
Supplementary Materials: Precision matrix estimation under data contamination with an application to portfolio selection

Authors

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Simulation Study (Complete)

In this section, we evaluate the performance of our proposals for Models 2-4. Tables 1-19 provide the results. We observe that the proposed covariance matrices provide robust results compared to standard approaches based on the sample covariance matrix and Ledoit-Wolf covariance estimator when the sample data is contaminated. However, as it was expected, sample covariance matrix provides better performance under the absence of contamination. In general, the obtained results are similar to those for Model 1. We observe that Spearman correlation performs better than Kendall correlation for both estimators GLasso and DTrace in terms of statistical losses. In terms of the GGM prediction measures, we obtain mixed results. The precision matrix estimators based on Kendall coefficient mostly provide higher MCC than the estimators based on Spearman coefficient for most of the experiments. We can also see that MAD performs better than Q_n and S_n under high contamination level, whereas Q_n and S_n perform better under low level of contamination.

Table 1. Average KLL (with standard deviations) over 100 replications: Model 2

Estimation Methods ($n = 75, p = 100$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	22.165 (2.691)	24.591 (3.215)	36.259 (2.050)	44.547 (1.300)
		S_n	20.932 (2.534)	23.421 (3.176)	37.228 (2.191)	48.037 (1.225)
		Q_n	21.062 (2.781)	23.354 (2.871)	38.509 (2.514)	48.788 (1.122)
	Kendall	MAD	25.558 (0.915)	27.614 (0.952)	38.789 (3.431)	49.006 (1.332)
		S_n	24.959 (0.799)	27.214 (0.877)	40.024 (3.502)	53.115 (1.237)
		Q_n	25.298 (0.793)	27.918 (0.879)	41.704 (3.589)	53.898 (1.805)
	Sample Covariance		17.756 (1.477)	40.334 (2.093)	92.394 (1.986)	129.44 (1.478)
	Ledoit Wolf		31.596 (0.341)	39.231 (0.684)	78.694 (0.949)	112.28 (1.009)
GLasso	Spearman	MAD	30.621 (2.373)	33.599 (2.294)	38.618 (1.750)	46.094 (1.358)
		S_n	29.173 (1.823)	33.291 (2.339)	40.404 (2.147)	50.467 (1.697)
		Q_n	29.642 (2.322)	34.161 (1.792)	42.080 (1.958)	50.999 (1.549)
	Kendall	MAD	34.616 (1.433)	36.030 (1.428)	42.393 (2.023)	50.018 (2.159)
		S_n	33.970 (1.291)	35.701 (1.412)	44.467 (1.824)	54.363 (1.534)
		Q_n	34.111 (1.277)	36.475 (1.454)	46.085 (1.282)	55.121 (1.314)
	Sample Covariance		26.775 (0.700)	46.963 (2.311)	97.909 (1.829)	134.30 (1.568)
	Ledoit Wolf		31.909 (0.421)	40.301 (0.631)	80.364 (0.916)	113.88 (0.981)
Estimation Methods ($n = 150, p = 200$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	39.609 (0.783)	43.744 (0.977)	59.733 (1.122)	78.279 (1.104)
		S_n	38.585 (0.753)	43.288 (0.845)	62.117 (1.037)	85.054 (1.110)
		Q_n	38.503 (0.701)	43.676 (0.778)	63.998 (0.974)	85.543 (1.040)
	Kendall	MAD	46.932 (0.909)	50.947 (0.998)	67.100 (1.062)	85.487 (1.193)
		S_n	46.167 (0.780)	50.874 (0.823)	70.114 (0.963)	93.145 (1.172)
		Q_n	46.286 (0.793)	51.607 (0.830)	72.330 (0.893)	93.856 (1.045)
	Sample Covariance		27.119 (0.486)	71.227 (0.745)	183.01 (0.708)	257.05 (0.637)
	Ledoit Wolf		62.874 (0.301)	77.665 (0.540)	155.90 (0.685)	224.46 (0.497)
GLasso	Spearman	MAD	50.401 (0.409)	52.433 (0.447)	66.379 (0.575)	82.269 (2.082)
		S_n	49.778 (0.369)	52.106 (0.389)	68.059 (1.022)	90.439 (1.196)
		Q_n	49.411 (0.376)	51.981 (0.358)	68.881 (1.155)	90.333 (1.047)
	Kendall	MAD	53.801 (0.559)	55.847 (0.531)	70.264 (0.646)	89.743 (0.901)
		S_n	53.383 (0.451)	55.957 (1.060)	72.635 (1.511)	95.558 (0.822)
		Q_n	53.113 (0.408)	56.068 (1.248)	74.641 (2.502)	95.597 (0.809)
	Sample Covariance		45.130 (2.345)	84.512 (2.036)	186.57 (0.496)	259.53 (0.495)
	Ledoit Wolf		62.982 (0.192)	78.328 (0.742)	157.23 (0.808)	224.77 (0.835)

Table 2. Average ℓ_2 norm loss (with standard deviations) over 100 replications: Model 2

Estimation Methods ($n = 75, p = 100$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	10.928 (0.334)	11.231 (0.354)	12.268 (0.154)	12.691 (0.050)
		S_n	10.917 (0.311)	11.219 (0.339)	12.365 (0.150)	12.836 (0.039)
		Q_n	11.000 (0.322)	11.269 (0.291)	12.446 (0.161)	12.881 (0.036)
	Kendall	MAD	11.484 (0.089)	11.674 (0.086)	12.436 (0.202)	12.880 (0.045)
		S_n	11.509 (0.078)	11.711 (0.075)	12.519 (0.193)	13.012 (0.034)
		Q_n	11.570 (0.074)	11.788 (0.070)	12.602 (0.185)	13.045 (0.060)
	Sample Covariance		10.531 (0.231)	12.276 (0.157)	13.628 (0.032)	13.978 (0.012)
Ledoit Wolf		12.049 (0.051)	12.616 (0.031)	13.547 (0.013)	13.887 (0.008)	
GLasso	Spearman	MAD	11.965 (0.186)	12.190 (0.173)	12.493 (0.089)	12.817 (0.046)
		S_n	11.905 (0.139)	12.206 (0.173)	12.594 (0.106)	12.963 (0.056)
		Q_n	11.959 (0.173)	12.278 (0.128)	12.675 (0.096)	12.988 (0.050)
	Kendall	MAD	12.273 (0.088)	12.358 (0.084)	12.678 (0.093)	12.949 (0.069)
		S_n	12.264 (0.080)	12.363 (0.085)	12.771 (0.078)	13.075 (0.042)
		Q_n	12.281 (0.079)	12.412 (0.083)	12.834 (0.050)	13.102 (0.035)
	Sample Covariance		11.744 (0.057)	12.779 (0.101)	13.735 (0.021)	14.020 (0.010)
Ledoit Wolf		12.096 (0.059)	12.664 (0.027)	13.569 (0.012)	13.899 (0.007)	
Estimation Methods ($n = 150, p = 200$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	15.562 (0.073)	15.919 (0.076)	16.943 (0.058)	17.736 (0.039)
		S_n	15.573 (0.068)	15.964 (0.066)	17.090 (0.048)	17.946 (0.034)
		Q_n	15.609 (0.062)	16.027 (0.059)	17.190 (0.043)	17.979 (0.031)
	Kendall	MAD	16.212 (0.062)	16.476 (0.061)	17.318 (0.045)	17.977 (0.034)
		S_n	16.220 (0.054)	16.522 (0.050)	17.456 (0.038)	18.175 (0.029)
		Q_n	16.254 (0.054)	16.584 (0.048)	17.546 (0.034)	18.205 (0.026)
	Sample Covariance		14.240 (0.078)	17.206 (0.049)	19.375 (0.009)	19.827 (0.004)
Ledoit Wolf		17.063 (0.035)	17.871 (0.018)	19.193 (0.007)	19.686 (0.003)	
GLasso	Spearman	MAD	16.472 (0.026)	16.597 (0.024)	17.327 (0.024)	17.920 (0.068)
		S_n	16.480 (0.024)	16.618 (0.022)	17.402 (0.041)	18.143 (0.033)
		Q_n	16.477 (0.024)	16.627 (0.021)	17.437 (0.046)	18.150 (0.029)
	Kendall	MAD	16.696 (0.028)	16.809 (0.025)	17.488 (0.022)	18.139 (0.021)
		S_n	16.707 (0.024)	16.841 (0.054)	17.573 (0.056)	18.259 (0.016)
		Q_n	16.707 (0.022)	16.858 (0.064)	17.646 (0.095)	18.268 (0.016)
	Sample Covariance		16.187 (0.191)	17.899 (0.076)	19.424 (0.006)	19.844 (0.003)
Ledoit Wolf		17.081 (0.023)	17.893 (0.024)	19.206 (0.008)	19.687 (0.004)	

Table 3. Average ℓ_{spec} norm loss (with standard deviations) over 100 replications: Model 2

