Reference:

Li, H.-C., & Stylianides, A. J. (accepted for publication). An examination of the roles of the teacher and students during a problem-based learning intervention: lessons learned from a study in a Taiwanese primary mathematics classroom. *Interactive Learning Environments.*

An examination of the roles of the teacher and students during a problem-based learning intervention: lessons learned from a study in a Taiwanese primary mathematics classroom

Hui-Chuan Li
Sultan Hassanal Bolkiah Institute of Education, Universiti Brunei Darussalam, Brunei
huichuan.li@ubd.edu.bn

Andreas J. Stylianides
Faculty of Education, University of Cambridge, UK
as899@cam.ac.uk

The benefits of problem-based learning (PBL) to student learning have prompted researchers to investigate this pedagogical approach over the past few decades. However, little research has examined how PBL can be applied to mathematics learning and teaching, especially in countries like Taiwan, where the majority of teachers are accustomed to lecture methods and students are used to this style of teaching. This study examines the actions of a teacher and her class of 35 fifth-grade students (10–11-year-olds) as they tried to take on and respond to the demands of their new roles as "facilitator" and "constructors", respectively, during a one-year PBL intervention in a Taiwanese mathematics classroom. Our findings provide insights into classroom participants' role transition, from a customary role to a new role, when engaging with PBL. We identify an interrelationship between the teacher and student roles and discuss implications for the implementation of PBL at the primary education level.

Keywords: Problem-based learning (PBL); primary education; mathematics; facilitator; constructor; role transition in PBL

Introduction

Mathematics teaching in typical classrooms has been criticised for being teacher-centred and content-oriented, with students being highly passive in their learning and having difficulties in constructing meaning and understanding (e.g., Schoenfeld & Kilpatrick, 2013). Many studies highlight the importance of constructivist perspectives on learning and teaching (Cobb & Yackel, 2011). One of the most popular pedagogies that challenge traditional lecture methods is problem-based learning (PBL) (Huang, Huang, Wu, Chen, & Chang, 2015). A key feature of PBL is the teacher's role as "facilitator" rather "instructor", and the students' role as "constructors" rather "recipients" (Barrows & Tamblyn, 1980). The roots of PBL can be traced back to Dewey's (1910, 1944) ideas of inquiry-based learning and can also be found in Piaget's (1954, 1965) cognitive constructivism and Vygotsky's (1962, 1978) social constructivism.

PBL has been one of the most influential pedagogical innovations since Barrows and his colleagues first used it in medical education in the 1960s (Dochy, Segers, Van den Bossche, & Gijbels, 2003; Gijbels, Dochy, Van den Bossche, & Segers, 2005; Hmelo-Silver, Duncan, & Chinn, 2007; Smith & Hung, in press). Over the past few decades, PBL, which stresses a problem-driven content structure, a student-centred approach and an elaboration of knowledge through social interaction (Barrows & Tamblyn, 1980), has seen a huge growth in its various adaptions and across many subject areas (Wijnia, Loyens, van Gog, Derous, & Schmidt, 2014). It has been "one of the few curriculum-wide educational innovations surviving the 60s" (Schmidt, Van der Molen, Te Winkel, & Wijnen, 2009, p. 228).

Research has shown that students who received pedagogy that aligned with PBL performed better in reasoning, communication, long-term retention and ability to apply new material, than students who received traditional lecture pedagogies (Wijnen, Loyens, & Schaap, 2015). The benefits of PBL to student learning have prompted researchers to investigate its potential "for much more widespread use in K–12 [ages 5-18] education" (Wirkala & Kuhn, 2011, p. 1159). However, its implementation is still rather scattered and unsystematic in K-12 educational settings (Hung, 2011). Smith and Hung (in press) also noted that "Taiwanese studies using PBL outside a medical context are sparse".

One reason for which PBL is rather uncommon in mathematics and other subject areas is that lessons, which encourage or allow students to engage in enquiry-based activities and discussion about what they are learning, are difficult to teach, even for experienced teachers or teachers who have aspirations for student-centred pedagogy (Stylianides & Stylianides, 2014). Also, and partly due to the teachers' responsibility to keep order in the classroom and cover the curriculum, primary school teachers in Taiwan and elsewhere often feel challenged by and reluctant to use group work or create an interactive learning environment in their classrooms (Weinstein, Romano, & Mignano, 2010), particularly in mathematics classrooms (Schoenfeld & Kilpatrick, 2013).

Research has focused on PBL's theoretical conception and students' learning outcomes and has paid less attention to understanding how teachers and students interpret or adapt to the new roles expected of them during the introduction of PBL in their classrooms. Without more light cast on this issue, teacher professional development will be limited in its potential to support teachers for the specific demands of PBL. There is a pressing need "to contribute further interactional data and analysis on PBL-in-action to support theory building" (Bridges, Botelho, Green, & Chau, 2012, p. 99). This paper takes a step towards addressing this research gap by reporting findings from a study that investigated the actions of the teacher and the students in a Taiwanese primary mathematics classroom as they tried to take on and respond to the demands of their new roles as "facilitator" and "constructors", respectively, during a one-year PBL intervention.

