Factoring Integers with Large-Prime Variations of the Quadratic Sieve

Henk Boender and Herman J. J. te Riele

CONTENTS

- 1. Introduction
- 2. The Basic Idea
- 3. The Multipolynomial Quadratic Sieve
- 4. Efficient Calculation of The Polynomials
- 5. The Large-Prime Variation of MPQS
- 6. The Double Large-Prime Variation
- 7. Implementation and Experiments

Acknowledgements

References

This article is concerned with the large-prime variations of the multipolynomial quadratic sieve factorization method: the PMPQS (one large prime) and the PPMPQS (two). We present the results of many factorization runs with the PMPQS and PPMPQS on SGI workstations and on a Cray C90 vector computer. Experiments show that for our Cray C90 implementations PPMPQS beats PMPQS for numbers of more than 80 digits, and that this crossover point goes down with the amount of available central memory.

For PMPQS we give a formula to predict the total running time based on a short test run. The accuracy of the prediction is within 10% of the actual running time. For PPMPQS we do not have such a formula. Yet in order to provide measurements to help determining a good choice of the parameters in PPMPQS, we factored *many* numbers. In addition we give an experimental prediction formula for PPMPQS suitable if one wishes to factor many large numbers of about the same size.

1. INTRODUCTION

Let n be an odd positive integer to be factored and suppose that n is not a prime power. If we can find two integers X and Y such that

$$X^2 \equiv Y^2 \bmod n, \tag{1.1}$$

then the greatest common divisor of X-Y and n is a nontrivial factor of n if $X \not\equiv \pm Y \pmod{n}$. If X and Y are randomly chosen subject to (1.1), then this yields a proper factor of n in at least 50% of the tries. This principle is the basis for the best known general factorization methods, namely, the multipolynomial quadratic sieve, or MPQS [Bressoud 1989; Pomerance 1985; Pomerance et al. 1988; Silverman 1987; te Riele et al. 1989], and the number field sieve, or NFS [Lenstra and Lenstra 1993].

AMS Subject Classification: 11A51, 11Y05.

Keywords: factorization, multiple polynomial quadratic sieve, vector supercomputer, cluster of workstations.

In this paper we discuss and compare the single large-prime variation (PMPQS) and the double large-prime variation (PPMPQS) of MPQS. An introduction to each of these methods is given starting in Section 2. We factor many numbers ranging from 66 to 88 decimal digits, mainly with PPM-PQS, on either SGI workstations or a Cray C90 vector computer.

PPMPQS is known to be faster than PMPQS "by approximately a factor of 2.5 for sufficiently large n" [Lenstra and Manasse 1994], but the crossover point depends heavily on the choice of the parameters in the two methods, the computer, the available memory, and the implementation. It is stated further in [Lenstra and Manasse 1994] that PPMPQS was found to be faster than PMPQS for numbers of at least 75 decimal digits, and that the speed-up factor of 2.5 was obtained for numbers of more than 90 digits. As a comparison, a 106-digit number was factored with PMPQS in about 140 mips years, and a 107-digit number with PPM-PQS in about 60 mips years, both with a factor base size of 65,500. A 116-digit number was factored with PPMPQS in about 400 mips years, with a factor base size of 120,000. No actual results for smaller numbers were given. Thomas Denny reports in his Master's Thesis [1993] various experiments with PPMPQS for numbers in the range of 75-95 decimal digits. From these experiments it is not clear where the crossover point for Denny's implementation lies. The largest numbers presently factored with PPMPQS are a 120-digit number done in about 825 mips years [Denny et al. 1994]. and the 129-digit RSA challenge described by Martin Gardner, done in about 5000 mips years with a factor base size of 524,339 [Atkins et al. 1995].

A theoretical and practical problem with PPM-PQS is the determination of the optimal parameters for a number of a given size. Since it only pays to use PPMPQS for rather large numbers, and since it is difficult to accurately predict the total running time of PPMPQS on the basis of a short test run (as contrasted with PMPQS), the precise effect of one specific choice of the parame-

ters can only be measured accurately by carrying out the complete sieve part of the job. So in order to find the optimal parameter choice for a given number, that would minimize the CPU time, one would have to repeat the complete sieve job for several (10, say) different choices of the parameters. Of course, this does not make much sense since one sieve job will do to factor the number, so we decided to adopt the strategy to factor as many as possible different numbers in a not too wide decimal digits range, thus providing extensive experience with PPMPQS for many different numbers on the one hand, and contributing to a table of unfactored numbers [Brent and te Riele 1992] on the other hand. The price to pay for this strategy is that we can only give an *indication* of the optimal parameter choice for PPMPQS for numbers in the 65-90 decimal digits range.

We have implemented PPMPQS on an SGI workstation, and on a Cray vector computer. Some comparative experiments with PMPQS and PPMPQS on a Cray C90 indicated that for our implementation on that machine the crossover point lies around numbers having 80–85 decimal digits. For several different choices of the parameters in PPMPQS, we have factored eight numbers in the 66–83 digit range on an SGI workstation, and more than 70 numbers in the 67–88 digit range on a Cray C90 vector computer, as a contribution to the table in [Brent and te Riele 1992]. Most of these numbers were already tried before with the elliptic curve method (ECM), without success.

Section 2 discusses Dixon's algorithm. MPQS is described in Section 3. In Section 4 we treat the efficient generation of the polynomials in MPQS. In Section 5 the single large-prime variation of MPQS (PMPQS) is described. A known theoretical formula is worked out that helps to predict the total sieve time on the basis of a short test run. In this test run (of a few minutes CPU time, say) the speed is determined by which so-called complete and partial relations are generated during the sieve step of the algorithm; this speed is approximately constant during the whole sieve step. The

accuracy of the prediction formula is within 10% of the actual sieve time. In Section 6 the double large-prime variation of MPQS is described, and an experimental prediction formula is given that has a restricted scope in the sense that it only applies to numbers of roughly the same size, and for a fixed choice of the parameters of the algorithm. In addition, for one particular number of 80 decimal digits, we have determined the optimal choice of one (of the four) parameters in PPMPQS as an illustration of the fact that this optimum is attained for a rather wide range of this parameter. Section 7 covers implementation aspects and discusses our experiments, including a comparison of our PMPQS- and PPMPQS-implementations for 71-, 87-, and 99-digit numbers. The paper closes with data from 81 factorizations.

2. THE BASIC IDEA

The algorithm described now is due to Dixon, who based it on the continued fraction method of Morrison and Brillhart [1975]. It is not efficient in practice compared to almost any other method, but it shows clearly the idea behind finding X and Y. So we mention it mainly for didactical reasons.

For $x \in \mathbb{Z}$ such that $|x| > \sqrt{n}$, define

$$g(x) :\equiv x^2 \mod n$$
.

(The notation \sqrt{n} means the positive square root of n.) Suppose that we have a finite subset $J\subset\mathbb{Z}$ such that $\prod_{x\in J}g(x)$ is a square. Then we can take

$$X = \sqrt{\prod_{x \in J} g(x)}, \qquad Y = \prod_{x \in J} x.$$

A problem is how to determine J.

Choose a positive integer B_1 , let $\pi = \pi(B_1)$ be the number of primes $\leq B_1$, and let $\{p_1, p_2, \ldots, p_{\pi}\}$ be the set of such primes. Suppose that we have a set T of $t > \pi$ numbers g(x) only composed of primes $\leq B_1$, that is,

$$g(x) = p_1^{e_1(x)} p_2^{e_2(x)} \dots p_{\pi}^{e_{\pi}(x)},$$

where $e_i(x)$ is the exponent of p_i in g(x). Then

$$\prod_{x \in J} g(x) = \prod_{i=1}^{\pi} p_i^{\sum_{x \in J} e_i(x)}.$$

This is a square if and only if

$$\sum_{x \in I} e_i(x) \equiv 0 \mod 2$$
, for $i = 1, 2, \dots, \pi$.

Since $|T| = t > \pi$, there exists a nontrivial solution of this linear system of equations over GF(2). A solution can be found using Gaussian elimination. This yields at least $t - \pi$ useful subsets J.

3. THE MULTIPOLYNOMIAL QUADRATIC SIEVE

Dixon's algorithm does not tell us how to find T efficiently. Building on previous work of Kraitchik [1929], Lehmer and Powers [1931], Morrison and Brillhart [1975], and Schroeppel, Carl Pomerance [1982] introduced the *quadratic sieve algorithm*. It works with the quadratic polynomial

$$g(x) = (x + |\sqrt{n}|)^2 - n,$$

where x runs over the integers in $(-n^{\varepsilon}, n^{\varepsilon})$, so that $g(x) = O(n^{1/2+\epsilon})$. With this g(x) the set T may be built up, where some of the numbers g(x) can be factored completely by a cheap sieve process because q(x) is a polynomial (this is much more efficient than trial division or any other factoring method). We could also use a sieve process in Dixon's algorithm if we choose random numbers xin an arithmetic progression like $x, x+1, x+2, \ldots$ However, in practice this single polynomial g(x)(or an arithmetic progression in Dixon's algorithm) does not give rise to a sufficiently large set T (with $t > \pi$ elements) in a reasonable amount of time. The reason for this is that the interval $(-n^{\epsilon}, n^{\epsilon})$ is large when n is large and, since $g(x) = O(n^{1/2+\epsilon})$ (which is large), most numbers g(x) are not likely to factor over a set of small primes. P. L. Montgomery found an efficient way to use several polynomials (thus introducing a simple way to run the algorithm in parallel), so that the numbers x can be taken from much smaller intervals rather than from one single very large interval. The average polynomial values then are smaller than the average value of g and are thus more likely to factor over small primes than the g(x)-values. If all the numbers in a small interval have been considered, we can pass to a next polynomial and try again. We describe here the resulting multipolynomial quadratic sieve method. We remark that Davis and Holdridge [1983] already had a multipolynomial version before Montgomery came up with his new idea. In fact, Montgomery's method is based on that of Davis and Holdridge, but it is slightly more efficient.

Suppose that we have integer numbers x, U(x), V(x), and W(x) such that

$$U^2(x) \equiv V^2(x)W(x) \bmod n$$
 for all $x \in \mathbb{Z}$. (3.1)

If $J \subset \mathbb{Z}$ is a finite subset such that $\prod_{x \in J} W(x)$ is a square, we can take

$$X = \prod_{x \in J} V(x) \sqrt{\prod_{x \in J} W(x)}, \qquad Y = \prod_{x \in J} U(x).$$

In practice we choose $U(x) = a^2x + b$, V(x) = a, and $W(x) = a^2x^2 + 2bx + c$, with $|x| \le M$ (where M is a parameter we choose beforehand) and where a, b and c are integers that satisfy the following conditions [Bressoud 1989, p. 117]:

$$a^2 \approx \sqrt{2n}/M,$$
 (3.2)

$$b^2 - n = a^2 c, (3.3)$$

$$|b| < a^2/2. (3.4)$$

In the next section we describe how a, b and c are to be calculated.

W(x) plays the role of g(x) in Dixon's algorithm. In order to determine the subset J, we choose an upper bound B_1 for the primes. We want to have many W(x)-values that consist of primes $\leq B_1$. However, only roughly half of the primes below B_1 can occur as a prime divisor of W(x). Namely, if a prime p divides W(x), then $p \mid a^2W(x)$ and thus $p \mid (a^2x+b)^2-n$, which means that n is a quadratic

residue modulo p. This leads to the definition of the factor base \mathcal{F} :

$$\mathfrak{F} = \{ \text{prime powers } q = p^k \leq B_1 : \left(\frac{n}{p}\right) = 1 \}.$$

(Of course, a prime can divide W(x) more than once, so we also have to account for prime powers.) Note that \mathcal{F} is independent of the choices of a, b and c, so we can use the same factor base for every proper choice of a, b and c.

Since W(x) is more likely to be divisible by small primes than by large primes, it is advantageous that the factor base contains many small primes. We can construct such a factor base by multiplying the number n to be factored by a suitable small integer m, called the *multiplier*, and factoring mn rather than n [Pomerance et al. 1988, p. 391].

For a given $q \in \mathcal{F}$ the values of x for which q divides W(x) can be found as follows. Compute the solution $t = t_q$ of the congruence equation

$$t^2 \equiv n \bmod q, \quad \text{for } 0 < t \leq q/2$$

[Riesel 1985, pp. 212 and 287–288]. This has to be done only once during the algorithm. Now, if $q \mid W(x_0)$, then $q \mid (a^2x_0 + b)^2 - n$ and thus

$$x_0 \equiv a^{-2}(\pm t_q - b) \bmod q, \tag{3.5}$$

provided that gcd(a,q) = 1. This is guaranteed by the choice of a (see Section 4). For each proper choice of a we compute $a^{-2} \mod q$ for all $q \in \mathcal{F}$. In the next section we describe how these computations can be done. Furthermore, since W(x) is a quadratic polynomial, q divides $W(x_0 + lq)$, for $l \in \mathbb{Z}$. So we can calculate efficiently the places where an element of \mathcal{F} divides the W-values. This idea originated from Schroeppel.

Define the report threshold RT as the average of $\log |W(x)|$ on the interval [-M,M], which is approximately $\log(\frac{1}{2}M\sqrt{n/2})$. Initialize a sieve array $\mathrm{SI}(-M,M)$ to zero and sieve with each $q=p^k\in\mathcal{F}$, i.e., add $\log p$ to $\mathrm{SI}(x_0+lq)$ for all $l\in\mathbb{Z}$ such that x_0+lq is in the interval [-M,M]. For those numbers x for which $\mathrm{SI}(x)\geq\mathrm{RT},\,W(x)$ is a good candidate for fully factoring over the factor base. In

general, the time spent on sieving takes more than 85% of the total computing time.

Since sieving with small primes is expensive, it is customary not to sieve with the primes and prime powers \le QT, where QT is some suitably chosen threshold value. In order not to lose W(x)-values divisible by such small primes, the report threshold RT will be lowered by the amount $\sum_{p^k \leq QT} \log p$. After the sieve step and the selection of those x for which $SI(x) \geq RT$, the prime factors of the corresponding W(x) are found by comparison, for all $q \in \mathcal{F}$, of x with the two values of x_0 in (3.5) (which are computed and stored after the factor base has been computed). In this way, W(x)-values divisible by one or more of the small primes omitted during the sieving phase are not lost. If QT is suitably chosen, this can save a considerable amount of sieve time. This refinement of MPQS is known as the small-prime variation.

4. EFFICIENT CALCULATION OF THE POLYNOMIALS

Choose integers r and k such that 1 < k < r (typical choices are r = 30 and k = 3, for example). Generate primes g_1, g_2, \ldots, g_r , the so-called g-primes, such that (i) $g_i \approx (\sqrt{2n}/M)^{1/(2k)}$, (ii) $(\frac{n}{g_i}) = 1$, and (iii) $\gcd(g_i, q) = 1$, for $i = 1, 2, \ldots, r$ and for all $q \in \mathcal{F}$. Let

$$a = g_i, g_i, \dots g_{i_k},$$

be the product of k g-primes, with $1 \le i_1 < i_2 < \cdots < i_k \le r$. Because of (i), this a satisfies condition (3.2).

Let b_i be a solution of the congruence equation

$$t^2 \equiv n \mod q_i^2$$

where i = 1, 2, ..., r. Solve the system of congruence equations (for a specific choice of the signs)

$$x \equiv b_{i_1} \mod g_{i_1}^2,$$
 $x \equiv \pm b_{i_2} \mod g_{i_2}^2,$
 \vdots
 $x \equiv \pm b_{i_k} \mod g_{i_k}^2,$

$$(4.1)$$

by means of the Chinese Remainder Theorem. Let b be the solution of this system of equations. Then $b^2 \equiv n \mod a^2$, so that condition (3.3) holds with

$$c = (b^2 - n)/a^2.$$

If $b \ge a^2/2$, we replace b by $b-a^2$ in order to satisfy condition (3.4). Since there are 2^{k-1} possible combinations of signs in (4.1), the number of polynomials that can be calculated with one set of r g-primes and a fixed k is $2^{k-1}\binom{r}{k}$.

If a new a has to be chosen, new sieve numbers x_0 subject to (3.5) must be computed. Since $a = g_{i_1}g_{i_2}\ldots g_{i_k}$, we can use

$$a^{-2} \mod q = g_{i_1}^{-2} g_{i_2}^{-2} \dots g_{i_k}^{-2} \mod q.$$

Therefore, with the generation of the g-primes we also compute and store the numbers $g_i^{-2} \mod q$, where $i = 1, 2, \ldots, r$, for all the prime powers q in the factor base.

For a fixed a, Alford and Pomerance [1995] developed a method to compute iteratively all the other values of b (and thus c) from a given initial value of b (see also [Peralta ≥ 1996]). They also pointed out how the two solutions in the interval [0,q) of the congruence equation $W(x) \equiv 0 \mod q$ can be calculated from the zeros mod q of a "previous" polynomial. With this improvement we obtain the self-initializing variation of MPQS. It has the advantage that it can change polynomials cheaply, so a shorter sieve interval can be used.

We have implemented this variation on an SGI workstation and on a Cray C90 vector computer. Some speed-up was observed on an SGI workstation when we reduced the length of the sieve interval, but other effects like an increasing loop overhead in the sieving step interfere with this in the opposite direction.

On a vector computer such as the Cray C90, reducing the length of the sieve interval reduces the vector lengths in the sieving step and, consequently, the efficiency of the vectorization. Therefore, we decided not to use the self-initializing variation of the quadratic sieve in our experiments.

5. THE LARGE-PRIME VARIATION OF MPQS

The large-prime variation of MPQS incorporates the following improvement, which is based on a step in the continued fraction algorithm of Morrison and Brillhart [1975]. W(x) is allowed to have a factor $R > B_1$ that is not composed of primes from the factor base. If the cofactor R (after dividing out all factor base primes in W(x)) is less than or equal to B_1^2 , it must be a prime. In order to restrict the amount of disk space needed for storage of the relations (3.1), we only accept factors $R \leq B_2$, where B_2 is a parameter we choose beforehand. In practice we choose B_2 in such a way that B_2/B_1 is a number between 10 and 100. We have to lower the report threshold by $\log(B_2)$ in order to find these W(x)-values after sieving.

If we have found two W(x)-values with the same R, multiplication of the corresponding relations (3.1) yields a relation of the form (3.1), where W(x) only consists of prime powers $q \in \mathcal{F}$ (and R is moved to V(x)).