Estimation Methods ($n = 75, p = 100$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	3.348 (0.072)	3.410 (0.073)	3.619 (0.029)	3.696 (0.010)
		S_n	3.347 (0.066)	3.408 (0.070)	3.637 (0.027)	3.721 (0.008)
		Q_n	3.364 (0.068)	3.419 (0.060)	3.651 (0.030)	3.729 (0.007)
	Kendall	MAD	3.459 (0.020)	3.498 (0.018)	3.648 (0.040)	3.730 (0.010)
		S_n	3.465 (0.018)	3.506 (0.017)	3.663 (0.038)	3.752 (0.008)
		Q_n	3.477 (0.017)	3.521 (0.015)	3.678 (0.036)	3.758 (0.011)
	Sample Covariance		3.270 (0.050)	3.636 (0.032)	3.860 (0.006)	3.910 (0.002)
	Ledoit Wolf		3.574 (0.011)	3.682 (0.006)	3.839 (0.002)	3.893 (0.001)
GLasso	Spearman	MAD	3.552 (0.037)	3.598 (0.035)	3.658 (0.018)	3.718 (0.010)
		S_n	3.541 (0.028)	3.601 (0.035)	3.677 (0.021)	3.743 (0.010)
		Q_n	3.552 (0.035)	3.615 (0.025)	3.691 (0.019)	3.748 (0.010)
	Kendall	MAD	3.613 (0.018)	3.630 (0.017)	3.691 (0.018)	3.741 (0.012)
		S_n	3.611 (0.016)	3.631 (0.017)	3.708 (0.015)	3.762 (0.008)
		Q_n	3.614 (0.016)	3.640 (0.017)	3.719 (0.010)	3.767 (0.006)
	Sample Covariance		3.509 (0.013)	3.717 (0.017)	3.872 (0.003)	3.914 (0.002)
	Ledoit Wolf		3.583 (0.012)	3.691 (0.005)	3.843 (0.002)	3.894 (0.001)
Estimation Methods ($n = 150, p = 200$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	3.370 (0.014)	3.424 (0.014)	3.567 (0.011)	3.673 (0.007)
		S_n	3.370 (0.014)	3.428 (0.012)	3.586 (0.010)	3.699 (0.006)
		Q_n	3.375 (0.012)	3.437 (0.010)	3.598 (0.009)	3.703 (0.005)
	Kendall	MAD	3.460 (0.010)	3.499 (0.010)	3.615 (0.008)	3.703 (0.005)
		S_n	3.460 (0.009)	3.504 (0.008)	3.633 (0.007)	3.727 (0.005)
		Q_n	3.465 (0.009)	3.512 (0.008)	3.644 (0.006)	3.731 (0.004)
	Sample Covariance		3.171 (0.015)	3.625 (0.011)	3.871 (0.002)	3.919 (0.001)
	Ledoit Wolf		3.579 (0.005)	3.690 (0.002)	3.848 (0.001)	3.903 (0.000)
GLasso	Spearman	MAD	3.493 (0.004)	3.510 (0.004)	3.613 (0.004)	3.694 (0.010)
		S_n	3.494 (0.004)	3.513 (0.004)	3.622 (0.006)	3.722 (0.005)
		Q_n	3.494 (0.004)	3.514 (0.004)	3.627 (0.007)	3.722 (0.004)
	Kendall	MAD	3.523 (0.004)	3.539 (0.004)	3.634 (0.004)	3.722 (0.003)
		S_n	3.524 (0.004)	3.543 (0.008)	3.644 (0.008)	3.736 (0.003)
		Q_n	3.524 (0.004)	3.546 (0.009)	3.654 (0.013)	3.737 (0.003)
	Sample Covariance		3.452 (0.028)	3.698 (0.009)	3.876 (0.001)	3.920 (0.001)
	Ledoit Wolf		3.582 (0.003)	3.693 (0.003)	3.850 (0.001)	3.903 (0.000)

Table 4. Average ℓ_1 norm loss (with standard deviations) over 100 replications: Model 2

Estimation Methods ($n = 75, p = 100$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	3.671 (0.048)	3.709 (0.050)	3.811 (0.022)	3.854 (0.016)
		S_n	3.643 (0.047)	3.687 (0.046)	3.811 (0.022)	3.866 (0.016)
		Q_n	3.637 (0.046)	3.687 (0.043)	3.810 (0.018)	3.850 (0.013)
	Kendall	MAD	3.694 (0.028)	3.728 (0.026)	3.817 (0.023)	3.862 (0.016)
		S_n	3.674 (0.025)	3.714 (0.025)	3.816 (0.021)	3.871 (0.014)
		Q_n	3.672 (0.025)	3.715 (0.023)	3.815 (0.020)	3.858 (0.011)
	Sample Covariance		3.560 (0.041)	3.901 (0.024)	3.947 (0.009)	3.958 (0.004)
	Ledoit Wolf		3.624 (0.013)	3.735 (0.015)	3.859 (0.004)	3.908 (0.002)
GLasso	Spearman	MAD	3.734 (0.032)	3.750 (0.027)	3.801 (0.024)	3.846 (0.018)
		S_n	3.702 (0.029)	3.732 (0.025)	3.802 (0.023)	3.855 (0.017)
		Q_n	3.695 (0.026)	3.732 (0.021)	3.801 (0.017)	3.841 (0.015)
	Kendall	MAD	3.745 (0.025)	3.759 (0.021)	3.810 (0.014)	3.853 (0.017)
		S_n	3.724 (0.020)	3.741 (0.019)	3.811 (0.017)	3.860 (0.016)
		Q_n	3.718 (0.018)	3.742 (0.021)	3.809 (0.014)	3.846 (0.012)
	Sample Covariance		3.649 (0.021)	3.913 (0.031)	3.943 (0.008)	3.957 (0.004)
	Ledoit Wolf		3.630 (0.014)	3.748 (0.012)	3.865 (0.004)	3.911 (0.002)
Estimation Methods ($n = 150, p = 200$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	3.593 (0.028)	3.636 (0.026)	3.746 (0.017)	3.815 (0.016)
		S_n	3.565 (0.025)	3.620 (0.024)	3.749 (0.019)	3.829 (0.016)
		Q_n	3.554 (0.025)	3.618 (0.022)	3.747 (0.017)	3.813 (0.012)
	Kendall	MAD	3.637 (0.023)	3.675 (0.021)	3.763 (0.015)	3.824 (0.015)
		S_n	3.616 (0.022)	3.661 (0.018)	3.768 (0.016)	3.837 (0.013)
		Q_n	3.607 (0.019)	3.660 (0.017)	3.766 (0.015)	3.823 (0.011)
	Sample Covariance		3.409 (0.032)	3.866 (0.019)	3.932 (0.005)	3.950 (0.003)
	Ledoit Wolf		3.613 (0.008)	3.723 (0.006)	3.855 (0.001)	3.907 (0.001)
GLasso	Spearman	MAD	3.648 (0.024)	3.680 (0.026)	3.751 (0.023)	3.809 (0.019)
		S_n	3.629 (0.022)	3.661 (0.023)	3.757 (0.027)	3.824 (0.015)
		Q_n	3.613 (0.021)	3.649 (0.020)	3.752 (0.023)	3.807 (0.012)
	Kendall	MAD	3.666 (0.024)	3.688 (0.023)	3.759 (0.018)	3.815 (0.015)
		S_n	3.648 (0.018)	3.671 (0.017)	3.762 (0.020)	3.831 (0.017)
		Q_n	3.633 (0.016)	3.663 (0.018)	3.759 (0.016)	3.813 (0.013)
	Sample Covariance		3.572 (0.018)	3.883 (0.027)	3.931 (0.007)	3.949 (0.003)
	Ledoit Wolf		3.611 (0.005)	3.727 (0.007)	3.858 (0.002)	3.908 (0.001)

Table 5. Average Sensitivity (with standard deviations) over 100 replications: Model 2

Estimation Methods ($n = 75, p = 100$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	0.058 (0.013)	0.053 (0.015)	0.027 (0.008)	0.020 (0.001)
		S_n	0.059 (0.012)	0.055 (0.013)	0.027 (0.008)	0.020 (0.001)
		Q_n	0.058 (0.012)	0.056 (0.012)	0.027 (0.009)	0.020 (0.001)
	Kendall	MAD	0.046 (0.002)	0.045 (0.002)	0.028 (0.011)	0.016 (0.001)
		S_n	0.046 (0.002)	0.045 (0.002)	0.029 (0.011)	0.016 (0.001)
		Q_n	0.046 (0.002)	0.045 (0.002)	0.029 (0.011)	0.017 (0.004)
	Sample Covariance		0.054 (0.008)	0.038 (0.009)	0.022 (0.005)	0.018 (0.002)
	Ledoit Wolf		0.011 (0.000)	0.010 (0.000)	0.010 (0.000)	0.010 (0.000)
GLasso	Spearman	MAD	0.040 (0.009)	0.032 (0.009)	0.027 (0.005)	0.020 (0.002)
		S_n	0.043 (0.007)	0.031 (0.010)	0.025 (0.006)	0.018 (0.004)
		Q_n	0.041 (0.009)	0.030 (0.008)	0.024 (0.007)	0.018 (0.004)
	Kendall	MAD	0.032 (0.004)	0.030 (0.005)	0.022 (0.006)	0.018 (0.003)
		S_n	0.032 (0.004)	0.030 (0.005)	0.021 (0.004)	0.017 (0.002)
		Q_n	0.032 (0.004)	0.030 (0.004)	0.021 (0.003)	0.016 (0.002)
	Sample Covariance		0.041 (0.004)	0.032 (0.009)	0.017 (0.004)	0.016 (0.002)
	Ledoit Wolf		0.011 (0.001)	0.010 (0.000)	0.010 (0.000)	0.010 (0.000)
Estimation Methods ($n = 150, p = 200$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	0.018 (0.000)	0.017 (0.000)	0.016 (0.000)	0.014 (0.000)
		S_n	0.018 (0.000)	0.017 (0.000)	0.016 (0.000)	0.014 (0.000)
		Q_n	0.018 (0.000)	0.017 (0.000)	0.016 (0.000)	0.014 (0.000)
	Kendall	MAD	0.016 (0.000)	0.016 (0.000)	0.015 (0.000)	0.012 (0.000)
		S_n	0.016 (0.000)	0.016 (0.000)	0.015 (0.000)	0.012 (0.000)
		Q_n	0.016 (0.000)	0.016 (0.000)	0.015 (0.000)	0.012 (0.000)
	Sample Covariance		0.027 (0.001)	0.026 (0.001)	0.009 (0.000)	0.007 (0.000)
	Ledoit Wolf		0.005 (0.000)	0.005 (0.000)	0.005 (0.000)	0.005 (0.000)
GLasso	Spearman	MAD	0.022 (0.001)	0.023 (0.001)	0.019 (0.001)	0.014 (0.002)
		S_n	0.021 (0.001)	0.023 (0.001)	0.021 (0.001)	0.014 (0.001)
		Q_n	0.021 (0.001)	0.023 (0.001)	0.021 (0.001)	0.014 (0.001)
	Kendall	MAD	0.022 (0.001)	0.023 (0.001)	0.019 (0.001)	0.012 (0.001)
		S_n	0.022 (0.001)	0.023 (0.001)	0.020 (0.002)	0.014 (0.001)
		Q_n	0.021 (0.001)	0.023 (0.002)	0.020 (0.003)	0.014 (0.001)
	Sample Covariance		0.021 (0.005)	0.022 (0.003)	0.008 (0.000)	0.007 (0.000)
	Ledoit Wolf		0.006 (0.000)	0.005 (0.000)	0.005 (0.000)	0.005 (0.000)