Developing an implementation model of PBL for this study

PBL perspectives on teaching and learning

Over the past three decades, various models have been developed and implemented to meet specific instructional goals or subjects (Barrows & Myers, 1993; Huang, Huang, Wu, Chen, & Chang, 2015; Hung, 2011; Savery & Duffy, 1995). For example, Barrows and Myers (1993) proposed five steps: starting a new class, starting a new problem, problem follow-up, performance presentation, and after conclusion of problem; and Huang et al. (2015) proposed five: problem posing, problem analysis and group division of the students, problem solving, results from group interaction, and reflection and evaluation.

These variations of PBL models involved several common features that can be divided into three areas: (i) the problem-driven content structure, (ii) the inquiry-based collaborative learning process, and (iii) the student-centred approach.

The problem-driven content structure

In PBL, the content is organised as a problem or a series of problems (Barrows & Tamblyn, 1980). The features of a good PBL problem generally involves ill-structured, contextualised and real life situations (Gijbels et al., 2005; Goodnough & Hung, 2008). An ill-structured problem helps provide opportunities for students to examine the problem from multiple perspectives and is not a well-defined question that can easily be solved with prior content knowledge (Torp & Sage, 2002). The relevance, authenticity and complexity of the problem embedded in the real-life context can help students construct "their own knowledge in a context which is similar to the context in which they would apply that knowledge" (Savery & Duffy, 2001, p. 14).

Problem solving has been a focus in mathematics education for many years (Stylianides & Stylianides, 2014). Not only is teaching students to solve problems important to them learning mathematics, learning about mathematics through problem solving is equally important (Lampert, 2001). However, at school, problem solving is often based on a narrower spectrum of story or word problems, which function more like an exercise for students to perform rather than as a challenge for them to solve real-life problem situations (Singer & Voica, 2013). This problem-driven feature of PBL distinguishes it "from other related instructional methods that are not necessarily problem focused, such as project-based learning […], inquiry learning, and cooperative learning" (Wirkala & Kuhn, 2011, p. 1158).

The inquiry-based collaborative learning process

Central to PBL are social constructivist perspectives (Barrows & Tamblyn, 1980), according to which learners are expected to be "interactive agents in communicative, socially situated relationships" (Vygotsky, 1962, p. 20). A key feature of learning in social constructivism is the creation of a "Zone of Proximal Development" (ZPD) (Vygotsky, 1962). The creation of the ZPD is, as suggested by Goos (2004, p. 282), "a process of negotiating personal meanings and comparing these with conventional interpretations from the community".

The social interactions between the learners and the teacher, and among the learners themselves, that typically happen during small-group work and whole-class discussions in a PBL environment can lead to negotiation and co-construction of students' ZPD (Wertsch, 2007). The inquiry-based collaborative learning process, therefore, is essential in transforming a learner's potential cognitive development to an actual development (Moll, 2014). This, for students, is a developmental process that largely lies in collaboration and engagement, as well as in the ability to do so (Vardi, & Ciccarelli, 2008). Teachers play an important role in developing a high degree of group collaboration; the more classroom conditions the teacher can create for students to ask questions, express opinions and pursue interests, the more high-quality engagement in collaboration the students will display (Reeve, 2012).

Student-centred approach

In PBL, teachers are expected to follow a student-centred approach to teaching, whereby students have opportunities to initiate and discuss with each other their own ideas (Barrows &

Myers, 1993). These opportunities, placed in the context of a problem-solving situation and small-group collaborative work, can allow students to “generate a common frame of reference in order to coordinate their actions and the ways they explain their actions and proposals to each other” (Kanselaar, Erkens, Jaspers, & Schijf, 2001, p. 126).

The shift from traditional teacher-centred approaches to interactive student-centred approaches, plays a critical role in developing students’ reasoning, critical thinking, creativity and argumentation skills (Beaumont, Savin-Baden, Conradi, & Poulton, 2014; Torp & Sage, 2002). Hmelo-Silver et al. (2007, p. 101) also noted that “PBL environments make disciplinary strategies explicit in students’ interactions with the tasks”. Consequently, when “students’ knowledge has been elaborated more, they have better recall of that knowledge” (Dochy et al., 2003, p. 545).

The PBL model used in this study

In line with the previous sections, an implementation model for this PBL intervention was formulated and is summarised in Figure 1. This PBL model includes (i) the features of PBL approaches, (ii) the features of PBL problems and (iii) the procedure of PBL. This model served as a theoretical basis for designing and implementing this intervention.

In considering that teachers can feel challenged by many steps in the procedure of implementing PBL in their classrooms, the simple form of Lubienski’s (2000) “Launch, Explore, Summarize” was adapted for this study. Also, in line with Barrows and Myers (1993) and Huang et al. (2015), the typical procedure for carrying out a PBL problem in the classroom involved three main steps: (i) launching the problem – the teacher launched a PBL problem which served as a stimulus for students to work in groups, communicate their ideas and take decisions; (ii) exploring the problem through small-group discussion – students were given opportunities to work on the problem together with their group members and without being led by the teacher; and (iii) closing the problem by conducting a whole-class discussion – after the small-group discussion, the teacher facilitated a whole-class discussion for students to share their work on the problem and to highlight key issues.