A relation of the form (3.1), where W(x) only consists of primes $q \in \mathcal{F}$, is called a *complete relation*. If W(x) has one prime factor $R \leq B_2$ (and the others are in \mathcal{F}), then the relation is called a partial relation.

We wish to compute E, the expected number of complete relations coming from a given number of r partial relations. Let

$$Q = \left\{ \text{primes } q : B_1 < q \le B_2, \ \left(\frac{n}{q}\right) = 1 \right\}.$$

The elements of Ω are called *large primes*. Let P_q be the probability that a large prime q occurs in a partial relation. Lenstra and Manasse [1994] assume that

$$P_q \approx q^{-\alpha} / \sum_{p \in \Omega} p^{-\alpha} \tag{5.1}$$

for some positive constant $\alpha < 1$ that should be determined experimentally. They report that $\alpha \in [\frac{2}{3}, \frac{3}{4}]$ gives a reasonable fit with their experimental results. Denny [1993, pp. 44–49] takes $\alpha = 0.775$.

From [Lenstra and Manasse 1994] it follows that

$$E = r - \#\Omega + \sum_{q \in \Omega} (1 - P_q)^r.$$

We apply the binomial formula of Newton and use approximation (5.1) to find

$$E \approx \sum_{i=2}^{r} (-1)^{i} {r \choose i} \left(\sum_{q \in \Omega} q^{-\alpha} \right)^{-i} \sum_{q \in \Omega} q^{-\alpha i}. \quad (5.2)$$

Since $\pi(t) \sim t/\log t$ as $t \to \infty$, we have

$$\sum_{p \le x} p^{-u} \approx \int_2^x t^{-u} \, d(t/\log t)$$

with p prime, $x \in \mathbb{R}_{\geq 2}$, $u \in \mathbb{R}_{>0}$. Hence for u > 0 we have

$$\sum_{q \in \Omega} q^{-u} \approx \frac{1}{2} \int_{B_1}^{B_2} t^{-u} \, d(t/\log t).$$

To compute the last integral we first use partial integration and then substitute $s = (1 - u) \log t$. We get

$$\int_{B_1}^{B_2} t^{-u} d(t/\log t) = B_2^{1-u}/\log B_2 - B_1^{1-u}/\log B_1 + u\{\operatorname{Ei}((1-u)\log B_2) - \operatorname{Ei}((1-u)\log B_1)\},\$$

where $\operatorname{Ei}(x) = \int_{-\infty}^{x} (e^{s}/s) ds$ is the exponential integral. Now combine the last three displayed equations for the appropriate choices of u to get an approximation for E. In approximation (5.2) we sum from i=2 to i=5 and forget about the higher-order terms to get a formula for an approximation of E that we can use in practice (given B_1 , B_2 , r, and α).

The experiments summarized in Table 1 show that our approximation works well if $\alpha = 0.73$. The table shows, for each example run, the number r of partial relations, the actual number of complete relations derived from these partial relations, and the estimated number of complete relations. An approximation of E can be used to predict the computing time.

n	$B_1/10^4$	B_2/B_1	r	actual	estim.
C75	30	20.0	37472	4790	4966
C80	10	60.0	15918	1121	1209
C80	30	167	68195	4113	4150
C84	80	25.0	96138	10894	11148
C88	50	100	94651	6605	6736
C88	75	100	148403	11455	11211
C88	75	100	158214	12830	12657
C88	75	100	146983	11051	11008
C88	75	100	150327	11498	11488
C88	70	100	148016	12116	11827

TABLE 1. For ten composite numbers and bounds B_1 , B_2 , we list the number r of partial relations, and the actual and estimated number of complete relations (last two columns). As usual, Cx denotes a composite number with x decimal digits.

To determine the best value of α , we wrote a program in Maple that, given α , computes the absolute value of the difference of the actual number of complete relations and the estimated number of complete relations for each of fifteen test numbers. Then we summed the fifteen absolute values of the differences, thus obtaining for each α a sum of absolute values. It turned out that $\alpha=0.73$ gave rise to the smallest sum.

6. THE DOUBLE LARGE-PRIME VARIATION

In the large-prime variation of MPQS we allow W(x) in (3.1) to have a prime factor R with B_1 $R \leq B_2$. In the double large-prime variation of MPQS we also let W(x) have a factor $R \leq B_2^2$ composed of two primes $> B_1$. In this case we call such a relation a partial-partial relation (pp-relation for short). Now the problem of finding combinations of partial and partial-partial relations that yield a complete relation can be formulated as finding cycles in an undirected graph: the vertices are the large primes and two vertices (primes) are connected by an edge if there is a pp-relation in which both primes occur. A partial relation is represented by adding 1 as a vertex to the graph. We consider this partial relation as a pp-relation where one of the large primes is 1. So an edge in the graph

corresponds to a partial or partial-partial relation and a cycle corresponds to a set of relations with the following property: if we multiply these relations, then all the large primes in the product occur to an even power. Hence, for the linear algebra step this set can be viewed as a complete relation. To avoid dependent relations one only has to find the basic cycles of the graph.

The number of complete relations coming from the pp-relations is much more difficult to predict than that coming from the partial relations. One has to know how the number of basic cycles in a graph with given vertices varies when edges are added more or less randomly. Having a basic cycle is a monotone increasing property [Bollobás 1985, p. 33] that can appear rather suddenly [Erdős and Rényi 1959; 1960; 1961]. An algorithm for finding the basic cycles in a graph can be found in [Paton 1969].

If R is prime then we require $R < B_2$ in order to restrict the total number of relations (in our experience partial relations with $B_2 \le R < B_1^2$ do not contribute much to the total number of complete relations). If R is composite, its large prime factors can be found, e.g., by using Shanks' SQUFOF algorithm [Riesel 1985, pp. 191–199]. This algorithm has the advantage that most numbers that occur during its execution are in absolute value not larger than $2\sqrt{R}$.

We want to estimate the time that PPMPQS spends on the sieve step for numbers n of about d decimal digits, given B_1, M, B_2 , and QT. To that end, let

 $n_f = \text{number of elements in the factor base},$

 n_c = number of complete relations,

 $f_1 = n_c/n_f$

 $n_1 = \text{number of partial relations},$

 $n_2 = \text{number of pp-relations},$

 $f_2 = n_2/n_1,$

 $T_s = \text{sieve time.}$

During the sieve step, the numbers n_c , n_1 and n_2 grow (more or less) linearly with the time, so that

#	33	35	37	44	42	38	48	40	47	34	39	36	46	41	32	43
																41
f_1	0.243	0.244	0.255	0.269	0.275	0.297	0.301	0.310	0.320	0.325	0.331	0.346	0.348	0.349	0.352	0.363
f_2	5.98	5.79	4.04	3.68	2.75	2.37	2.13	2.29	1.70	1.64	1.14	0.906	0.961	0.862	0.760	0.798

TABLE 2. Values of f_1 and f_2 measured for 16 numbers n from Table 7 (identified by the number in the first row). We used d = 86, $B_1 = 5 \times 10^5$, $M = 1.5 \times 10^6$, $B_2/B_1 = 20$, and QT = 40, with multiplier m.

also the fraction f_1 grows linearly, and f_2 stays more or less constant (after the sieve step has been running for a short time). We observed that the values of the fractions f_1 and f_2 , measured after completion of the sieve step, seem to be connected; see Table 2.

The table suggests that f_2 is an exponential function of f_1 , that is,

$$f_2 = ae^{bf_1}$$

for some constants a and b. Based on the table, we estimated a = 315 and b = -16.5. Since $\log f_2 = \log a + bf_1$, it follows that

$$n_c = rac{1}{b}(\log f_2 - \log a) \cdot n_f.$$

If u is the time needed to generate one complete relation, we obtain the following approximation for the sieve time T_s :

$$T_s \approx (0.349 - 0.061 \log f_2) \cdot u \cdot n_f.$$
 (6.1)

We can estimate u and f_2 by letting the program run for a short while, five minutes say. The measurements shown in Table 3, pertaining to runs on

#	m	u	f_2	n_f	T_s	approx.
21	19	$5.140\mathrm{s}$	1.1945	20741	9.8 h	10.0 h
22	1	$4.518\mathrm{s}$	0.7646	20744	$9.8\mathrm{h}$	$9.50\mathrm{h}$
24	1	$3.357\mathrm{s}$	1.4378	20930	$6.0\mathrm{h}$	$6.37\mathrm{h}$
31	1	$4.226\mathrm{s}$	1.0866	24641	$10.0\mathrm{h}$	$9.94\mathrm{h}$
48	5	$8.785\mathrm{s}$	2.1364	20911	15.4 h	15.4 h

TABLE 3. Tests of approximation (6.1). For five composite numbers from Table 7 (identified by the number in the first column), we measured the actual value of T_s and computed the value predicted by the approximation (last column).

the Cray C90 of several 85- and 86-digit numbers, suggest that the estimate works well.

Consequently, approximation (6.1) can be used to obtain a good estimate of T_s in the PPMPQS algorithm for numbers of about the same size, and fixed parameters B_1 , M, B_2 , and QT. For numbers in another range, or if we wish to change the parameters, some experiments have to be done to determine the total sieve time under these new conditions, by which the coefficients in (6.1) can be estimated.

In order to test the dependency of T_s on B_2 , we carried out on the Cray C90 the *complete* sieve step of PPMPQS for the 80-digit number

$$\frac{75^{64}+1}{2\cdot 224914177\cdot 151113908786421917036806943723393},$$

(6.2)

which has the two prime factors

68799038786512319388821350925569 and 215768091527974049646247615957101365677594246657.

We kept $B_1 = 10^5$, $M = 3 \times 10^6$, and QT = 50 fixed, and tried various values of B_2 . The statistics are shown in Table 4.

In the partial relations we allowed the large prime R to be less than B_1^2 . (We get these relations free, because $R < B_1^2$ implies that R is prime.) For $B_1 = 10^5$ the number of elements in the factor base is 4806. The sieving was continued until the total number of complete relations, including those generated by the partial relations and the partial-partial relations, surpassed this number. We only measure the total number of complete relations obtained so far at selected points in our program, so the actual total number of complete relations is

B_2/B	T_s	n_c	n_1	$n_{c,1}$	n_2	$n_{c,2}$	total
30	$8.64\mathrm{h}$	1036	129318	1661	29143	2121	4818
60	$7.06\mathrm{h}$	871	117532	1249	51929	2739	4859
100	$6.49\mathrm{h}$	775	109506	1025	76324	3070	4870
200	$6.02\mathrm{h}$	685	99474	795	123001	3339	4819
400	$5.67\mathrm{h}$	618	91332	634	193278	3598	4850
600	$5.71\mathrm{h}$	578	87265	568	243015	3698	4844
800	$5.62\mathrm{h}$	563	84926	531	291177	3766	4869
1000	$5.75\mathrm{h}$	546	83082	501	333726	3796	4843
1600	6.19 h	521	79960	464	445526	3860	4845

TABLE 4. Number of relations as a function of B_2 , for the factorization of (6.2) with $B_1 = 10^5$, $M = 3 \times 10^6$, and QT = 50. The column $n_{c,1}$ is the number of complete relations generated by the n_1 partial relations, and $n_{c,2}$ is the number of complete relations generated by combining the partial relations (with different large primes) and the n_2 pp-relations. "Total" is the sum $n_c + n_{c,1} + n_{c,2}$.

usually somewhat larger than the number of elements in the factor base.

As we increase B_2/B_1 , the program generates more partial-partial relations and less complete and less partial relations in a given amount of sieve time. For $30 \le B_2/B_1 \le 400$, the gain in complete relations $(n_{c,2})$ generated by the pp-relations (n_2) more than sufficiently compensates for the loss of complete relations directly found by the sieve (n_c) and the loss of complete relations $(n_{c,1})$ generated by the partial relations (n_1) . As a result, the total sieve time T_s goes down. For $B_2/B_1 > 1000$, however, the increase in size of the large primes in the partial and partial-partial relations is responsible for a decrease in the number of complete relations derived from these relations, and also the time that SQUFOF needs to find the two large primes in a pp-relation increases, so now the resulting total sieve time increases. Consequently, the minimal sieve time is reached if we choose B_2/B_1 in the interval $400 < B_2/B_1 < 1000$. In that interval the total sieve time is only slightly varying. We conclude that, in order also to minimize the amount of memory for storage of the relations, the optimal choice of B_2/B_1 is about 400.

7. IMPLEMENTATION AND EXPERIMENTS

For our PMPQS-experiments we used the implementation described in [te Riele et al. 1989]. Almost all our subroutines are written in Fortran.

We originally implemented the PPMPQS algorithm on a supercomputer like the Cray C90 vector computer. We used the same implementation on Silicon Graphics workstations. (We now have written a program especially designed for workstations).

The sieve operations (i.e., additions of $\log p$ to an element of the sieve array) are done in 64-bits floating-point arithmetic on Cray and in 32-bits on SGI. The maximum speed we obtained (in millions of sieve operations per second) was 3.3 on the Silicon Graphics, 110 on the Cray Y-MP [te Riele et al. 1991] and 270 on the Cray C90. The maximum speed was 5.7 when we used the workstation version of our program.

We used a package of Winter in order to carry out multiprecision integer arithmetic.

The large prime R occurring in the partial relations was accepted if $B_1 < R < B_2$ and rejected if $B_2 \le R < B_1^2$.

We have implemented Paton's cycle-finding algorithm [1969] and used it as a preprocessing step for the Gaussian elimination step in PPMPQS.

An algorithm for just counting (not finding) the basic cycles [Lenstra and Manasse 1994, pp. 789–790; Denny 1993, pp. 61–64] was implemented by us as a tool to check during the sieve part of PPM-PQS whether sufficiently many relations (complete, partial, and partial-partial) were collected.

The method used to do the Gaussian elimination modulo 2 is described in [Parkinson and Wunderlich 1984]. The elements of the bit-array are packed in words of 64 bits (on the Cray computers) or 32 bits (on the Silicon Graphics). This allows the use of the exclusive-or operation with the column vectors of the array, which is very efficient. The total Gaussian elimination step (including finding basic cycles) accounts for less than 0.6% of the total work of the PPMPQS algorithm.

			***************************************				PMF	PQS				PPM	PQS		
	B_1	n_f	B_2/B	1	M	T_s	n_c	n_1	$n_{c,1}$	T_s	n_c	n_1	$n_{c,1}$	n_2	$n_{c,2}$
C71	3×10^5	12979	20	5.0	$\times 10^5$	0.58 h	10204	17993	2784	$0.55\mathrm{h}$	5063	36468	4709	42617	3400
C71	6×10^5	24510	20	5.0	0×10^{5}	0.56 h	20827	23794	3703	0.00				70395	
C71	6×10^5	24510	40	5.0	$) \times 10^{5}$	$0.55{\rm h}$	20312	30399	4209	1.28 h				132290	
C71	6×10^5	24510	40	2.5	6×10^6	0.29 h	20196	31034	4359	1.21 h	9803	81612	7499	138147	7969
C80	10^{5}	4806	400	3.0	0×10^{6}	13.4 h	1580	49143	3229	$5.67\mathrm{h}$	618	91332	634	193278	3598
C87	5×10^5	20838	20	2.5	5×10^6	16.4 h	9902	70029	10940	11.9 h	7009	63089	8220	57513	5620

TABLE 5. Comparison of PMPQS and PPMPQS. The C71 and C87 are listed on this page, the C80 in (6.2) on page 264.

In order to compare PMPQS with PPMPQS we have run our implementations of these algorithms on the Cray C90 for the 71-digit number

$$C71 = (10^{71} - 1)/9$$

and for the 87-digit cofactor

C87 = 1360245925758378639396610479463908049304-23542841197990430220444148923901462079070640121

of $72^{99} + 1$. For C71, four experiments with different combinations of B_1 , B_2/B_1 , and M were carried out where in the second, third and fourth experiment only one of the three parameters was changed compared with the previous experiment. The value of QT was kept fixed at 40. For C80 from (6.2), which was treated in the previous section with PPMPQS, we made a comparison run with PMPQS for $B_1 = 10^5$, $M = 3 \times 10^6$, QT = 50, and $B_2/B_1 = 400$ (the optimal choice for PPMPQS). The results are given in Table 5.

For C71, the parameter choice $B_1=3\times 10^5$, $B_2/B_1=20$, and $M=5\times 10^5$ yields a somewhat smaller sieve time for PPMPQS (0.55 CPU hours) than for PMPQS (0.58), but if we allow more memory use by choosing $B_1=6\times 10^5$ and $M=2.5\times 10^6$ (and $B_2/B_1=40$), then PMPQS beats PPMPQS (0.29 vs. 1.21). Increasing the length of the sieve interval (M from 5×10^5 to 2.5×10^6) particularly improves the efficiency of PMPQS (and, to a lesser extent, of PPMPQS). For C87, with the parameter choice $B_1=5\times 10^5$, $B_2/B_1=20$, and $M=2.5\times 10^6$, PPMPQS is faster than PMPQS (11.9 vs. 16.4).

We conclude that for our implementations PPM-PQS can beat PMPQS for numbers of more than 80 (say) decimal digits, but the crossover point strongly depends on the amount of available central memory. For practical reasons (like throughput) it can be profitable to reduce the size of a sieve job on the Cray C90, so even though such a computer has a very large central memory, it is still worthwhile to restrict the size of the upper bound on the primes in the factor base and to have an efficient implementation of a memory-economic method like PPMPQS. This is even more important on workstations, particularly when there are primary and secondary cache memories (as is usual on workstations).

Furthermore, with our PMPQS program we have factored the 99-digit cofactor

 $1684830849783397621153043603997266025308430041776 \\ 92574904043633682183896384221755952112008347771913$

of the "more wanted" C133 with code 2,914M in the Cunningham table [Wagstaff 1993]. This C133 is the number $(2^{457} + 2^{229} + 1)/(5 \times 71293)$; Peter Montgomery had found the 34-digit prime factor

6196333979234679466021864314534473

with ECM, and left the 99-digit composite factor. We decomposed it into the product of the 49- and a 50-digit primes,

 $5845296257595668545524969937697507923682374822769 \times \\ 28823703291241135239378075616078003806433692452377$

with the help of an eight-processor IBM 9076 SP1, and 69 Silicon Graphics workstations (63 at CWI and 6 at Leiden University). The factor base size was 56976 with $B_1 = 1.5 \times 10^6$, $B_2/B_1 = 50$, $M = 2 \times 10^6$, and QT = 30. Parallel processing with good load balancing was effectuated by assigning different polynomials to different workstations. The total amount of sieve time was about 19,500 workstation CPU hours. The physical time for this factorization was about four weeks. This means that we consumed about 40% of the total CPU capacity of these workstations during that period (assuming that they all are equally fast: in fact, an RS 6000 processor of the IBM SP1 sieved about twice as fast as an SGI workstation). The Gaussian elimination step was carried out on a Cray C90; it required about 0.5 Gbytes of central memory, and one hour CPU time.