Table 6. Average KLL (with standard deviations) over 100 replications: Model 3

Estimation Methods ($n = 75, p = 100$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	30.336 (1.452)	33.122 (1.554)	50.728 (0.978)	58.055 (1.331)
		S_n	29.120 (0.924)	32.178 (0.947)	51.796 (1.576)	61.654 (1.318)
		Q_n	28.984 (0.913)	32.333 (0.897)	53.250 (1.030)	62.256 (1.162)
	Kendall	MAD	38.351 (3.167)	43.947 (4.963)	55.450 (1.280)	63.084 (1.701)
		S_n	37.837 (3.370)	43.015 (4.808)	57.096 (1.175)	67.255 (1.624)
		Q_n	38.141 (3.316)	44.075 (5.047)	58.896 (1.130)	68.161 (1.510)
	Sample Covariance		22.480 (3.349)	49.659 (3.521)	101.051 (2.147)	135.552 (1.612)
	Ledoit Wolf		46.432 (0.585)	53.702 (0.953)	88.064 (1.071)	118.878 (1.064)
GLasso	Spearman	MAD	46.693 (2.281)	48.787 (2.199)	54.530 (1.968)	61.017 (1.708)
		S_n	45.983 (2.249)	49.019 (2.347)	56.273 (1.832)	65.233 (1.695)
		Q_n	46.088 (2.441)	49.419 (2.546)	57.653 (1.678)	66.069 (1.537)
	Kendall	MAD	51.718 (1.891)	53.360 (1.905)	57.926 (1.824)	64.730 (1.906)
		S_n	51.502 (2.631)	53.392 (2.090)	60.029 (1.575)	69.102 (1.531)
		Q_n	51.814 (2.857)	54.221 (1.876)	61.918 (1.576)	69.997 (1.566)
	Sample Covariance		41.111 (3.760)	61.633 (1.616)	105.133 (1.265)	140.238 (1.628)
	Ledoit Wolf		46.965 (0.598)	54.595 (0.906)	89.592 (1.038)	120.760 (1.028)
Estimation Methods ($n = 150, p = 200$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	38.764 (0.863)	45.551 (1.001)	69.505 (2.967)	107.151 (1.211)
		S_n	37.400 (0.798)	44.726 (0.874)	71.207 (2.235)	114.019 (1.139)
		Q_n	37.014 (0.808)	44.800 (0.836)	72.625 (2.250)	114.431 (1.037)
	Kendall	MAD	46.233 (0.981)	52.413 (1.003)	82.111 (10.816)	114.956 (1.393)
		S_n	45.337 (0.858)	52.083 (0.889)	84.335 (10.869)	122.572 (1.384)
		Q_n	45.228 (0.829)	52.549 (0.867)	84.684 (10.199)	123.172 (1.309)
	Sample Covariance		33.295 (0.846)	90.613 (1.049)	199.177 (0.771)	268.778 (0.706)
	Ledoit Wolf		93.709 (0.422)	107.043 (1.453)	174.381 (0.912)	236.052 (0.909)
GLasso	Spearman	MAD	66.605 (0.669)	71.325 (3.968)	94.527 (2.656)	116.114 (2.479)
		S_n	65.689 (0.599)	71.575 (4.490)	97.067 (2.758)	122.537 (1.785)
		Q_n	65.228 (0.582)	72.142 (4.746)	98.064 (3.293)	122.550 (1.648)
	Kendall	MAD	73.552 (4.320)	81.586 (2.856)	101.083 (3.183)	121.189 (1.014)
		S_n	72.067 (3.718)	81.615 (2.328)	105.199 (3.135)	129.352 (3.277)
		Q_n	71.523 (3.569)	81.413 (2.414)	106.461 (2.979)	129.375 (3.159)
	Sample Covariance		59.709 (3.612)	114.444 (3.044)	202.270 (2.225)	270.446 (0.985)
	Ledoit Wolf		93.561 (0.696)	107.708 (0.590)	176.264 (1.090)	238.018 (0.872)

Table 7. Average ℓ_2 norm loss (with standard deviations) over 100 replications: Model 3

Estimation Methods ($n = 75, p = 100$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	9.247 (0.140)	9.603 (0.149)	11.027 (0.054)	11.358 (0.054)	
		S_n	9.255 (0.085)	9.643 (0.087)	11.129 (0.085)	11.509 (0.046)
		Q_n	9.328 (0.091)	9.736 (0.083)	11.221 (0.046)	11.553 (0.038)
	Kendall	MAD	10.280 (0.220)	10.665 (0.306)	11.288 (0.056)	11.560 (0.055)
		S_n	10.308 (0.225)	10.666 (0.287)	11.382 (0.047)	11.694 (0.044)
		Q_n	10.366 (0.215)	10.756 (0.289)	11.461 (0.041)	11.734 (0.039)
	Sample Covariance		8.061 (0.766)	10.788 (0.239)	12.241 (0.035)	12.593 (0.013)
	Ledoit Wolf		10.783 (0.065)	11.282 (0.044)	12.150 (0.016)	12.495 (0.009)
GLasso	Spearman	MAD	10.893 (0.121)	11.005 (0.115)	11.282 (0.088)	11.524 (0.059)
		S_n	10.890 (0.116)	11.047 (0.117)	11.372 (0.075)	11.662 (0.050)
		Q_n	10.915 (0.121)	11.084 (0.119)	11.434 (0.064)	11.696 (0.043)
	Kendall	MAD	11.173 (0.089)	11.244 (0.086)	11.427 (0.071)	11.647 (0.056)
		S_n	11.184 (0.117)	11.265 (0.091)	11.515 (0.052)	11.769 (0.037)
		Q_n	11.208 (0.125)	11.311 (0.075)	11.582 (0.049)	11.798 (0.038)
	Sample Covariance		10.620 (0.227)	11.518 (0.061)	12.334 (0.015)	12.638 (0.011)
	Ledoit Wolf		10.849 (0.060)	11.325 (0.040)	12.172 (0.015)	12.511 (0.008)
Estimation Methods ($n = 150, p = 200$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	11.395 (0.111)	12.248 (0.109)	14.255 (0.163)	15.928 (0.038)
		S_n	11.385 (0.102)	12.299 (0.096)	14.431 (0.118)	16.140 (0.030)
		Q_n	11.418 (0.100)	12.380 (0.089)	14.546 (0.114)	16.171 (0.027)
	Kendall	MAD	12.524 (0.103)	13.114 (0.093)	14.965 (0.472)	16.165 (0.039)
		S_n	12.532 (0.089)	13.173 (0.078)	15.106 (0.449)	16.360 (0.032)
		Q_n	12.571 (0.085)	13.254 (0.074)	15.144 (0.406)	16.388 (0.030)
	Sample Covariance		10.773 (0.106)	15.159 (0.065)	17.384 (0.011)	17.850 (0.004)
	Ledoit Wolf		15.307 (0.037)	15.972 (0.049)	17.196 (0.010)	17.691 (0.005)
GLasso	Spearman	MAD	14.401 (0.019)	14.634 (0.179)	15.571 (0.089)	16.215 (0.065)
		S_n	14.410 (0.017)	14.699 (0.204)	15.694 (0.083)	16.378 (0.040)
		Q_n	14.408 (0.016)	14.747 (0.215)	15.742 (0.096)	16.389 (0.036)
	Kendall	MAD	14.782 (0.188)	15.128 (0.122)	15.806 (0.094)	16.345 (0.023)
		S_n	14.758 (0.163)	15.169 (0.094)	15.953 (0.087)	16.524 (0.066)
		Q_n	14.750 (0.157)	15.178 (0.102)	15.999 (0.080)	16.533 (0.063)
	Sample Covariance		14.078 (0.251)	16.148 (0.081)	17.429 (0.019)	17.865 (0.005)
	Ledoit Wolf		15.307 (0.052)	15.997 (0.020)	17.216 (0.011)	17.703 (0.005)