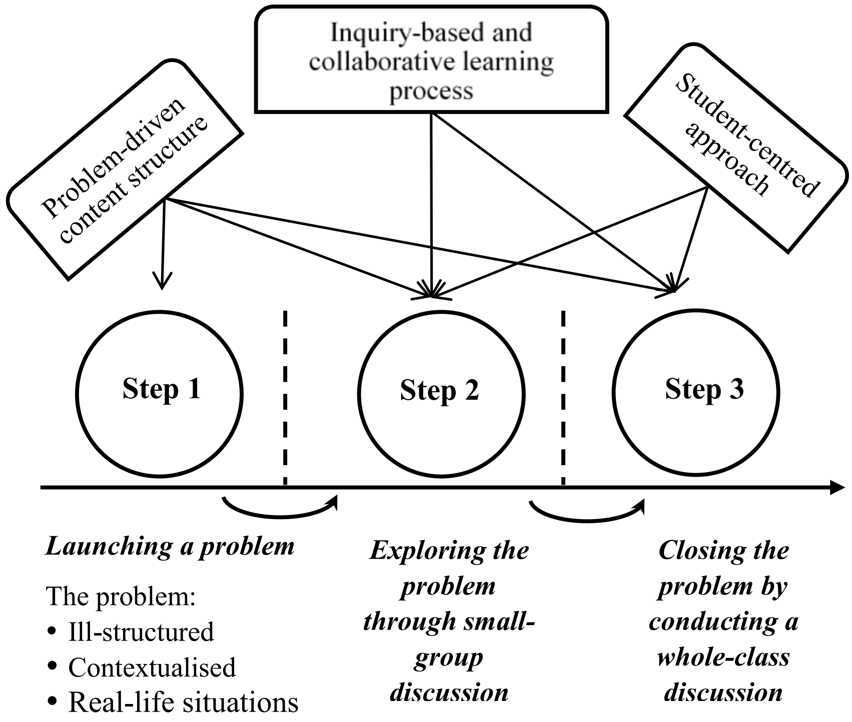

Figure 1. The PBL model used in this study

In summary, following the PBL model above, a typical lesson in this study started with an ill-structured and contextualised problem that reflected a real-life situation and could not easily be solved with prior content knowledge (Cunningham & Corderio, 2000). Students were expected to engage in the problem solving process through an inquiry-based and collaborative learning process, while the teacher was expected to adopt a student-centred approach and support students' learning by giving them opportunities for exploration, articulation and reflection. Classroom interactions were intended to lead students towards a genuine commitment to engage with one another in solving the PBL problem (Barrett, 2010).

Aim and research questions

We should clarify that our aim with this intervention was not to directly control the classroom participants' behaviours in order to measure their roles in PBL. Rather, we aimed to support the participating teacher in understanding what a PBL environment involved, equip her with appropriate PBL problems, and observe what actually happened during the implementation of these problems in the classroom. We followed Thompson and William's (2008) suggestion for a "tight but loose" framework, with the intention of understanding "[a] complex social phenomen[on]" (Yin, 2003, p. 2), in this case, the implementation of a pedagogical innovation in a primary mathematics classroom. Thompson and William (2008) also noted that the "tight but loose" framework affords effective teacher professional development, as it involves teachers in active participation rather than in the form of one-day workshops.

With regard to its "tight" part, the intervention was designed by synthesising both the specialised educational literature on mathematics teaching and learning and the more general literature on PBL. With regard to its "loose" part, the intervention offered flexibility to the teacher to make any changes she considered necessary to the design of the problems and to manage and control the implementation of these problems in her classroom. In doing so, we sought to understand how the participants interpreted and enacted their roles in PBL and, in particular, to examine the extent to which there were changes in the teacher's role from instructor to facilitator and in the students' role from recipients to constructors. Specifically, we aimed to address the following two research questions:

1. What teacher and student actions were observed relating to the instructor and facilitator roles for the teacher, and the recipient and constructor roles for the students, during the PBL intervention?

2. What changes, if any, were observed in the roles of the teacher and students during the PBL intervention?

Methods

The research participants were a Grade 5 teacher, Miss Lee (a pseudonym), and her class of 35 students (19 boys and 16 girls). This intervention focused on the area of fractions, which is recognised to be a hard-to-learn and hard-to-teach topic in primary mathematics in many countries (Charalambous & Pitta-Pantazi, 2007) including Taiwan (Li, 2014).

The intervention was designed by the first author and implemented by Miss Lee who, like many primary school teachers in Taiwan, had no prior experience with PBL. This was confirmed by the first author's lesson observations in Miss Lee's class prior to the implementation of the intervention and was consistent with the typical lecture methods of mathematics teaching in Taiwan that are reported in the literature (Chin & Lin, 2013). Miss Lee voluntarily participated in this intervention because she wanted (i) to change her teaching practice in a way that approximated the PBL perspectives and (ii) to help her students learn about fractions in more meaningful ways. Miss Lee's case gave us a good opportunity to investigate whether, or how, a traditionally trained teacher and her class of fifth-grade students changed their customary ways to adapt to the new roles expected in PBL. A four-week period of preparation with the teacher was carried out before the implementation of the intervention. During the preparation phase, the teacher and the first author discussed the features of PBL, the purpose and structure of the intervention, the objectives that underpinned the PBL problems, and other details. The first author also met with the teacher before and after each PBL lesson to provide her with ongoing opportunities for reflection and learning.

