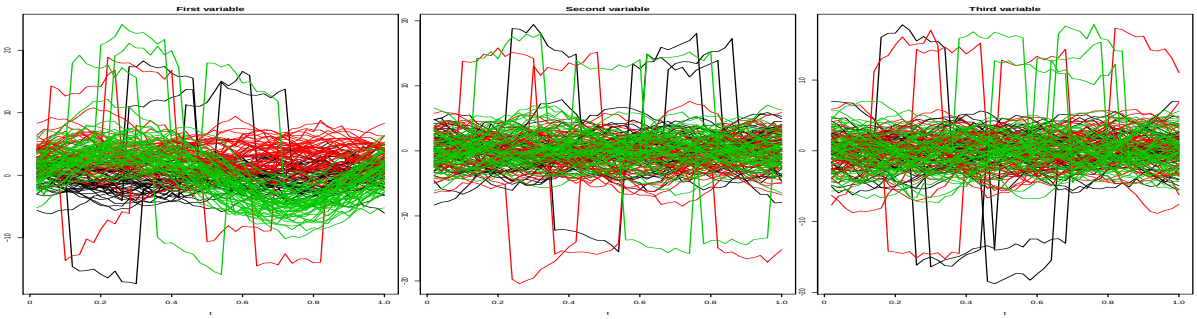

Figure 5: Realizations of three functional variables for Setup D1 with 10% of isolated outliers and $L = 3$ groups, $n_i = 50$, $i = 1, 2, 3$.

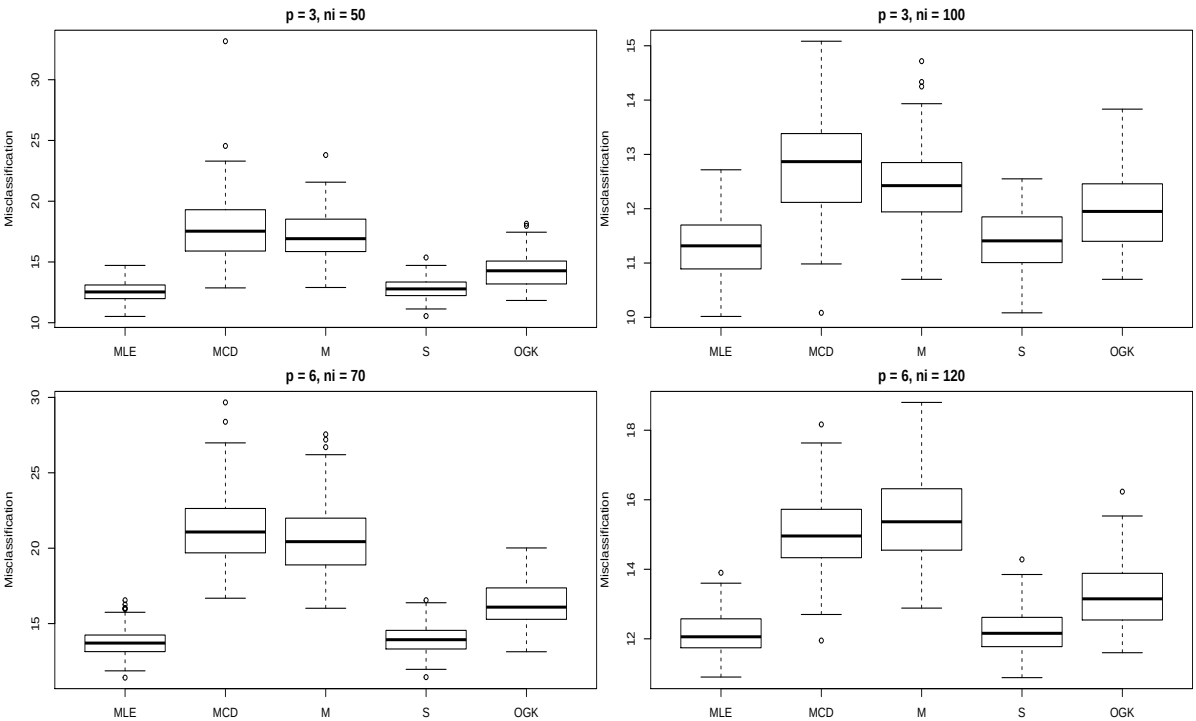

Figure 6: Mean probability of misclassification (as percentages) for Setup D1, $M = 100$ and $L = 3$ groups.

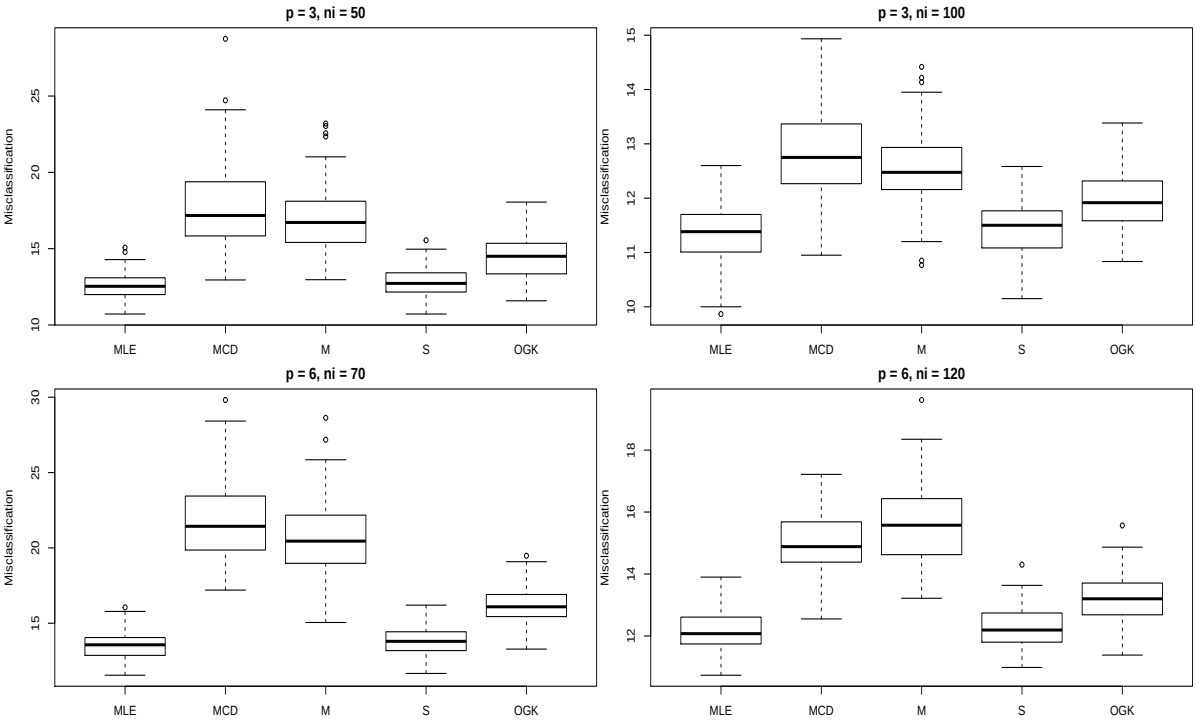

Figure 7: Mean probability of misclassification (as percentages) for Setup D1, $M = 150$ and $L = 3$ groups.

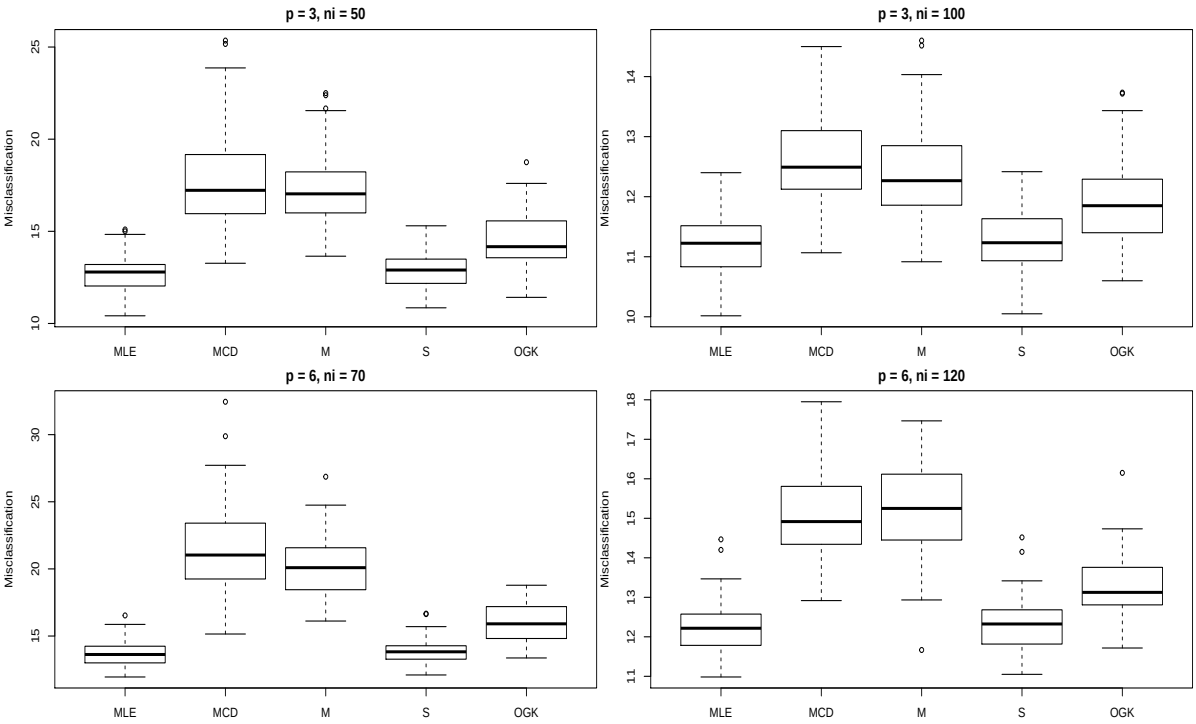

Figure 8: Mean probability of misclassification (as percentages) for Setup D1, $M = 50$ unequally spaced design time points and $L = 3$ groups.

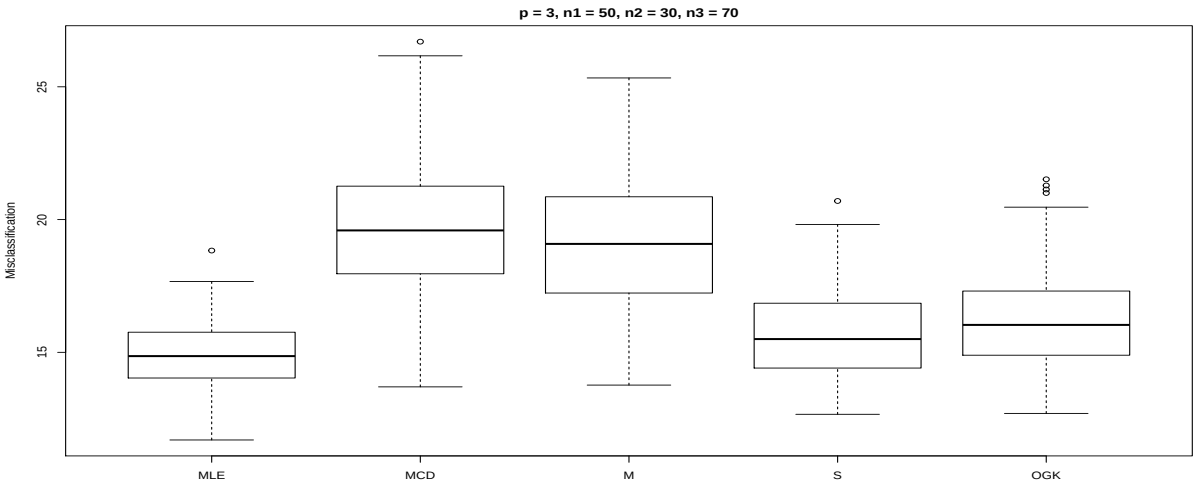

Figure 9: Mean probability of misclassification (as percentages) for Setup D1, $M = 50$, $n_1 = 50$, $n_2 = 30$, $n_3 = 70$ and $L = 3$ groups.

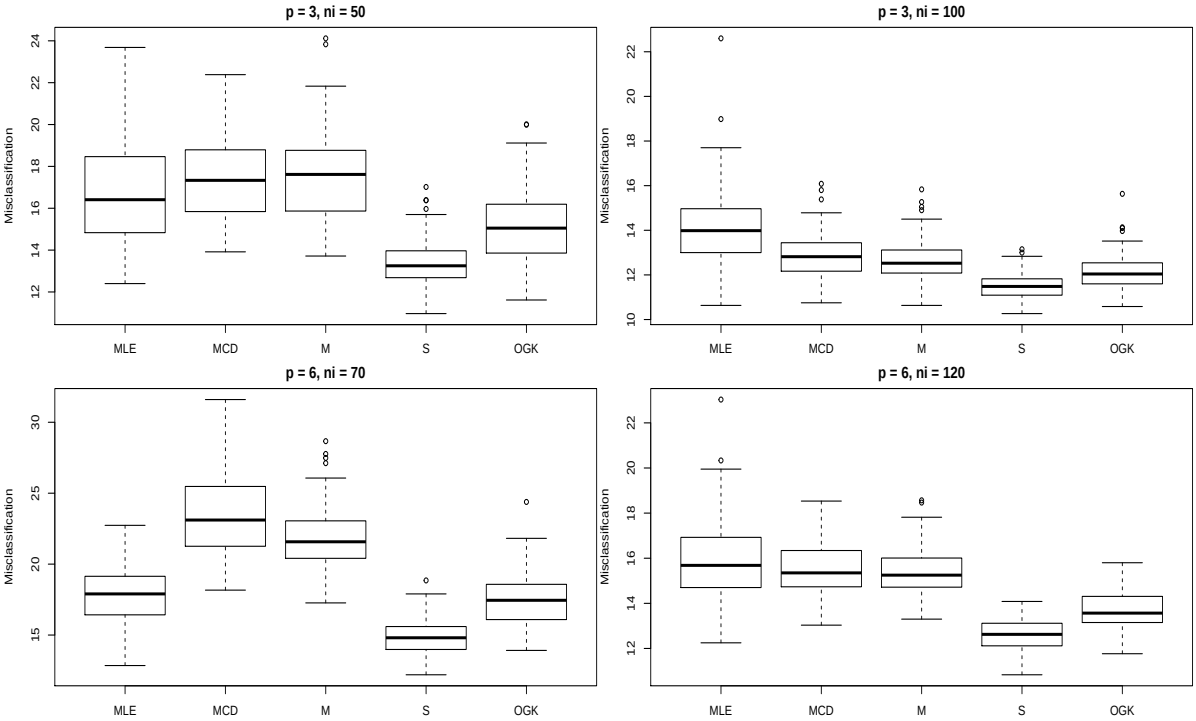

Figure 10: Mean probability of misclassification (as percentages) for Setup D1 with 40% multivariate t_3-distribution contamination, $M = 100$ and $L = 3$ groups.