Table 8. Average ℓ_{spec} norm loss (with standard deviations) over 100 replications: Model 3

Estimation Methods ($n = 75, p = 100$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	2.123 (0.032)	2.196 (0.033)	2.418 (0.013)	2.464 (0.011)
		S_n	2.120 (0.025)	2.198 (0.026)	2.430 (0.014)	2.483 (0.009)
		Q_n	2.130 (0.027)	2.212 (0.025)	2.442 (0.011)	2.487 (0.008)
	Kendall	MAD	2.283 (0.042)	2.355 (0.051)	2.449 (0.011)	2.486 (0.009)
		S_n	2.285 (0.041)	2.352 (0.048)	2.461 (0.009)	2.503 (0.008)
		Q_n	2.293 (0.039)	2.365 (0.047)	2.471 (0.008)	2.508 (0.007)
	Sample Covariance		1.912 (0.141)	2.413 (0.027)	2.584 (0.004)	2.625 (0.002)
Ledoit Wolf		2.357 (0.011)	2.434 (0.007)	2.559 (0.002)	2.607 (0.001)	
GLasso	Spearman	MAD	2.382 (0.019)	2.399 (0.018)	2.442 (0.014)	2.477 (0.009)
		S_n	2.381 (0.018)	2.404 (0.018)	2.454 (0.012)	2.496 (0.008)
		Q_n	2.384 (0.019)	2.410 (0.018)	2.463 (0.010)	2.500 (0.006)
	Kendall	MAD	2.423 (0.014)	2.434 (0.013)	2.462 (0.011)	2.493 (0.008)
		S_n	2.424 (0.018)	2.436 (0.014)	2.474 (0.009)	2.510 (0.006)
		Q_n	2.428 (0.019)	2.443 (0.012)	2.483 (0.008)	2.514 (0.006)
	Sample Covariance		2.339 (0.036)	2.481 (0.008)	2.590 (0.003)	2.629 (0.001)
Ledoit Wolf		2.367 (0.010)	2.441 (0.006)	2.562 (0.002)	2.609 (0.001)	

Estimation Methods ($n = 150, p = 200$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	1.882 (0.029)	2.018 (0.029)	2.273 (0.022)	2.447 (0.007)
		S_n	1.878 (0.025)	2.022 (0.028)	2.290 (0.019)	2.466 (0.006)
		Q_n	1.880 (0.024)	2.031 (0.029)	2.301 (0.018)	2.468 (0.005)
	Kendall	MAD	2.012 (0.020)	2.107 (0.024)	2.340 (0.052)	2.467 (0.006)
		S_n	2.010 (0.019)	2.112 (0.020)	2.353 (0.049)	2.486 (0.006)
		Q_n	2.013 (0.018)	2.120 (0.019)	2.355 (0.045)	2.487 (0.005)
	Sample Covariance		1.795 (0.027)	2.406 (0.016)	2.584 (0.002)	2.625 (0.001)
Ledoit Wolf		2.360 (0.005)	2.434 (0.006)	2.559 (0.001)	2.608 (0.001)	
GLasso	Spearman	MAD	2.262 (0.006)	2.293 (0.021)	2.398 (0.010)	2.467 (0.007)
		S_n	2.263 (0.006)	2.300 (0.024)	2.411 (0.010)	2.482 (0.005)
		Q_n	2.263 (0.006)	2.305 (0.026)	2.416 (0.011)	2.483 (0.005)
	Kendall	MAD	2.305 (0.023)	2.348 (0.015)	2.422 (0.010)	2.479 (0.004)
		S_n	2.303 (0.020)	2.352 (0.011)	2.438 (0.009)	2.497 (0.006)
		Q_n	2.302 (0.019)	2.353 (0.013)	2.442 (0.008)	2.497 (0.006)
	Sample Covariance		2.224 (0.033)	2.463 (0.007)	2.586 (0.002)	2.626 (0.001)
Ledoit Wolf		2.360 (0.006)	2.436 (0.002)	2.561 (0.001)	2.609 (0.001)	

Table 9. Average ℓ_1 norm loss (with standard deviations) over 100 replications: Model 3

Estimation Methods ($n = 75, p = 100$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	2.426 (0.045)	2.477 (0.047)	2.559 (0.020)	2.585 (0.018)	
		S_n	2.401 (0.042)	2.466 (0.044)	2.557 (0.017)	2.591 (0.015)
		Q_n	2.398 (0.047)	2.464 (0.044)	2.551 (0.016)	2.580 (0.012)
	Kendall	MAD	2.471 (0.037)	2.515 (0.030)	2.560 (0.017)	2.587 (0.013)
		S_n	2.455 (0.033)	2.499 (0.029)	2.558 (0.014)	2.593 (0.012)
		Q_n	2.451 (0.030)	2.504 (0.024)	2.558 (0.014)	2.583 (0.011)
	Sample Covariance		2.269 (0.079)	2.625 (0.027)	2.649 (0.007)	2.660 (0.004)
	Ledoit Wolf		2.409 (0.015)	2.491 (0.014)	2.570 (0.005)	2.611 (0.002)
GLasso	Spearman	MAD	2.514 (0.023)	2.525 (0.029)	2.551 (0.018)	2.577 (0.018)
		S_n	2.494 (0.021)	2.510 (0.019)	2.550 (0.021)	2.585 (0.019)
		Q_n	2.490 (0.021)	2.508 (0.021)	2.545 (0.016)	2.570 (0.014)
	Kendall	MAD	2.526 (0.021)	2.536 (0.019)	2.558 (0.019)	2.580 (0.016)
		S_n	2.508 (0.018)	2.523 (0.018)	2.556 (0.019)	2.590 (0.016)
		Q_n	2.505 (0.017)	2.520 (0.015)	2.553 (0.014)	2.576 (0.011)
	Sample Covariance		2.462 (0.022)	2.603 (0.020)	2.646 (0.008)	2.659 (0.005)
	Ledoit Wolf		2.411 (0.013)	2.495 (0.011)	2.575 (0.004)	2.615 (0.002)
Estimation Methods ($n = 150, p = 200$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	2.195 (0.050)	2.311 (0.042)	2.498 (0.029)	2.556 (0.014)
		S_n	2.170 (0.047)	2.299 (0.044)	2.499 (0.025)	2.566 (0.014)
		Q_n	2.163 (0.047)	2.302 (0.040)	2.501 (0.025)	2.554 (0.013)
	Kendall	MAD	2.263 (0.037)	2.349 (0.037)	2.512 (0.026)	2.561 (0.013)
		S_n	2.240 (0.034)	2.342 (0.036)	2.514 (0.023)	2.570 (0.010)
		Q_n	2.235 (0.032)	2.344 (0.032)	2.510 (0.021)	2.560 (0.010)
	Sample Covariance		2.056 (0.045)	2.586 (0.017)	2.633 (0.004)	2.651 (0.003)
	Ledoit Wolf		2.399 (0.007)	2.477 (0.008)	2.566 (0.002)	2.610 (0.001)
GLasso	Spearman	MAD	2.460 (0.030)	2.486 (0.024)	2.521 (0.019)	2.554 (0.015)
		S_n	2.438 (0.026)	2.469 (0.020)	2.524 (0.018)	2.562 (0.015)
		Q_n	2.421 (0.025)	2.460 (0.021)	2.522 (0.017)	2.550 (0.011)
	Kendall	MAD	2.469 (0.024)	2.482 (0.019)	2.525 (0.017)	2.555 (0.010)
		S_n	2.451 (0.023)	2.469 (0.016)	2.526 (0.014)	2.567 (0.011)
		Q_n	2.441 (0.022)	2.463 (0.013)	2.523 (0.013)	2.556 (0.009)
	Sample Covariance		2.375 (0.023)	2.582 (0.021)	2.635 (0.008)	2.651 (0.004)
	Ledoit Wolf		2.396 (0.008)	2.479 (0.008)	2.569 (0.002)	2.611 (0.001)

Table 10. Average Specificity (with standard deviations) over 100 replications: Model 3