Through synthesis of the relevant literature on fractions and on knowledge of the Taiwanese curriculum relating to fractions, fifteen fractions-related, ill-structured and contextualised PBL problems were designed to encompass the learning objectives of fractions (an example of PBL problems can be found in Appendix A). The intervention comprised 19 lessons that spread over a school year. All of the PBL lessons were observed by the first author and were recorded using three sets of videotaping equipment, which were placed at the front, back right and back centre sides of the room. The teacher's and students' actions were examined during the intervention, particularly the extent to which their actions shifted, respectively, from the instructor role and recipient role, as it was customary in the class prior to the intervention, to the facilitator role and constructor role, as it is expected in PBL environments (cf. Reynolds & Hancock, 2010).

Creswell's (2012) "inductive coding" process was adopted to analyse the classroom observation data. Four steps were undertaken to codify the data: (i) organising the raw data for analysis; (ii) identifying and labelling; (iii) reducing; and (iv) summarising categories. Accordingly, a coding scheme for the teacher's and students' actions during small-group and whole-class discussions was developed (Table 1), which was used to categorise those actions according to whether they related to (i) a facilitator or an instructor role for the teacher and (ii) a recipient or a constructor role for the students. A preliminary version of the coding scheme was applied throughout all 19 lessons and was revised to better fit the data before it took its final form as shown in Table 1.

Table 1. Coding scheme for the teacher and student roles during small-group and whole-class discussions

Categories of teacher actions relating to different roles		Categories of student actions relating to different roles	
Key category	**Specific actions within each category**	**Key category**	**Specific actions within each category**
During small-group discussion		***During small-group discussion***	
▪ Teacher actions relating to an instructor role	▪ Directly give students correct answers. ▪ Demonstrate solutions. ▪ Ask for answers, rather than explanations. ▪ Rush to finish group work/not allow enough group work time.	▪ Student actions relating to a recipient role	▪ Keep asking the teacher for direction. ▪ Make little effort to solve the problem but wait for the teacher's instruction. ▪ Rely on only one or two group members to solve the problem. ▪ Present off-task behaviour.
▪ Teacher actions relating to a facilitator role	▪ Encourage students to further think. ▪ Allow students to make mistakes. ▪ Listen to students' explanations rather than merely the answers. ▪ Value group work discussion.	▪ Student actions relating to a constructor role	▪ Ask group members for help to understand what was being discussed. ▪ Help other group members to engage with discussion. ▪ Evaluate the group solution to reach a consensus. ▪ Create a positive group work situation.
During whole-class discussion		***During whole-class discussion***	
▪ Teacher actions relating to an instructor role	▪ Provide whole-class direct instruction to demonstrate solutions. ▪ Not allow enough time for groups to present their solutions. ▪ Refer to mathematical algorithms to finish discussion. ▪ Use yes-no as an approach to explicitly or implicitly guide students to solutions.	▪ Student actions relating to a recipient role	▪ Read out answers without explanations. ▪ Respond to the teacher with yes-no answers when she used a yes-no strategy to direct the discussion. ▪ Mainly listen to the teacher's instruction. ▪ Put little effort into preparing a presentation, relying on the teacher's help.
▪ Teacher actions relating to a facilitator role	▪ Allow enough time for groups to present their solutions. ▪ Help initiate a new discussion. ▪ Encourage groups to reflect on what they have learned. ▪ Articulate and stress the advantages of discussion.	▪ Student actions relating to a constructor role	▪ Try to explain their solutions to the class, rather than simply read out their answers. ▪ Show a willingness (e.g. raising their hands) to share ideas and provide feedback. ▪ Show less reliance on the teacher to help present solutions.

Results and discussion

In this section we report the frequencies we obtained based on the coding scheme (Table 1) of the teacher actions relating to an instructor or a facilitator role and of the student actions relating to a recipient or a constructor role during small-group and whole-class discussions. Also, with a focus on the patterns in the teacher and student roles when engaging with PBL during the course of the intervention, we discuss implications for implementing PBL at the primary education level.

The teacher's and students' roles during small-group discussions

Figure 2 shows the frequencies of the teacher actions relating to an instructor or a facilitator role and of the student actions relating to a recipient or a constructor role during small-group discussions. There is a clear association between the roles of the teacher and those of the students; for example, the frequencies of the facilitator for the teacher and constructor for the students both presented a U-shaped curve in Figure 2. The teacher played more a facilitative role in the beginning part of the intervention (lessons 1-5), but her commitment to that role declined in the middle part (lessons 6-12) when she reverted to more directive teaching. Yet, in the last part of the intervention (lessons 13-19) the teacher resumed again a more facilitative role. Likewise, the students played more a constructor role in the beginning part of the intervention, which declined gradually in the middle part at the expense of an increasing recipient role. In the last part of the intervention the students' role became again predominantly that of a constructor.