As a comparison with a vector computer [te Riele et al. 1991], on a Cray Y-MP we factored a 101-digit more wanted Cunningham number with PM-PQS in 475 CPU hours, using $B_1 = 1300000$, with 50179 primes in the factor base, $B_2/B_1 = 50$, $M = 4.5 \times 10^6$, and QT = 40 (our PMPQS implementation runs about twice as fast on the Cray C90 as on the Cray Y-MP).

As a comparison with PPMPQS, from the results listed on the right in Table 5 we estimate (based on the assumption that the computing time of PPMPQS approximately doubles if the size of the number increases by three decimal digits) that we would roughly need 10,000 CPU hours of an SGI workstation to factor the 99-digit cofactor of 2,914M C133, yielding a speed-up factor of about 2 compared to PMPQS. If we would take a factor 1.64 (see the next paragraph) instead of 2, then the time would be less than 4000 CPU hours.

Tables 6 and 7, on pages 268–271, list the results of our experiments with PPMPQS on eight numbers in the 66–83 digit range on an SGI workstation, and 73 numbers in the 67–88 digit range on a Cray C90 vector computer. Most of these numbers fill gaps in the table found in [Brent and te Riele 1992], and are difficult to factor, having

been tried before with ECM without success. The factorizations of some numbers of the form $a^n \pm 1$ that are outside the range covered by that reference are also given in Table 7.

We have varied the parameters B_1 , B_2/B_1 , and M on different numbers (but not in a very systematic way) and kept QT = 40 fixed. We observe that the average CPU time for numbers in the 67–88 digit range varies between 0.4 and 12 CPU hours, so that increasing the number of digits by three gives an increase of the sieve time by a factor of about 1.64. This is smaller than the factor of 2 that is usually observed for PMPQS.

ACKNOWLEDGEMENTS

We thank Arjen Lenstra, Walter Lioen and Rob Tijdeman for reading the paper and for suggesting several improvements. Walter Lioen helped us with the implementation of our programs on SGI workstations and on Cray vector computers. We gratefully acknowledge the Dutch National Computing Facilities Foundation NCF for the provision of computer time on Cray Y-MP and Cray C90 vector processors. Finally, we acknowledge the help of IBM and the Academic Computing Center Amsterdam (SARA) for providing access to and CPU time on the IBM SP1 at SARA.

REFERENCES

[Alford and Pomerance 1995] W. R. Alford and C. Pomerance, "Implementing the self-initializing quadratic sieve on a distributed network", pp. 163– 174 in Number-theoretic and algebraic methods in computer science (NTAMCS '93: Moscow, 1993), edited by A. J. van der Poorten et al., World Scientific, River Edge, NJ, 1995.

[Atkins et al. 1995] D. Atkins, M. Graff, A. K. Lenstra, and P. C. Leyland, "The magic words are squeamish ossifrage", pp. 263-277 in Advances in cryptology (ASIACRYPT '94: Wollongong, 1994), edited by J. Pieprzyk and R. Safavi-Naini, Lecture Notes in Comp. Sci. 917, Springer, Berlin, 1995.

#	\overline{n}	prime factor(s)
1	C66 from $77^{53} + 1 = P31 \cdot P35$	P31 = 8508101816450689975658227843439
2	C67 from $58^{88} + 1 = P26 \cdot P41$	P26 = 62057338333442627487392257
3	C67 from $62^{89} - 1 = P31 \cdot P37$	P31 = 3916898265747514256035560079891
4	C75 from $70^{87} + 1 = P29 \cdot P46$	P29 = 56476537654063551106920429541
5	C79 from $72^{118} + 1 = P38 \cdot P42$	P38 = 16059490907009321225480347480687832441
6	C82 from $84^{71} + 1 = P33 \cdot P50$	P33 = 133184106044570646620234096956423
7	C82 from $80^{99} + 1 = P32 \cdot P51$	P32 = 11935171798229644025656192643827
8	C83 from $92^{87} + 1 = P23 \cdot P61$	P23 = 10127992394070979564027

TABLE 6. Parameter choices, timings, and factors for numbers ranging from 66 to 83 decimal digits, factored with PPMPQS on a SGI workstation. Key: n = number to be factored ("Cx from y" means a composite factor of y having x decimal digits); $d = \log_{10} n$; $B_1 =$ upper bound for the primes in the factor base; $B_2^2 =$ upper bound for the input R to SQUFOF (yielding a pp-relation); $n_f =$ number of primes in the factor base;

#	n	prime factor(s)
1	C67 from $89^{64} + 1 = P24 \cdot P44$	P24 = 153316525308739316934017
2	C69 from $50^{122} + 1 = P30 \cdot P40$	P30 = 276832194921994230575098974137
3	C75 from $101^{41} + 1 = P32 \cdot P43$	P32 = 21587227703328821952030527314507
4	C75 from $110^{41} + 1 = P16 \cdot P25 \cdot P35$	P16 = 3850561614882023 P25 = 7797598239853074057655219
5	C75 from $110^{47} + 1 = P24 \cdot P51$	P24 = 728424414211828929294823
6	C75 from $35^{147} + 1 = P35 \cdot P40$	P35 = 86052439411099140168070862933143801
7	C75 from $53^{59} - 1 = P24 \cdot P51$	P24 = 943970114867362247759443
8	C78 from $19^{165} + 1 = P28 \cdot P50$	P28 = 2481953419044452308291386601
9	C78 from $51^{102} + 1 = P30 \cdot P48$	P30 = 459028910227193494771112394289
10	C80 from $86^{58} + 1 = P33 \cdot P47$	P33 = 129094951090723152084884804969621
11	C80 from $75^{64} + 1 = P32 \cdot P48$	P32 = 68799038786512319388821350925569
12	C80 from $59^{85} - 1 = P36 \cdot P44$	P36 = 192052183634195717382812875959337681
13	C80 from $76^{123} + 1 = P28 \cdot P53$	P28 = 1602475801546350975094860307
14	C80 from $84^{87} - 1 = P40 \cdot P41$	P40 = 2904043752413366850400636076474517615769
15	C81 from $18^{103} - 1 = P35 \cdot P47$	P35 = 15936754604932361311519937275763087
16	C83 from $82^{68} + 1 = P40 \cdot P43$	P40 = 9241855378580566956862595601843404638609
17	C83 from $93^{71} + 1 = P34 \cdot P50$	P34 = 1871598891695207952802939248474557
18	C84 from $89^{67} - 1 = P41 \cdot P44$	P41 = 17345460386856072657168883886351357651503
19	C84 from $74^{91} - 1 = P31 \cdot P54$	P31 = 6300454649733691099786120178647
20	C85 from $69^{117} + 1 = P42 \cdot P43$	P42 = 553775456930001686459646662784000439421893
21	C85 from $98^{91} + 1 = P39 \cdot P47$	P39 = 150856027763097994901861400756223948651
22	C85 from $80^{58} + 1 = P42 \cdot P44$	P42 = 587407531780545617292693056474932755332969
23	C85 from $56^{64} + 1 = P43 \cdot P43$	P43 = 1120971223480359091305712645673434758493441
24		P32 = 38661901037861787717347412050407
25	C85 from $77^{95} - 1 = P34 \cdot P52$	P34 = 1254200040785197567017611121581711
26	C86 from $18^{111} + 1 = P35 \cdot P51$	P35 = 57095169829153516132919139336069139
27	C86 from $76^{59} + 1 = P39 \cdot P47$	P39 = 471586815074704431240140019672222092489
28	C86 from $20^{97} + 1 = P34 \cdot P52$	P34 = 2645332912014287669339495089951567

TABLE 7. Parameter choices, timings, and factorizations for numbers ranging from 67 to 88 decimal digits,

#	d	$B_1/10^5$	n_f	B_2/B_1	$M/10^5$	n_c	n_1	$n_{c,1}$	n_2	$n_{c,2}$	T_s
1	65.56	0.8	3911	11.25	2	1493	9753	1715	4102	710	5.8 h
2	66.17	0.8	3908	10	1.5	1452	9433	1766	3697	693	4.8 h
3	66.83	0.8	3984	10	2	1214	9952	2139	4238	637	14.2 h
4	74.15	3	13045	20	6	4840	37472	4790	26391	3424	$55.4\mathrm{h}$
5	78.76	3	12898	30	5	4444	44583	5104	29653	3355	123 h
6	81.54	5	20812	20	5	7992	63176	8471	33614	4351	173 h
7	81.70	4.5	18961	20	4.5	6796	55435	7229	38950	4942	198 h
8	82.89	5	20861	20	8	7387	62346	8229	40035	5250	273 h

[-M,M] = sieve interval; n_c = number of complete relations found immediately; n_1 = number of partial relations; $n_{c,1}$ = number of complete relations coming from partial relations; n_2 = number of pp-relations; $n_{c,2}$ = number of complete relations coming from pp-relations; T_s = sieve CPU time. The small-prime variation parameter QT is always 40.

#	d	$B_1/10^5$	n_f	B_2/B_1	$M/10^{5}$	n_c	n_1	$n_{c,1}$	n_2	$n_{c,2}$	T_s
1	66.80	2	8881	30	25	2945	27673	2762	31855	3347	0.36 h
2	68.74	2.5	11086	20	5	3988	30107	3631	27746	3476	0.46 h
3	74.20	3.16	13623	20	6.31	4921	38371	4889	29855	3822	1.22 h
4	74.51	3.16	13625	20	6.31	5503	42284	5844	17604	2297	1.16 h
5	74.69	1	4790	60	5	1005	17630	1320	29502	2465	$2.42\mathrm{h}$
6	74.83	3	12892	17	25	4697	37137	5388	19447	2820	1.20 h
7	74.92	2.5	11086	36	25	3339	35899	3335	43531	4382	1.91 h
8	77.37	5	20972	20	25	7152	54706	6444	60361	7393	1.84 h
9	77.56	5	20888	30	30	7518	65930	6980	60042	6453	1.41 h
10	79.04	5	20597	30	30	6596	61563	6201	76295	7828	2.43 h
11	79.17	4	16927	20	1	5619	45717	5584	48399	6279	$3.29\mathrm{h}$
12	79.17	5	20895	20	3	6457	72272	11650	37114	2802	2.68 h
13	79.39	3	13001	166.7	3	3739	68195	4113	72708	5157	$2.27\mathrm{h}$
14	79.87	3	13011	166.7	3	3323	64308	3624	91150	6084	$3.41\mathrm{h}$
15	80.86	5	20819	20	6	6925	57619	7050	55281	6877	$3.36\mathrm{h}$
16	82.82	6	24598	20	2.5	8522	68723	8378	59901	7713	4.82 h
17	82.91	7	28413	20	2.5	11451	87010	11694	40636	5271	4.38 h
18	83.66	8	32104	25	2.5	11419	96138	10894	85260	9807	5.46 h
19	83.98	7	27980	25.7	2.5	10594	93766	11327	51233	6070	6.59 h
20	84.10	5	20713	20	2.5	6175	51592	5808	76377	8732	5.6 h
21	84.35	5	20741	20	2.5	6865	60444	7638	72201	6256	9.8 h
22	84.80	5	20744	20	2.5	7809	57576	7457	44022	5481	9.8 h
	04.05	5	20790	20	2.5	7153	61546	7923	43044	5721	8.4 h
23	84.87	5	20790	40	2.5	6412	73385	6960	75133	7427	8.4 h
24	84.92	5	20930	20	2.5	6434	52315	5865	75217	8614	6.0 h
25	84.99	5	20749	20	2.5	7106	58607	7259	53507	6389	6.8 h
26	85.02	5	20675	20	2.5	6982	61080	7920	64746	5774	9.8 h
27	85.02	5	20792	20	2.5	6679	58782	7268	81258	6853	11. h
28	85.05	5	20887	20	2.5	7754	65228	8990	46265	4178	8.4 h

factored with PPMPQS on a Cray C90 vector computer. Key as in Table 6. Continued overleaf.