Estimation Methods ($n = 75, p = 100$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	0.952 (0.004)	0.954 (0.005)	0.989 (0.002)	0.992 (0.001)
		S_n	0.953 (0.004)	0.954 (0.003)	0.989 (0.004)	0.993 (0.002)
		Q_n	0.953 (0.004)	0.954 (0.003)	0.990 (0.002)	0.992 (0.001)
	Kendall	MAD	0.971 (0.007)	0.979 (0.011)	0.994 (0.001)	0.996 (0.001)
		S_n	0.971 (0.007)	0.977 (0.011)	0.994 (0.001)	0.996 (0.001)
		Q_n	0.971 (0.007)	0.978 (0.011)	0.994 (0.001)	0.996 (0.001)
	Sample Covariance		0.942 (0.023)	0.971 (0.012)	0.990 (0.005)	0.992 (0.003)
Ledoit Wolf		0.999 (0.000)	1.000 (0.000)	1.000 (0.000)	1.000 (0.000)	
GLasso	Spearman	MAD	0.983 (0.007)	0.987 (0.007)	0.992 (0.003)	0.994 (0.002)
		S_n	0.983 (0.006)	0.988 (0.006)	0.993 (0.003)	0.995 (0.002)
		Q_n	0.983 (0.007)	0.988 (0.007)	0.993 (0.003)	0.996 (0.002)
	Kendall	MAD	0.990 (0.005)	0.992 (0.003)	0.994 (0.003)	0.996 (0.002)
		S_n	0.991 (0.006)	0.992 (0.004)	0.995 (0.002)	0.996 (0.001)
		Q_n	0.990 (0.006)	0.992 (0.003)	0.995 (0.002)	0.997 (0.002)
	Sample Covariance		0.974 (0.014)	0.991 (0.004)	0.992 (0.002)	0.994 (0.003)
Ledoit Wolf		0.999 (0.001)	0.999 (0.000)	1.000 (0.000)	1.000 (0.000)	
Estimation Methods ($n = 150, p = 200$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	0.967 (0.001)	0.968 (0.001)	0.970 (0.003)	0.994 (0.000)
		S_n	0.967 (0.001)	0.968 (0.001)	0.970 (0.002)	0.994 (0.000)
		Q_n	0.967 (0.001)	0.968 (0.001)	0.970 (0.002)	0.994 (0.000)
	Kendall	MAD	0.969 (0.001)	0.969 (0.001)	0.978 (0.011)	0.996 (0.000)
		S_n	0.969 (0.001)	0.969 (0.001)	0.978 (0.011)	0.996 (0.000)
		Q_n	0.969 (0.001)	0.969 (0.001)	0.976 (0.010)	0.996 (0.000)
	Sample Covariance		0.983 (0.001)	0.982 (0.001)	0.997 (0.000)	0.998 (0.000)
Ledoit Wolf		1.000 (0.000)	1.000 (0.000)	1.000 (0.000)	1.000 (0.000)	
GLasso	Spearman	MAD	0.972 (0.002)	0.973 (0.006)	0.988 (0.003)	0.997 (0.002)
		S_n	0.972 (0.002)	0.973 (0.006)	0.988 (0.003)	0.996 (0.001)
		Q_n	0.972 (0.002)	0.974 (0.007)	0.987 (0.003)	0.996 (0.001)
	Kendall	MAD	0.975 (0.006)	0.982 (0.004)	0.991 (0.003)	0.997 (0.000)
		S_n	0.974 (0.005)	0.982 (0.003)	0.991 (0.003)	0.997 (0.002)
		Q_n	0.974 (0.005)	0.981 (0.003)	0.991 (0.003)	0.997 (0.002)
	Sample Covariance		0.974 (0.008)	0.992 (0.003)	0.996 (0.001)	0.997 (0.000)
Ledoit Wolf		0.999 (0.000)	1.000 (0.000)	1.000 (0.000)	1.000 (0.000)	

Table 11. Average Sensitivity (with standard deviations) over 100 replications: Model 3

Estimation Methods ($n = 75, p = 100$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	0.916 (0.033)	0.867 (0.042)	0.431 (0.025)	0.326 (0.022)
		S_n	0.920 (0.018)	0.870 (0.024)	0.429 (0.041)	0.324 (0.024)
		Q_n	0.919 (0.018)	0.870 (0.023)	0.426 (0.033)	0.326 (0.020)
	Kendall	MAD	0.806 (0.099)	0.643 (0.160)	0.359 (0.021)	0.286 (0.015)
		S_n	0.804 (0.109)	0.666 (0.157)	0.359 (0.020)	0.283 (0.016)
		Q_n	0.805 (0.108)	0.655 (0.159)	0.358 (0.020)	0.284 (0.015)
	Sample Covariance		0.963 (0.024)	0.585 (0.086)	0.228 (0.015)	0.210 (0.006)
Ledoit Wolf		0.220 (0.013)	0.205 (0.004)	0.202 (0.000)	0.202 (0.000)	
GLasso	Spearman	MAD	0.529 (0.089)	0.461 (0.082)	0.344 (0.050)	0.285 (0.029)
		S_n	0.545 (0.087)	0.455 (0.086)	0.336 (0.047)	0.276 (0.026)
		Q_n	0.554 (0.097)	0.467 (0.095)	0.344 (0.049)	0.274 (0.024)
	Kendall	MAD	0.427 (0.068)	0.515 (0.027)	0.324 (0.041)	0.268 (0.027)
		S_n	0.424 (0.097)	0.523 (0.029)	0.315 (0.035)	0.264 (0.020)
		Q_n	0.434 (0.104)	0.526 (0.031)	0.310 (0.041)	0.262 (0.023)
	Sample Covariance		0.667 (0.108)	0.303 (0.044)	0.213 (0.007)	0.207 (0.005)
Ledoit Wolf		0.225 (0.014)	0.207 (0.005)	0.202 (0.000)	0.202 (0.000)	
Estimation Methods ($n = 150, p = 200$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	0.999 (0.002)	0.995 (0.004)	0.921 (0.034)	0.495 (0.017)
		S_n	0.999 (0.002)	0.995 (0.004)	0.923 (0.025)	0.495 (0.018)
		Q_n	0.999 (0.002)	0.995 (0.004)	0.924 (0.024)	0.494 (0.017)
	Kendall	MAD	0.998 (0.002)	0.993 (0.004)	0.825 (0.142)	0.439 (0.018)
		S_n	0.998 (0.002)	0.993 (0.004)	0.831 (0.141)	0.439 (0.018)
		Q_n	0.998 (0.002)	0.993 (0.004)	0.851 (0.133)	0.438 (0.018)
	Sample Covariance		0.999 (0.001)	0.777 (0.018)	0.225 (0.008)	0.204 (0.002)
Ledoit Wolf		0.211 (0.007)	0.211 (0.010)	0.201 (0.000)	0.201 (0.000)	
GLasso	Spearman	MAD	0.944 (0.011)	0.915 (0.037)	0.665 (0.049)	0.354 (0.052)
		S_n	0.954 (0.009)	0.922 (0.037)	0.687 (0.057)	0.392 (0.033)
		Q_n	0.957 (0.009)	0.922 (0.038)	0.701 (0.069)	0.396 (0.034)
	Kendall	MAD	0.925 (0.036)	0.864 (0.028)	0.614 (0.071)	0.343 (0.019)
		S_n	0.942 (0.029)	0.878 (0.025)	0.619 (0.070)	0.354 (0.061)
		Q_n	0.947 (0.027)	0.888 (0.023)	0.635 (0.062)	0.357 (0.062)
	Sample Covariance		0.971 (0.011)	0.484 (0.075)	0.218 (0.008)	0.205 (0.003)
Ledoit Wolf		0.235 (0.016)	0.208 (0.006)	0.201 (0.000)	0.201 (0.000)	

Table 12. Average MCC (with standard deviations) over 100 replications: Model 3

Estimation Methods ($n = 75, p = 100$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	0.657 (0.016)	0.632 (0.019)	0.523 (0.022)	0.457 (0.021)
		S_n	0.659 (0.017)	0.634 (0.019)	0.521 (0.021)	0.457 (0.021)
		Q_n	0.659 (0.017)	0.635 (0.018)	0.521 (0.022)	0.458 (0.023)
	Kendall	MAD	0.672 (0.031)	0.612 (0.044)	0.505 (0.020)	0.457 (0.018)
		S_n	0.672 (0.036)	0.619 (0.045)	0.506 (0.020)	0.455 (0.019)
		Q_n	0.674 (0.036)	0.616 (0.046)	0.505 (0.020)	0.456 (0.019)
	Sample Covariance		0.657 (0.065)	0.525 (0.020)	0.339 (0.034)	0.332 (0.030)
GLasso	Ledoit Wolf		0.449 (0.011)	0.438 (0.007)	0.440 (0.003)	0.440 (0.001)
	Spearman	MAD	0.555 (0.026)	0.531 (0.022)	0.476 (0.021)	0.439 (0.019)
		S_n	0.569 (0.029)	0.537 (0.027)	0.479 (0.020)	0.441 (0.018)
		Q_n	0.573 (0.030)	0.542 (0.029)	0.485 (0.019)	0.449 (0.017)
	Kendall	MAD	0.529 (0.023)	0.393 (0.062)	0.477 (0.020)	0.439 (0.017)
		S_n	0.531 (0.034)	0.399 (0.068)	0.479 (0.018)	0.438 (0.018)
		Q_n	0.537 (0.035)	0.401 (0.070)	0.479 (0.023)	0.445 (0.017)
DTrace	Sample Covariance		0.602 (0.019)	0.423 (0.023)	0.332 (0.021)	0.352 (0.029)
	Ledoit Wolf		0.451 (0.012)	0.435 (0.008)	0.439 (0.003)	0.441 (0.001)
Estimation Methods ($n = 150, p = 200$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	0.648 (0.007)	0.651 (0.008)	0.627 (0.013)	0.565 (0.015)
		S_n	0.648 (0.007)	0.651 (0.008)	0.627 (0.011)	0.565 (0.015)
		Q_n	0.648 (0.007)	0.651 (0.008)	0.628 (0.012)	0.565 (0.015)
	Kendall	MAD	0.660 (0.008)	0.659 (0.009)	0.642 (0.020)	0.552 (0.016)
		S_n	0.661 (0.008)	0.660 (0.009)	0.641 (0.019)	0.552 (0.016)
		Q_n	0.661 (0.008)	0.660 (0.009)	0.638 (0.017)	0.552 (0.016)
	Sample Covariance		0.763 (0.009)	0.624 (0.013)	0.372 (0.013)	0.381 (0.009)
GLasso	Ledoit Wolf		0.449 (0.006)	0.446 (0.007)	0.443 (0.001)	0.444 (0.000)
	Spearman	MAD	0.649 (0.012)	0.640 (0.023)	0.618 (0.015)	0.502 (0.017)
		S_n	0.654 (0.011)	0.649 (0.029)	0.625 (0.014)	0.516 (0.017)
		Q_n	0.659 (0.011)	0.656 (0.032)	0.629 (0.016)	0.522 (0.016)
	Kendall	MAD	0.662 (0.026)	0.680 (0.017)	0.611 (0.019)	0.500 (0.013)
		S_n	0.663 (0.025)	0.688 (0.017)	0.618 (0.021)	0.502 (0.024)
		Q_n	0.667 (0.024)	0.690 (0.018)	0.626 (0.017)	0.507 (0.026)
GLasso	Sample Covariance		0.680 (0.041)	0.537 (0.021)	0.358 (0.025)	0.360 (0.013)
	Ledoit Wolf		0.460 (0.010)	0.444 (0.006)	0.443 (0.001)	0.444 (0.000)