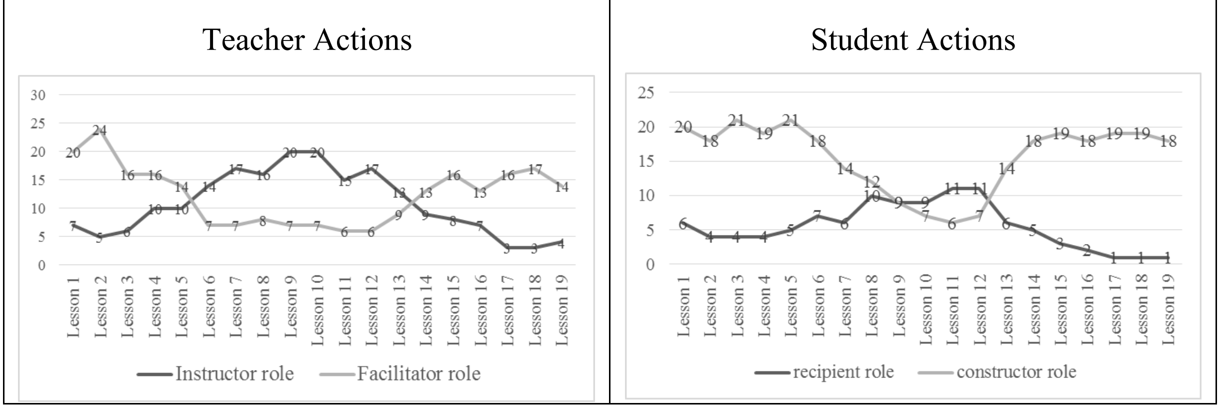

Figure 2. Frequencies of the teacher and student actions during small-group discussions

The similar U-shaped patterns of the facilitator and constructor roles for the teacher and students, respectively, showed the interconnection between their roles during the implementation of the PBL intervention. The frequencies of the facilitator and constructor roles declined to their lowest during the middle part of the intervention. This decline might be explained by our prior findings from the analysis of the challenges faced by the teacher during the implementation of the intervention (Li, 2012): there were increased incidents of student misbehaviour in the middle part that appeared to have triggered the teacher's shift towards an instructor role, with a parallel shift in students' role towards a recipient role.

The shift observed in the teacher's role during the middle part of the intervention echoes the findings of Kirschner, Sweller and Clark (2006) that it is tempting for teachers to give up in the face of the challenges they encounter when implementing PBL. On a positive note, by the end of the intervention the teacher developed, or changed, her classroom management strategies thus becoming more successful in dealing with students' disruptive behaviour (Li, 2012; Li & Tsai, in press). This helped alleviate some of the challenges faced by the teacher during the middle part and presumably explains the reversal in the teacher's role back to the facilitator role towards the end of the intervention, again with a parallel shift in students' role back to the constructor role.

The teacher's and students' roles during whole-class discussions

Figure 3 shows the frequencies of the teacher actions relating to an instructor or a facilitator role and of the student actions relating to a recipient or a constructor role during whole-class discussions.

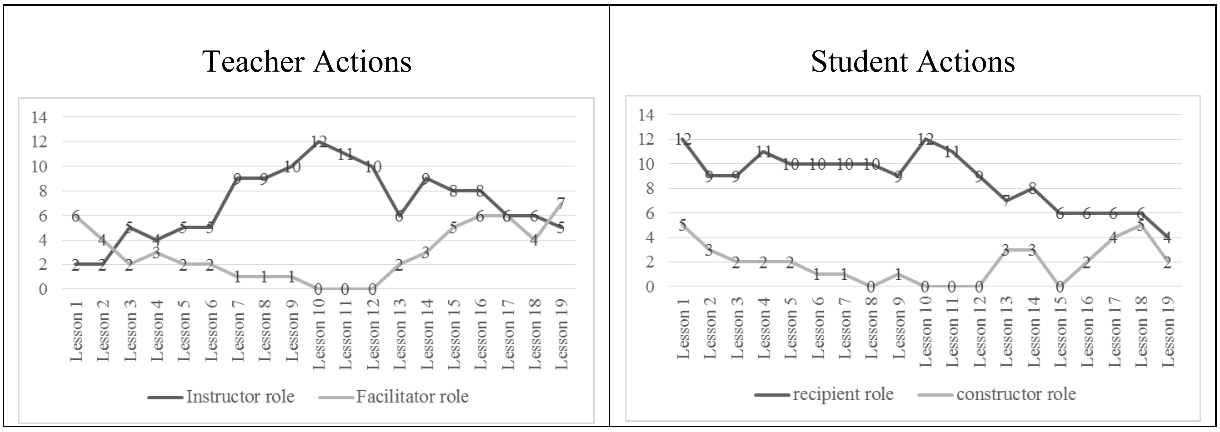

Figure 3. Frequencies of the teacher and student actions during whole-class discussions

On the whole the teacher played a predominantly instructor role, with the facilitator role being marginally more prominent during only three lessons (1, 2 and 19). Like it happened during small-group discussions (Figure 2), the teacher made a positive start with actions during whole-group discussion that tended to align with PBL. Yet, in the middle part of the intervention she reverted to a role that was predominantly (and, in some lessons, exclusively) that of an instructor. It is interesting to note, though, that in the last part of the intervention the teacher started to assume an increasingly facilitator role, following a similar trend as in small-group discussions. The frequency of the student actions relating to the recipient role for all lessons was higher than the frequency of those relating to the constructor role, suggesting that the students tended to be more recipients than constructors during whole-class discussions. This trend matched the previous observation that the role of the teacher during whole-class discussions was predominantly that of an instructor.