# n prime factor(s) prime factor(s) 29 C86 from $33^{90} - 1 = P31 - P55 - P31 = 3466732593888008254791613360081$ 30 C86 from $56^{93} - 1 = P32 \cdot P54$ 31 C86 from $56^{94} + 1 = P39 \cdot P47$ 32 C86 from $56^{94} + 1 = P39 \cdot P47$ 32 C86 from $56^{94} + 1 = P39 \cdot P47$ 33 C86 from $56^{94} + 1 = P34 \cdot P43$ 48 P43 = 246515271565874842883088099424343633019833 33 C86 from $56^{94} - 1 = P34 \cdot P52$ 34 C86 from $56^{94} - 1 = P34 \cdot P52$ 35 C86 from $56^{94} - 1 = P34 \cdot P52$ 36 C86 from $56^{94} - 1 = P31 \cdot P55$ 37 C86 from $56^{94} - 1 = P31 \cdot P55$ 38 C86 from $33^{83} - 1 = P36 \cdot P50$ 38 C86 from $33^{83} - 1 = P36 \cdot P50$ 38 C86 from $33^{83} - 1 = P36 \cdot P50$ 38 C86 from $33^{81} - 1 = P36 \cdot P50$ 39 C86 from $33^{81} - 1 = P36 \cdot P50$ 40 C86 from $33^{81} - 1 = P36 \cdot P50$ 40 C86 from $36^{96} - 1 = P34 \cdot P51$ 41 C86 from $36^{96} + 1 = P24 \cdot P54$ 42 C86 from $36^{96} + 1 = P23 \cdot P55$ 42 C86 from $36^{96} + 1 = P23 \cdot P55$ 43 C86 from $36^{96} + 1 = P23 \cdot P55$ 44 C86 from $36^{96} + 1 = P23 \cdot P50$ 45 C86 from $36^{96} + 1 = P23 \cdot P50$ 46 C86 from $36^{96} + 1 = P23 \cdot P50$ 47 C86 from $36^{96} + 1 = P33 \cdot P50$ 48 C86 from $36^{96} + 1 = P33 \cdot P50$ 49 C86 from $36^{96} + 1 = P33 \cdot P50$ 40 C86 from $36^{96} + 1 = P33 \cdot P50$ 41 C86 from $36^{96} + 1 = P33 \cdot P50$ 42 C86 from $36^{96} + 1 = P33 \cdot P50$ 43 C86 from $36^{96} + 1 = P34 \cdot P51$ 44 C86 from $36^{96} + 1 = P34 \cdot P51$ 45 C86 from $36^{96} + 1 = P34 \cdot P51$ 46 C86 from $36^{96} + 1 = P34 \cdot P51$ 47 C86 from $36^{96} + 1 = P34 \cdot P51$ 48 C86 from $36^{96} + 1 = P34 \cdot P54$ 49 C86 from $36^{96} + 1 = P34 \cdot P54$ 40 C86 from $36^{96} + 1 = P34 \cdot P54$ 41 C86 from $36^{96} + 1 = P34 \cdot P54$ 42 C86 from $36^{96} + 1 = P34 \cdot P54$ 43 C86 from $36^{96} + 1 = P34 \cdot P54$ 44 C86 from $36^{96} + 1 = P34 \cdot P54$ 45 C86 from $36^{96} + 1 = P34 \cdot P54$ 46 C86 from $36^{96} + 1 = P34 \cdot P54$ 47 C86 from $36^{96} + 1 = P34 \cdot P54$ 48 C86 from $36^{96} + 1 = P34 \cdot P54$ 49 C86 from $36^{96} + 1 = P34 \cdot P54$ 40 C86 from $36^{96} + 1 = P34 \cdot P54$ 41				
30 C86 from 56 ⁸⁹ - 1 = P32 - P54 P32 = 7570185042739143157590250368211 31 C86 from 67 ⁸⁶ - 1 = P34 - P43 P39 = 232559086557404752033343407938321409 32 C86 from 92 ⁸⁴ - 1 = P34 - P43 P43 = 246515271565874842883088994824343639019833 33 C86 from 67 ⁸⁶ - 1 = P34 - P52 P34 = 2515208214206285121254951932641469 34 C86 from 13 ⁸³⁴ 1 = P39 - P57 P29 = 5485636771655203699971812089 35 C86 from 38 ⁸¹ - 1 = P31 - P55 P31 = 26899414244883480323848649808389 36 C86 from 38 ⁸¹ - 1 = P36 - P50 P36 = 511662075163970762060417539436484323 37 C86 from 38 ⁸¹ - 1 = P36 - P50 P36 = 511662075163970762060417539436484323 38 C86 from 30 ⁸¹ - 1 = P38 - P50 P36 = 511662075163970762060417539436484323 39 C86 from 50 ⁶¹ - 1 = P38 - P50 P36 = 684989928641194001785075922656446841 40 C86 from 96 ⁸⁵ + 1 = P38 - P50 P36 = 684989928641194001785075922656446841 41 C86 from 96 ⁸⁵ + 1 = P32 - P58 P38 = 1919269986955025383980959785510672821873 42 C86 from 80 ⁸⁵ + 1 = P31 - P55 P31 = 3416871674919158699528742801241 43 C86 from 80 ⁸⁵ + 1 = P31 - P55 P31 = 3416871674919158699528742801241 44 C86 from 80 ⁸⁵ + 1 = P31 - P55 P31 = 3416871674919158699528742801241 45 C86 from 28 ⁸³ - 1 = P38 - P39 P38 = 1919295350603466060783986052760977025136897 P44 = 6575767424035583516769248181741955409999833473 46 C86 from 476 ⁷⁷ - 1 = P32 - P55 P32 = 21270964162538889103104983761851 47 C86 from 67 ⁶⁶ + 1 = P40 - P47 P40 = 488869958817220522890920460964327586529 48 C86 from 76 ¹⁶⁷ - 1 = P32 - P55 P32 = 21270964162538089013014983761851 50 C86 from 28 ⁸⁵ - 1 = P33 - P39 P38 = 7315367592181313518059 51 C86 from 28 ⁸⁶ - 1 = P34 - P52 P3 P3 = 3443003616566663099989827441133518059 51 C86 from 76 ¹⁶⁷ - 1 = P42 - P45 P42 = 315618216027848486834301078445774290254513 50 C86 from 76 ¹⁶⁷ - 1 = P42 - P45 P42 = 315618216027848486834301078445774290254513 51 C86 from 76 ¹⁶⁷ - 1 = P42 - P45 P42 = 315618216027848486834301078445774290254513 52 C86 from 76 ¹⁶⁷ - 1 = P42 - P45 P42 = 315618216027848486834301078445774290254513 53 C86 from 76 ¹⁶⁷ - 1 = P34 - P52 P59 P38 = 8	#			prime factor(s)
31 C86 from 92 ⁸⁴ + 1 = P43 · P47 P38 = 23255908655740746776290133340798321409 32 C86 from 92 ⁸⁴ + 1 = P43 · P43 R4 = 2465152715658748428830880994824343639019833 33 C86 from 67 ⁹⁰ - 1 = P34 · P52 A C86 from 67 ⁹⁰ - 1 = P34 · P52 R4 C86 from 56 ⁸⁶ - 1 = P34 · P52 R5 C86 from 56 ⁸⁶ - 1 = P34 · P52 R5 C86 from 56 ⁸⁶ - 1 = P34 · P52 R5 C86 from 21 ¹²³ + 1 = P39 · P47 R5 C86 from 31 ¹³⁸ - 1 = P36 · P50 R5 C86 from 31 ¹³⁸ - 1 = P36 · P50 R5 C86 from 31 ¹³⁷ - 1 = P39 · P47 R5 C86 from 31 ¹³⁷ - 1 = P39 · P47 R5 C86 from 31 ¹³⁷ - 1 = P39 · P47 R5 C86 from 31 ¹³⁷ - 1 = P39 · P47 R5 C86 from 31 ¹³⁷ - 1 = P39 · P47 R5 C86 from 31 ¹³⁷ - 1 = P39 · P47 R5 C86 from 31 ¹³⁷ - 1 = P39 · P47 R5 C86 from 56 ⁸⁶ + 1 = P35 · P51 R5 C86 from 98 ⁸⁶ + 1 = P38 · P59 R5 C86 from 98 ⁸⁶ + 1 = P38 · P59 R5 C86 from 98 ⁸⁶ + 1 = P38 · P49 R5 C86 from 98 ⁸⁶ + 1 = P38 · P49 R5 C86 from 98 ⁸⁶ + 1 = P31 · P55 R5 C86 from 88 ⁷⁷ - 7 = P42 · P44 R5 C86 from 88 ⁷⁸ - 7 + 2 ⁷² R5 C86 from 24 ⁸³ - 1 = P38 · P49 R5 C86 from 88 ⁷⁸ - 1 = P31 · P55 R5 C86 from 88 ⁷⁸ - 1 = P31 · P55 R5 C86 from 88 ⁷⁸ - 1 = P31 · P55 R5 C86 from 88 ⁷⁸ - 1 = P31 · P55 R5 C86 from 88 ⁷⁸ - 1 = P31 · P55 R5 C86 from 88 ⁷⁸ - 1 = P31 · P55 R5 C86 from 88 ⁷⁸ - 1 = P31 · P55 R5 C86 from 88 ⁷⁸ - 1 = P31 · P55 R5 C86 from 88 ⁷⁸ - 1 = P31 · P55 R5 C86 from 88 ⁷⁸ - 1 = P31 · P55 R5 C86 from 88 ⁷⁸ - 1 = P31 · P55 R5 C86 from 88 ⁷⁸ - 1 = P31 · P55 R5 C86 from 88 ⁷⁸ - 1 = P31 · P55 R5 C86 from 98 ⁸⁸ - 1 = P31 · P54 R5 C86 from 98 ⁸⁸ - 1 = P31 · P55 R5 C86 from 98 ⁸⁸ - 1 = P31 · P55 R5 C86 from 98 ⁸⁸ - 1 = P31 · P55 R5 C86 from 98 ⁸⁸ - 1 = P31 · P59 R5 C86 from 98 ⁸⁸ - 1 = P31 · P50 R5 C86 from 98 ⁸⁸ - 1 = P31 · P50 R5 C86 from 98 ⁸⁸ - 1 = P31 · P50 R5 C86 from 98 ⁸⁸ - 1 = P31 · P50 R5 C86 from 98 ⁸⁸ - 1 = P31 · P50 R5 C86 from 98 ⁸⁸ - 1 = P31 · P50 R5 C86 from 98 ⁸⁸ - 1 = P31 · P50 R5 C86 from 98 ⁸⁸ - 1 = P31 · P50 R5 C86 from 98 ⁸⁸ - 1 = P31 · P50 R5 C86 from 98 ⁸⁸ - 1 = P31 · P50 R	29			
33 C86 from 9284 + 1 = P43 - P43 P43 P43 = 2465152715658748428830880904824343639019833 33 C86 from 13 ¹³⁸ 1 = P29 - P57 35 C86 from 13 ¹³⁸ 1 = P29 - P57 35 C86 from 21 ¹²³ 1 = P39 - P47 36 C86 from 21 ¹²³ 1 = P39 - P47 37 C86 from 38 ³⁴ - 1 = P36 - P50 38 C86 from 31 ¹¹⁷ 1 = P39 - P47 39 C86 from 50 ⁹⁶ - 1 = P31 - P55 40 C86 from 50 ⁹⁶ - 1 = P35 - P51 40 C86 from 98 ³⁵ 1 = P38 - P51 41 C86 from 98 ³⁵ 1 = P38 - P51 42 C86 from 98 ³⁵ 1 = P38 - P55 43 C86 from 98 ³⁵ 1 = P38 - P55 44 C86 from 98 ³⁵ 1 = P38 - P57 45 C86 from 98 ³⁵ 1 = P38 - P58 46 C86 from 98 ³⁵ 1 = P38 - P58 47 C86 from 98 ³⁵ 1 = P38 - P59 48 C86 from 98 ³⁵ 1 = P38 - P59 49 C86 from 98 ³⁵ 1 = P38 - P59 40 C86 from 98 ³⁵ 1 = P38 - P59 40 C86 from 98 ³⁵ 1 = P38 - P59 41 C86 from 98 ³⁵ 1 = P38 - P59 42 C86 from 98 ³⁵ 1 = P38 - P59 43 C86 from 98 ³⁵ 1 = P38 - P59 44 C86 from 98 ³⁵ 1 = P38 - P59 45 C86 from 98 ³⁵ 1 = P38 - P59 46 C86 from 98 ³⁵ 1 = P38 - P59 47 C86 from 28 ³⁵ 1 = P38 - P49 48 C86 from 28 ³⁵ 1 = P38 - P49 49 C86 from 67 ⁷⁶ 1 = P32 - P55 50 C86 from 98 ³⁵ 1 = P38 - P49 50 C86 from 98 ³⁵ 1 = P38 - P49 51 C86 from 98 ³⁵ 1 = P38 - P49 52 C86 from 98 ³⁵ 1 = P38 - P49 53 C86 from 98 ³⁵ 1 = P38 - P49 54 C86 from 98 ³⁵ 1 = P39 - P50 55 C86 from 98 ³⁵ 1 = P39 - P50 56 C86 from 98 ³⁵ 1 = P39 - P50 57 C87 from 98 ³⁵ 1 = P39 - P50 58 C86 from 98 ³⁵ 1 = P39 - P50 59 C86 from 98 ³⁵ 1 = P39 - P50 50 C86 from 98 ³⁵ 1 = P39 - P50 50 C86 from 98 ³⁵ 1 = P39 - P50 50 C86 from 98 ³⁵ 1 = P39 - P50 50 C86 from 98 ³⁵ 1 = P39 - P50 50 C86 from 98 ³⁵ 1 = P39 - P50 50 C86 from 98 ³⁵ 1 = P39 - P50 50 C86 from 98 ³⁵ 1 = P39 - P50 50 C86 from 98 ³⁵ 1 = P39 - P50 50 C86 from 98 ³⁵ 1 = P39 - P50 50 C86 from 98 ³⁵ 1 = P39 - P50 50 C86 from 98 ³⁵ 1 = P39 - P50 50 C86 from 98 ³⁵ 1 = P39 - P50 50 C86 from 98 ³⁵ 1 = P39 - P50 50 C86 from 98 ³⁵ 1 = P39 - P50 50 C86 from 98 ³⁵ 1 = P39 - P50 50 C86 from 98 ³⁵ 1 = P39 - P50 50 C86 from 98 ³⁵ 1 = P39 - P50 50 C86 fr	I .			
33 C86 from 67** 1 = P34 · P52 34 C86 from 131** 1 = P39 · P57 35 C86 from 159** 1 = P31 · P52 36 C86 from 59** 1 = P31 · P52 37 C86 from 31** 1 = P39 · P47 38 C86 from 31** 1 = P39 · P47 39 C86 from 31** 1 = P39 · P47 39 C86 from 31** 1 = P39 · P47 39 C86 from 31** 1 = P35 · P51 30 C86 from 31** 1 = P35 · P51 31 C86 from 31** 1 = P35 · P51 32 C86 from 31** 1 = P36 · P50 33 C86 from 31** 1 = P36 · P50 34 C86 from 31** 1 = P36 · P50 35 C86 from 36** 1 = P36 · P50 36 C86 from 59** 1 = P35 · P51 37 C86 from 59** 1 = P38 · P47 39 C86 from 59** 1 = P38 · P51 30 C86 from 59** 1 = P38 · P51 31 C86 from 24** 1 = P36 · P50 32 C86 from 98** 1 = P38 · P49 42 C86 from 98** 1 = P38 · P49 43 C86 from 80** 1 = P31 · P55 44 C86 from 80** 1 = P31 · P55 45 C86 from 80** 1 = P31 · P55 46 C86 from 80** 1 = P31 · P55 47 C86 from 80** 1 = P31 · P55 48 C86 from 47** 7 - 1 = P32 · P55 49 C86 from 47** 7 - 1 = P32 · P55 51 C86 from 98** 1 = P34 · P55 51 C86 from 76** 1 = P42 · P45 52 C86 from 38** 1 = P34 · P55 53 C86 from 38** 1 = P34 · P55 54 C86 from 40** 1 = P40 · P41 · P41 · P41 · P44 · E86809568817205928002040694327586529 54 C86 from 76** 1 = P42 · P45 · P42 · E86809568817205928002040694327586529 55 C86 from 76** 1 = P42 · P45 · P42 · E868056881705898091049833761851 56 C86 from 76** 1 = P42 · P45 · P42 · E86809568817205928002040694327586529 56 C87 from 72** 1 = P38 · P59 · P32 · E2170964162538089013014983761851 57 C87 from 22** 1 = P38 · P59 · P32 · E317058500255270742194544605533987 58 C87 from 72** 1 = P32 · P55 · P32 · E317058500255270742194544605533987 58 C87 from 72** 1 = P32 · P56 · P32 · E4285278844357974752432939513571 58 C87 from 30** 1 = P38 · P59 · P38 · E3087580997186890210686588841 58 C87 from 30** 1 = P38 · P59 · P38 · E3087580997186890210686588841 59 C87 from 30** 1 = P38 · P59 · P32 · E4285278844357974752432939513571 57 C87 from 30** 1 = P32 · P56 · P32 · E4285278844357974752432939513571 58 C87 from 30** 1 = P33 · P56 · P32 · E4285278844357974752432939513571 59 C87 from 30** 1 = P33 · P56 · P33 · E3456929891901068	l .			
34 C86 from 13 ¹³⁸ , 1 = P29 · P57 P29 = 54836637716450236909071812089 35 C86 from 50 ⁸⁹ - 1 = P31 · P55 P31 = 268994142448848643803348643803381 36 C86 from 32 ¹¹⁷ · 1 = P39 · P47 P39 = 3807706353969474313312691529545132713 37 C86 from 33 ¹¹⁷ · 1 = P39 · P47 P39 = 250630033376957433234617073114910871767 39 C86 from 50 ¹⁶ + 1 = P35 · P51 P35 = 36774112300765382067961168652800897 40 C86 from 96 ⁸⁵ + 1 = P28 · P58 P28 = 241847699068876014581890831 41 C86 from 93 ¹³ + 1 = P36 · P50 P36 = 684898928644194001785075922656446841 42 C86 from 93 ¹³ + 1 = P31 · P55 P31 = 3146871674919185699528742801241 42 C86 from 98 ¹⁹ + 1 = P32 · P55 P32 = 29037047448209810589475647292291 44 C86 from 80 ⁸⁵ + 1 = P31 · P55 P31 = 3416871674919185699528742801241 45 C86 from 82 ²⁷ + 7 ²⁷ = P42 · P44 P44 E6757674240355835167624181741955409969883473 46 C86 from 23 ²⁵ - 1 = P38 · P49 P38 = 27736074503263071062950777025136897 P44 = 657576742403558351676241817419554099698833473 P38 = 2773607450326307106295077708805902164759 47 C86 from 76 ⁵⁶ + 1 = P40 · P47 P40 = 486869568817220592890920460964327586529 P32 = 21270964162538089013014983761851 50 C86 from 39 ⁵¹ + 1 = P37 · P50 P31 = 315618216073843458633430174845774290254513 51 C86 from 76 ¹⁵¹ - 1 = P42 · P45 P42 = 315618216073843458633430174845774290254513 52 C86 from 76 ¹⁵¹ - 1 = P42 · P45 P34 = 31561821607384345863343017844574290254513 53 C86 from 76 ¹⁵⁷ - 1 = P33 · P55 P32 = 21270964162538089013014983761851 54 C87 from 76 ¹⁵⁹ + 1 = P38 · P49 P38 = 27888050052552707442194584400553987 55 C87 from 76 ¹⁵⁹ + 1 = P38 · P49 P38 = 44606228871058500552570742194584400553987 56 C87 from 30 ¹⁵⁶ + 1 = P31 · P50 P32 = 445602288710580500552570742194584400553987 57 C87 from 30 ⁵⁶ + 1 = P31 · P56 P32 = 445602288710580005867600753931750000886762077392077601 56 C87 from 35 ⁵⁶ + 1 = P33 · P56 P32 = 445602082871058000580760055155451 57 C87 from 35 ⁵⁶ + 1 = P33 · P56 P32 = 2495569219891628050055151257001 58 C87 from 35 ⁵⁶ + 1 = P31 · P36 P3 · P38 = 249556921893123939613571 58 C87 from 35 ⁵⁶	32	C86 from	$92^{84} + 1 = P43 \cdot P43$	P43 = 2465152715658748428830880994824343639019833