Table 13. Average KLL (with standard deviations) over 100 replications: Model 4

Estimation Methods ($n = 75, p = 100$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	28.340 (1.033)	30.128 (1.497)	38.034 (2.064)	48.474 (2.607)
		S_n	27.578 (1.001)	29.670 (1.376)	38.691 (1.814)	50.879 (2.502)
		Q_n	27.722 (0.878)	30.062 (1.230)	39.712 (1.728)	50.457 (2.563)
	Kendall	MAD	36.430 (1.287)	37.868 (1.171)	44.836 (1.444)	53.483 (1.848)
		S_n	36.021 (1.216)	37.764 (1.189)	46.429 (1.340)	57.057 (1.694)
		Q_n	36.560 (1.194)	38.743 (1.064)	47.837 (1.406)	57.166 (1.442)
	Sample Covariance		22.476 (2.133)	28.311 (1.824)	48.723 (2.313)	64.663 (2.257)
	Ledoit Wolf		44.723 (0.900)	48.260 (1.023)	59.068 (0.827)	67.756 (0.808)
GLasso	Spearman	MAD	39.944 (2.125)	41.807 (2.033)	47.875 (2.158)	56.397 (2.140)
		S_n	40.095 (1.814)	41.846 (1.717)	49.383 (2.041)	59.442 (1.955)
		Q_n	40.535 (1.815)	42.434 (1.538)	50.601 (1.894)	59.216 (1.631)
	Kendall	MAD	44.926 (1.935)	46.449 (1.861)	52.927 (2.093)	61.290 (2.452)
		S_n	44.764 (1.833)	46.471 (1.863)	54.797 (2.037)	64.664 (1.953)
		Q_n	45.268 (1.859)	47.471 (2.042)	55.927 (1.807)	64.339 (1.978)
	Sample Covariance		35.576 (1.940)	41.148 (1.942)	57.789 (1.542)	71.639 (1.387)
	Ledoit Wolf		48.164 (1.007)	51.283 (0.907)	60.757 (0.781)	69.183 (0.834)
Estimation Methods ($n = 150, p = 200$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	48.469 (0.709)	52.843 (0.875)	69.973 (1.036)	90.909 (1.312)
		S_n	48.469 (0.709)	52.843 (0.875)	69.973 (1.036)	90.909 (1.312)
		Q_n	48.475 (0.645)	53.309 (0.853)	71.133 (0.919)	89.780 (1.045)
	Kendall	MAD	58.556 (1.250)	62.265 (1.475)	77.236 (1.950)	94.669 (2.133)
		S_n	57.936 (1.165)	62.438 (1.406)	80.507 (1.898)	101.707 (2.050)
		Q_n	58.111 (1.118)	63.077 (1.303)	81.760 (1.746)	100.416 (1.813)
	Sample Covariance		36.860 (0.473)	50.177 (0.927)	90.373 (1.001)	122.664 (0.838)
	Ledoit Wolf		83.563 (2.115)	91.981 (1.457)	116.518 (1.275)	134.229 (0.715)
GLasso	Spearman	MAD	68.924 (2.359)	76.290 (0.621)	84.008 (1.904)	101.894 (1.292)
		S_n	68.946 (2.665)	76.451 (0.559)	90.009 (4.386)	109.868 (3.922)
		Q_n	68.360 (2.254)	76.466 (0.516)	90.536 (4.383)	106.598 (3.009)
	Kendall	MAD	79.297 (0.679)	81.072 (0.666)	94.357 (4.235)	109.460 (3.295)
		S_n	79.196 (0.604)	81.396 (0.612)	99.584 (1.601)	119.761 (2.585)
		Q_n	79.013 (0.554)	81.499 (0.561)	100.091 (1.546)	116.618 (3.700)
	Sample Covariance		61.664 (0.871)	74.875 (1.823)	106.104 (0.567)	133.574 (0.599)
	Ledoit Wolf		90.497 (1.335)	97.550 (1.081)	118.729 (1.095)	135.866 (1.015)

Table 14. Average ℓ_2 norm loss (with standard deviations) over 100 replications: Model 4

Estimation Methods ($n = 75, p = 100$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	2.618 (0.041)	2.677 (0.042)	2.866 (0.044)	3.022 (0.034)
		S_n	2.626 (0.033)	2.690 (0.036)	2.893 (0.034)	3.061 (0.031)
		Q_n	2.643 (0.031)	2.711 (0.033)	2.920 (0.031)	3.068 (0.030)
	Kendall	MAD	2.887 (0.030)	2.913 (0.027)	3.022 (0.025)	3.117 (0.022)
		S_n	2.893 (0.028)	2.923 (0.025)	3.049 (0.021)	3.157 (0.019)
		Q_n	2.908 (0.026)	2.945 (0.022)	3.070 (0.021)	3.165 (0.016)
	Sample Covariance		2.339 (0.103)	2.616 (0.064)	3.093 (0.035)	3.261 (0.017)
GLasso	Ledoit Wolf		3.091 (0.012)	3.139 (0.013)	3.249 (0.008)	3.312 (0.006)
	Spearman	MAD	3.010 (0.036)	3.039 (0.030)	3.112 (0.026)	3.193 (0.021)
		S_n	3.022 (0.029)	3.047 (0.024)	3.134 (0.023)	3.220 (0.017)
		Q_n	3.032 (0.028)	3.058 (0.021)	3.150 (0.021)	3.223 (0.015)
	Kendall	MAD	3.090 (0.026)	3.109 (0.024)	3.174 (0.021)	3.238 (0.019)
		S_n	3.094 (0.024)	3.114 (0.023)	3.194 (0.018)	3.263 (0.014)
		Q_n	3.102 (0.023)	3.128 (0.024)	3.205 (0.016)	3.264 (0.014)
DTrace	Sample Covariance		2.946 (0.038)	3.040 (0.030)	3.229 (0.013)	3.323 (0.008)
	Ledoit Wolf		3.140 (0.014)	3.176 (0.011)	3.263 (0.007)	3.321 (0.005)
Estimation Methods ($n = 150, p = 200$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	3.478 (0.030)	3.573 (0.031)	3.850 (0.024)	4.069 (0.026)
		S_n	3.482 (0.026)	3.591 (0.026)	3.900 (0.021)	4.140 (0.018)
		Q_n	3.493 (0.022)	3.611 (0.025)	3.927 (0.017)	4.143 (0.015)
	Kendall	MAD	3.770 (0.028)	3.835 (0.032)	4.053 (0.032)	4.227 (0.024)
		S_n	3.772 (0.027)	3.850 (0.030)	4.097 (0.029)	4.291 (0.021)
		Q_n	3.781 (0.025)	3.867 (0.027)	4.117 (0.025)	4.289 (0.019)
	Sample Covariance		2.924 (0.027)	3.503 (0.050)	4.279 (0.021)	4.541 (0.010)
GLasso	Ledoit Wolf		4.263 (0.021)	4.352 (0.013)	4.543 (0.008)	4.639 (0.003)
	Spearman	MAD	4.067 (0.032)	4.164 (0.007)	4.235 (0.019)	4.381 (0.010)
		S_n	4.076 (0.036)	4.172 (0.007)	4.296 (0.042)	4.437 (0.029)
		Q_n	4.072 (0.030)	4.175 (0.006)	4.304 (0.042)	4.420 (0.022)
	Kendall	MAD	4.212 (0.008)	4.229 (0.007)	4.344 (0.038)	4.444 (0.022)
		S_n	4.217 (0.007)	4.237 (0.007)	4.388 (0.014)	4.508 (0.016)
		Q_n	4.217 (0.007)	4.240 (0.006)	4.394 (0.013)	4.493 (0.024)
GLasso	Sample Covariance		3.957 (0.016)	4.160 (0.028)	4.461 (0.007)	4.618 (0.005)
	Ledoit Wolf		4.341 (0.013)	4.407 (0.009)	4.558 (0.007)	4.646 (0.005)

Table 15. Average ℓ_{spec} norm loss (with standard deviations) over 100 replications: Model 4