In particular, the nil frequency of both the facilitator role for teacher and the constructor role for the students during lessons 10-12 implies that, when the role of the teacher was exclusively that of an instructor, the role of the students was also exclusively that of a recipient (Figure 3). However, when the teacher started from lesson 13 onwards to assume an increasingly facilitator role during whole-class discussions, there were some sparks of students assuming a constructor role, thus lessening the gap between students' recipient and constructor roles during the lessons. These changes in the teacher's role appeared to have encouraged the students to explain their solution methods rather than simply focus on the answers (cf. Table 1), actions that are consistent with the way in which we and others (e.g., Watkins, Carnell, & Lodge, 2007) have operationalised students' constructor role in PBL. Moreover, students appeared convinced that they could learn through communicating with peers. For example, in a student interview at the end of lesson 15, the student mentioned: "You can't just open your brain and put in the information, you need to have a good conversation about it".

Implications for the implementation of PBL at the primary education level

The findings presented in this paper together with those reported by others (e.g., Watkins et al., 2007) have implications for the implementation of PBL at the primary education level. The process of a teacher changing from instructor to facilitator, as necessitated by PBL, is not straightforward and involves a steep learning curve (Donnelly, 2013), even when the teacher believes in the benefits of PBL (Reeve, 2012). The variations in the teacher's role during small-group and whole-class discussions, as shown in Figures 2 and 3, suggest further that a teacher's performance in PBL "is not a stable teacher characteristic, but rather it is partly situation specific" (Dolmans et al., 2002, p. 173), socially negotiated and tested in the classroom (Hack, McKillop, Sweetman, & McCormack, 2015). Consequently, the implementation of PBL is not simply a matter of controlling the nature or level of the teacher's actions, nor can it be taken for granted and systematised into theory (Kiemer, Gröschner, Pehmer, & Seidel, 2015; Vardi & Ciccarelli, 2008).

Another implication for the implementation of PBL is the need for teachers to be supported when implementing PBL. Teachers can revert to their customary teaching styles in the face of challenges when adopting PBL (Kirschner et al., 2006), particularly at the primary education level where students are of young age and are often inexperienced with participating in discussions and offering explanations (Li, 2012). Probably because of the collaborative relationship between the authors and the teacher, by the end of the intervention, the teacher in our study became more competent in dealing with the challenges that emerged for her during the implementation of the intervention and made an effort to change her role from an instructor back again to a facilitator.

The mechanism that might have underpinned the changes observed in Miss Lee's pedagogical practice during the study appeared to be related to the framework of "teachers as learners" that is afforded by PBL (McPhee, 2002). For example, in an interview, the teacher mentioned: "That [students' behaviour] is really frustrating, […] it means that I need to change my classroom management" and "In the end, I have learned different teaching pedagogies". This may relate to what Ball (2009, p. 47) referred to as "generativity": "the teachers' ability to continually add to their understanding by connecting their personal and professional knowledge to the knowledge they gain from their students". Teacher education programmes have a responsibility to support this process of teacher development so as to increase the possibility that the challenges emerging from teachers' initial efforts to implement PBL will not result in its abandonment.

There are several limitations of this study that should be acknowledged. As a convenience sampling strategy and a small sample size were used, it is inappropriate to generalise the results beyond the given population pool. Also, while a school-year long, prolonged engagement with the participants helped enhance the trustworthiness of the study (Yin, 2003), the implementation of PBL in the classroom is a complex process and the actions of the teacher and students in the intervention may well have been interwoven with other variables, which were not identified in the study. Further research is needed to follow up on those issues.

Conclusions

While our findings are subject to the limitations of the methodology used, the study offers some useful insights into the teacher's and students' roles when implementing a PBL intervention in a primary classroom. The roles of the teacher and students in this intervention were intricately intertwined in creating a classroom environment that, at different stages during the intervention, either aligned with or distanced itself from main principles of PBL. Our findings provided evidence of an interesting variation in the degree of proximity (or lack thereof) between the classroom environment during the course of the intervention and main principles of PBL. The documentation of this variation shows that the implementation of PBL requires a fundamental restructuring of learning perspectives (Kiemer et al., 2015).

Another key point that was learned from our experience in this study is that inquiry into the learning and teaching of mathematics through PBL approaches at primary education level must take greater account of how established classroom and cultural norms relate to these approaches and what can be done to bridge the two so that the classroom participants face fewer challenges and the potential of these approaches is not compromised.

Effective adoption of pedagogical innovations such as PBL in school classrooms is a demanding endeavour that requires a dynamic blending of theory, research and practice, as well as coordinated efforts from researchers, teacher educators and teachers (Stylianides & Stylianides, 2013). As much of the PBL-related research over the past few decades has been conducted at the university level (Wirkala & Kuhn, 2011), more research is needed to explore the potential of PBL in primary school classrooms and to understand what is involved for teachers and students as they engage with PBL.

References

Ball, A. F. (2009). Toward a theory of generative change in culturally and linguistically complex classrooms. *American Educational Research Journal, 46*(1), 45-72.

Barrett, T. (2010). The problem-based learning process as finding and being in flow. *Innovations in Education and Teaching International, 47*(2), 165-174.