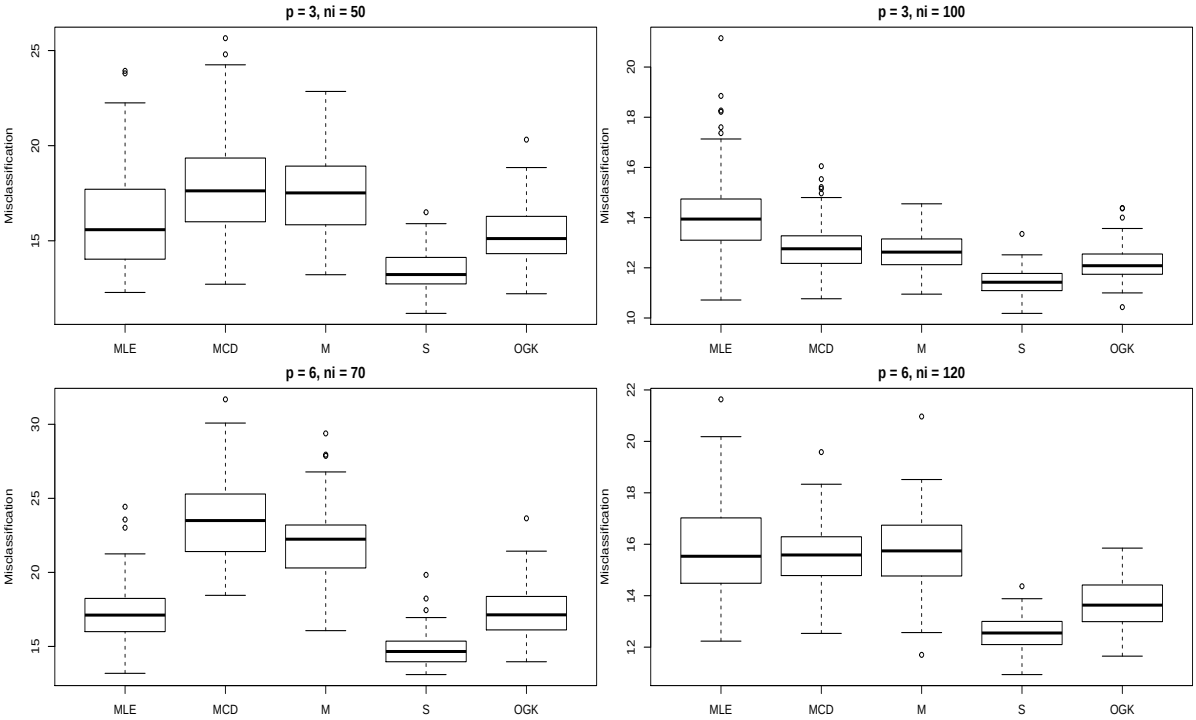

Figure 11: Mean probability of misclassification (as percentages) for Setup D1 with 40% multivariate t_3-distribution contamination, $M = 150$ and $L = 3$ groups.

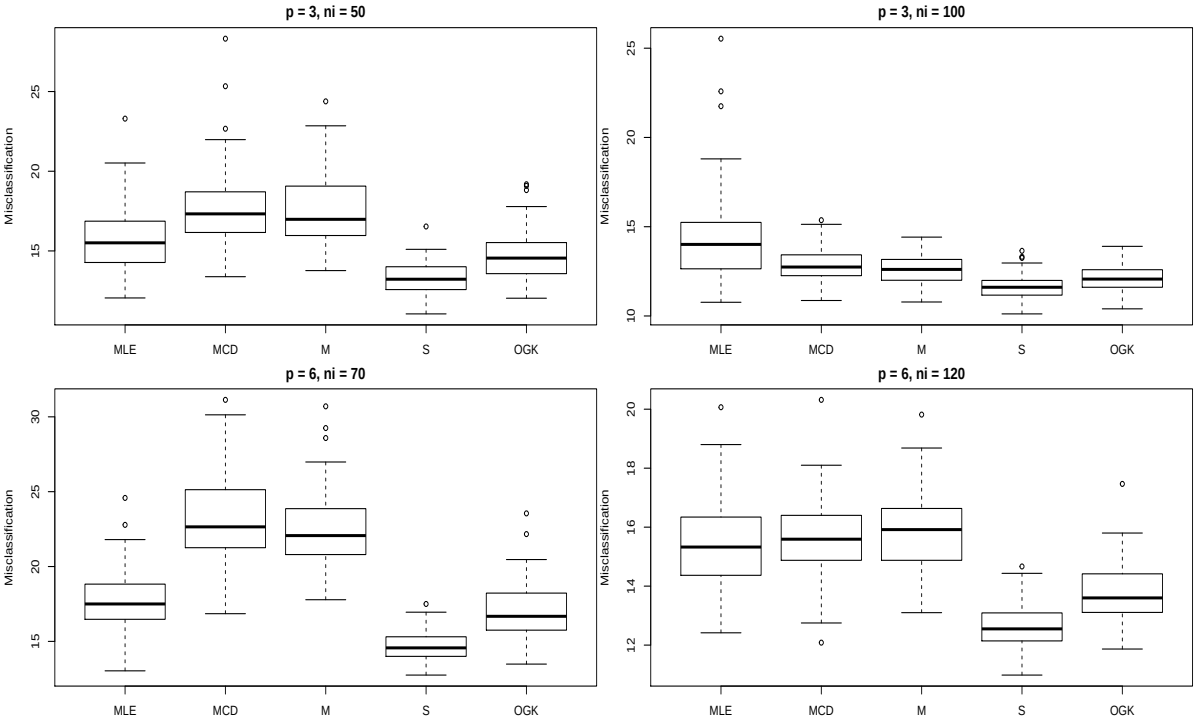

Figure 12: Mean probability of misclassification (as percentages) for Setup D1 with 40% multivariate t_3-distribution contamination, $M = 50$ unequally spaced design time points and $L = 3$ groups.

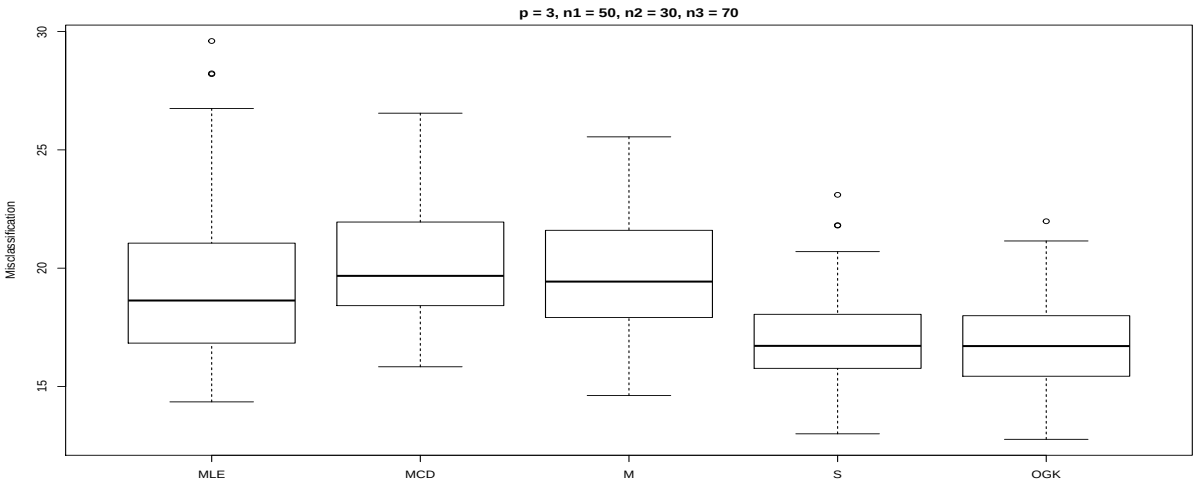

Figure 13: Mean probability of misclassification (as percentages) for Setup D1 with 40% multivariate t_3-distribution contamination, $M = 50$, $n_1 = 50$, $n_2 = 30$, $n_3 = 70$ and $L = 3$ groups.

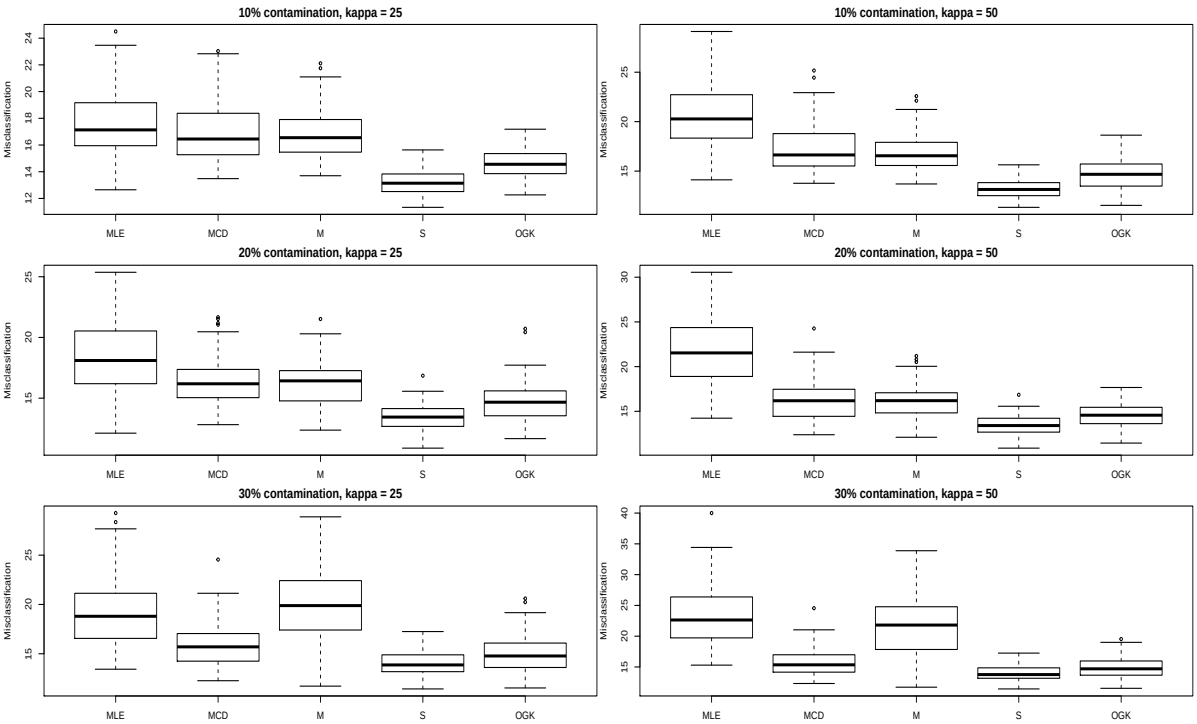

Figure 14: Mean probability of misclassification (as percentages) for Setup D1 with scale contamination and $p = 3$, $M = 100$, $n_i = 50$, $i = 1, 2, 3$, $L = 3$ groups.

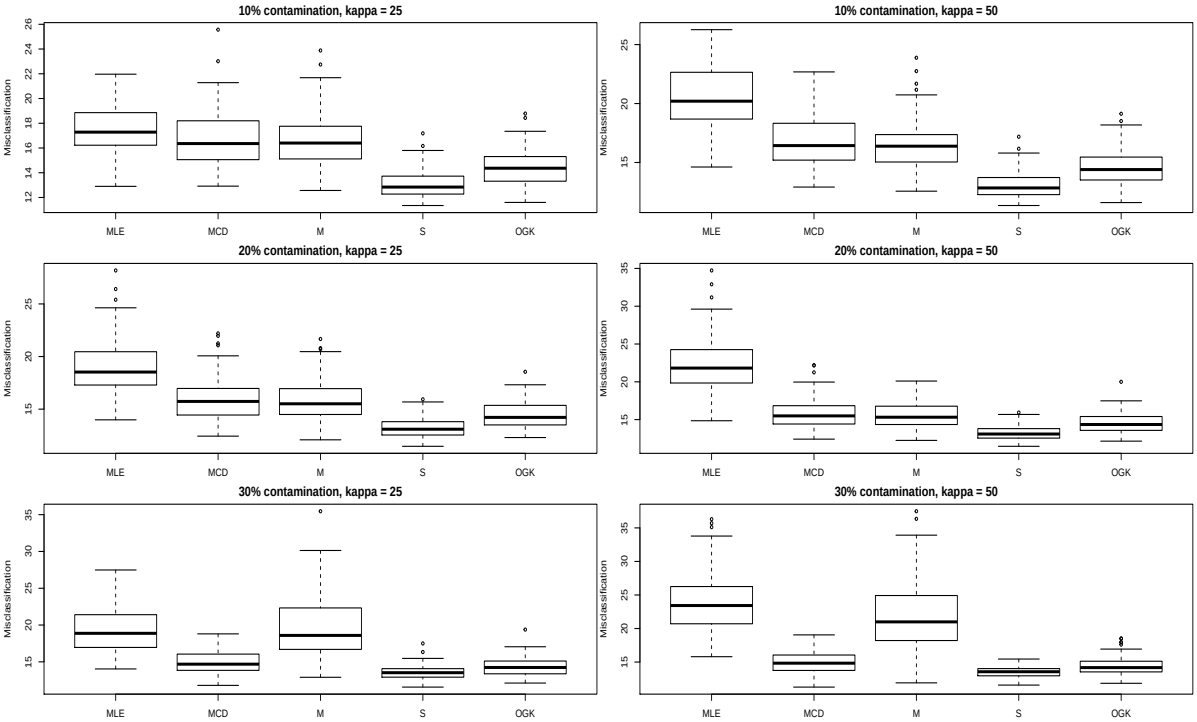

Figure 15: Mean probability of misclassification (as percentages) for Setup D1 with scale contamination and $p = 3$, $M = 150$, $n_i = 50$, $i = 1, 2, 3$, $L = 3$ groups.