Estimation Methods ($n = 75, p = 100$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	0.818 (0.034)	0.843 (0.033)	0.891 (0.026)	0.918 (0.020)
		S_n	0.821 (0.027)	0.842 (0.029)	0.896 (0.023)	0.923 (0.017)
		Q_n	0.826 (0.023)	0.846 (0.026)	0.901 (0.021)	0.925 (0.015)
	Kendall	MAD	0.887 (0.017)	0.894 (0.019)	0.918 (0.015)	0.933 (0.014)
		S_n	0.887 (0.014)	0.895 (0.016)	0.922 (0.013)	0.938 (0.011)
		Q_n	0.890 (0.012)	0.900 (0.013)	0.925 (0.012)	0.940 (0.010)
	Sample Covariance		0.746 (0.041)	0.822 (0.046)	0.927 (0.018)	0.948 (0.010)
GLasso	Ledoit Wolf		0.922 (0.004)	0.928 (0.004)	0.945 (0.002)	0.953 (0.001)
	Spearman	MAD	0.909 (0.009)	0.913 (0.009)	0.926 (0.009)	0.940 (0.007)
		S_n	0.911 (0.007)	0.915 (0.007)	0.930 (0.008)	0.943 (0.006)
		Q_n	0.912 (0.007)	0.917 (0.006)	0.933 (0.007)	0.944 (0.006)
	Kendall	MAD	0.921 (0.007)	0.925 (0.007)	0.936 (0.006)	0.946 (0.007)
		S_n	0.921 (0.006)	0.926 (0.007)	0.939 (0.006)	0.950 (0.005)
		Q_n	0.923 (0.006)	0.928 (0.006)	0.941 (0.005)	0.950 (0.005)
DTrace	Sample Covariance		0.898 (0.009)	0.913 (0.009)	0.943 (0.007)	0.957 (0.005)
	Ledoit Wolf		0.928 (0.003)	0.933 (0.003)	0.947 (0.002)	0.955 (0.001)
Estimation Methods ($n = 150, p = 200$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	0.838 (0.014)	0.851 (0.013)	0.893 (0.010)	0.915 (0.009)
		S_n	0.837 (0.012)	0.853 (0.012)	0.896 (0.009)	0.921 (0.008)
		Q_n	0.838 (0.011)	0.855 (0.012)	0.898 (0.009)	0.922 (0.008)
	Kendall	MAD	0.863 (0.011)	0.871 (0.009)	0.903 (0.010)	0.925 (0.008)
		S_n	0.863 (0.009)	0.873 (0.008)	0.907 (0.010)	0.931 (0.008)
		Q_n	0.864 (0.008)	0.875 (0.007)	0.909 (0.009)	0.931 (0.007)
	Sample Covariance		0.791 (0.012)	0.845 (0.020)	0.918 (0.010)	0.944 (0.007)
GLasso	Ledoit Wolf		0.910 (0.003)	0.921 (0.003)	0.943 (0.002)	0.953 (0.001)
	Spearman	MAD	0.894 (0.006)	0.907 (0.004)	0.917 (0.005)	0.935 (0.004)
		S_n	0.895 (0.006)	0.908 (0.003)	0.924 (0.006)	0.940 (0.005)
		Q_n	0.895 (0.005)	0.908 (0.003)	0.925 (0.006)	0.939 (0.004)
	Kendall	MAD	0.910 (0.003)	0.912 (0.003)	0.927 (0.005)	0.940 (0.004)
		S_n	0.910 (0.002)	0.913 (0.003)	0.932 (0.003)	0.946 (0.003)
		Q_n	0.910 (0.002)	0.914 (0.003)	0.932 (0.003)	0.945 (0.003)
GLasso	Sample Covariance		0.884 (0.004)	0.905 (0.005)	0.937 (0.004)	0.953 (0.003)
	Ledoit Wolf		0.920 (0.002)	0.928 (0.002)	0.944 (0.001)	0.953 (0.001)

Table 16. Average ℓ_1 norm loss (with standard deviations) over 100 replications: Model 4

Estimation Methods ($n = 75, p = 100$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	1.717 (0.033)	1.743 (0.039)	1.803 (0.037)	1.842 (0.024)
		S_n	1.715 (0.029)	1.743 (0.034)	1.804 (0.032)	1.849 (0.022)
		Q_n	1.715 (0.028)	1.744 (0.034)	1.809 (0.028)	1.849 (0.018)
	Kendall	MAD	1.798 (0.018)	1.806 (0.021)	1.837 (0.017)	1.860 (0.017)
		S_n	1.795 (0.017)	1.806 (0.018)	1.842 (0.016)	1.866 (0.015)
		Q_n	1.798 (0.016)	1.810 (0.016)	1.845 (0.016)	1.866 (0.013)
	Sample Covariance		1.618 (0.057)	1.739 (0.061)	1.857 (0.024)	1.886 (0.012)
GLasso	Ledoit Wolf		1.853 (0.004)	1.864 (0.005)	1.886 (0.003)	1.897 (0.002)
	Spearman	MAD	1.830 (0.012)	1.838 (0.011)	1.855 (0.010)	1.871 (0.008)
		S_n	1.833 (0.009)	1.839 (0.009)	1.858 (0.008)	1.877 (0.009)
		Q_n	1.834 (0.008)	1.841 (0.007)	1.860 (0.008)	1.876 (0.007)
	Kendall	MAD	1.848 (0.009)	1.852 (0.008)	1.867 (0.008)	1.878 (0.007)
		S_n	1.848 (0.007)	1.852 (0.008)	1.870 (0.008)	1.882 (0.006)
		Q_n	1.849 (0.007)	1.855 (0.008)	1.872 (0.006)	1.883 (0.005)
	Sample Covariance		1.814 (0.011)	1.839 (0.012)	1.877 (0.008)	1.897 (0.007)
	Ledoit Wolf		1.864 (0.003)	1.871 (0.003)	1.888 (0.002)	1.899 (0.002)
Estimation Methods ($n = 150, p = 200$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	1.887 (0.025)	1.899 (0.026)	1.954 (0.019)	1.990 (0.018)
		S_n	1.886 (0.024)	1.901 (0.024)	1.960 (0.017)	1.999 (0.015)
		Q_n	1.887 (0.023)	1.903 (0.022)	1.964 (0.016)	1.999 (0.013)
	Kendall	MAD	1.927 (0.020)	1.932 (0.021)	1.973 (0.016)	2.000 (0.014)
		S_n	1.926 (0.017)	1.935 (0.018)	1.979 (0.016)	2.007 (0.012)
		Q_n	1.927 (0.015)	1.937 (0.017)	1.981 (0.014)	2.007 (0.011)
	Sample Covariance		1.831 (0.018)	1.893 (0.033)	1.995 (0.016)	2.027 (0.009)
GLasso	Ledoit Wolf		1.989 (0.005)	2.001 (0.004)	2.026 (0.002)	2.037 (0.001)
	Spearman	MAD	1.967 (0.010)	1.984 (0.008)	1.994 (0.008)	2.015 (0.007)
		S_n	1.968 (0.009)	1.985 (0.006)	2.002 (0.009)	2.021 (0.008)
		Q_n	1.967 (0.008)	1.985 (0.006)	2.002 (0.009)	2.020 (0.006)
	Kendall	MAD	1.987 (0.006)	1.989 (0.006)	2.007 (0.009)	2.020 (0.007)
		S_n	1.987 (0.005)	1.991 (0.005)	2.013 (0.006)	2.027 (0.005)
		Q_n	1.987 (0.005)	1.991 (0.005)	2.013 (0.006)	2.026 (0.006)
	Sample Covariance		1.954 (0.007)	1.982 (0.009)	2.017 (0.007)	2.036 (0.005)
	Ledoit Wolf		2.003 (0.002)	2.011 (0.002)	2.028 (0.002)	2.038 (0.001)

Table 17. Average Specificity (with standard deviations) over 100 replications: Model 4

Estimation Methods ($n = 75, p = 100$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	0.984 (0.003)	0.985 (0.004)	0.987 (0.005)	0.991 (0.006)
		S_n	0.984 (0.003)	0.985 (0.004)	0.987 (0.004)	0.991 (0.006)
		Q_n	0.984 (0.003)	0.985 (0.003)	0.987 (0.004)	0.991 (0.005)
	Kendall	MAD	0.996 (0.001)	0.996 (0.003)	0.996 (0.001)	0.995 (0.002)
		S_n	0.996 (0.001)	0.996 (0.002)	0.996 (0.001)	0.996 (0.001)
		Q_n	0.996 (0.001)	0.996 (0.001)	0.997 (0.001)	0.997 (0.001)
	Sample Covariance		0.977 (0.009)	0.980 (0.007)	0.989 (0.005)	0.993 (0.004)
Ledoit Wolf		1.000 (0.000)	1.000 (0.000)	1.000 (0.000)	1.000 (0.000)	
GLasso	Spearman	MAD	0.975 (0.008)	0.977 (0.007)	0.980 (0.006)	0.983 (0.005)
		S_n	0.978 (0.006)	0.979 (0.006)	0.982 (0.006)	0.984 (0.005)
		Q_n	0.978 (0.007)	0.978 (0.005)	0.983 (0.005)	0.985 (0.004)
	Kendall	MAD	0.982 (0.005)	0.983 (0.004)	0.984 (0.005)	0.986 (0.004)
		S_n	0.982 (0.005)	0.983 (0.005)	0.986 (0.004)	0.988 (0.004)
		Q_n	0.982 (0.005)	0.984 (0.005)	0.986 (0.004)	0.989 (0.003)
	Sample Covariance		0.974 (0.008)	0.979 (0.007)	0.988 (0.004)	0.992 (0.003)
Ledoit Wolf		0.997 (0.001)	0.998 (0.001)	0.999 (0.001)	1.000 (0.000)	
Estimation Methods ($n = 150, p = 200$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	0.992 (0.001)	0.992 (0.001)	0.993 (0.001)	0.993 (0.001)
		S_n	0.992 (0.001)	0.993 (0.001)	0.993 (0.001)	0.994 (0.001)
		Q_n	0.993 (0.001)	0.993 (0.001)	0.993 (0.001)	0.994 (0.001)
	Kendall	MAD	0.997 (0.001)	0.997 (0.001)	0.997 (0.001)	0.997 (0.001)
		S_n	0.997 (0.001)	0.997 (0.001)	0.997 (0.001)	0.998 (0.001)
		Q_n	0.997 (0.001)	0.997 (0.001)	0.998 (0.001)	0.998 (0.000)
	Sample Covariance		0.980 (0.002)	0.986 (0.002)	0.994 (0.001)	0.996 (0.001)
Ledoit Wolf		1.000 (0.000)	1.000 (0.000)	1.000 (0.000)	1.000 (0.000)	
GLasso	Spearman	MAD	0.984 (0.003)	0.990 (0.001)	0.986 (0.002)	0.990 (0.001)
		S_n	0.985 (0.003)	0.990 (0.001)	0.989 (0.004)	0.991 (0.003)
		Q_n	0.984 (0.003)	0.990 (0.001)	0.989 (0.004)	0.991 (0.002)
	Kendall	MAD	0.991 (0.001)	0.990 (0.001)	0.991 (0.004)	0.990 (0.002)
		S_n	0.991 (0.001)	0.990 (0.001)	0.993 (0.002)	0.994 (0.002)
		Q_n	0.991 (0.001)	0.990 (0.001)	0.993 (0.002)	0.993 (0.003)
	Sample Covariance		0.986 (0.002)	0.991 (0.002)	0.994 (0.001)	0.996 (0.001)
Ledoit Wolf		0.999 (0.001)	0.999 (0.000)	1.000 (0.000)	1.000 (0.000)	