Barrows, H. S., & Myers, A. C. (1993). *Problem-based learning in secondary schools*. Springfield: Problem-Based Learning Institute, Lanipher High School, and Southern Illinois Medical School.

Barrows, H. S., & Tamblyn, R. M. (1980). *Problem-based learning: an approach to medical education*. New York: Springer Publishing Company.

Beaumont, C., Savin-Baden, M., Conradi, E., & Poulton, T. (2014). Evaluating a second life problem-based learning (PBL) demonstrator project: what can we learn? *Interactive Learning Environments, 22*(1), 125-141

Bridges, S., Botelho, M., Green, J., & Chau, A. M. (2012). Multimodality in problem-based learning (PBL): an interactional ethnography. In S. Bridges, C. McGrath & T. L. Whitehill (Eds.), *Problem-based learning in clinical education* (Vol. 8, pp. 99-120). The Netherlands: Springer.

Charalambous, C. Y., & Pitta-Pantazi, D. (2007). Drawing on a theoretical model to study students' understandings of fractions. *Educational Studies in Mathematics, 64*(4), 293-316.

Chin, E.-T., & Lin, F.-L. (2013). A survey of the practice of a large-scale implementation of inquiry-based mathematics teaching: from Taiwan's perspective. *ZDM-International Journal on Mathematics Education, 45*, 919-923.

Creswell, J. W. (2012). *Qualitative enquiry and research design: choosing among five approaches* (3rd ed.). Thousand Oaks, CA: Sage.

Cunningham, W. G., & Corderio, P. A. (2000). *Educational administration: a problem based aproach*. London: Allyn & Bacon.

Dewey, J. (1910). *How we think*. Boston, DC: Heath & Co.

Dewey, J. (1944). *Democracy and education*. New York: Macmillan.

Dochy, F., Segers, M., Van den Bossche, P., & Gijbels, D. (2003). Effects of problem-based learning: a meta-analysis. *Learning and Instruction, 13*(5), 533-568.

Dolmans, D. H., Gijselaers, W. H., Moust, J. H., de Grave, W. S., Wolfhagen, I. H., & van der Vleuten, C. P. (2002). Trends in research on tutor in problem-based learnining: conclusion and implication for educational practice and research. *Medical Teacher, 24*(2), 173-180.

Donnelly, R. (2013). The role of the PBL tutor within blended academic development. *Innovations in Education and Teaching International, 50*(2), 133-143.

Gijbels, D., Dochy, F., Van den Bossche, P., & Segers, M. (2005). Effects of problem-based learning: a meta-analysis from the angle of assessment. *Review of educational research, 75*(1), 27-61.

Goodnough, K. C., & Hung, W. (2008). Engaging teachers' pedagogical content knowledge: adopting a nine-step problem-based learning model. *Interdisciplinary Journal of Problem-based Learning, 2*(2), 61-90.

Cobb, P., & Yackel, E. (2011). An introduction to part II. In E. Yackel, K. Gravemeijer & A. Sfard (Eds.), *A journey in mathematics education research - Insights from the work of Paul Cobb* (pp. 33-40). New York: Springer.

Goos, M. (2004). Learning mathematics in a classroom community of inquiry. *Journal for Research in Mathematics Education, 35*(4), 258-291.

Hack, C., McKillop, A., Sweetman, S., & McCormack, J. (2015). An evaluation of resource development and dissemination activities designed to promote problem-based learning at the University of Ulster. *Innovations in Education and Teaching International, 52*(2), 218-228.

Hmelo-Silver, C. E., Duncan, R. G., & Chinn, C. A. (2007). Scaffolding and achievement in problem-based and inquiry learning: a response to Kirschner, Sweller, and Clark (2006). *Educational Psychologist, 42*(2), 99-107.

Huang, S.-H., Huang, Y.-M., Wu, T.-T., Chen, H.-R., & Chang, S.-M. (2015). Problem-based learning effectiveness on micro-blog and blog for students: a case study. *Interactive Learning Environments*, 1-21.

Hung, W. (2011). Theory to reality: a few issues in implementing problem-based learning. *Educational Technology Research and Development, 59*(4), 529-552.

Kanselaar, G., Erkens, G., Jaspers, J., & Schijf, H. (2001). Computer supported collaborative learning. *Teaching and Teacher Education, 17*(1), 123-129.

Kiemer, K., Gröschner, A., Pehmer, A.-K., & Seidel, T. (2015). Effects of a classroom discourse intervention on teachers' practice and students' motivation to learn mathematics and science. *Learning and Instruction, 35*, 94-103.

Kirschner, P. A., Sweller, J., & Clark, R. E. (2006). Why minimal guidance during instruction does not work: an analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry-based teaching. *Educational Psychologist, 4*(2), 75-86.

Lampert, M. (2001). *Teaching problems and the problems of teaching*. New Haven, CT: Yale University Press.

Li, H.-C. (2012). Implementing problem-based learning in a Taiwanese elementary classroom: a case study of challenges and strategies. *Research in Mathematics Education, 14*(1), 89-90.

Li, H.-C. (2014). A comparative analysis of British and Taiwanese students' conceptual and procedural knowledge of fraction addition. *International Journal of Mathematical Education in Science and Technology, 45*(7), 968-979.