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	33	C86 from	$67^{99} - 1 = P34 \cdot P52$	P34 = 2515208214206285121254951932641469
36 C86 from $21^{123} + 1 = P39 \cdot P47$ P39 = $38077006353966947431312691529545132713$ 37 C86 from $38^{11} - 1 = P39 \cdot P50$ P36 = $5116650751639707620604175384684323$ 38 C86 from $50^{96} + 1 = P35 \cdot P51$ P35 = $256030033376957435234617073114910871767$ 39 C86 from $96^{95} + 1 = P35 \cdot P51$ P35 = $36774112300765382067961168652800897$ 40 C86 from $96^{95} + 1 = P35 \cdot P51$ P36 = $6643989928641940017785075922856446841$ 41 C86 from $93^{35} + 1 = P38 \cdot P58$ P38 = $19192699869550253389099978550167828173$ P32 = $97374744820981059478647292291$ 43 C86 from $98^{59} + 1 = P32 \cdot P55$ P31 = $34168716749191158699528742801241$ 45 C86 from $80^{57} + 7^{27} = P42 \cdot P44$ P42 = $519975935060346660783986052760977025136897$ P44 = $657576742440555835167624181741955409969833473$ P38 = $7277860745023630710629507770850992164759$ P44 = $65757674240355835167624181741955409969833473$ P38 = $7277860745023603710629507770850992164759$ P46 = $86660000000000000000000000000000000000$	34			P29 = 54836637716450236990971812089
37 C86 from $38^{51} - 1 = P36 \cdot P50$ P36 = $511662075163970762060417538436484323$ 38 C86 from $31^{117} - 1 = P39 \cdot P47$ P39 = $250630033376957433234617073114910871767$ 39 C86 from $96^{95} + 1 = P38 \cdot P59$ P28 = $2418476990688796014581890831$ 41 C86 from $96^{95} + 1 = P36 \cdot P50$ P36 = $6849899286411940017850758922856446841$ 42 C86 from $33^{53} + 1 = P38 \cdot P49$ P38 = $19192699869550253389095978550167828173$ 43 C86 from $89^{59} + 1 = P32 \cdot P55$ P32 = $290370474448209810589475647292291$ 44 C86 from $80^{55} + 1 = P31 \cdot P55$ P31 = $3416871674919158699528742801241$ 45 C86 from $32^{55} - 1 = P38 \cdot P49$ P38 = $27736074503263071062950778805992164759$ 46 C86 from $32^{55} - 1 = P38 \cdot P49$ P38 = $27736074503263071062950778805992164759$ 47 C86 from $47^{67} - 1 = P32 \cdot P55$ P32 = $21270964162538089013014983761851$ 48 C86 from $47^{67} - 1 = P32 \cdot P55$ P32 = $21270964162538089013014983761851$ 49 C86 from $32^{53} - 1 = P34 \cdot P50$ P37 = $2443003616566663069989278441133518059$ 51 C86 from $32^{51} - 1 = P34 \cdot P50$ P37 = $2443003616566663069989278441133518059$ 51 C86 from $32^{51} - 1 = P34 \cdot P50$ P38 = $4508975809971867831637486244759667041$ 52 C86 from $32^{50} - 1 = P34 \cdot P50$ P38 = $4508975809971867831637486244759667041$ 54 C87 from $26^{55} + 1 = P34 \cdot P50$ P38 = $4508975809971867831637486244759667041$ 55 C87 from $32^{55} - 1 = P34 \cdot P50$ P38 = $44089529025292424484110155444391$ 56 C87 from $32^{55} - 1 = P34 \cdot P50$ P38 = $44089529025292424484110155444391$ 57 C87 from $32^{55} - 1 = P34 \cdot P50$ P38 = $44089529025292424484110155444391$ 58 C87 from $32^{55} - 1 = P34 \cdot P50$ P38 = $44089529025292424484110155444391$ 59 C87 from $32^{55} - 1 = P34 \cdot P50$ P38 = $44089529025292424484110155444391$ 50 C87 from $32^{55} - 1 = P34 \cdot P50$ P38 = $4408952902592629242448411015544591$ 57 C87 from $33^{55} + 1 = P34 \cdot P50$ P38 = $44089529029292083745643939513571$ 58 C87 from $33^{55} - 1 = P34 \cdot P50$ P39 P38 = $44089529029292083745649391313771$ 59 C87 from $33^{55} - 1 = P35 \cdot P53$ P38 = 44089529029916086588841 60 C87 from $33^{$				
38 C86 from $50^{66} + 1 = 193 \cdot P47$ P39 = 250630033376957433234617073114910871767 39 C86 from $50^{66} + 1 = 1928 \cdot P58$ P28 = 2418476990688796014581890831 41 C86 from $95^{56} + 1 = 1928 \cdot P58$ P28 = 2418476990688796014581890831 41 C86 from $95^{55} + 1 = 1928 \cdot P58$ P36 = 684989928644194001785075922656446841 42 C36 from $95^{55} + 1 = 1928 \cdot P58$ P38 = 19192699869550253389095978550167828173 43 C36 from $80^{56} + 1 = 191 \cdot P55$ P31 = 3416871674919158699528742801241 45 C86 from $80^{55} + 1 = 191 \cdot P55$ P31 = 3416871674919158699528742801241 45 C86 from $80^{55} + 1 = 191 \cdot P55$ P31 = 3416871674919158699528742801241 46 C86 from $65^{56} + 1 = 191 \cdot P55$ P32 = 21770674240355835167624181741955409969833473 46 C86 from $47^{67} - 1 = 193 \cdot P55$ P32 = 21270964162538089013014983761851 48 C86 from $47^{67} - 1 = 193 \cdot P55$ P32 = 21270964162538089013014983761851 49 C36 from $47^{67} - 1 = 193 \cdot P55$ P32 = 21270964162538089013014983761851 50 C36 from $39^{51} + 1 = 193 \cdot P55$ P32 = 243003616566663069989278441133518059 51 C36 from $25^{50} - 1 = 1934 \cdot P52$ P34 = 962435791906840355091512367141261 52 C36 from $76^{50} + 1 = 1934 \cdot P52$ P34 = 962435791906840355091512367141261 53 C36 from $95^{50} + 1 = 1938 \cdot P49$ P38 = 45089758099791867831637486244759667041 54 C37 from $20^{50} + 1 = 1938 \cdot P59$ P38 = 45089758099791867831637486244759667041 55 C37 from $95^{50} + 1 = 1938 \cdot P59$ P35 = 804519119969344448365372156040931 56 C37 from $30^{56} + 1 = 193 \cdot P56$ P32 = 142852788443179330500886762077329077601 59 C37 from $30^{56} + 1 = 193 \cdot P56$ P32 = 142852788443193371930500886762077329077601 60 C37 from $30^{56} + 1 = 193 \cdot P56$ P35 = 804519119969344448365372156040931 57 C37 from $30^{56} + 1 = 193 \cdot P56$ P32 = 142852788443179330500886762077320777017 60 C37 from $30^{56} + 1 = 193 \cdot P56$ P33 = 32447309094137193434426100841739 61 C37 from $30^{56} + 1 = 193 \cdot P56$ P31 = 3076814278757622888317626405309 62 C37 from $30^{56} + 1 = 193 \cdot P56$ P31 = 30768142787576228883176226403309 63 C37 from $30^{56} + 1 = 19$	36			P39 = 380770063539669474313312691529545132713
$\begin{array}{c} 39 & \text{C86 from} & 50^{96} + 1 = \text{P35} \cdot \text{P51} \\ 40 & \text{C86 from} & 69^{56} + 1 = \text{P28} \cdot \text{P58} \\ 80^{56} & \text{P28} & \text{P28} = 2418476990688796014581890831 \\ 41 & \text{C86 from} & 24^{130} + 1 = \text{P36} \cdot \text{P50} \\ 42 & \text{C86 from} & 98^{56} + 1 = \text{P38} \cdot \text{P50} \\ 43 & \text{C86 from} & 98^{56} + 1 = \text{P32} \cdot \text{P55} \\ 44 & \text{C86 from} & 98^{56} + 1 = \text{P32} \cdot \text{P55} \\ 44 & \text{C86 from} & 98^{56} + 1 = \text{P31} \cdot \text{P55} \\ 44 & \text{C86 from} & 98^{56} + 1 = \text{P31} \cdot \text{P55} \\ 44 & \text{C86 from} & 80^{66} + 1 = \text{P31} \cdot \text{P55} \\ 44 & \text{C86 from} & 80^{66} + 1 = \text{P31} \cdot \text{P55} \\ 45 & \text{C86 from} & 80^{66} + 1 = \text{P31} \cdot \text{P55} \\ 46 & \text{C86 from} & 23^{85} - 1 = \text{P38} \cdot \text{P49} \\ 47 & \text{C86 from} & 23^{85} - 1 = \text{P38} \cdot \text{P49} \\ 48 & \text{C86 from} & 76^{56} + 1 = \text{P40} \cdot \text{P47} \\ 49 & \text{C86 from} & 76^{56} + 1 = \text{P40} \cdot \text{P47} \\ 49 & \text{C86 from} & 76^{56} + 1 = \text{P40} \cdot \text{P47} \\ 40 & \text{486869956881722059289920460964327586529} \\ 48 & \text{C86 from} & 76^{56} + 1 = \text{P40} \cdot \text{P47} \\ 49 & \text{C86 from} & 67^{76} + 1 = \text{P42} \cdot \text{P45} \\ 49 & \text{C86 from} & 36^{36} + 1 = \text{P37} \cdot \text{P55} \\ 51 & \text{C86 from} & 39^{31} + 1 = \text{P37} \cdot \text{P55} \\ 51 & \text{C86 from} & 39^{31} + 1 = \text{P37} \cdot \text{P55} \\ 51 & \text{C86 from} & 29^{36} - 1 = \text{P34} \cdot \text{P52} \\ 52 & \text{C86 from} & 39^{31} + 1 = \text{P37} \cdot \text{P50} \\ 53 & \text{C86 from} & 30^{56} + 1 = \text{P34} \cdot \text{P53} \\ 54 & \text{C87 from} & 29^{56} - 1 = \text{P34} \cdot \text{P53} \\ 55 & \text{C87 from} & 29^{56} - 1 = \text{P34} \cdot \text{P53} \\ 56 & \text{C87 from} & 30^{56} + 1 = \text{P34} \cdot \text{P53} \\ 56 & \text{C87 from} & 30^{56} + 1 = \text{P35} \cdot \text{P52} \\ 728 & 142852788443579476524399910886053574424914584405533987 \\ 56 & \text{C87 from} & 30^{56} + 1 = \text{P35} \cdot \text{P52} \\ 728 & 1428527844357947452432939513571 \\ 76 & \text{C87 from} & 30^{56} + 1 = \text{P35} \cdot \text{P52} \\ 728 & 142852784435794752432939513571 \\ 77 & \text{C87 from} & 30^{56} + 1 = \text{P35} \cdot \text{P52} \\ 78 & 142852784435794752432939513571 \\ 78 & \text{C87 from} & 30^{56} + 1 = \text{P35} \cdot \text{P52} \\ 78 & 142852784435794752432939513571 \\ 78 & \text{C87 from} & 30^{56} + 1 = \text{P35} \cdot \text{P53} \\ 78 & 12485279607444483683182$	37	C86 from	$38^{81} - 1 = P36 \cdot P50$	P36 = 511662075163970762060417538436484323
$ \begin{array}{c} 40 \text{C86 from} 96^{95} + 1 = \text{P28} \cdot \text{P58} \\ 41 \text{C86 from} 24^{130} + 1 = \text{P36} \cdot \text{P50} \\ 266 \text{F8489992864419401785075922656446841} \\ 42 \text{C86 from} 93^{83} + 1 = \text{P38} \cdot \text{P50} \\ 43 \text{C86 from} 98^{89} + 1 = \text{P32} \cdot \text{P55} \\ 44 \text{C86 from} 80^{85} + 1 = \text{P31} \cdot \text{P55} \\ 44 \text{C86 from} 80^{85} + 1 = \text{P31} \cdot \text{P55} \\ 45 \text{C86 from} 80^{85} + 1 = \text{P31} \cdot \text{P55} \\ 46 \text{C86 from} 80^{85} + 1 = \text{P31} \cdot \text{P55} \\ 47 \text{C86 from} 82^{87} + 72^{7} = \text{P42} \cdot \text{P44} \\ 48 \text{C86 from} 23^{83} - 1 = \text{P38} \cdot \text{P49} \\ 47 \text{C86 from} 23^{83} - 1 = \text{P38} \cdot \text{P49} \\ 48 \text{C86 from} 76^{86} + 1 = \text{P40} \cdot \text{P47} \\ 48 \text{C86 from} 47^{87} - 1 = \text{P32} \cdot \text{P55} \\ 49 \text{C86 from} 47^{87} - 1 = \text{P32} \cdot \text{P55} \\ 49 \text{C86 from} 39^{81} + 1 = \text{P37} \cdot \text{P50} \\ 51 \text{C86 from} 39^{81} + 1 = \text{P37} \cdot \text{P50} \\ 52 \text{C86 from} 39^{81} + 1 = \text{P37} \cdot \text{P50} \\ 52 \text{C86 from} 39^{81} + 1 = \text{P37} \cdot \text{P50} \\ 52 \text{C86 from} 39^{81} + 1 = \text{P37} \cdot \text{P50} \\ 52 \text{C86 from} 95^{80} + 1 = \text{P34} \cdot \text{P53} \\ 52 \text{C86 from} 95^{80} + 1 = \text{P34} \cdot \text{P53} \\ 53 \text{C86 from} 95^{80} + 1 = \text{P34} \cdot \text{P53} \\ 54 \text{C87 from} 92^{85} - 1 = \text{P34} \cdot \text{P53} \\ 55 \text{C87 from} 92^{85} - 1 = \text{P32} \cdot \text{P55} \\ 52 \text{P32} = 1438069290252824244341105544391} \\ 55 \text{C87 from} 92^{85} - 1 = \text{P32} \cdot \text{P56} \\ 52 \text{P32} = 14390692902528242443411055444391} \\ 54 \text{C87 from} 92^{85} - 1 = \text{P32} \cdot \text{P55} \\ 52 \text{P32} = 1428527884357974752432939513571} \\ 57 \text{C87 from} 30^{85} + 1 = \text{P38} \cdot \text{P59} \\ 928 = 809754078916899910886588841} \\ 56 \text{C87 from} 92^{85} - 1 = \text{P32} \cdot \text{P56} \\ 922 = 1428527884357974752432939513571} \\ 57 \text{C87 from} 30^{85} + 1 = \text{P33} \cdot \text{P55} \\ 923 = 1428507894391444438653727156040931} \\ 58 \text{C87 from} 30^{85} + 1 = \text{P33} \cdot \text{P55} \\ 925 = 5245647644318663182854571} \\ 61 \text{C87 from} 33^{85} + 1 = \text{P33} \cdot \text{P55} \\ 923 = 243709908891689601908085818819 \\ 9207973735910608989100886588841} \\ 62 \text{C87 from} 42^{89$	38			P39 = 250630033376957433234617073114910871767