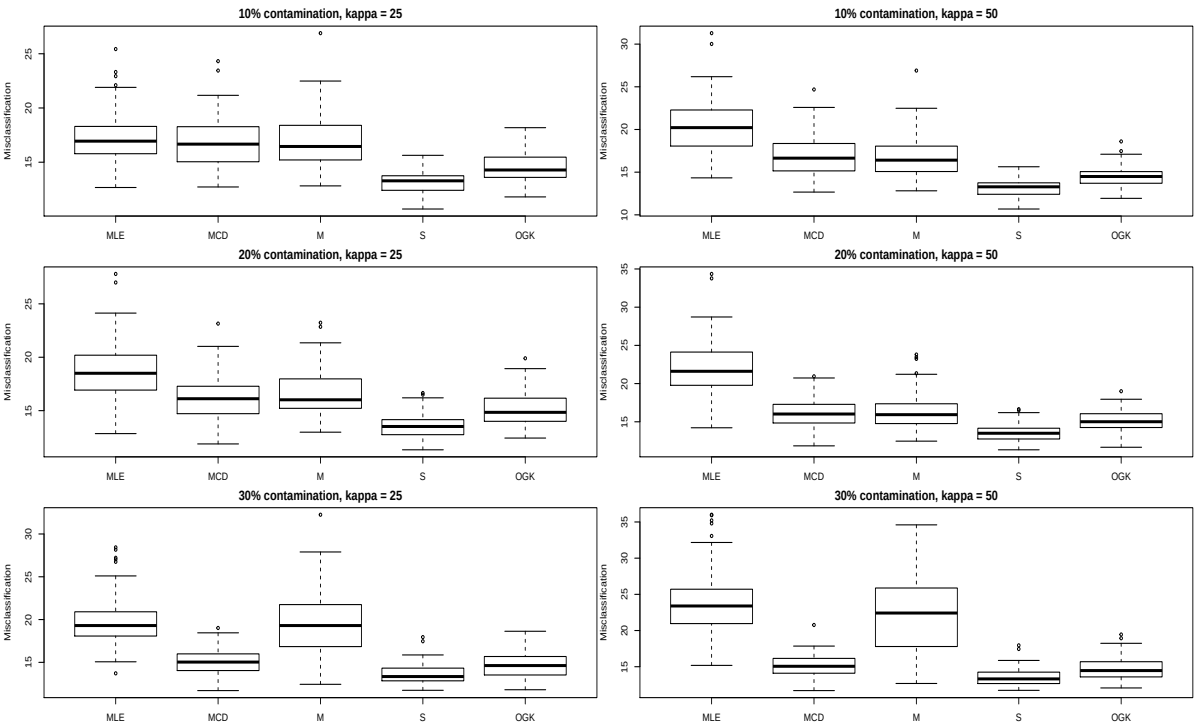

Figure 16: Mean probability of misclassification (as percentages) for Setup D1 with scale contamination and $p = 3$, $M = 50$ unequally spaced design time points, $n_i = 50$, $i = 1, 2, 3$, $L = 3$ groups.

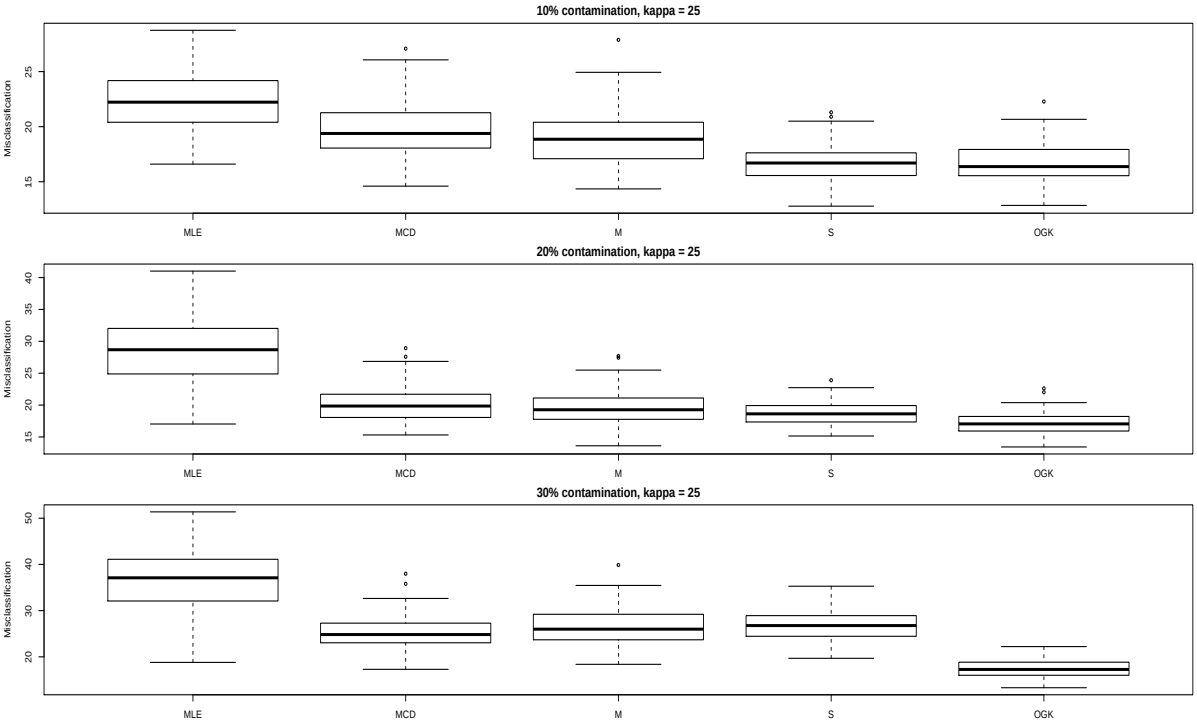

Figure 17: Mean probability of misclassification (as percentages) for Setup D1 with scale contamination and $p = 3$, $M = 50$, $n_1 = 50$, $n_2 = 30$, $n_3 = 70$, $i = 1, 2, 3$, $L = 3$ groups.

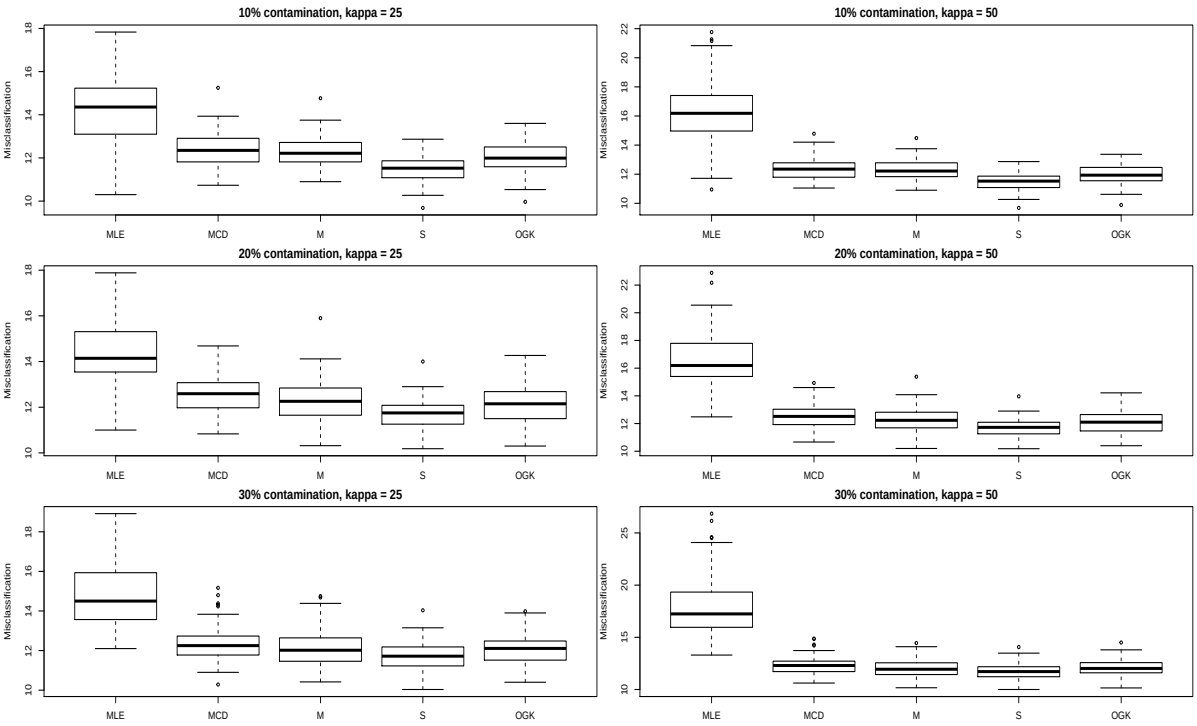

Figure 18: Mean probability of misclassification (as percentages) for Setup D1 with scale contamination and $p = 3$, $M = 50$, $n_i = 100$, $i = 1, 2, 3$, $L = 3$ groups.

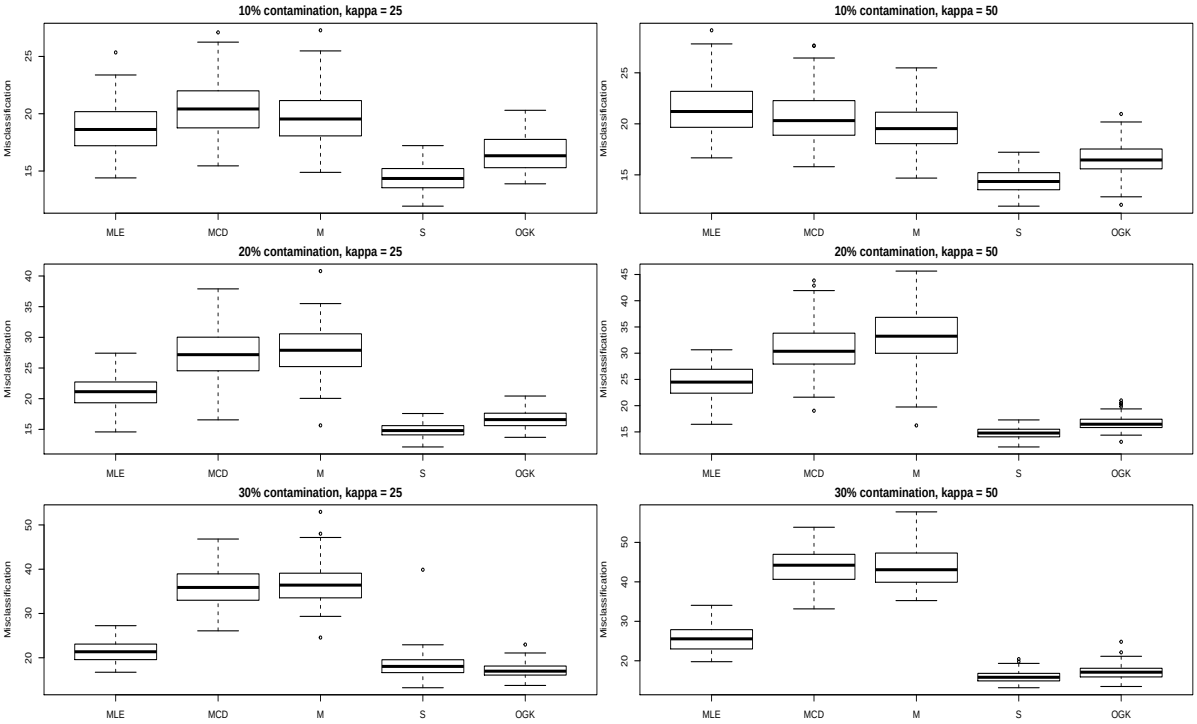

Figure 19: Mean probability of misclassification (as percentages) for Setup D1 with scale contamination and $p = 6$, $M = 50$, $n_i = 70$, $i = 1, 2, 3$, $L = 3$ groups.

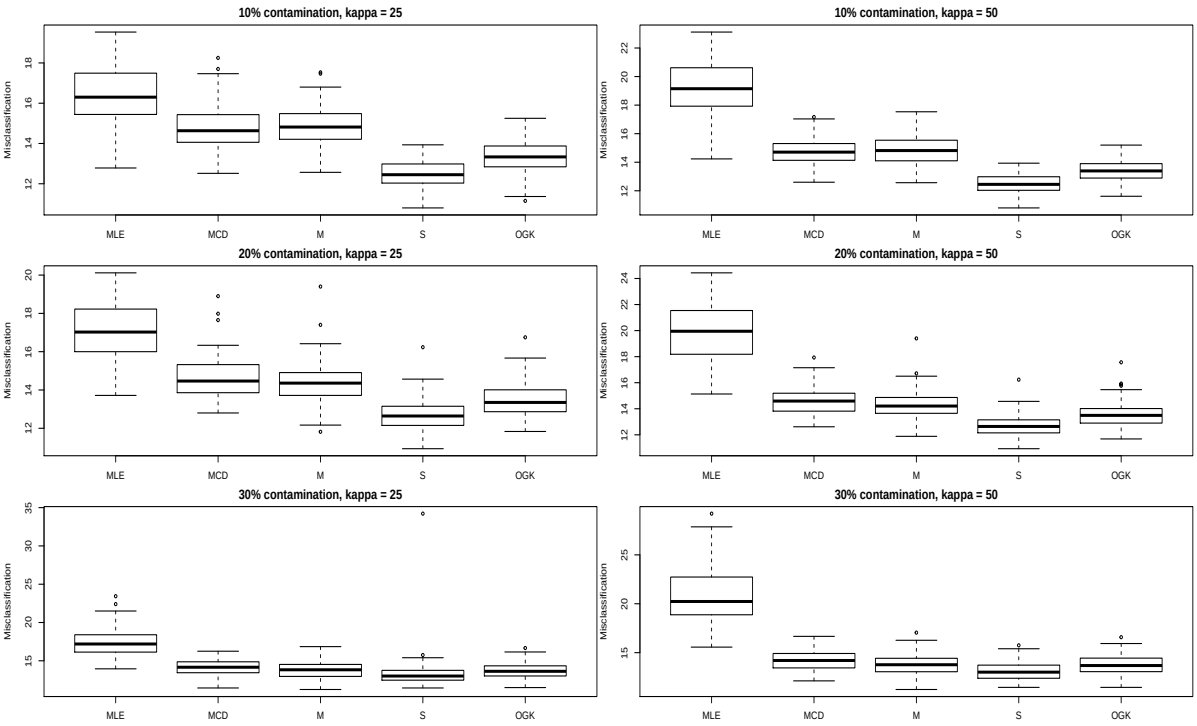

Figure 20: Mean probability of misclassification (as percentages) for Setup D1 with scale contamination and $p = 6$, $M = 50$, $n_i = 120$, $i = 1, 2, 3$, $L = 3$ groups.