Li, H.-C. & Tsai, T.-L. (in press). The implementation of problem-based learning in a Taiwanese primary mathematics classroom: lessons learned from the students' side of the story. *Educational Studies*.

Lubienski, S. T. (2000). Problem solving as a means toward mathematics for all: an exploratory look through a class lens. *Journal for Research in Mathematics Education, 31*(4), 454-482.

McPhee, A. (2002). Problem-based learning in initial teacher education: taking the agenda forward. *Journal of Educational Enquiry, 3*(1), 60-78.

Moll, L. C. (2014). *L.S. Vygotsky and education*. Oxford: Taylor & Francis.

Piaget, J. (1954). *The construction of reality in the child*. New York: Basic Books.

Piaget, J. (1965). *The child's conception of number*. England: W. W. Norton.

Plowright, D., & Watkins, M. (2004). There are no problems to be solved, only inquiries to be made, in social work education. *Innovations in Education and Teaching International, 41*(2), 185-206.

Reeve, J. (2012). A self-determination theory perspective on student engagement. In S. L. Christenson, A. L. Reschly & C. Wylie (Eds.), *Handbook of research on student engagement* (pp. 149-171). New York: Springer.

Reynolds, J. M., & Hancock, D. R. (2010). Problem-based learning in a higher education environmental biotechnology course. *Innovations in Education and Teaching International, 47*(2), 175-186.

Savery, J., & Duffy, T. (2001). Problem based learning: an instructional model and its constructivist framework *CRLT Technical Report. Retrieved from* http://crlt.indiana.edu/publications/journals/TR16-01.pdf

Savery, J. R., & Duffy, T. M. (1995). Problem based learning: an instructional model and its constructivist framework. *Educational Technology, 35*(5), 31-37.

Schmidt, H. G., Van der Molen, H. T., Te Winkel, W. W. R., & Wijnen, W. H. F. W. (2009). Constructivist, problem-based learning does work: a meta-analysis of curricular comparisons involving a single medical school. *Educational Psychologist, 44*, 227-249.

Schoenfeld, A., & Kilpatrick, J. (2013). A US perspective on the implementation of inquiry-based learning in mathematics. *ZDM-International Journal on Mathematics Education, 45*(6), 901-909.

Singer, F. M., & Voica, C. (2013). A problem-solving conceptual framework and its implications in designing problem-posing tasks. *Educational Studies in Mathematics, 83*(1), 9-26.

Smith, C. S., & Hung, L.-C. (in press). Using problem-based learning to increase computer self-efficacy in Taiwanese students. *Interactive Learning Environments*.
Stylianides, A. J., & Stylianides, G. J. (2013). Seeking research-grounded solutions to problems of practice: classroom-based interventions in mathematics education. *ZDM-International Journal on Mathematics Education, 45*(3), 333-341.
Stylianides, A. J., & Stylianides, G. J. (2014). Impacting positively on students' mathematical problem solving beliefs: an instructional intervention of short duration. *The Journal of Mathematical Behavior, 33*, 8-29.
Thompson, M., & Wiliam, D. (2008). Tight but loose: a conceptual framework for scaling up school reforms. In C. Wylie (Ed.), *Tight but loose: scaling up teacher professional development in diverse contexts* (pp. 1-44). Princeton: Educational Testing.
Torp, L., & Sage, S. (2002). *Problems as possibilities: problem-based learning for k-16 education* (2nd ed.). Alexandria, WA: Association for Supervision and Curriculum Development.
Vardi, I., & Ciccarelli, M. (2008). Overcoming problems in problem-based learning: a trial of strategies in an undergraduate unit. *Innovations in Education and Teaching International, 45*(4), 345-354.
Vygotsky, L. (1962). *Thought and language*. Cambridge, MA: MIT Press.
Vygotsky, L. (1978). *Mind in society: the development of higher psychological processes*. Cambridge, MA: Harvard University Press.
Watkins, C., Carnell, E., & Lodge., C. (2007). *Effective learning in classrooms*. Thousand Oaks: Paul Chapman Publishing.
Weinstein, C. S., Romano, M., & Jr. Mignano, A. (2010). *Elementary classroom management: lessons from research and practice*. New York: McGraw-Hill.
Wertsch, J. V. (2007). Mediation. In H. Daniels, M. Cole & J. V. Wertsch (Eds.), *The Cambridge companion to Vygotsky*. Cambridge: Cambridge University Press.
Wijnen, M., Loyens, S. M. M., & Schaap, L. (2015). Experimental evidence of the relative effectiveness of problem-based learning for knowledge acquisition and retention. *Interactive Learning Environments*, 1-15.
Wijnia, L., Loyens, S. M. M., van Gog, T., Derous, E., & Schmidt, H. G. (2014). Is there a role for direct instruction in problem-based learning? Comparing student-constructed versus integrated model answers. *Learning and Instruction, 34*, 22-31.
Wirkala, C., & Kuhn, D. (2011). Problem-based learning in K–12 education: is it effective and how does it achieve its effects? *American Educational Research Journal, 48*(5), 1157-1186.
Yin, R. K. (2003). *Applications of case study research* (2nd ed.). Thousand Oaks, CA: Sage.