$\begin{array}{c} 41 \text{C86 from} 24^{130} + 1 = \text{P36} \cdot \text{P50} \\ 42 \text{C86 from} 93^{33} + 1 = \text{P38} \cdot \text{P49} \\ 83 19192699869550523389095978550167828173 \\ 43 \text{C86 from} 98^{59} + 1 = \text{P32} \cdot \text{P55} \\ 44 \text{C86 from} 89^{59} + 1 = \text{P31} \cdot \text{P55} \\ 81 191926998695505253389095978550167828173 \\ 44 \text{C86 from} 89^{59} + 1 = \text{P31} \cdot \text{P55} \\ 81 3416871674919158699528742801241 \\ 45 \text{C86 from} 8^{27} + 7^{27} = \text{P42} \cdot \text{P44} \\ 46 \text{C86 from} 8^{27} + 7^{27} = \text{P42} \cdot \text{P44} \\ 46 \text{C86 from} 8^{28} - 1 = \text{P38} \cdot \text{P49} \\ 47 \text{C86 from} 76^{56} + 1 = \text{P40} \cdot \text{P47} \\ 49 \text{P42} = 519975935060346660783986052760977025136897 \\ 48 \text{C86 from} 77^{66} + 1 = \text{P40} \cdot \text{P47} \\ 40 \text{P40} = 4868699568817220592890920460964327586529 \\ 48 \text{C86 from} 47^{67} - 1 = \text{P32} \cdot \text{P55} \\ 932 = 21270964162538089013014983761851 \\ 49 \text{C86 from} 67^{76} + 1 = \text{P42} \cdot \text{P45} \\ 49 \text{C86 from} 39^{81} + 1 = \text{P37} \cdot \text{P50} \\ 51 \text{C86 from} 22^{95} - 1 = \text{P34} \cdot \text{P52} \\ 734 = 9624357919068403555091512367414261 \\ 52 \text{C86 from} 76^{17} - 1 = \text{P42} \cdot \text{P45} \\ 42 60620289710585002552707442194584005533987 \\ 53 \text{C86 from} 75^{89} + 1 = \text{P38} \cdot \text{P49} \\ 54 \text{C87 from} 62^{29} + 1 = \text{P34} \cdot \text{P53} \\ 55 \text{C87 from} 72^{29} + 1 = \text{P38} \cdot \text{P59} \\ 728 = 809754078916890910686588841 \\ 56 \text{C87 from} 30^{25} - 1 = \text{P32} \cdot \text{P56} \\ 60 \text{C87 from} 66^{36} + 1 = \text{P32} \cdot \text{P56} \\ 732 = 14285278844357974752432939513571 \\ 57 \text{C87 from} 66^{36} + 1 = \text{P33} \cdot \text{P54} \\ 61 \text{C87 from} 66^{36} + 1 = \text{P33} \cdot \text{P54} \\ 913 = 249536921989169261065035112257901 \\ 62 \text{C87 from} 66^{36} + 1 = \text{P33} \cdot \text{P54} \\ 63 \text{C87 from} 30^{35} + 1 = \text{P33} \cdot \text{P54} \\ 64 \text{C87 from} 66^{36} + 1 = \text{P34} \cdot \text{P53} \\ 913 = 24953692189169261065035112257901 \\ 62 \text{C87 from} 66^{36} + 1 = \text{P34} \cdot \text{P54} \\ 63 \text{C87 from} 86^{36} + 1 = \text{P35} \cdot \text{P52} \\ 913 = 24953692189169261065035112257901 \\ 63 \text{C87 from} 86^{35} + 1 = \text{P33} \cdot \text{P55} \\ 913 = 234537309903413$	39			P35 = 36774112300765382067961168652800897
$\begin{array}{c} 42 \text{C86 from} 93^{53} + 1 = \text{P38} \cdot \text{P49} \\ 42 \text{C86 from} 89^{59} + 1 = \text{P32} \cdot \text{P55} \\ 44 \text{C86 from} 80^{65} + 1 = \text{P31} \cdot \text{P55} \\ 44 \text{C86 from} 80^{65} + 1 = \text{P31} \cdot \text{P55} \\ 44 \text{C86 from} 80^{65} + 1 = \text{P31} \cdot \text{P55} \\ 45 \text{C86 from} 80^{65} + 1 = \text{P31} \cdot \text{P55} \\ 46 \text{C86 from} 82^{7} + 72^{7} = \text{P42} \cdot \text{P44} \\ 46 \text{C86 from} 23^{83} - 1 = \text{P38} \cdot \text{P49} \\ 47 \text{C86 from} 23^{83} - 1 = \text{P38} \cdot \text{P49} \\ 48 \text{C86 from} 47^{65} + 1 = \text{P40} \cdot \text{P47} \\ 48 \text{C86 from} 47^{67} - 1 = \text{P32} \cdot \text{P55} \\ 49 \text{C86 from} 67^{65} + 1 = \text{P40} \cdot \text{P47} \\ 48 \text{C86 from} 67^{65} + 1 = \text{P42} \cdot \text{P45} \\ 49 \text{C86 from} 67^{67} + 1 = \text{P32} \cdot \text{P55} \\ 792 = 21270964162538089013014983761851 \\ 49 \text{C86 from} 39^{81} + 1 = \text{P37} \cdot \text{P50} \\ 50 \text{C86 from} 39^{81} + 1 = \text{P37} \cdot \text{P50} \\ 51 \text{C86 from} 39^{81} + 1 = \text{P37} \cdot \text{P50} \\ 52 \text{C86 from} 39^{81} + 1 = \text{P37} \cdot \text{P50} \\ 52 \text{C86 from} 22^{95} - 1 = \text{P34} \cdot \text{P52} \\ 724 = 8062028971058502552707442194584803533987 \\ 53 \text{C86 from} 95^{80} + 1 = \text{P38} \cdot \text{P49} \\ 783 = 45089758099791867831637486244759667041 \\ 54 \text{C87 from} 62^{65} + 1 = \text{P34} \cdot \text{P53} \\ 55 \text{C87 from} 92^{85} - 1 = \text{P32} \cdot \text{P50} \\ 792 = 1 + \text{P32} \cdot \text{P50} \\ 792 = 1 + \text{P32} \cdot \text{P50} \\ 792 = 1 + \text{P33} \cdot \text{P51} \\ 793 = 14285278844357974752432939513571 \\ 757 \text{C87 from} 30^{65} + 1 = \text{P35} \cdot \text{P52} \\ 793 = 14285278843457974752432939513571 \\ 757 \text{C87 from} 60^{66} + 1 = \text{P34} \cdot \text{P46} \\ 741 = 5895147887531307193050086762077392077601 \\ 759 \text{C87 from} 60^{66} + 1 = \text{P42} \cdot \text{P46} \\ 742 = 15084588791990106865388841 \\ 750 \text{C87 from} 60^{65} + 1 = \text{P33} \cdot \text{P55} \\ 753 = 2343730909341371934534426100841739 \\ 759 \text{C87 from} 30^{85} + 1 = \text{P33} \cdot \text{P55} \\ 759 \text{P33} = 244366219801923052389300803276301 \\ 750 \text{C87 from} 60^{65} + 1 = \text{P42} \cdot \text{P46} \\ 741 = 589514788531307193050086762077392077601 \\ 750 \text{C87 from} 60^{65} + 1 = \text{P42} \cdot \text{P46} \\ 741 = 75024943248441$	40	C86 from	$96^{95} + 1 = P28 \cdot P58$	P28 = 2418476990688796014581890831
$\begin{array}{c} 43 \text{C86 from} 89^{89} + 1 = \text{P32} \cdot \text{P55} \\ 44 \text{C86 from} 80^{65} + 1 = \text{P31} \cdot \text{P55} \\ 81 = 3416871674919158699528742801241 \\ \hline \\ 45 \text{C86 from} 8^{2^7} + 7^{2^7} = \text{P42} \cdot \text{P44} \\ 46 \text{C86 from} 8^{2^7} + 7^{2^7} = \text{P42} \cdot \text{P44} \\ 46 \text{C86 from} 23^{33} - 1 = \text{P38} \cdot \text{P49} \\ 47 \text{C86 from} 76^{56} + 1 = \text{P40} \cdot \text{P44} \\ 48 \text{C86 from} 76^{56} + 1 = \text{P40} \cdot \text{P44} \\ 49 \text{C86 from} 47^{67} - 1 = \text{P32} \cdot \text{P55} \\ 50 \text{C86 from} 39^{31} + 1 = \text{P37} \cdot \text{P50} \\ 50 \text{C86 from} 39^{31} + 1 = \text{P37} \cdot \text{P50} \\ 51 \text{C86 from} 29^{56} - 1 = \text{P34} \cdot \text{P52} \\ 52 \text{C86 from} 29^{56} - 1 = \text{P34} \cdot \text{P52} \\ 52 \text{C86 from} 29^{56} - 1 = \text{P34} \cdot \text{P52} \\ 52 \text{C86 from} 29^{56} - 1 = \text{P34} \cdot \text{P52} \\ 52 \text{C86 from} 95^{80} + 1 = \text{P38} \cdot \text{P49} \\ 53 \text{C86 from} 95^{80} + 1 = \text{P38} \cdot \text{P49} \\ 54 \text{C87 from} 29^{56} - 1 = \text{P34} \cdot \text{P52} \\ 55 \text{C87 from} 29^{56} - 1 = \text{P34} \cdot \text{P52} \\ 56 \text{C87 from} 92^{56} - 1 = \text{P34} \cdot \text{P52} \\ 66 \text{C87 from} 29^{56} - 1 = \text{P38} \cdot \text{P59} \\ 725 \text{P34} = 360899791867831637486244759667041} \\ 54 \text{C87 from} 92^{56} - 1 = \text{P34} \cdot \text{P53} \\ 75 \text{C87 from} 92^{56} - 1 = \text{P34} \cdot \text{P53} \\ 75 \text{C87 from} 92^{56} - 1 = \text{P32} \cdot \text{P56} \\ 75 \text{P32} = 1428527884357974752432939513571} \\ 57 \text{C87 from} 30^{56} + 1 = \text{P34} \cdot \text{P52} \\ 75 \text{P35} = 80451911996934444483653727156040931} \\ 58 \text{C87 from} 30^{56} + 1 = \text{P34} \cdot \text{P46} \\ 742 = 1530557322480390417869992078374592702017 \\ 760 \text{C87 from} 30^{56} + 1 = \text{P34} \cdot \text{P52} \\ 75 \text{P35} = 244383093041786999207837452432393917515040931} \\ 61 \text{C87 from} 30^{56} + 1 = \text{P34} \cdot \text{P46} \\ 742 = 153055732248039041786999207837459270270017 \\ 760 \text{C87 from} 30^{56} + 1 = \text{P34} \cdot \text{P46} \\ 742 = 10841088997442568505957564739184105515451} \\ 781 = 307681427875622588317626405309 \\ 782 = 1428527884313719343426100841739 \\ 79 \text{C87 from} 33^{56} + 1 = \text{P34} \cdot \text{P46} \\ 794 = 130512269081399353915948072656199331337 \\ 79 C$	41	C86 from	$24^{130} + 1 = P36 \cdot P50$	P36 = 684989928644194001785075922656446841
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	42			P38 = 19192699869550253389095978550167828173
$ \begin{array}{c} 45 \text{C86 from} 8^{2^7} + 7^{2^7} = \text{P42} \cdot \text{P44} \\ 46 \text{C86 from} 23^{83} - 1 = \text{P38} \cdot \text{P49} \\ 47 \text{C86 from} 76^{86} + 1 = \text{P40} \cdot \text{P47} \\ 48 \text{C86 from} 76^{86} + 1 = \text{P40} \cdot \text{P47} \\ 49 \text{C86 from} 47^{67} - 1 = \text{P32} \cdot \text{P55} \\ 49 \text{C86 from} 47^{67} - 1 = \text{P32} \cdot \text{P55} \\ 49 \text{C86 from} 67^{76} + 1 = \text{P42} \cdot \text{P45} \\ 49 \text{C86 from} 67^{76} + 1 = \text{P42} \cdot \text{P45} \\ 49 \text{C86 from} 67^{76} + 1 = \text{P37} \cdot \text{P55} \\ 49 \text{C86 from} 39^{81} + 1 = \text{P37} \cdot \text{P55} \\ 49 \text{C86 from} 39^{81} + 1 = \text{P37} \cdot \text{P50} \\ 49 \text{C86 from} 39^{81} + 1 = \text{P37} \cdot \text{P50} \\ 40 \text{C86 from} 39^{81} + 1 = \text{P37} \cdot \text{P50} \\ 40 \text{C86 from} 39^{81} + 1 = \text{P37} \cdot \text{P50} \\ 40 \text{C86 from} 39^{81} + 1 = \text{P37} \cdot \text{P50} \\ 40 \text{C86 from} 39^{81} + 1 = \text{P37} \cdot \text{P50} \\ 40 \text{C86 from} 39^{81} + 1 = \text{P37} \cdot \text{P50} \\ 40 \text{C86 from} 39^{81} + 1 = \text{P37} \cdot \text{P50} \\ 40 \text{C86 from} 39^{81} + 1 = \text{P37} \cdot \text{P50} \\ 40 \text{C86 from} 39^{81} + 1 = \text{P37} \cdot \text{P50} \\ 40 \text{C86 from} 39^{81} + 1 = \text{P37} \cdot \text{P50} \\ 40 \text{C86 from} 39^{81} + 1 = \text{P37} \cdot \text{P50} \\ 40 \text{C86 from} 70^{117} - 1 = \text{P42} \cdot \text{P45} \\ 40 \text{C86 from} 70^{117} - 1 = \text{P42} \cdot \text{P45} \\ 40 \text{C80 from} 80^{80} + 1 = \text{P38} \cdot \text{P50} \\ 40 \text{C80 from} 80^{80} + 1 = \text{P38} \cdot \text{P49} \\ 40 \text{C80 from} 99^{80} - 1 = \text{P38} \cdot \text{P49} \\ 40 \text{C80 from} 99^{80} - 1 = \text{P38} \cdot \text{P49} \\ 40 \text{C80 from} 99^{80} - 1 = \text{P38} \cdot \text{P49} \\ 40 \text{C80 from} 99^{80} - 1 = \text{P38} \cdot \text{P50} \\ 40 \text{C80 from} 90^{80} + 1 = \text{P38} \cdot \text{P50} \\ 40 \text{C80 from} 90^{80} + 1 = \text{P39} \cdot \text{P50} \\ 40 \text{C80 from} 90^{80} + 1 = \text{P39} \cdot \text{P50} \\ 40 \text{C80 from} 90^{80} + 1 = \text{P39} \cdot \text{P50} \\ 40 \text{C80 from} 90^{80} + 1 = \text{P39} \cdot \text{P50} \\ 40 \text{C80 from} 90^{80} + 1 = \text{P41} \cdot \text{P46} \\ 40 \text{C80 from} 90^{80} + 1 = \text{P41} \cdot \text{P46} \\ 40 \text{C80 from} 90^{80} + 1 = \text{P41} \cdot \text{P46} \\ 40 \text{C80 from} 90^{80} + 1 = \text{P31} \cdot \text{P50} \\ 40 \text{C80 from} 30^{80} + 1 = \text{P32} \cdot$	43			P32 = 29037047448209810589475647292291
46 C86 from 82 + 72 = P42 - P44 46 C86 from 2383 - 1 = P38 - P49 47 C86 from 7656 + 1 = P40 · P47 48 C86 from 7656 + 1 = P40 · P47 48 C86 from 7656 + 1 = P40 · P47 48 C86 from 7656 + 1 = P40 · P47 48 C86 from 7656 + 1 = P40 · P47 48 C86 from 4767 - 1 = P32 · P55 49 C86 from 6776 + 1 = P42 · P45 49 C86 from 3981 + 1 = P37 · P50 50 C36 from 3981 + 1 = P37 · P50 51 C86 from 2956 - 1 = P34 · P52 52 C86 from 76 ¹¹⁷ - 1 = P42 · P45 53 C36 from 9580 + 1 = P38 · P49 53 C36 from 9580 + 1 = P38 · P49 54 C87 from 6255 + 1 = P34 · P53 55 C37 from 9285 - 1 = P35 · P55 56 C37 from 9285 - 1 = P35 · P52 57 C37 from 3085 + 1 = P35 · P52 58 C37 from 19101 - 1 = P42 · P46 59 C37 from 6365 + 1 = P42 · P46 60 C37 from 6365 + 1 = P31 · P57 60 C37 from 19101 - 1 = P42 · P46 61 C37 from 3385 + 1 = P33 · P54 62 C37 from 3385 + 1 = P33 · P54 63 C37 from 3385 + 1 = P33 · P54 64 C37 from 3385 + 1 = P33 · P55 65 C37 from 3385 + 1 = P33 · P54 66 C37 from 3385 + 1 = P33 · P55 67 from 3385 + 1 = P33 · P54 67 C37 from 3385 + 1 = P33 · P54 67 C37 from 3385 + 1 = P33 · P54 67 C37 from 3385 + 1 = P33 · P55 67 from 3385 + 1 = P33 · P54 67 C37 from 3385 + 1 = P33 · P55 67 from 3385 + 1 = P33 · P54 67 C37 from 3385 + 1 = P33 · P55 67 from 3385 + 1 = P33 · P54 67 C37 from 3385 + 1 = P33 · P55 67 from 3385 + 1 = P33 · P54 67 C37 from 3385 + 1 = P33 · P55 67 from 3385 + 1 = P33 · P55 67 from 3385 + 1 = P33 · P55 67 from 3385 + 1 = P33 · P55 67 from 3385 + 1 = P33 · P55 67 from 3385 + 1 = P33 · P55 67 from 3385 + 1 = P33 · P55 67 from 3385 + 1 = P35 · P53 60 C37 from 3385 + 1 = P35 · P53 60 C37 from 3385 + 1 = P35 · P53 60 C37 from 3385 + 1 = P35 · P53 60 C37 from 3385 + 1 = P35 · P53 60 C37 from 3385 + 1 = P35 · P53 60 C37 from 3385 + 1 = P35 · P53 60 C37 from 3385 + 1 = P35 · P53 60 C37 from 3385 + 1 = P35 · P53 60 C37 from 3385 + 1 = P35 · P53 60 C37 from 3385 + 1 = P35 · P53 60 C37 from 3385 + 1 = P35 · P53 60 C37 from 3385 + 1 = P35 · P53 60 C37 from 3385 + 1 = P35 · P53 60 C37 from 3385	44	C86 from	$80^{65} + 1 = P31 \cdot P55$	P31 = 3416871674919158699528742801241