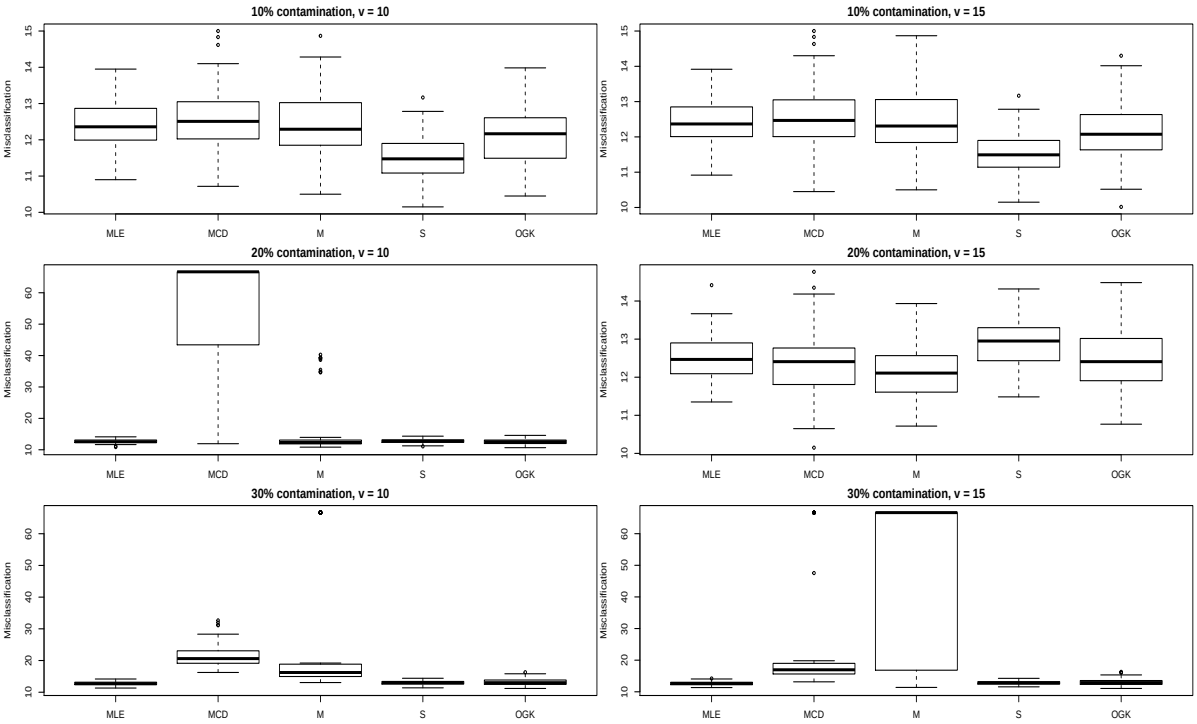

Figure 21: Mean probability of misclassification (as percentages) for Setup D1 with one-direction shift location contamination and $p = 3$, $M = 100$, $n_i = 100$, $i = 1, 2, 3$, $L = 3$ groups.

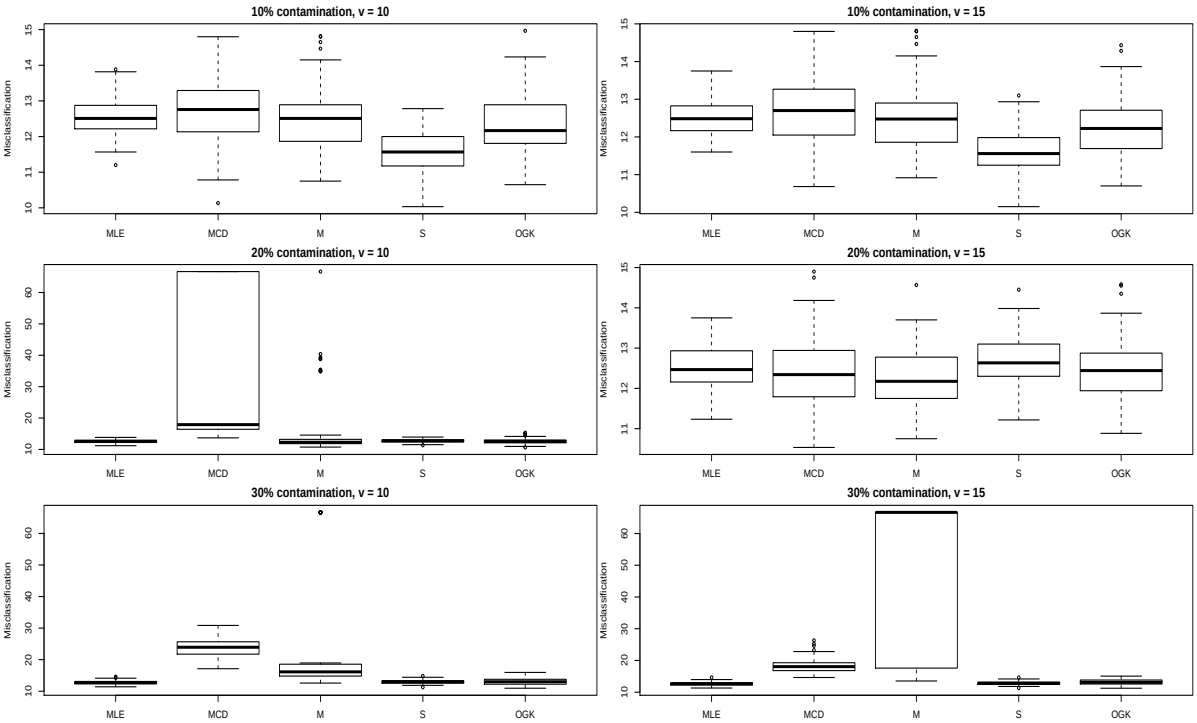

Figure 22: Mean probability of misclassification (as percentages) for Setup D1 with one-direction shift location contamination and $p = 3$, $M = 150$, $n_i = 100$, $i = 1, 2, 3$, $L = 3$ groups.

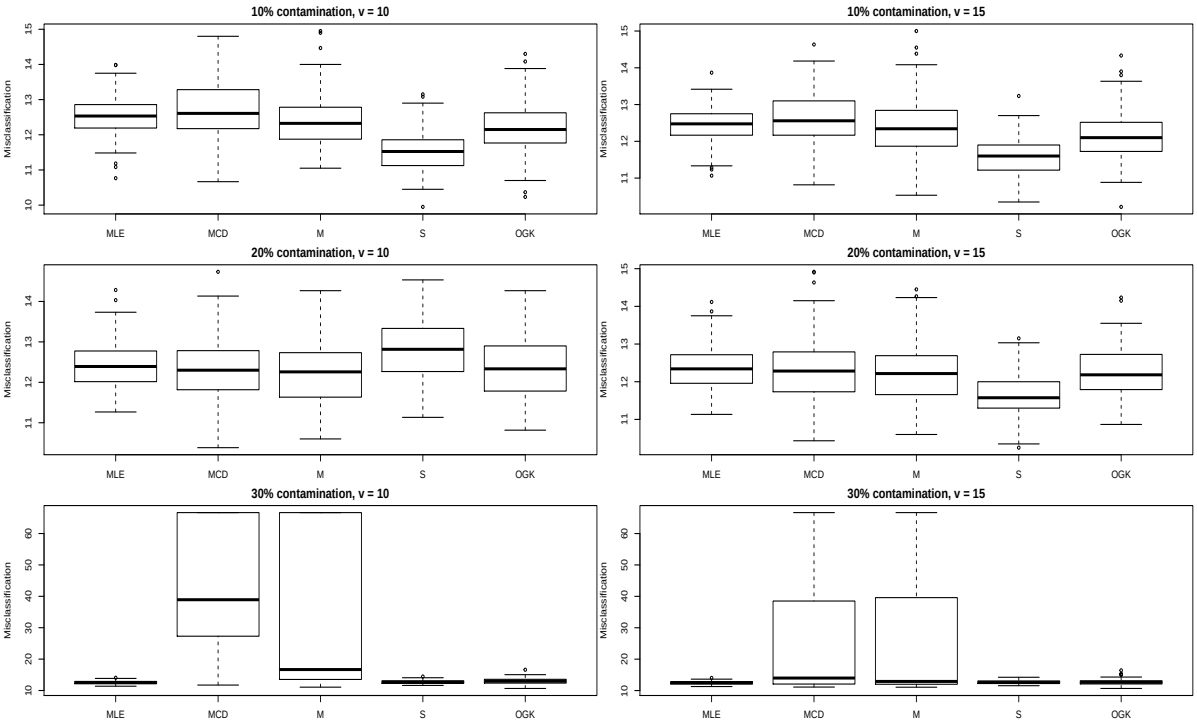

Figure 23: Mean probability of misclassification (as percentages) for Setup D1 with one-direction shift location contamination and $p = 3$, $M = 50$ unequally spaced design time points, $n_i = 100$, $i = 1, 2, 3$, $L = 3$ groups.

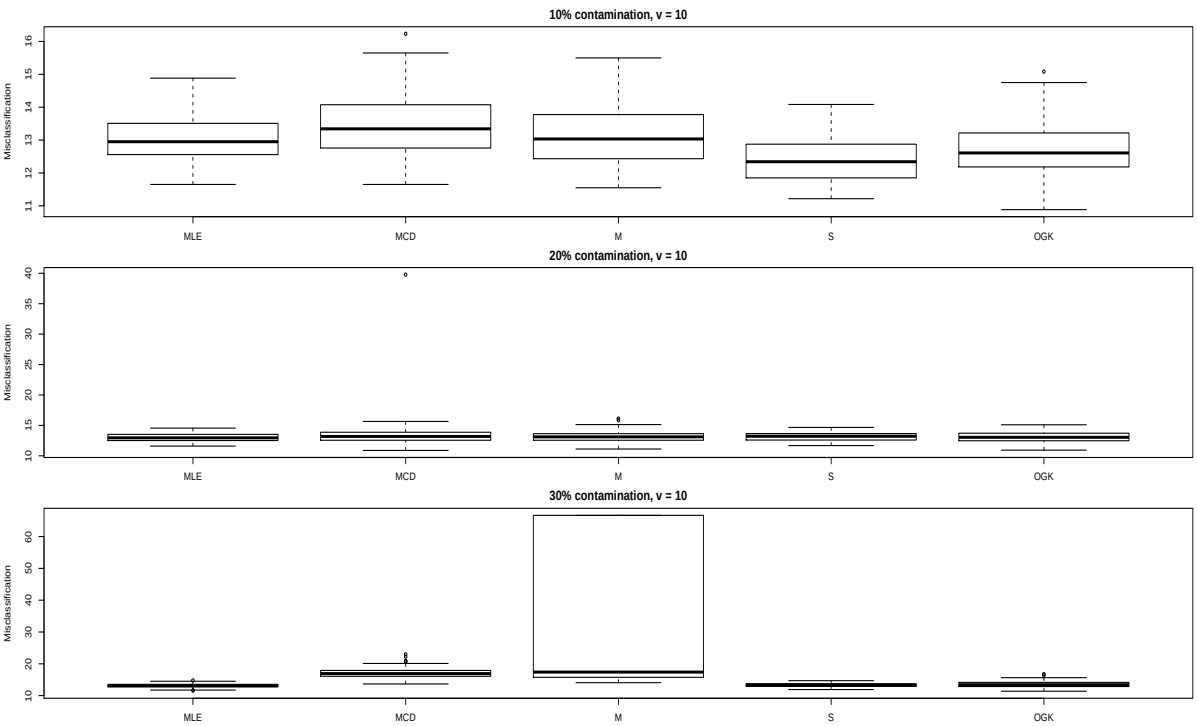

Figure 24: Mean probability of misclassification (as percentages) for Setup D1 with one-direction shift location contamination and $p = 3$, $M = 50$, $n_1 = 100$, $n_2 = 80$, $n_3 = 120$, $i = 1, 2, 3$, $L = 3$ groups.

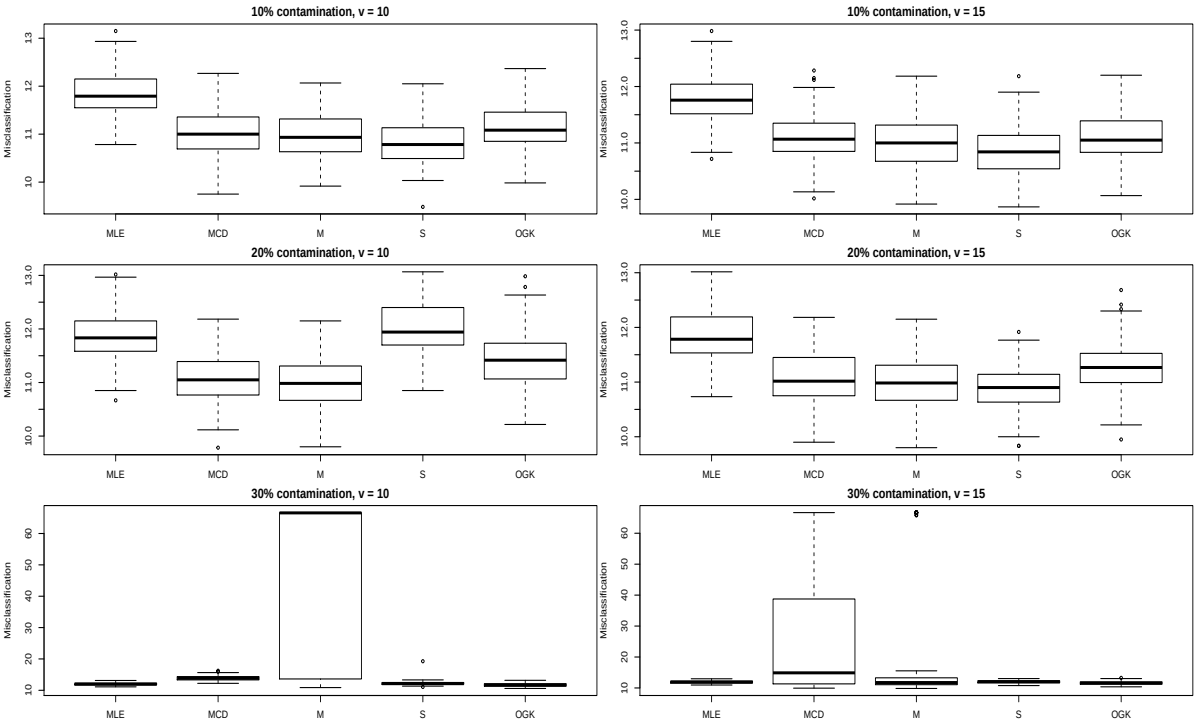

Figure 25: Mean probability of misclassification (as percentages) for Setup D1 with one-direction shift location contamination and $p = 3$, $M = 50$, $n_i = 200$, $i = 1, 2, 3$, $L = 3$ groups.

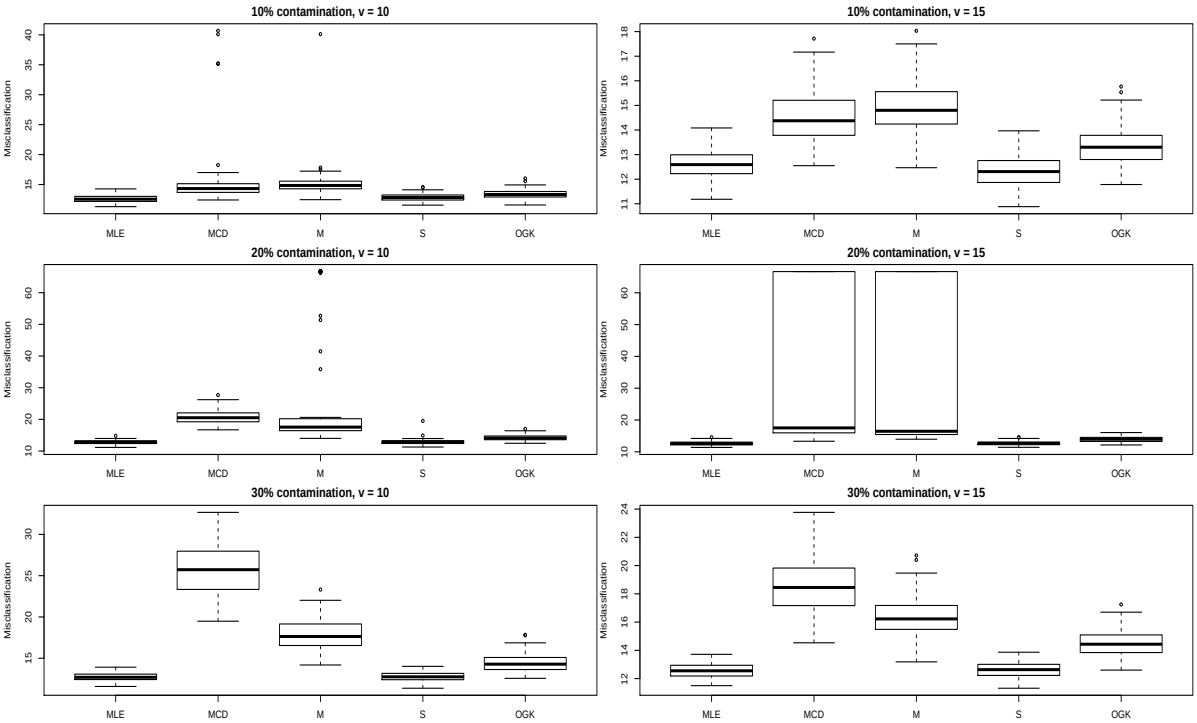

Figure 26: Mean probability of misclassification (as percentages) for Setup D1 with one-direction shift location contamination and $p = 6$, $M = 50$, $n_i = 120$, $i = 1, 2, 3$, $L = 3$ groups.