Table 18. Average Sensitivity (with standard deviations) over 100 replications: Model 4

Estimation Methods ($n = 75, p = 100$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	0.072 (0.005)	0.068 (0.008)	0.057 (0.009)	0.041 (0.010)
		S_n	0.072 (0.005)	0.068 (0.007)	0.058 (0.007)	0.042 (0.010)
		Q_n	0.072 (0.004)	0.068 (0.006)	0.058 (0.007)	0.044 (0.009)
	Kendall	MAD	0.052 (0.003)	0.050 (0.004)	0.043 (0.003)	0.037 (0.002)
		S_n	0.051 (0.002)	0.050 (0.004)	0.043 (0.002)	0.036 (0.002)
		Q_n	0.051 (0.002)	0.049 (0.002)	0.043 (0.002)	0.036 (0.002)
	Sample Covariance		0.083 (0.015)	0.074 (0.011)	0.047 (0.008)	0.035 (0.006)
GLasso	Ledoit Wolf		0.031 (0.003)	0.028 (0.003)	0.023 (0.001)	0.022 (0.001)
	Spearman	MAD	0.070 (0.011)	0.065 (0.009)	0.055 (0.008)	0.046 (0.006)
		S_n	0.066 (0.010)	0.063 (0.009)	0.054 (0.009)	0.045 (0.006)
		Q_n	0.066 (0.011)	0.065 (0.007)	0.053 (0.009)	0.044 (0.006)
	Kendall	MAD	0.060 (0.008)	0.057 (0.007)	0.049 (0.006)	0.043 (0.006)
		S_n	0.059 (0.008)	0.057 (0.009)	0.048 (0.006)	0.041 (0.005)
		Q_n	0.059 (0.008)	0.056 (0.008)	0.047 (0.006)	0.040 (0.005)
DTrace	Sample Covariance		0.076 (0.012)	0.064 (0.010)	0.043 (0.006)	0.033 (0.004)
	Ledoit Wolf		0.030 (0.003)	0.028 (0.002)	0.024 (0.001)	0.022 (0.001)
Estimation Methods ($n = 150, p = 200$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	0.043 (0.001)	0.041 (0.001)	0.034 (0.001)	0.028 (0.002)
		S_n	0.042 (0.001)	0.041 (0.001)	0.034 (0.001)	0.028 (0.001)
		Q_n	0.042 (0.001)	0.040 (0.001)	0.033 (0.001)	0.028 (0.001)
	Kendall	MAD	0.035 (0.001)	0.034 (0.001)	0.028 (0.001)	0.023 (0.001)
		S_n	0.035 (0.001)	0.033 (0.001)	0.028 (0.001)	0.023 (0.001)
		Q_n	0.035 (0.001)	0.033 (0.001)	0.028 (0.001)	0.023 (0.001)
	Sample Covariance		0.055 (0.002)	0.045 (0.002)	0.028 (0.001)	0.021 (0.001)
GLasso	Ledoit Wolf		0.022 (0.003)	0.020 (0.002)	0.013 (0.001)	0.012 (0.000)
	Spearman	MAD	0.049 (0.005)	0.037 (0.002)	0.041 (0.004)	0.032 (0.002)
		S_n	0.049 (0.006)	0.038 (0.002)	0.037 (0.008)	0.030 (0.006)
		Q_n	0.049 (0.005)	0.038 (0.002)	0.037 (0.008)	0.032 (0.004)
	Kendall	MAD	0.038 (0.002)	0.038 (0.002)	0.033 (0.007)	0.031 (0.004)
		S_n	0.038 (0.002)	0.039 (0.002)	0.030 (0.002)	0.027 (0.003)
		Q_n	0.038 (0.002)	0.039 (0.002)	0.030 (0.002)	0.027 (0.005)
GLasso	Sample Covariance		0.048 (0.003)	0.037 (0.004)	0.027 (0.001)	0.022 (0.001)
	Ledoit Wolf		0.020 (0.002)	0.018 (0.001)	0.014 (0.001)	0.012 (0.000)

Table 19. Average MCC (with standard deviations) over 100 replications: Model 4

Estimation Methods ($n = 75, p = 100$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	0.138 (0.010)	0.134 (0.010)	0.122 (0.010)	0.106 (0.011)
		S_n	0.138 (0.010)	0.135 (0.010)	0.123 (0.010)	0.110 (0.010)
		Q_n	0.138 (0.011)	0.135 (0.010)	0.124 (0.010)	0.110 (0.010)
	Kendall	MAD	0.146 (0.008)	0.142 (0.008)	0.130 (0.009)	0.115 (0.008)
		S_n	0.147 (0.007)	0.143 (0.007)	0.132 (0.008)	0.119 (0.007)
		Q_n	0.147 (0.007)	0.143 (0.007)	0.132 (0.008)	0.120 (0.007)
	Sample Covariance		0.135 (0.010)	0.129 (0.012)	0.110 (0.012)	0.099 (0.011)
	Ledoit Wolf		0.125 (0.006)	0.120 (0.006)	0.109 (0.003)	0.105 (0.002)
GLasso	Spearman	MAD	0.107 (0.012)	0.105 (0.012)	0.094 (0.014)	0.082 (0.015)
		S_n	0.108 (0.011)	0.106 (0.011)	0.097 (0.013)	0.087 (0.013)
		Q_n	0.109 (0.011)	0.107 (0.012)	0.098 (0.011)	0.089 (0.013)
	Kendall	MAD	0.108 (0.013)	0.106 (0.012)	0.094 (0.014)	0.086 (0.011)
		S_n	0.108 (0.011)	0.106 (0.012)	0.098 (0.012)	0.090 (0.011)
		Q_n	0.109 (0.011)	0.107 (0.011)	0.098 (0.013)	0.093 (0.011)
	Sample Covariance		0.113 (0.012)	0.109 (0.012)	0.097 (0.011)	0.091 (0.011)
	Ledoit Wolf		0.111 (0.007)	0.109 (0.006)	0.106 (0.005)	0.104 (0.003)
Estimation Methods ($n = 150, p = 200$)	Input matrices		Contamination			
	Correlation	Scale	0%	1%	5%	10%
DTrace	Spearman	MAD	0.113 (0.005)	0.109 (0.005)	0.096 (0.005)	0.083 (0.006)
		S_n	0.114 (0.005)	0.110 (0.005)	0.096 (0.005)	0.085 (0.006)
		Q_n	0.114 (0.005)	0.110 (0.005)	0.097 (0.005)	0.086 (0.006)
	Kendall	MAD	0.120 (0.003)	0.116 (0.004)	0.104 (0.004)	0.093 (0.005)
		S_n	0.120 (0.003)	0.117 (0.004)	0.105 (0.003)	0.095 (0.004)
		Q_n	0.121 (0.003)	0.117 (0.004)	0.106 (0.003)	0.095 (0.004)
	Sample Covariance		0.092 (0.006)	0.093 (0.006)	0.088 (0.006)	0.080 (0.005)
	Ledoit Wolf		0.106 (0.006)	0.101 (0.004)	0.084 (0.003)	0.078 (0.001)
GLasso	Spearman	MAD	0.096 (0.006)	0.093 (0.005)	0.085 (0.007)	0.077 (0.006)
		S_n	0.096 (0.006)	0.093 (0.005)	0.085 (0.007)	0.079 (0.005)
		Q_n	0.096 (0.006)	0.095 (0.005)	0.087 (0.006)	0.081 (0.005)
	Kendall	MAD	0.095 (0.007)	0.094 (0.006)	0.087 (0.006)	0.076 (0.006)
		S_n	0.096 (0.006)	0.094 (0.005)	0.088 (0.005)	0.080 (0.006)
		Q_n	0.096 (0.005)	0.094 (0.005)	0.089 (0.005)	0.082 (0.006)
	Sample Covariance		0.099 (0.006)	0.096 (0.004)	0.086 (0.006)	0.079 (0.005)
	Ledoit Wolf		0.095 (0.003)	0.093 (0.003)	0.083 (0.002)	0.078 (0.001)