### 65/5/6/424/335853/162950778805992164759 ### 676	45	Cocc	02 ⁷ + 72 ⁷ D40 D44	P42 = 519975935060346660783986052760977025136897
$\begin{array}{c} 47 \text{C86 from} 76^{56} + 1 = \text{P40} \cdot \text{P47} \\ 48 \text{C86 from} 47^{67} - 1 = \text{P32} \cdot \text{P55} \\ \text{P32} = 21270964162538089013014983761851 \\ \hline \\ 49 \text{C86 from} 67^{76} + 1 = \text{P42} \cdot \text{P45} \\ \text{P42} = 31561821602784848683430107844774290254513} \\ \text{50} \text{C86 from} 39^{81} + 1 = \text{P37} \cdot \text{P50} \\ \text{P37} = 244300361656663069989278441133518059 \\ \text{51} \text{C86 from} 22^{95} - 1 = \text{P34} \cdot \text{P52} \\ \text{P34} = 9624357919068403555901512367414261} \\ \text{52} \text{C86 from} 76^{117} - 1 = \text{P42} \cdot \text{P45} \\ \text{P42} = 606202897105850025527074421945484005533987 \\ \text{53} \text{C86 from} 95^{80} + 1 = \text{P38} \cdot \text{P49} \\ \text{P38} = 45089758099791867831637486244759667041} \\ \text{54} \text{C87 from} 62^{65} + 1 = \text{P34} \cdot \text{P53} \\ \text{P34} = 1439106922902522842484110155444391} \\ \text{55} \text{C87 from} 72^{99} + 1 = \text{P28} \cdot \text{P59} \\ \text{P28} = 8097540789168990910686588841} \\ \text{56} \text{C87 from} 30^{95} + 1 = \text{P35} \cdot \text{P52} \\ \text{P32} = 142852778844357974752432939513571} \\ \text{57} \text{C87 from} 30^{95} + 1 = \text{P35} \cdot \text{P52} \\ \text{P35} = 80451911996934444483653727156040931} \\ \text{58} \text{C87 from} 50^{100} + 1 = \text{P41} \cdot \text{P46} \\ \text{P41} = 58951478878513071930500886762077392077601} \\ \text{59} \text{C87 from} 60^{96} + 1 = \text{P42} \cdot \text{P46} \\ \text{P42} = 153055732248039041786999207837459270270017} \\ \text{60} \text{C87 from} 33^{85} + 1 = \text{P33} \cdot \text{P56} \\ \text{P33} = 249536921989169261065035112257901} \\ \text{61} \text{C87 from} 33^{85} + 1 = \text{P33} \cdot \text{P54} \\ \text{P33} = 249536921989169261065035112257901} \\ \text{62} \text{C87 from} 33^{85} + 1 = \text{P33} \cdot \text{P55} \\ \text{P33} = 234373090934137193434426100841739} \\ \text{64} \text{C87 from} 33^{45} + 1 = \text{P35} \cdot \text{P53} \\ \text{P33} = 234373090934137193434426100841739} \\ \text{64} \text{C87 from} 33^{11} - 1 = \text{P35} \cdot \text{P53} \\ \text{P35} = 117795480191223028083289208083263631} \\ \text{66} \text{C87 from} 33^{11} - 1 = \text{P35} \cdot \text{P53} \\ \text{P35} = 1177954801912230280832892080832631} \\ \text{67} \text{C87 from} 33^{11} - 1 = \text{P38} \cdot \text{P50} \\ \text{P38} = 2145793960589871224437297672972660829} \\ \text{68} \text{C87 from} 45^{85} + 1 = \text{P40} $	45	C86 from	$8^2 + 7^2 = P42 \cdot P44$	P44 = 65757674240355835167624181741955409969833473
$\begin{array}{c} 48 \text{C86 from} 47^{67} - 1 = \text{P32} \cdot \text{P55} \\ \text{P32} = 21270964162538089013014983761851} \\ 49 \text{C86 from} 67^{76} + 1 = \text{P42} \cdot \text{P45} \\ \text{P42} = 315618216027848486834301078445774290254513} \\ 50 \text{C86 from} 39^{81} + 1 = \text{P37} \cdot \text{P50} \\ \text{P37} = 244300361656663069989278441133518059} \\ 51 \text{C86 from} 22^{95} - 1 = \text{P34} \cdot \text{P52} \\ \text{P34} = 9624357919068403555091512367414261} \\ 52 \text{C86 from} 76^{117} - 1 = \text{P42} \cdot \text{P45} \\ \text{P42} = 606202897105850025527074421945484005533987} \\ 53 \text{C86 from} 95^{80} + 1 = \text{P38} \cdot \text{P49} \\ \text{P38} = 45089758099791867831637486244759667041} \\ 54 \text{C87 from} 62^{65} + 1 = \text{P34} \cdot \text{P53} \\ \text{P34} = 1439106922902522842484110155444391} \\ 55 \text{C87 from} 72^{99} + 1 = \text{P28} \cdot \text{P59} \\ \text{P28} = 8097540789168990910686588841} \\ 56 \text{C87 from} 92^{85} - 1 = \text{P32} \cdot \text{P56} \\ \text{P32} = 14285278844357974752432999513571} \\ 57 \text{C87 from} 30^{95} + 1 = \text{P35} \cdot \text{P52} \\ \text{P35} = 80451911996934444483653727156040931} \\ 58 \text{C87 from} 50^{100} + 1 = \text{P41} \cdot \text{P46} \\ \text{P41} = 58951478878513071930500886762077392077601} \\ 59 \text{C87 from} 66^{96} + 1 = \text{P42} \cdot \text{P46} \\ \text{P42} = 153055732248039041786999207837459270270017} \\ 60 \text{C87 from} 33^{85} + 1 = \text{P33} \cdot \text{P56} \\ \text{P35} = 5245647644316863182854571} \\ 61 \text{C87 from} 33^{85} + 1 = \text{P33} \cdot \text{P54} \\ \text{P33} = 249536921989169261065035112257901} \\ 62 \text{C87 from} 33^{85} + 1 = \text{P33} \cdot \text{P55} \\ \text{P33} = 234373090934137193434426100841739} \\ 64 \text{C87 from} 36^{45} - 1 = \text{P41} \cdot \text{P46} \\ \text{P42} = 108410889974425685059575647391841055155451} \\ 63 \text{C87 from} 84^{59} - 1 = \text{P33} \cdot \text{P55} \\ \text{P33} = 234373090934137193433426100841739} \\ 65 \text{C87 from} 84^{59} - 1 = \text{P33} \cdot \text{P55} \\ \text{P33} = 234373090934137193433426100841739} \\ 67 \text{C87 from} 84^{59} - 1 = \text{P33} \cdot \text{P55} \\ \text{P31} = 3076814278757622888317626405309} \\ 67 \text{C87 from} 86^{84} + 1 = \text{P30} \cdot \text{P55} \\ \text{P33} = 21457939605898871224437297672972660829} \\ 68 \text{C87 from} 86^{84} + 1 = \text{P40} \cdot \text{P48} \\ $	46			P38 = 27736074503263071062950778805992164759
$\begin{array}{c} 49 \text{C86 from} 67^{76} + 1 = \text{P42} \cdot \text{P45} \\ 50 \text{C86 from} 39^{81} + 1 = \text{P37} \cdot \text{P50} \\ 51 \text{C86 from} 39^{81} + 1 = \text{P37} \cdot \text{P50} \\ 52 \text{C86 from} 22^{95} - 1 = \text{P34} \cdot \text{P52} \\ \text{P34} = 9624357919068403555091512367414261 \\ \text{52} \text{C86 from} 76^{117} - 1 = \text{P42} \cdot \text{P45} \\ \text{P42} = 606202897105850025527074421945484005533987 \\ \text{53} \text{C86 from} 95^{80} + 1 = \text{P38} \cdot \text{P49} \\ \text{P38} = 45089758099791867831637486244759667041 \\ \text{54} \text{C87 from} 62^{25} + 1 = \text{P34} \cdot \text{P53} \\ \text{55} \text{C87 from} 62^{25} + 1 = \text{P34} \cdot \text{P53} \\ \text{56} \text{C87 from} 92^{85} - 1 = \text{P32} \cdot \text{P56} \\ \text{P32} = 14285278844357974752432939513571 \\ \text{57} \text{C87 from} 30^{85} + 1 = \text{P35} \cdot \text{P52} \\ \text{C87 from} 30^{85} + 1 = \text{P35} \cdot \text{P52} \\ \text{C87 from} 66^{66} + 1 = \text{P41} \cdot \text{P46} \\ \text{P41} = 15305573224803904178699207837459270270017 \\ \text{59} \text{C87 from} 19^{101} - 1 = \text{P25} \cdot \text{P62} \\ \text{P25} = 5245647644316863182854571 \\ \text{61} \text{C87 from} 33^{85} + 1 = \text{P33} \cdot \text{P54} \\ \text{62} \text{C87 from} 33^{85} + 1 = \text{P33} \cdot \text{P54} \\ \text{63} \text{C87 from} 33^{85} + 1 = \text{P33} \cdot \text{P54} \\ \text{64} \text{C87 from} 33^{85} + 1 = \text{P33} \cdot \text{P54} \\ \text{64} \text{C87 from} 37^{87} - 1 = \text{P41} \cdot \text{P46} \\ \text{64} \text{C87 from} 37^{87} - 1 = \text{P31} \cdot \text{P57} \\ \text{65} \text{C87 from} 37^{87} - 1 = \text{P31} \cdot \text{P57} \\ \text{66} \text{C87 from} 33^{85} + 1 = \text{P33} \cdot \text{P54} \\ \text{67} \text{C87 from} 37^{87} - 1 = \text{P31} \cdot \text{P57} \\ \text{68} \text{C87 from} 37^{87} - 1 = \text{P31} \cdot \text{P57} \\ \text{69} \text{C87 from} 84^{89} - 1 = \text{P33} \cdot \text{P55} \\ \text{69} \text{C87 from} 33^{11} - 1 = \text{P35} \cdot \text{P52} \\ \text{69} \text{C87 from} 84^{89} - 1 = \text{P35} \cdot \text{P53} \\ \text{69} \text{C87 from} 86^{84} + 1 = \text{P40} \cdot \text{P44} \\ \text{P41} = 75024943244441937370512643013155715853} \\ \text{65} \text{C87 from} 86^{85} + 1 = \text{P40} \cdot \text{P48} \\ \text{P40} = 1039512269081394539159468072656199331337} \\ \text{79} \text{C87 from} 86^{85} + 1 = \text{P40} \cdot \text{P48} \\ \text{P40} = 1039512269081394539159468072656199331337} \\ \text{79} \text{C87 from} 87^{83} + 1 = \text{P35} \cdot \text{P55} \\ 79$	47			P40 = 4868699568817220592890920460964327586529
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	48	C86 from	$47^{67} - 1 = P32 \cdot P55$	P32 = 21270964162538089013014983761851
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	49	C86 from	$67^{76} + 1 = P42 \cdot P45$	P42 = 315618216027848486834301078445774290254513
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	50			P37 = 2443003616566663069989278441133518059
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	51			
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	52	C86 from	$76^{117} - 1 = P42 \cdot P45$	P42 = 606202897105850025527074421945484005533987
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	53	C86 from	$95^{80} + 1 = P38 \cdot P49$	P38 = 45089758099791867831637486244759667041
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	54			P34 = 1439106922902522842484110155444391
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	55	C87 from	$72^{99} + 1 = P28 \cdot P59$	P28 = 8097540789168990910686588841
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	56	C87 from	$92^{85} - 1 = P32 \cdot P56$	P32 = 14285278844357974752432939513571
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	57	C87 from	$30^{95} + 1 = P35 \cdot P52$	P35 = 80451911996934444483653727156040931
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	1			
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	59	C87 from	$66^{96} + 1 = P42 \cdot P46$	P42 = 153055732248039041786999207837459270270017
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	60	C87 from	$19^{101} - 1 = P25 \cdot P62$	P25 = 5245647644316863182854571
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	61	C87 from	$33^{85} + 1 = P33 \cdot P54$	P33 = 249536921989169261065035112257901
$\begin{array}{llllllllllllllllllllllllllllllllllll$	1			
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	63	C87 from	$42^{99} - 1 = P33 \cdot P55$	
$\begin{array}{llllllllllllllllllllllllllllllllllll$	64	C87 from	$77^{67} - 1 = P41 \cdot P46$	P41 = 75024943244844149373705126243013155715853
$\begin{array}{llllllllllllllllllllllllllllllllllll$	65			P35 = 11779548019122302808328920808327631
$\begin{array}{llllllllllllllllllllllllllllllllllll$	1			
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	67	C87 from	$33^{111} - 1 = P38 \cdot P50$	
70 C87 from $45^{85} + 1 = P36 \cdot P52$ P36 = 218136090485068920975060625740020221 71 C87 from $87^{93} + 1 = P35 \cdot P53$ P35 = 65234702723152738657728499902597613 72 C87 from $45^{71} + 1 = P27 \cdot P61$ P27 = 692298161874034730813881603	68	C87 from	$86^{84} + 1 = P40 \cdot P48$	
70 C87 from $45^{85} + 1 = P36 \cdot P52$ P36 = 218136090485068920975060625740020221 71 C87 from $87^{93} + 1 = P35 \cdot P53$ P35 = 65234702723152738657728499902597613 72 C87 from $45^{71} + 1 = P27 \cdot P61$ P27 = 692298161874034730813881603	79			P40 = 4645176624103101144238593467706089788481
71 C87 from $87^{93} + 1 = P35 \cdot P53$ $P35 = 65234702723152738657728499902597613$ 72 C87 from $45^{71} + 1 = P27 \cdot P61$ $P27 = 692298161874034730813881603$	1			
100	71	C87 from	$87^{93} + 1 = P35 \cdot P53$	
73 C88 from $19^{168} + 1 = P42 \cdot P47$ $P42 = 261688712348581672325146786097393313497473$	1			P27 = 692298161874034730813881603
	73	C88 from	$19^{168} + 1 = P42 \cdot P47$	P42 = 261688712348581672325146786097393313497473