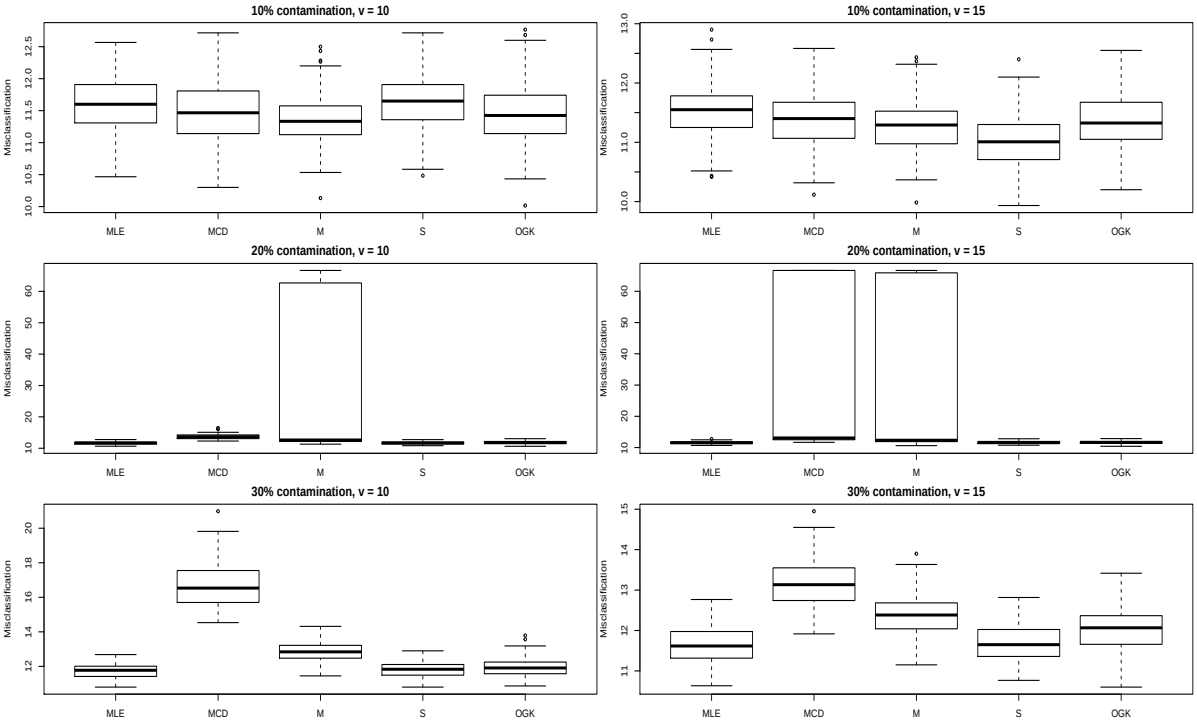

Figure 27: Mean probability of misclassification (as percentages) for Setup D1 with one-direction shift location contamination and $p = 6$, $M = 50$, $n_i = 300$, $i = 1, 2, 3$, $L = 3$ groups.

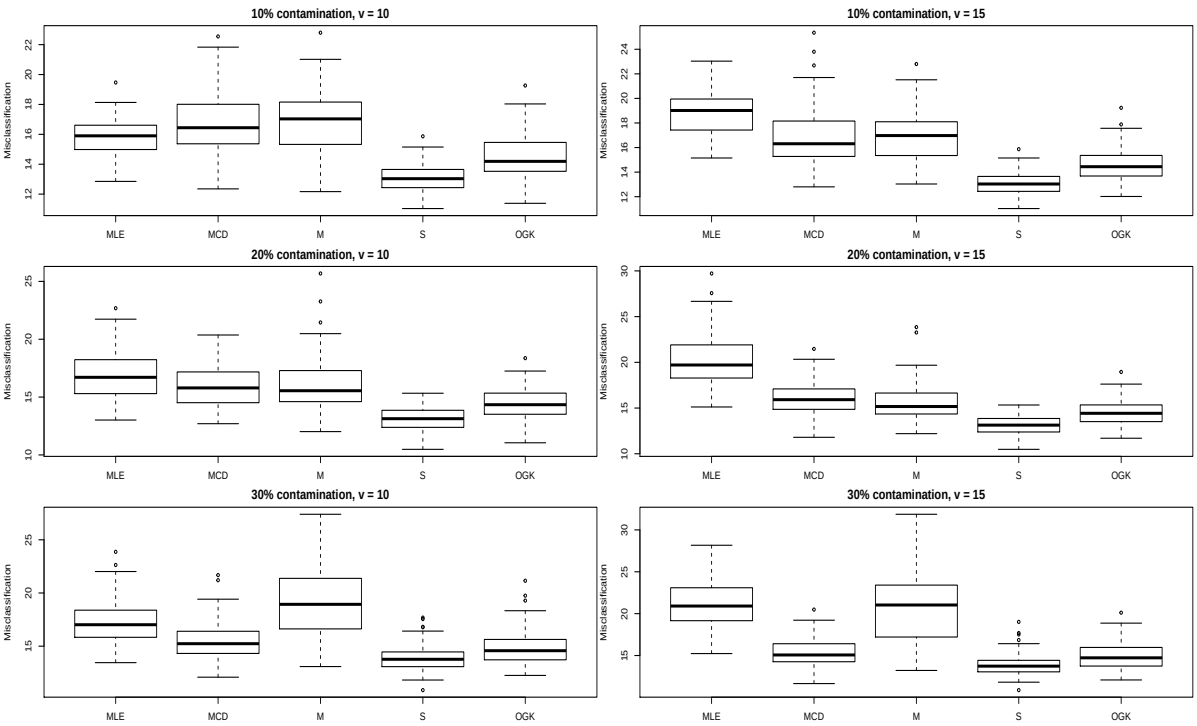

Figure 28: Mean probability of misclassification (as percentages) for Setup D1 with radial location contamination and $p = 3$, $M = 100$, $n_i = 50$, $i = 1, 2, 3$, $L = 3$ groups.

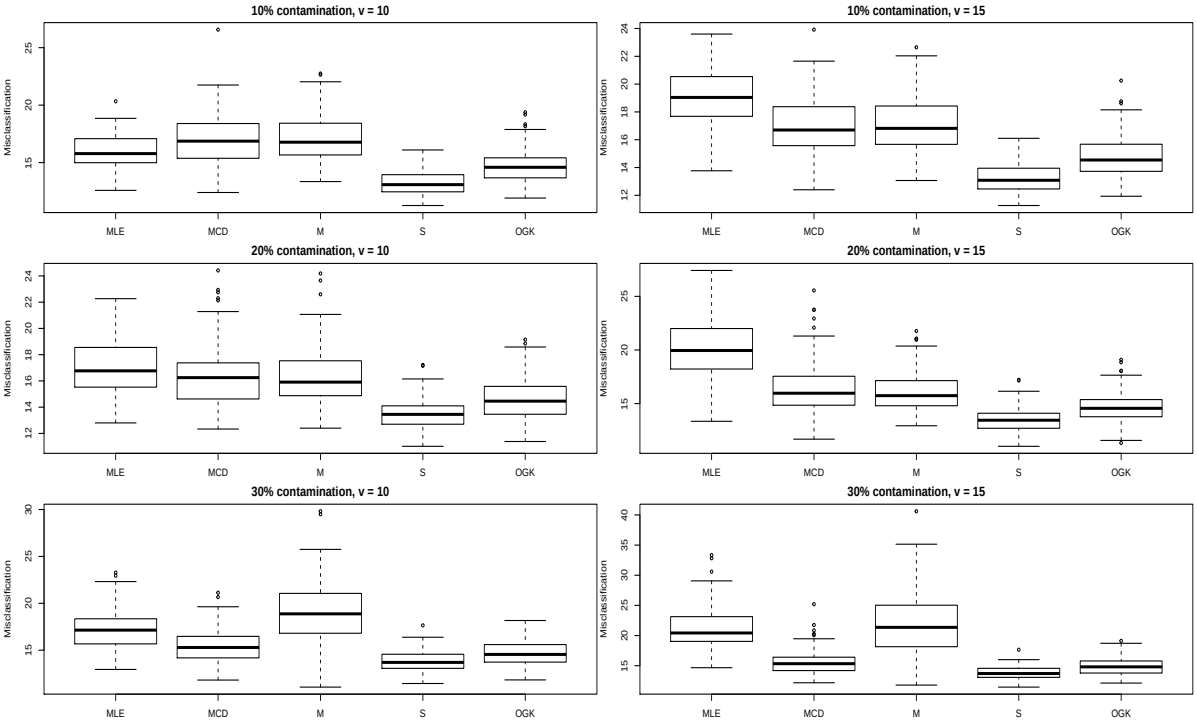

Figure 29: Mean probability of misclassification (as percentages) for Setup D1 with radial location contamination and $p = 3$, $M = 150$, $n_i = 50$, $i = 1, 2, 3$, $L = 3$ groups.

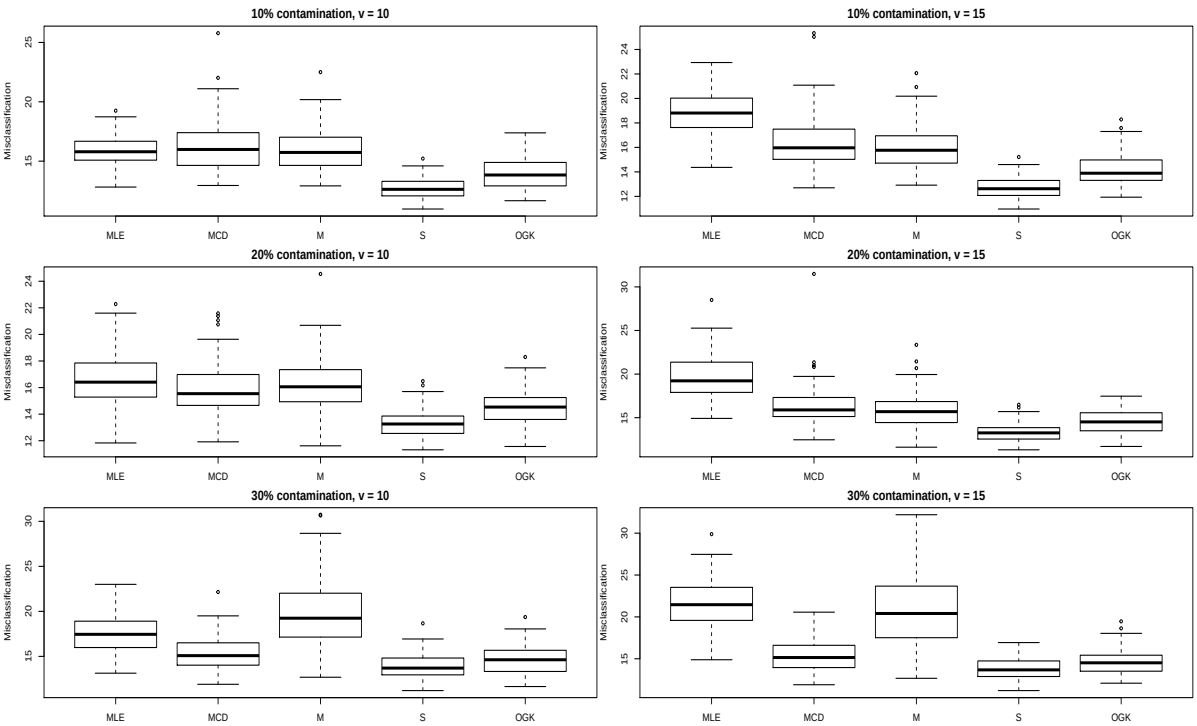

Figure 30: Mean probability of misclassification (as percentages) for Setup D1 with radial location contamination and $p = 3$, $M = 50$ unequally spaced design time points, $n_i = 50$, $i = 1, 2, 3$, $L = 3$ groups.

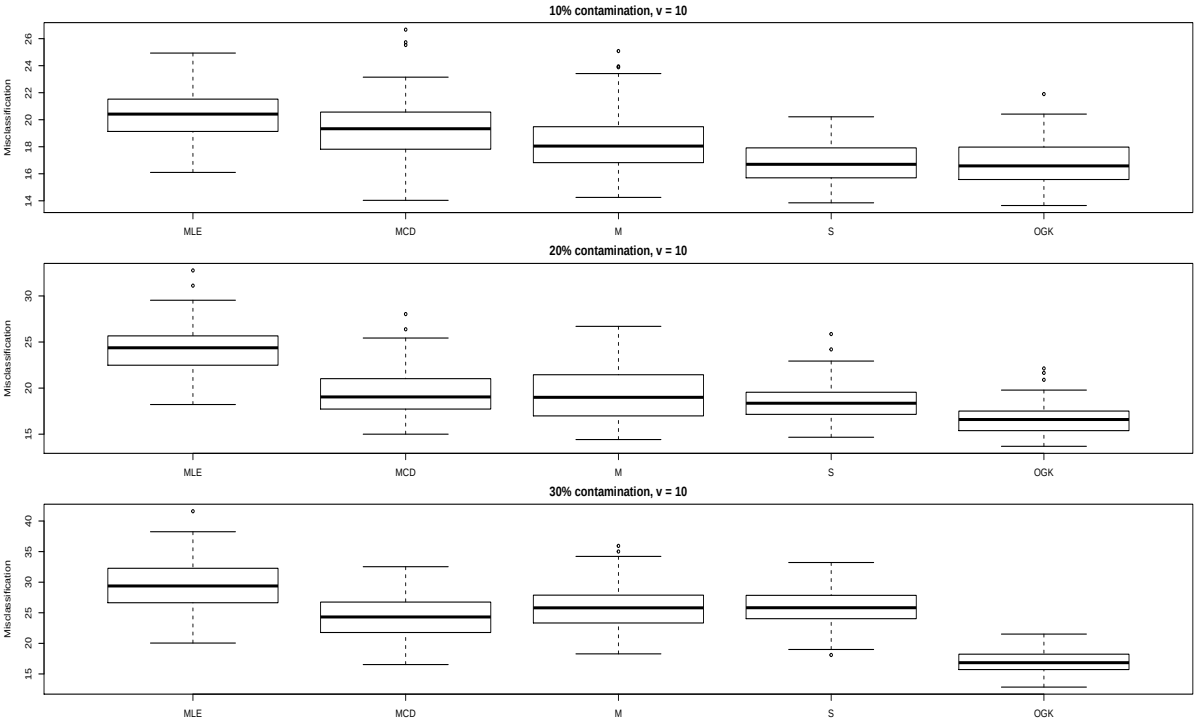

Figure 31: Mean probability of misclassification (as percentages) for Setup D1 with radial location contamination and $p = 3$, $M = 50$, $n_1 = 50$, $n_2 = 30$, $n_3 = 70$, $i = 1, 2, 3$, $L = 3$ groups.

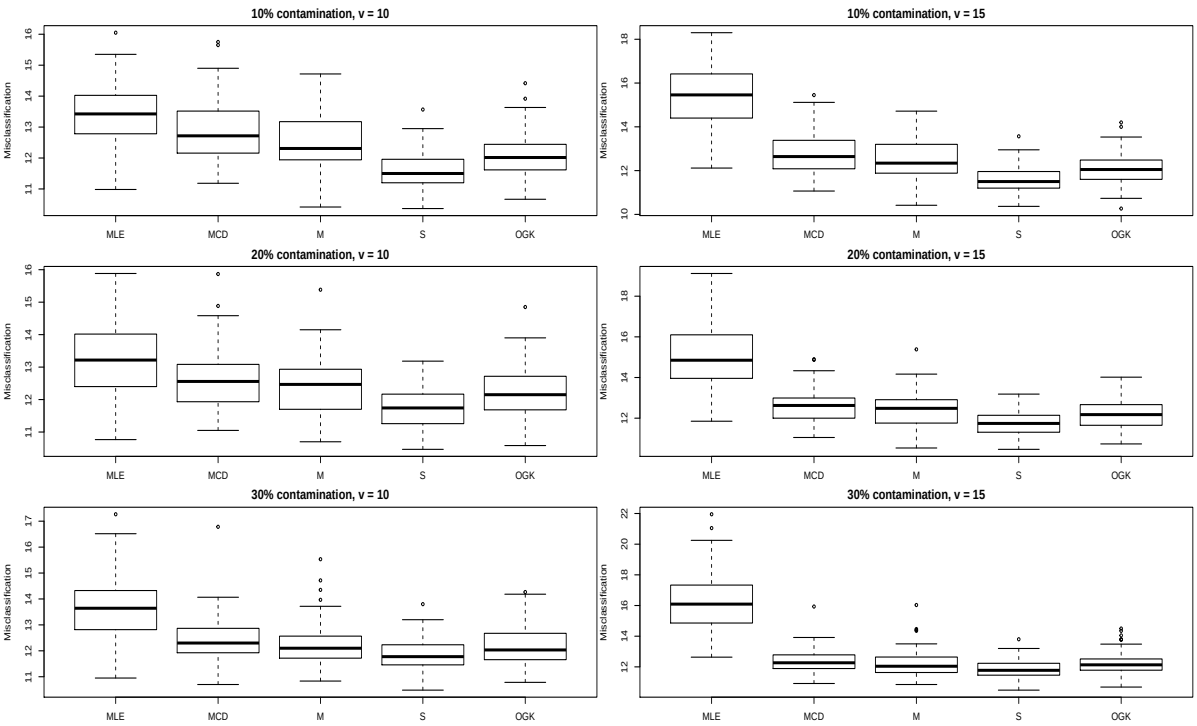

Figure 32: Mean probability of misclassification (as percentages) for Setup D1 with radial location contamination and $p = 3$, $M = 50$, $n_i = 100$, $i = 1, 2, 3$, $L = 3$ groups.

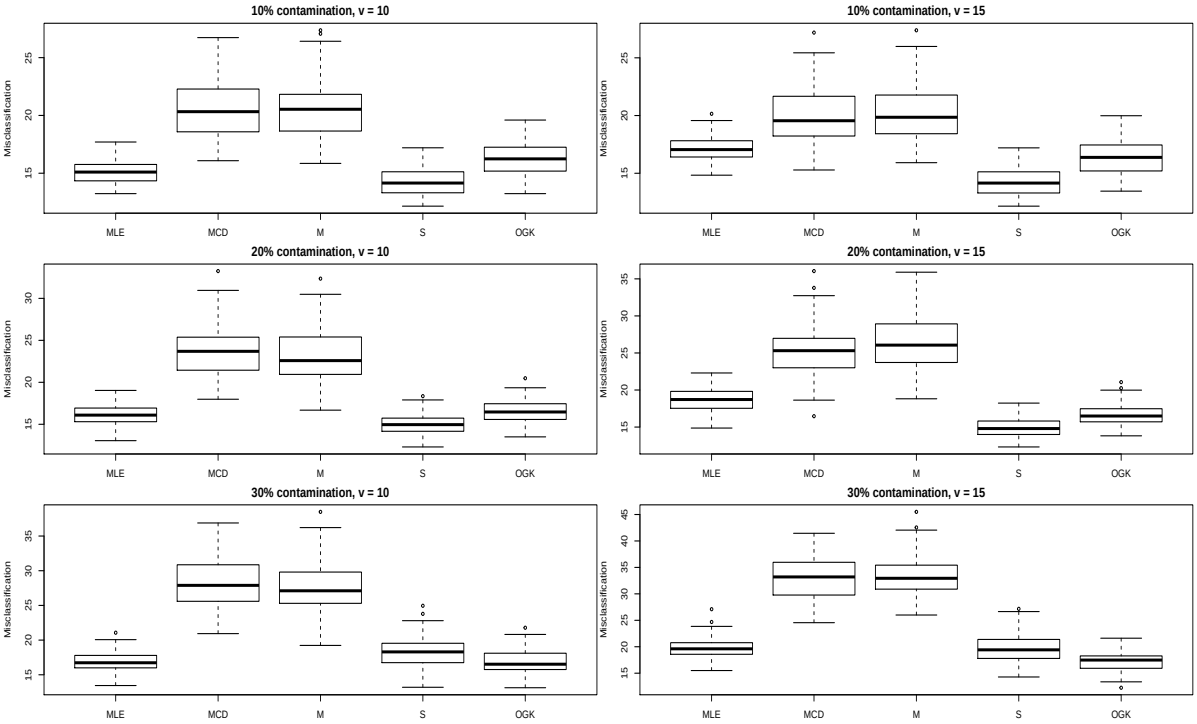

Figure 33: Mean probability of misclassification (as percentages) for Setup D1 with radial location contamination and $p = 6$, $M = 50$, $n_i = 70$, $i = 1, 2, 3$, $L = 3$ groups.

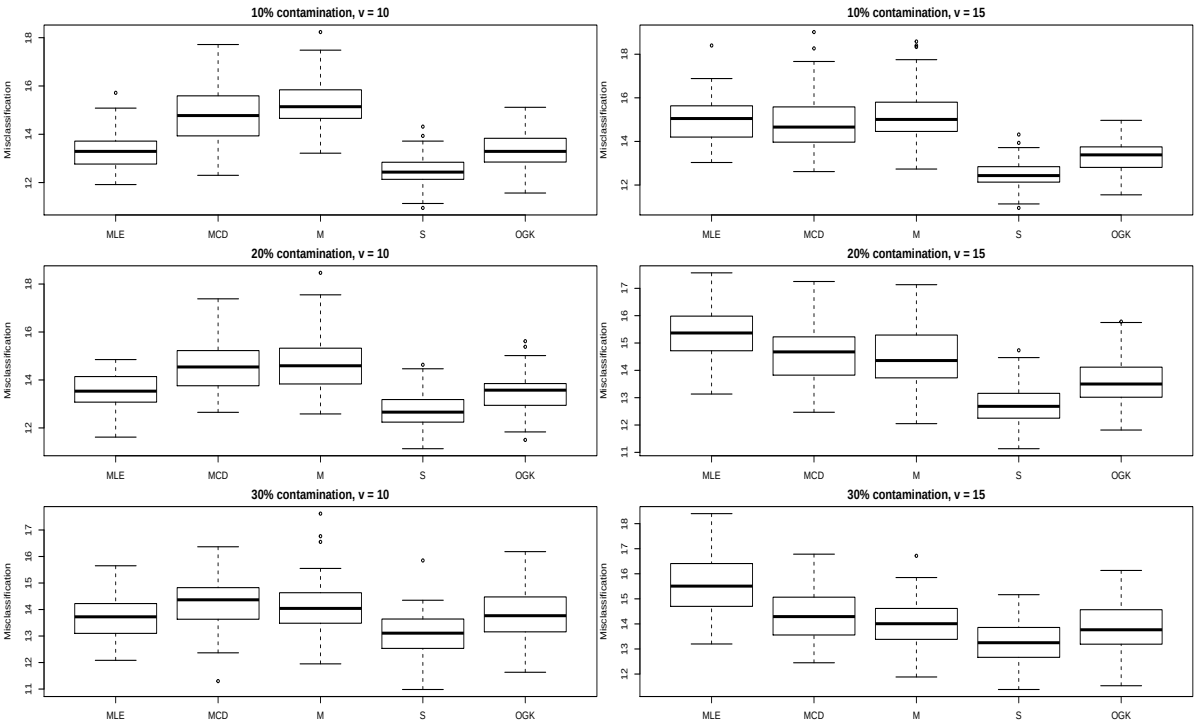

Figure 34: Mean probability of misclassification (as percentages) for Setup D1 with radial location contamination and $p = 6$, $M = 50$, $n_i = 120$, $i = 1, 2, 3$, $L = 3$ groups.

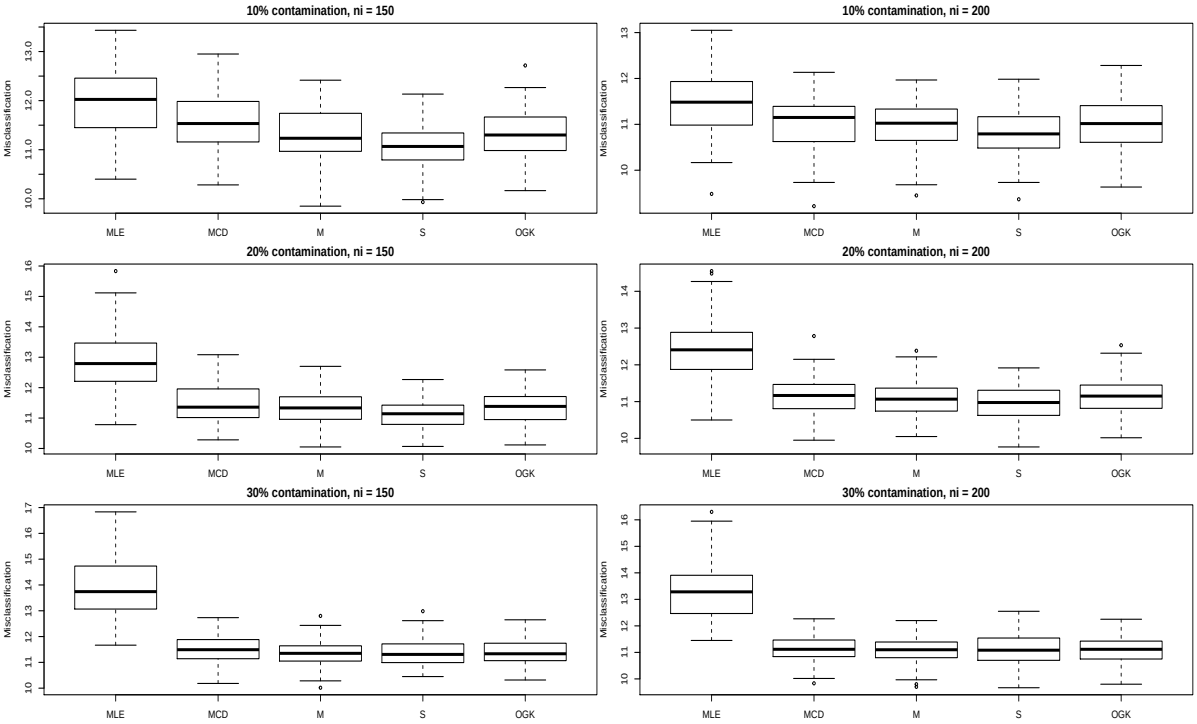

Figure 35: Mean probability of misclassification (as percentages) for Setup D1 with isolated outliers and $p = 3$, $M = 50$, $L = 3$ groups.

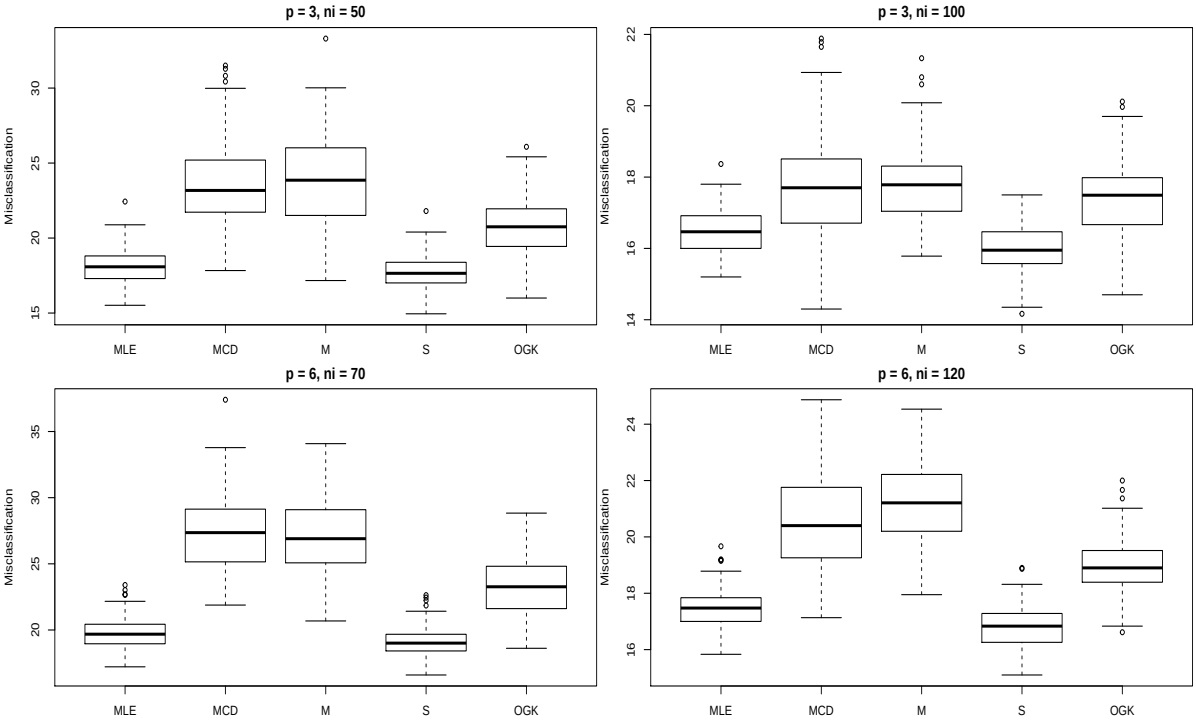

Figure 36: Mean probability of misclassification (as percentages) for Setup D2, $M = 50$ and $L = 3$ groups.

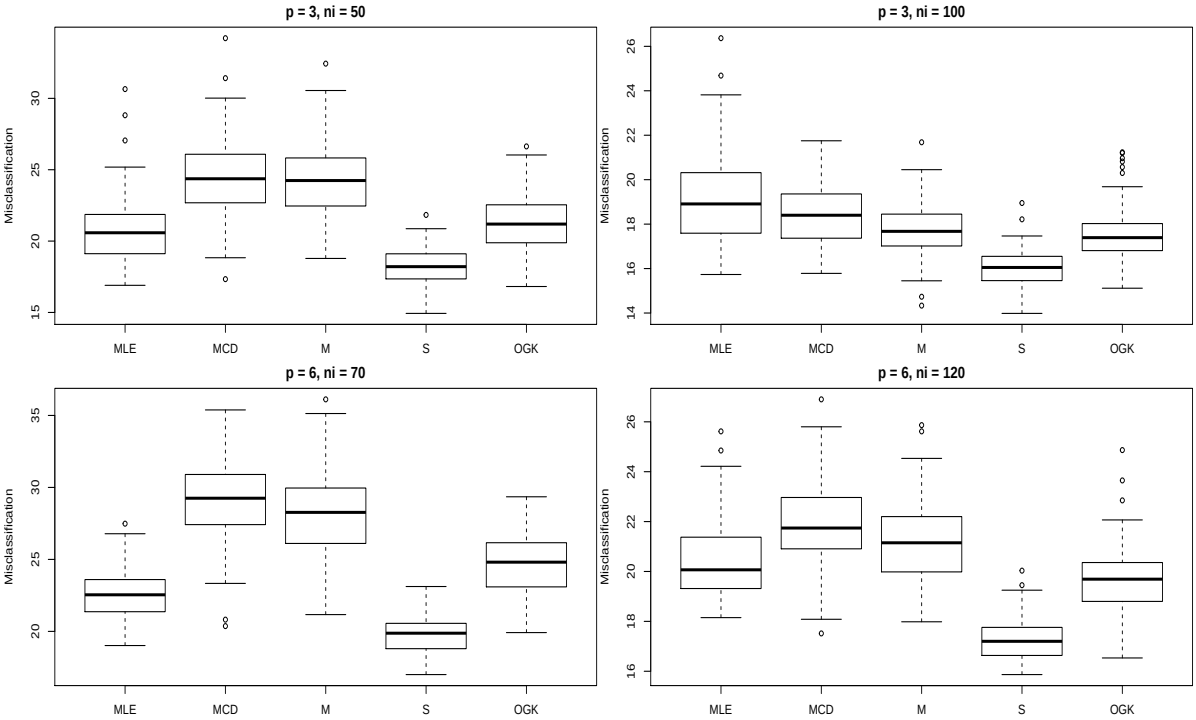

Figure 37: Mean probability of misclassification (as percentages) for Setup D2 with 40% multivariate t_3-distribution contamination, $M = 50$ and $L = 3$ groups.

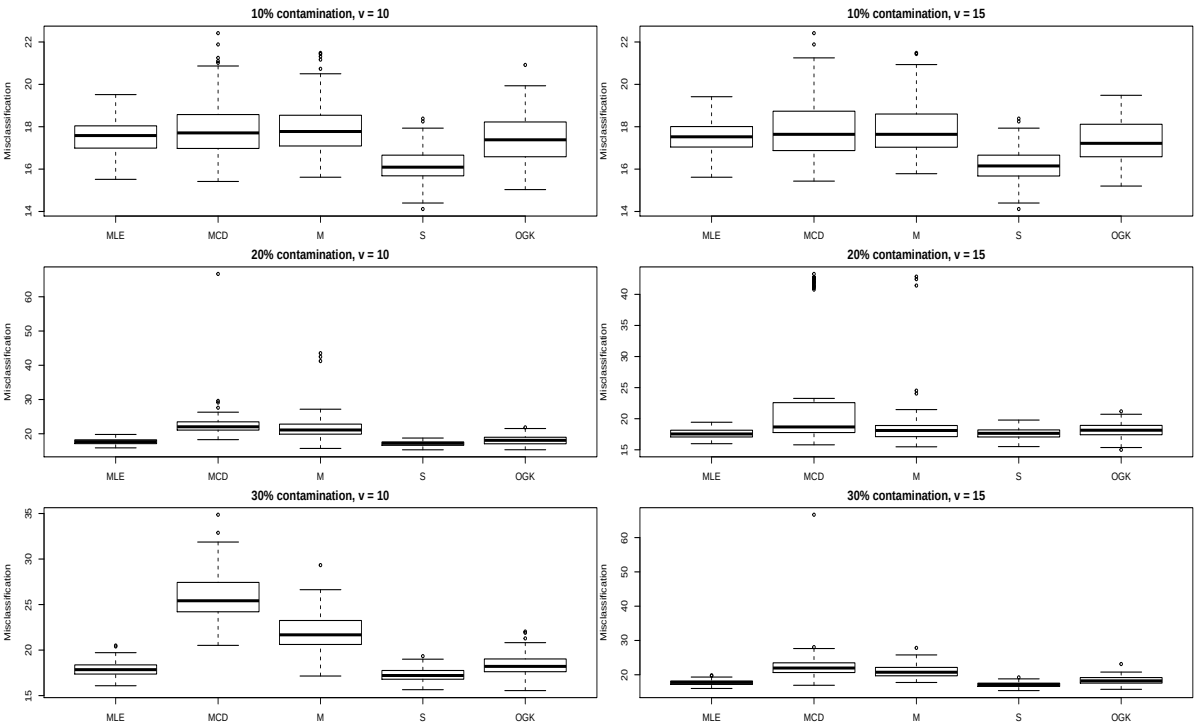

Figure 38: Mean probability of misclassification (as percentages) for Setup D2 with one-direction shift location contamination and $p = 3$, $M = 50$, $n_i = 100$, $i = 1, 2, 3$, $L = 3$ groups.

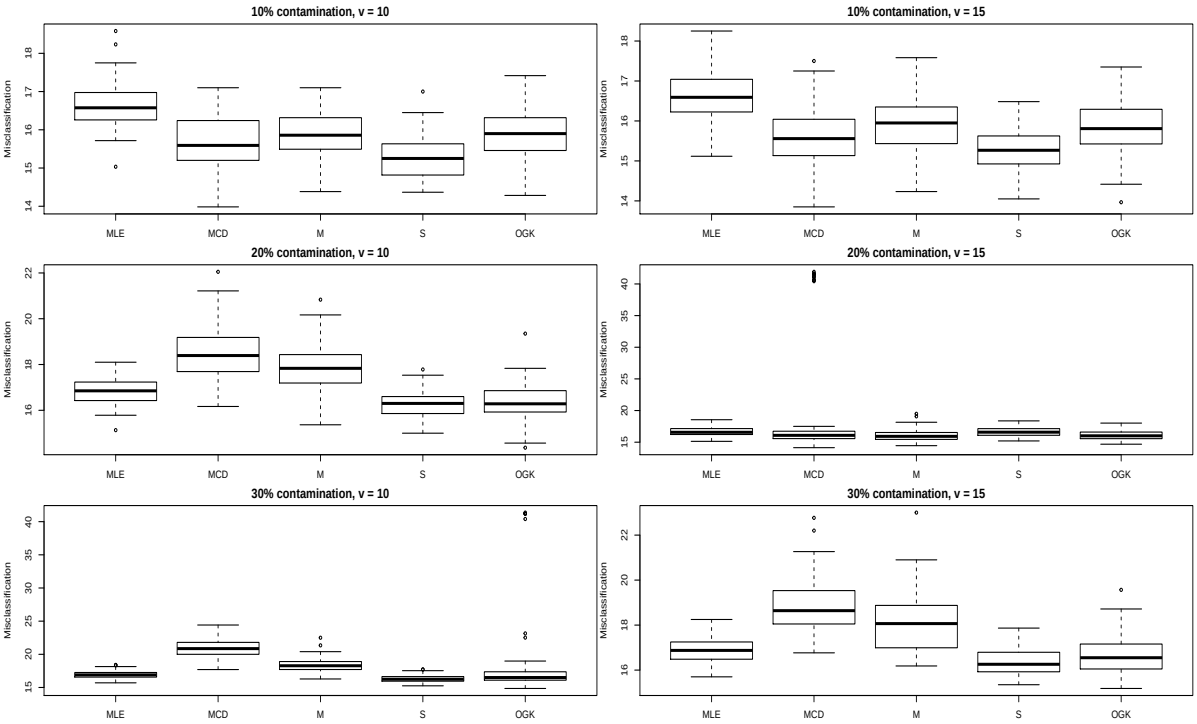

Figure 39: Mean probability of misclassification (as percentages) for Setup D2 with one-direction shift location contamination and $p = 3$, $M = 50$, $n_i = 200$, $i = 1, 2, 3$, $L = 3$ groups.

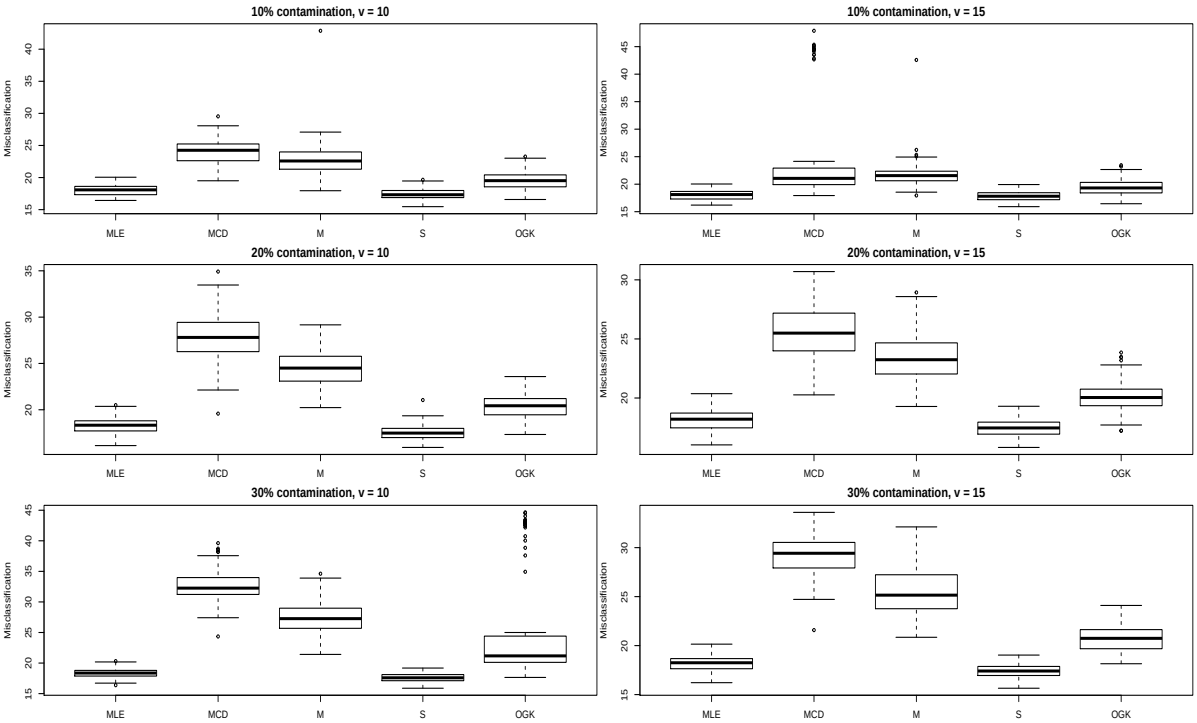

Figure 40: Mean probability of misclassification (as percentages) for Setup D2 with one-direction shift location contamination and $p = 6$, $M = 50$, $n_i = 120$, $i = 1, 2, 3$, $L = 3$ groups.

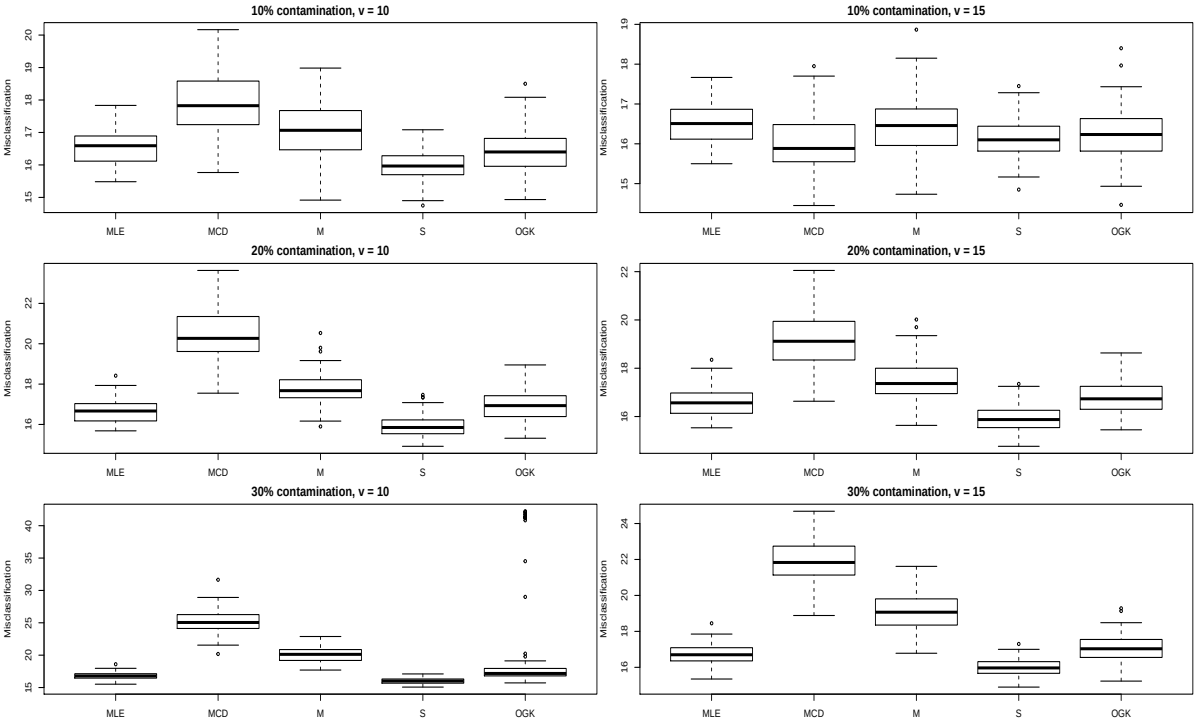

Figure 41: Mean probability of misclassification (as percentages) for Setup D2 with one-direction shift location contamination and $p = 6$, $M = 50$, $n_i = 300$, $i = 1, 2, 3$, $L = 3$ groups.

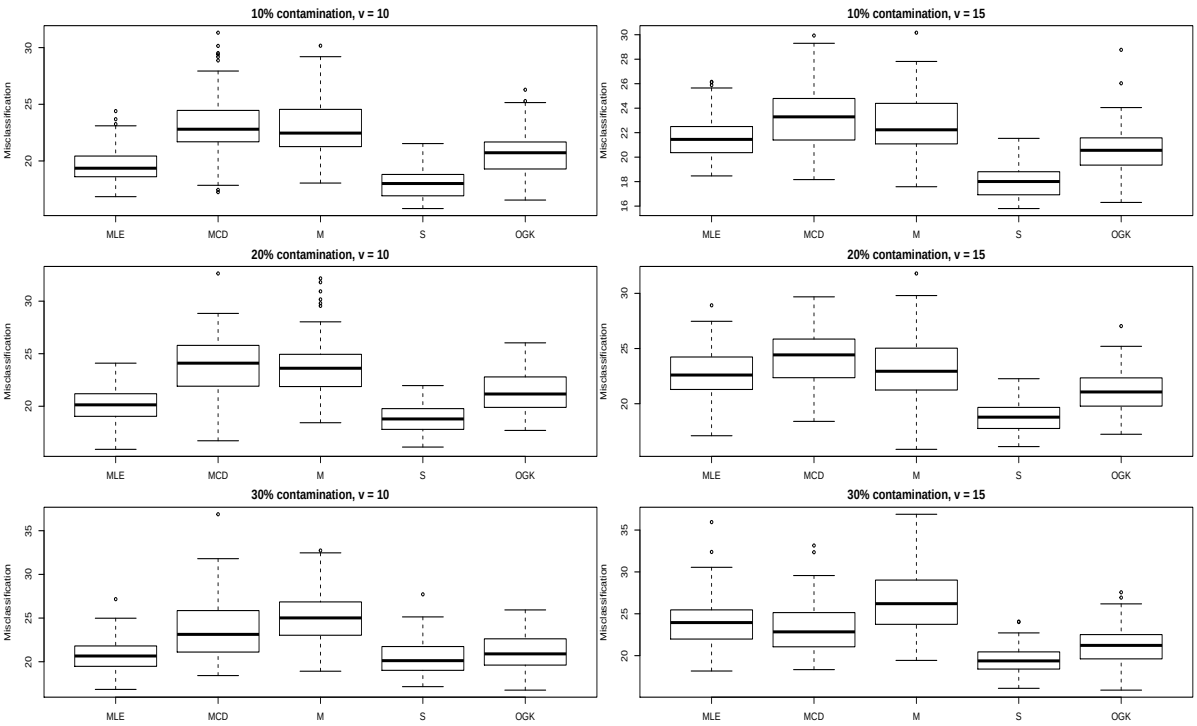

Figure 42: Mean probability of misclassification (as percentages) for Setup D2 with radial location contamination and $p = 3$, $M = 50$, $n_i = 50$, $i = 1, 2, 3$, $L = 3$ groups.

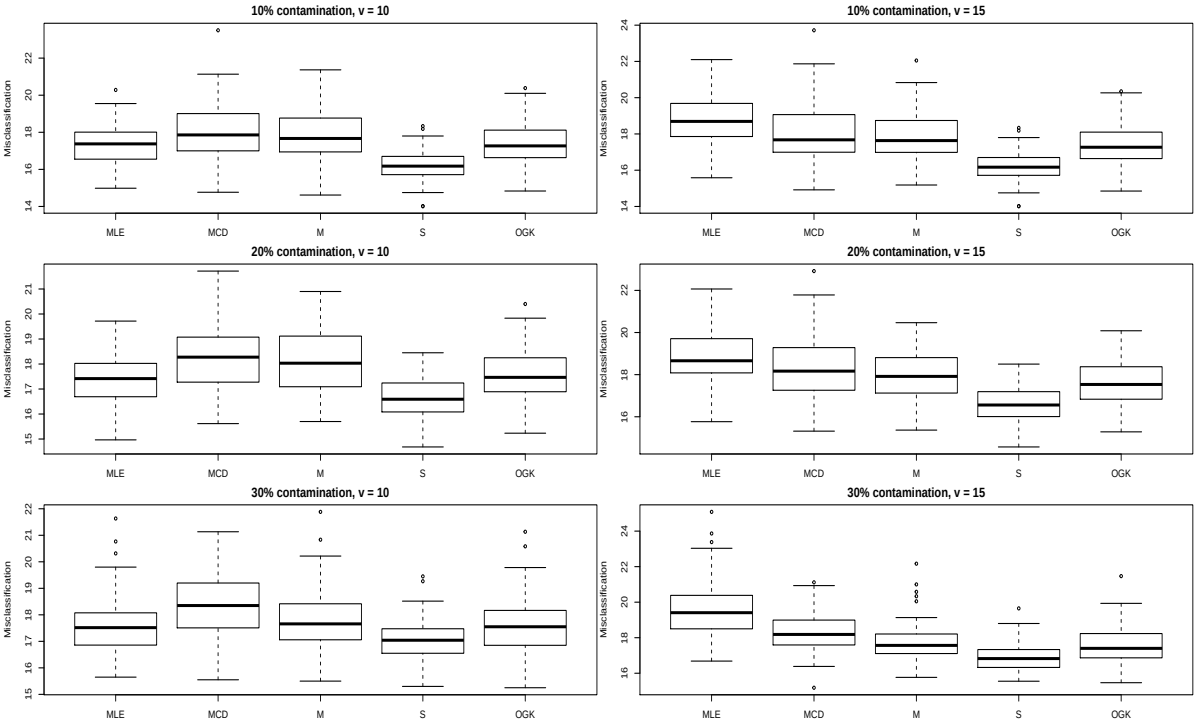

Figure 43: Mean probability of misclassification (as percentages) for Setup D2 with radial location contamination and $p = 3$, $M = 50$, $n_i = 100$, $i = 1, 2, 3$, $L = 3$ groups.

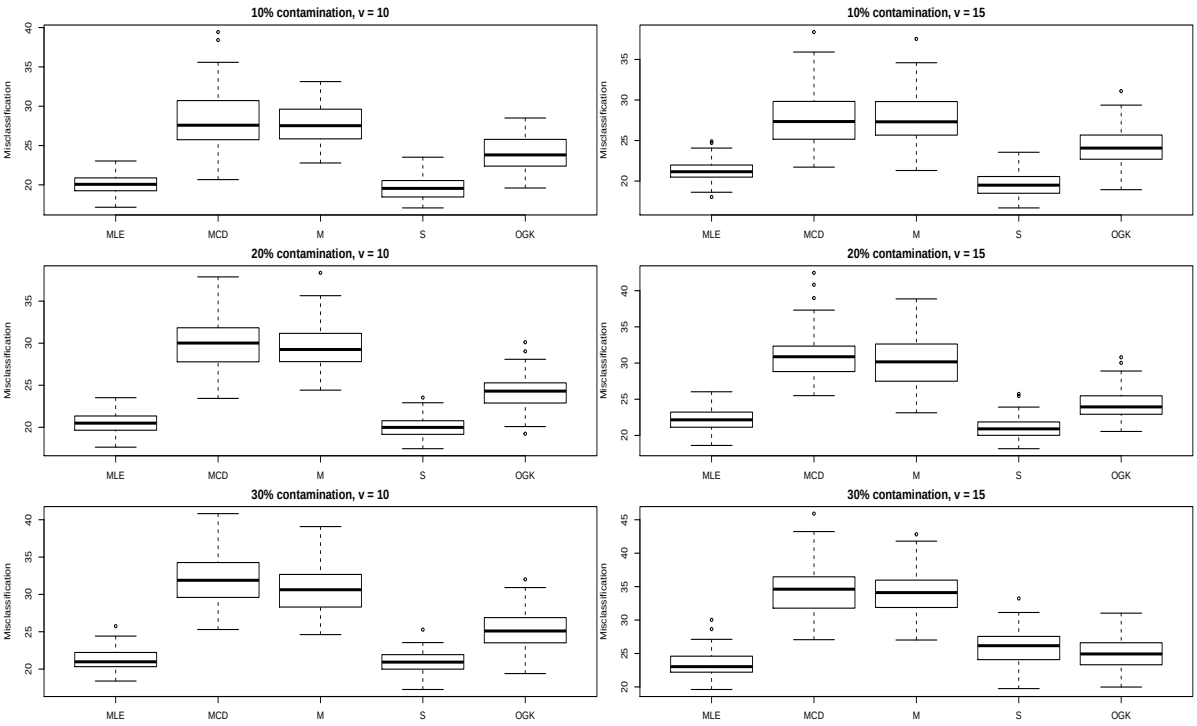

Figure 44: Mean probability of misclassification (as percentages) for Setup D2 with radial location contamination and $p = 6$, $M = 50$, $n_i = 70$, $i = 1, 2, 3$, $L = 3$ groups.

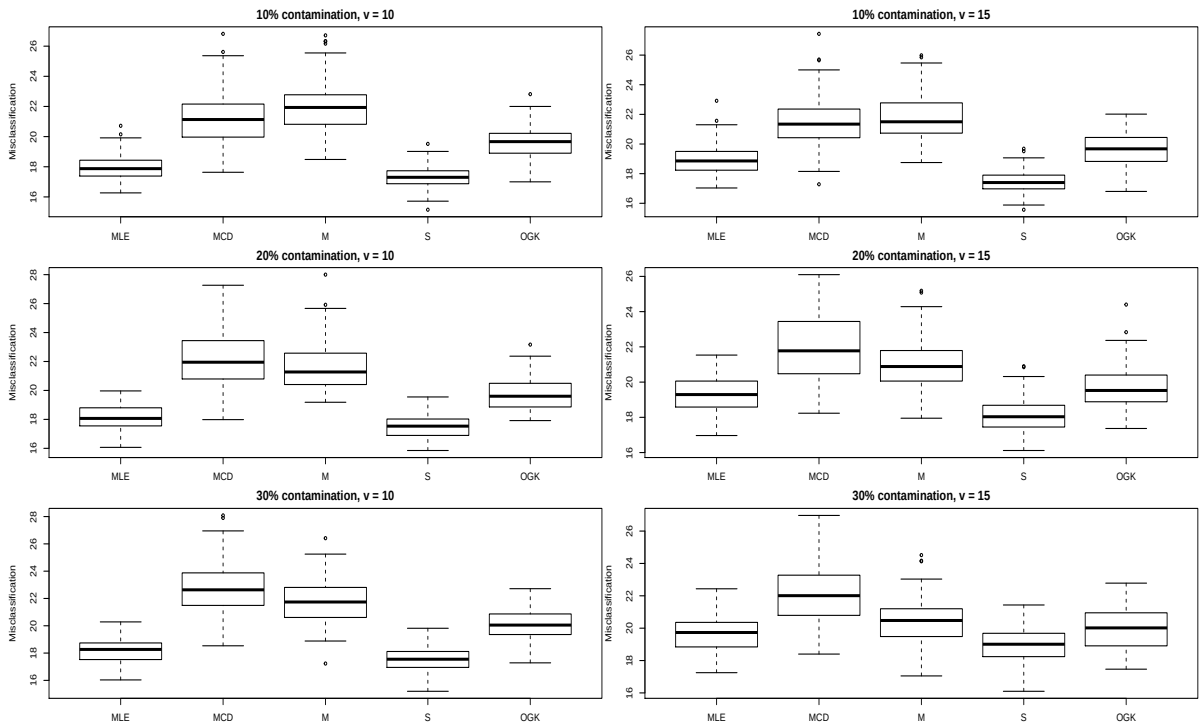

Figure 45: Mean probability of misclassification (as percentages) for Setup D2 with radial location contamination and $p = 6$, $M = 50$, $n_i = 120$, $i = 1, 2, 3$, $L = 3$ groups.

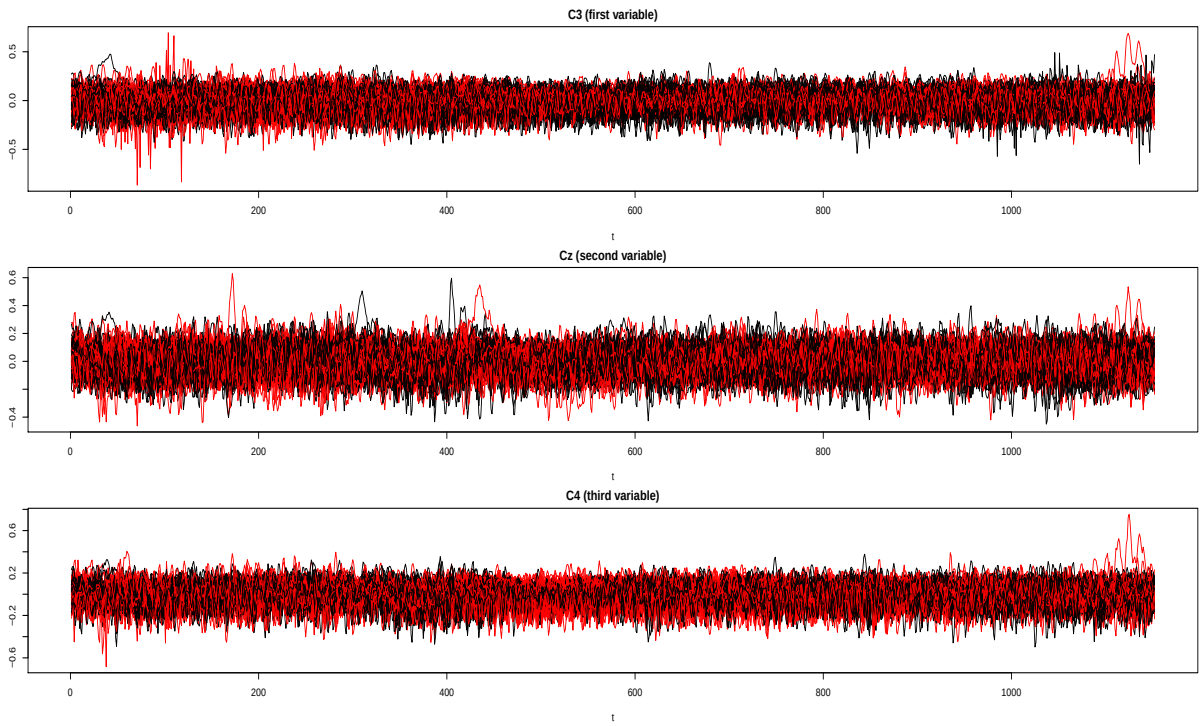

Figure 46: Graz data. The observations in the first (second) class are in black (resp. red).