$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$												
Section Sect	#	d	$B_1/10^5$	n_f	B_2/B_1	$M/10^{5}$	n_c	n_1	$n_{c,1}$	n_2	$n_{c,2}$	T_s
30 85.11 5 20841 20 2.5 5615 50851 5434 182705 9822 10.7 h 32 85.12 5 20651 20 1.5 7269 64239 8799 48843 6425 9.5 h 32 85.12 5 20651 20 1.5 7269 64239 8799 48843 6425 9.5 h 33 85.14 5 20812 20 1.5 5064 43981 4223 26349 11614 10.7 h 34 85.21 5 20709 20 1.5 6722 66788 6694 29.891 1136 8.27h 35 85.26 5 20859 20 1.5 5101 44412 4378 256996 11412 11.0 h 36 85.26 5 20869 20 1.5 5101 44412 4378 256996 11412 11.0 h 36 85.26 5 20869 20 1.5 5297 4484 57739 5154 12.4 h 378 85.31 5 20676 20 1.5 5297 45852 4384 85169 10967 7.45h 38 85.31 5 20576 20 1.5 66859 60552 7686 68840 6177 1.5 h 48 85.35 5 20923 20 1.5 6480 55476 6564 127990 7903 10.7 h 41 85.37 5 20672 20 1.5 5695 50790 5707 139604 9308 10.4 h 48 85.49 5 20772 20 1.5 55695 50790 5707 139604 9308 10.4 h 48 85.49 5 20772 20 1.5 5565 50383 5456 185347 9653 13.7 h 46 85.59 5 20674 20 1.5 56637 56910 6862 96604 7226 10.3 h 47 85.70 5 20711 20 2.5 5064 43759 4078 270308 11587 11.4 h 46 85.59 5 20797 20 1.5 6637 56910 6862 96604 7226 10.3 h 48 85.70 5 20712 20 1.5 6637 56910 6862 96604 7226 10.3 h 48 85.72 5 20911 20 1.5 6637 56910 6862 96604 7226 10.3 h 48 85.72 5 20911 20 1.5 6637 56910 6862 96604 7226 10.3 h 48 85.92 6 26363 2 3 16159 24478 9358 7417 6311 12.1 h 5 6637 56910 6862 96604 7226 10.3 h 5 6637 56910 6	29	85.11	5	20810	20	2.5	4923	43182	4064	280566	11857	8.4 h
31 85.12 6 24641 20 2.5 8518 67320 9253 73153 68953 10.0 h 32 85.12 5 20651 20 1.5 7269 64239 8799 48843 4625 9.5 h 33 85.14 5 20812 20 1.5 5064 43981 4223 263194 11614 10.7 h 34 85.21 5 20709 20 1.5 6722 66788 6924 92891 7136 8.27 h 35 85.26 5 20768 20 1.5 5101 44412 4378 256996 11412 11.0 h 36 85.26 5 20768 20 1.5 5101 44412 4378 256996 11412 11.0 h 36 85.31 5 20812 20 1.5 6107 5044 6362 130553 8115 13.1 h 38 85.33 5 20767 20 1.5 6107 55044 6362 130553 8115 13.1 h 39 85.33 5 20709 20 1.5 6480 55476 6564 12790 7903 10.7 h 41 85.37 5 20672 20 1.5 6480 55476 6546 12790 7903 10.7 h 41 85.37 5 20672 20 1.5 5695 30790 5707 139604 9308 10.4 h 43 85.49 5 20772 20 1.5 5695 30790 5707 139604 9308 10.4 h 44 85.55 5 20634 20 1.5 5556 50383 5456 185347 9565 18347 9565 13.1 h 45 85.53 5 20711 20 2.5 5054 43759 4078 270308 11587 11.4 h 46 85.59 5 20772 20 1.5 6385 6991 6882 96694 7226 10.3 h 48 85.70 5 20712 20 1.5 6311 56349 6710 120384 7895 15.4 h 48 85.73 6 26392 2 3 16359 24514 9487 3153 749 13.8 h 49 85.73 6 26392 2 3 16359 24514 9487 3153 749 13.8 h 50 85.92 6 26363 2 3 16359 24514 9487 3153 749 13.8 h 50 85.92 6 26363 2 3 16359 24514 9487 3153 749 13.8 h 51 85.93 3 13041 20 1.5 6311 56349 6710 120384 7895 15.4 h 52 85.95 3 13011 20 2.5 7153 6231 839 63273 5557 14.0 h 53 85.98 5 20766 2.4 2.5 10516 2204 7450 6892 3395 3713 17.4 h 51 85.93 3 13041 20 2.5 7153 6231 8139 63273 5557 14.0 h 52 86.04 5 20840 20 2.5 7153 6231 8139 63273 5557 14.0 h 53 86.25 5 20862 20 2.5 7009 63089 822 57513 5620 11.9 h 54 86.04 5 20868 20 2.5 7168 63987 8599 54478 3366 20.6 h 55 86.16 5 20877 40 2.5 6387 7288 609 116000 7520 11.1 h 56 86.29 5 20764 40 2.5 6387 7288 609 116000 7520 11.1 h 56 86.69 6 24458 100 3 7864 124510 9691 89594 7355 13.2 h 66 86.69 6 24458 100 3 7863 12288 9899 80444 6862 14.7 h 57 86.76 6 24465 100 3 6651 12088 9899 80444 6862 14.7 h 57 86.86 6 6 6 24658 100 3 7863 12288 9899 80444 6862 14.7 h 57 86.96 6 24668 100 3 7862 12187 8989 80444 6862 14.7 h 57 86.96 6 24668 100 3 7862 12187 8989 80444 6862 14.7 h	30	85.11	5	20841	20	2.5	5615	50651	5434	182705		
33 85.14 5 20651 20 1.5 7269 64239 8799 48843 4625 9.5 h 33 85.14 5 20812 20 1.5 5064 43981 4223 263194 11614 10.7 h 34 85.21 5 20709 20 1.5 6722 56788 6024 92891 7136 8.27 h 35 85.26 5 20859 20 1.5 5101 44412 4378 266996 11412 11.0 h 36 85.26 5 20768 20 1.5 5101 44412 4378 266996 11412 11.0 h 37 85.31 5 20812 20 1.5 5297 45852 4584 185169 10967 7.45 h 38 85.31 5 20876 20 1.5 6107 55044 6362 130553 8115 13.1 h 39 85.33 5 20709 20 1.5 6859 60552 7686 68840 6177 11.5 h 40 85.35 5 20923 20 1.5 6480 55476 6546 127090 7903 10.7 h 41 85.37 5 20672 20 1.5 5695 60552 7686 68840 6177 11.5 h 42 85.42 5 20672 20 1.5 5695 65790 5707 138604 9308 10.4 h 43 85.49 5 20772 20 1.5 5695 65790 5707 138604 9308 10.4 h 44 85.52 5 20634 20 1.5 5556 50383 5456 185347 9635 11.7 h 45 85.53 5 20711 20 2.5 5054 43759 4078 270308 11587 11.4 h 46 85.59 5 20797 20 1.5 6637 66682 96694 7226 10.3 h 48 85.70 5 20712 20 1.5 6637 56910 6862 96694 7226 10.3 h 48 85.70 5 20712 20 1.5 6637 56910 6862 96694 7226 10.3 h 49 85.73 6 26392 2 3 16159 24514 9487 3153 749 13.8 h 50 85.92 6 26363 2 3 16376 24473 9358 7417 6331 12.1 h 51 85.93 3 13041 20 1.5 6311 56349 6710 120384 7895 15.4 h 51 85.93 3 13041 20 2.5 7156 6224 7450 6610 2795 15.7 h 54 86.04 5 20840 20 2.5 7153 62231 8139 63273 5557 14.0 h 55 86.18 5 20868 20 2.5 7447 64368 854 47154 4419 10.7 h 58 86.22 5 20862 20 2.5 75367 63987 8599 24788 8909 11.0 h 56 86.16 5 20877 22 2.5 7367 63987 8599 54708 8909 11.0 h 58 86.22 5 20862 20 2.5 7367 63987 8599 54708 8909 11.0 h 58 86.22 5 20862 20 2.5 7367 63987 8599 54708 8909 11.0 h 58 86.22 5 20862 20 2.5 7447 6478 8836 47154 4419 10.7 h 58 86.62 5 20864 20 2.5 7447 6488 848 47544 6862 11.9 h 56 86.63 5 20902 80 2.5 7367 6388 9389 9390 7811 11.5 h 58 86.63 5 20902 80 2.5 7367 6388 7488 8876 13.8 h 59 86.63 5 20902 80 2.5 7447 6488 836 47154 4419 10.7 h 58 86.62 5 20861 40 2.5 6387 72888 899 80444 6862 11.1 h 68 86.70 6 24495 100 3 7863 12283 9899 80444 6862 11.1 h 68 86.70 6 24465 100 3 7863 12283 9899 80444 6862 11.1 h 68 86.70 6 24658 100 3 7863 1228	31	85.12	6	24641	20	2.5	8518	67320	9253			
34 85.21 5 20709 20 1.5 6722 56788 6924 92891 7136 8.27 h 35 85.26 5 20859 20 1.5 5101 44412 4378 256996 11412 11.0 h 36 85.26 5 20768 20 1.5 5101 44412 4378 256996 11412 11.0 h 37 85.31 5 20812 20 1.5 5297 45852 4584 185169 10967 7.45 h 38 85.31 5 20576 20 1.5 6859 60552 7686 68840 6177 11.5 h 40 85.35 5 20923 20 1.5 6859 66546 127090 7903 10.7 h 141 85.37 5 20672 20 1.5 7590 60522 7686 6840 6177 11.5 h 10.7 144412 85.2 520404 10.1 h 442	32	85.12	5	20651	20	1.5	7269	64239	8799			
35 85.26 5 20859 20 1.5 5101 44412 4378 256996 11412 11.0 h 36 85.26 5 20768 20 1.5 5101 63721 8449 57739 5154 12.4 h 37 85.31 5 20576 20 1.5 6107 55044 6362 130553 8115 13.1 h 39 85.33 5 20709 20 1.5 6869 60552 7666 68840 6177 11.5 14 40 85.35 5 20672 20 1.5 6865 55476 6646 127090 7903 10.7 h 41 85.37 5 20672 20 1.5 7521 62980 8435 54345 5029 9.65 h 42 85.42 5 20672 20 1.5 7530 68927 8600 51034 4056 11.9	33	85.14	5	20812	20	1.5	5064	43981	4223	263194	11614	10.7 h
36 85.26 5 20768 20 1.5 7186 63721 8449 57739 5154 12.4 h 37 85.31 5 20812 20 1.5 5297 48582 4884 185169 10967 7.45 h 39 85.33 5 20709 20 1.5 6859 60552 7686 68840 6177 11.5 h 40 85.35 5 20923 20 1.5 6480 55476 6546 127090 7903 10.7 h 41 85.37 5 20672 20 1.5 7298 6646 127090 7903 10.7 h 42 85.42 5 20672 20 1.5 7599 5070 159040 9308 10.4 h 43 85.49 5 20772 20 1.5 75556 50383 5456 185347 9653 13.7 h 45 85.53 5 20711		85.21	5	20709	20	1.5	6722	56788	6924	92891	7136	8.27 h
37 85.31 5 20812 20 1.5 5297 45852 4584 185169 10967 7.45 h 38 85.31 5 20576 20 1.5 6107 55044 6362 130553 8115 13.1 h 39 85.33 5 20709 20 1.5 6859 60552 7686 68840 6177 11.5 h 40 85.35 5 20923 20 1.5 6480 55476 6546 127090 7903 10.7 h 41 85.37 5 20672 20 1.5 7221 62980 8435 54345 5029 9.65 h 42 85.42 5 20672 20 1.5 5695 50790 5707 139604 9308 10.4 h 43 85.49 5 20772 20 1.5 5595 50790 8707 139604 9308 10.4 h 44 85.52 5 20634 20 1.5 5556 50383 5456 185347 9653 13.7 h 45 85.53 5 20711 20 2.5 5054 43759 4078 270308 11587 11.4 h 46 85.59 5 20772 20 1.5 6637 66910 6862 96694 7226 10.3 h 48 85.72 5 20911 20 1.5 6311 56349 6710 120384 7895 154 h 49 85.73 6 26392 2 3 16159 24514 9487 3153 749 13.8 h 50 85.92 6 26363 2 3 16376 24473 9358 7417 631 12.1 h 51 85.93 3 13041 20 1.5 4117 39602 5212 33935 3713 17.4 h 52 85.95 3 13011 20 2.5 4255 40517 5390 24478 3366 20.6 h 53 85.98 5 20756 2.4 2.5 10516 22044 7450 6610 2795 15.7 h 54 86.04 5 20840 20 2.5 7059 6387 8559 54708 4900 10.8 h 55 86.13 5 20838 20 2.5 7009 63089 8220 57513 5620 11.9 h 56 86.16 5 20787 22 2.5 6327 747 6478 8836 4715 6419 10.7 h 58 86.22 5 20947 20 2.5 6327 7476 6382 44769 8376 4419 10.7 h 58 86.22 5 20947 20 2.5 6387 7885 1896 83273 5557 14.0 h 55 86.13 5 20838 20 2.5 7009 63089 8220 57513 5620 11.9 h 56 86.16 5 20787 22 2.5 6362 54180 6282 14409 8376 11.6 h 57 86.18 5 20688 20 2.5 7099 63089 8220 57513 5620 11.9 h 56 86.25 5 20947 20 2.5 6327 7447 6478 8836 47154 4419 10.7 h 58 86.22 5 20947 20 2.5 6387 7886 6909 116000 7520 11.1 h 58 86.62 5 20947 20 2.5 6387 7886 6909 116000 7520 11.1 h 58 86.63 5 20787 40 2.5 6387 7886 6909 116000 7520 11.1 h 58 86.64 5 20681 40 2.5 6387 7886 6909 116000 7520 11.1 h 58 86.69 6 24573 100 3 7635 120888 9389 9930 7811 11.5 h 58 86.70 6 24434 100 3 7635 120888 9389 9930 7811 11.5 h 58 86.62 6 24658 100 3 7635 120888 9389 9930 7811 11.5 h 58 86.73 6 24658 100 3 7635 120888 9389 9930 7811 11.5 h 58 86.96 6 24658 100 3 7635 120888 9389 9930 7811 11.5 h 58 86.96 6 24658 100 3 7635 120888 9389 9930 7811 11.5 h	35		5	20859	20	1.5	5101	44412	4378	256996	11412	11.0 h
38 85.31 5 20709 20 1.5 6107 55044 6362 130553 8115 13.1 h 63 85.33 5 20709 20 1.5 6859 60552 7686 68840 6177 11.5 h 64 85.35 5 20923 20 1.5 6859 60552 7686 68840 6177 11.5 h 74 185.37 5 20672 20 1.5 6869 55476 6546 127090 7903 10.7 h 74 185.37 5 20672 20 1.5 5695 50790 5707 139604 9308 10.4 h 74 85.37 5 20672 20 1.5 5695 50790 5707 139604 9308 10.4 h 74 85.52 5 20634 20 1.5 5556 50383 5456 185347 9653 13.7 h 74 85.52 5 20634 20 1.5 5556 50383 5456 185347 9653 13.7 h 74 85.52 5 20634 20 1.5 5556 50383 5456 185347 9653 13.7 h 74 85.70 5 20712 20 1.5 6637 56910 6862 96694 7226 10.3 h 74 85.70 5 20712 20 1.5 6311 56349 6710 120384 7895 15.4 h 74 85.70 5 20712 20 1.5 6311 56349 6710 120384 7895 15.4 h 75 85.93 13011 20 1.5 6311 56349 6710 120384 7895 15.4 h 75 85.93 13011 20 2.5 4255 40517 5390 24478 3366 20.6 h 15 85.93 13011 20 2.5 4255 40517 5390 24478 3366 20.6 h 15 85.93 13011 20 2.5 4255 40517 5390 24478 3366 20.6 h 15 86.34 5248 13.9 h 15 86.34 13.9 h 15	36	85.26	5	20768	20	1.5	7186	63721	8449	57739	5154	12.4 h
39 85.33 5 20709 20 1.5 6859 60552 7686 68840 6177 11.5 h 40 85.35 5 20923 20 1.5 6480 55476 6646 127090 7903 10.7 h 41 85.37 5 20672 20 1.5 7521 62980 8435 54345 5029 9.656 h 42 85.42 5 20672 20 1.5 7530 63927 8600 51034 4656 11.9 h 44 85.52 5 20634 20 1.5 5556 50383 5456 185347 9653 13.7 h 45 85.53 5 20711 20 2.5 5054 43759 4078 270308 11587 11.4 h 46 85.59 5 20712 20 1.5 6637 56910 6862 96944 7226				20812	20	1.5	5297	45852	4584	185169	10967	7.45 h
40 85.35 5 20923 20 1.5 6480 55476 6546 127090 7903 10.7 h 41 85.37 5 20672 20 1.5 7221 62980 8435 54345 5029 9.65 h 42 85.42 5 20672 20 1.5 5695 50790 5707 139604 9308 10.4 h 43 85.49 5 20772 20 1.5 5556 50380 51034 4656 11.9 h 44 85.52 5 20634 20 1.5 5556 50383 5456 185347 9653 13.7 h 45 85.53 5 20711 20 2.5 5054 43759 4078 270308 11587 11.4 h 46 85.59 5 20797 20 1.5 7244 63668 8524 61191 5044 12.6 h 47 85.70 5 20712 20 1.5 6637 56910 6862 96694 7226 10.3 h 48 85.72 5 20911 20 1.5 6311 56349 6710 120384 7895 15.4 h 49 85.73 6 26392 2 3 16159 24514 9487 3153 749 13.8 h 49 85.93 3 13041 20 1.5 4117 39602 5212 39395 3713 17.4 h 51 85.93 3 13041 20 1.5 4117 39602 5212 39395 3713 17.4 h 52 85.95 3 13011 20 2.5 4255 40517 5390 24478 3366 20.6 h 53 85.98 5 20756 2.4 2.5 10516 22044 7450 6610 2795 15.7 h 54 86.04 5 20840 20 2.5 7153 62231 8139 63273 5557 14.0 h 55 86.13 5 20838 20 2.5 7009 63089 8220 57513 5620 11.9 h 56 86.16 5 20787 22 2.5 7622 5486 6999 116000 7520 11.1 h 58 86.22 5 20852 20 2.5 7029 63087 8529 54708 4900 10.8 h 57 86.18 5 20688 20 2.5 7447 64778 8836 47154 4419 10.7 h 58 86.22 5 20852 20 2.5 6202 54180 6282 144069 8376 11.6 h 59 86.22 5 20947 20 2.5 7522 63620 8412 5219 5091 9.35 h 60 86.43 5 20992 80 2.5 7447 64778 8836 47154 4419 10.7 h 58 86.22 5 20947 20 2.5 7522 63620 8412 52191 5091 9.35 h 60 86.43 5 20994 40 2.5 6773 79489 8184 75416 6035 14.2 h 61 86.29 5 20774 40 2.5 6387 72868 6909 116000 7520 11.1 h 62 86.38 5 20754 40 2.5 6387 72868 6909 116000 7520 11.1 h 62 86.38 5 20754 40 2.5 6387 72868 6909 116000 7520 11.1 h 63 86.43 5 20992 80 2.5 7447 6881 76961 7638 8691 75408 4900 10.8 h 64 86.45 5 20631 40 2.5 6387 72868 8999 116000 7520 11.1 h 65 86.66 6 24573 1000 3 7635 12088 9389 9930 7811 11.5 h 69 86.73 6 24538 100 3 7635 12088 9389 9930 7811 11.5 h 69 86.73 6 24538 100 3 7635 12088 9389 9930 7811 11.5 h 70 86.75 6 24495 100 3 6532 121187 9864 167590 8507 11.8 h 71 86.86 6 24658 100 3 7762 116033 8546 126334 8798 7.766 h 72 86.66 6 24658 100 3 7762 11								55044	6362	130553	8115	13.1 h
41 85.37 5 20672 20 1.5 7221 62980 8435 54345 5029 9.65 h 42 85.42 5 20672 20 1.5 5695 50790 5707 139604 9308 10.4 h 43 85.49 5 20772 20 1.5 7530 63927 8600 51034 4656 11.9 h 44 85.52 5 26634 20 1.5 5556 50383 5456 185347 9653 13.7 h 45 85.53 5 20711 20 2.5 5054 43759 4078 270308 11587 11.4 h 46 85.59 5 20797 20 1.5 7244 63668 8524 61191 5044 12.6 h 47 85.70 5 20712 20 1.5 6637 56910 6862 96694 7226 10.3 h 48 85.72 5 20911 20 1.5 6311 56349 6710 120384 7895 15.4 h 49 85.73 6 26392 2 3 16159 24514 9487 3153 749 13.8 h 50 85.92 6 26363 2 3 16376 24473 9358 7417 631 12.1 h 51 85.93 3 13041 20 1.5 4117 39602 5212 39395 3713 17.4 h 52 85.95 3 13011 20 2.5 4255 40517 5390 24478 3366 20.6 h 53 85.98 5 20756 2.4 2.5 10516 22044 7450 6610 2795 15.7 h 54 86.04 5 20840 20 2.5 7153 62231 8139 63273 5557 14.0 h 55 86.13 5 20838 20 2.5 7009 63089 8220 57513 5620 11.9 h 56 86.16 5 20787 22 2.5 7367 63987 8559 54708 4900 10.8 h 57 86.18 5 20688 20 2.5 7447 64778 8856 47154 4419 10.7 h 58 86.22 5 20852 20 2.5 6202 54180 6282 144069 8376 11.6 h 59 86.22 5 20947 20 2.5 7522 63620 8412 5219 5091 5091 9.35 h 66 86.64 5 20979 40 2.5 6737 79489 8184 75416 6035 14.2 h 66 86.64 6 24404 100 3 7564 12451 9691 88959 77513 12.1 h 67 86.68 5 20757 6 24573 100 3 7564 12451 9691 88959 7753 13.2 h 68 86.70 6 24495 100 3 7635 12088 9389 9930 7811 11.5 h 69 86.73 6 24588 100 3 7635 12088 9389 9930 7811 11.5 h 69 86.73 6 2458 100 3 7652 1189 899 80444 6862 14.7 h 69 86.75 6 24475 100 3 6531 1228 9899 80444 6862 14.7 h 60 86.75 6 24475 100 3 7635 12088 9389 9930 7811 11.5 h 60 86.75 6 24475 100 3 7635 12088 9389 9930 7811 11.5 h 60 86.75 6 24475 100 3 7635 12088 9389 9930 7811 11.5 h 60 86.75 6 24475 100 3 7635 12088 9389 9930 7811 11.5 h 61 86.82 6 24658 100 3 7662 116023 8846 126334 8788 7.666 11.6 h 62 86.86 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6									7686			11.5 h
42 85.42 5 20672 20 1.5 5695 50790 5707 139604 9308 10.4 h 43 85.49 5 20772 20 1.5 7530 63927 8600 51034 4656 11.9 h 44 85.52 5 20634 20 1.5 5556 50383 5456 185347 9653 13.7 h 45 85.53 5 20711 20 2.5 5054 43759 4078 270308 11587 11.4 h 46 85.59 5 20797 20 1.5 7244 63668 8524 61191 5044 12.6 h 48 85.72 5 20911 20 1.5 6311 56349 6710 120384 7895 15.4 h 49 85.73 6 26392 2 3 16159 24514 9487 3153 749 13.8 h 50 85.92 6 26363 2 3 16376 24473 9358 7417 631 12.	40	85.35	5	20923	20	1.5	6480	55476	6546	127090	7903	10.7 h
43 85.49 5 20772 20 1.5 7530 63927 8600 51034 4656 11.9 h 44 85.52 5 20634 20 1.5 5556 50383 5456 185347 9653 13.7 h 45 85.53 5 20711 20 2.5 5054 43759 4078 270308 11587 11.4 h 46 85.59 5 20797 20 1.5 6244 63668 8524 61191 5044 12.6 h 48 85.72 5 20911 20 1.5 6311 56349 6710 120344 7895 15.4 h 49 85.73 6 26392 2 3 16159 24514 9487 3153 749 13.8 h 50 85.92 6 26363 2 3 16159 24514 9487 3153 749 13.8 h 50 85.92 6 26363 2 3 1615	41	85.37	5	20672	20	1.5	7221	62980	8435	54345	5029	9.65 h
44 85.52 5 20634 20 1.5 5556 50383 5456 185347 9653 13.7 h 45 85.53 5 20711 20 2.5 5054 43759 4078 270308 11587 11.4 h 46 85.59 5 20797 20 1.5 7244 63668 8524 61191 5044 12.6 h 47 85.70 5 20712 20 1.5 6637 56910 6862 96694 7226 10.3 h 48 85.72 5 20911 20 1.5 6311 56349 6710 120384 7895 15.4 h 49 85.73 6 26363 2 3 16159 24514 9487 3153 749 13.8 h 50 85.92 6 26363 2 3 16376 24473 3935 7417 631 12.1 h 51 51 52 85.95 3 13011 20 2.5 4255 40517 5390 24478 3366 <td>42</td> <td>85.42</td> <td>5</td> <td>20672</td> <td>20</td> <td>1.5</td> <td>5695</td> <td>50790</td> <td>5707</td> <td>139604</td> <td>9308</td> <td>10.4 h</td>	42	85.42	5	20672	20	1.5	5695	50790	5707	139604	9308	10.4 h
45 85.53 5 20711 20 2.5 5054 43759 4078 270308 11587 11.4 h 46 85.59 5 20797 20 1.5 7244 63668 8524 61191 5044 12.6 h 47 85.70 5 20712 20 1.5 6637 56910 6862 96694 7226 10.3 h 48 85.72 5 20911 20 1.5 6311 56349 6710 120384 7895 15.4 h 49 85.73 6 26392 2 3 16159 24514 9487 3153 749 13.8 h 50 85.92 6 26363 2 3 16376 24473 9358 7417 631 12.1 h 51 85.93 3 13041 20 1.5 4117 39602 5212 39395 3713 17.4 h 52 85.95 3 13011 20 2.5 4255 40517 5390 24478 3366 20.6 h 53 85.98 5 20756 2.4 2.5 10516 22044 7450 6610 2795 15.7 h 54 86.04 5 20840 20 2.5 7153 62231 8139 63273 5557 14.0 h 55 86.13 5 20838 20 2.5 7009 63089 8220 57513 5620 11.9 h 56 86.16 5 20787 22 2.5 7367 63987 8559 54708 4900 10.8 h 57 86.18 5 20688 20 2.5 7447 64778 8836 47154 4419 10.7 h 58 86.22 5 20852 20 2.5 6202 54180 6282 144069 8376 11.6 h 59 86.22 5 20947 20 2.5 7522 63620 8412 52191 5091 9.35 h 60 86.27 5 20978 40 2.5 6387 7286 6909 116000 7520 11.1 h 61 86.29 5 20754 40 2.5 6387 7286 6909 116000 7520 11.1 h 62 86.38 5 20754 40 2.5 6387 7286 6909 116000 7520 11.1 h 62 86.38 5 20754 40 2.5 6881 76861 7638 86915 6253 6.72 h 63 86.43 5 20920 40 2.5 7177 76854 7706 82085 6054 8.81 h 64 86.45 5 20631 40 2.5 6339 74885 7262 92362 7046 14.6 h 65 86.66 6 24573 100 3 7564 124510 9691 89594 7355 13.2 h 66 86.67 6 24573 100 3 7563 122935 9614 89375 7369 14.1 h 68 86.70 6 24495 100 3 6571 105037 7010 149698 11272 12.1 h 69 86.73 6 24404 100 3 7567 124510 9691 89594 7355 13.2 h 69 86.75 6 24474 100 3 7563 122935 9614 89375 7369 14.1 h 69 86.75 6 24573 100 3 7635 12088 9389 99900 7811 11.5 h 69 86.75 6 24574 100 3 7655 12088 9389 99900 7811 11.5 h 69 86.75 6 24574 100 3 7655 12088 9389 99900 7811 11.5 h 69 86.75 6 24574 100 3 7655 12088 9389 99900 7811 11.5 h 69 86.75 6 24615 100 3 6532 121187 9864 167590 8507 11.8 h 60 86.66 6 24658 100 3 7762 116023 8546 126334 8798 7.66 h	43	85.49	5	20772	20	1.5	7530	63927	8600	51034	4656	
46 85.59 5 20797 20 1.5 7244 63668 8524 61191 5044 12.6 h 47 85.70 5 20712 20 1.5 6637 56910 6862 96694 7226 10.3 h 48 85.72 5 20911 20 1.5 6311 56349 6710 120384 7895 15.4 h 49 85.73 6 26392 2 3 16159 24514 9487 3153 749 13.8 h 50 85.92 6 26363 2 3 16376 24473 9358 7417 631 12.1 h 51 85.93 3 13041 20 1.5 4255 40517 5390 24478 3366 20.6 h 53 85.98 5 20756 2.4 2.5 10516 22044 7450 6610 2795 15.7 h 54 86.04 5 20840 20 2.5 715	44	85.52	5	20634	20	1.5	5556	50383	5456	185347	9653	13.7 h
47 85.70 5 20712 20 1.5 6637 56910 6862 96694 7226 10.3 h 48 85.72 5 20911 20 1.5 6311 56349 6710 120384 7895 15.4 h 49 85.73 6 26392 2 3 16159 24514 9487 3153 749 13.8 h 50 85.92 6 26363 2 3 16376 24473 9358 7417 631 12.1 h 51 85.93 3 13041 20 1.5 4117 39602 5212 39395 3713 17.4 h 52 85.95 3 13011 20 2.5 4255 40517 5390 24478 3366 20.6 h 53 85.98 5 20756 2.4 2.5 10516 22044 7450 6610 2795 15.7 h 54 86.04 5 20840 20 2.5 705	45	85.53	5	20711	20	2.5	5054	43759	4078	270308	11587	11.4 h
48 85.72 5 20911 20 1.5 6311 56349 6710 120384 7895 15.4 h 49 85.73 6 26392 2 3 16159 24514 9487 3153 749 13.8 h 50 85.92 6 26363 2 3 16376 24473 9358 7417 631 12.1 h 51 85.93 3 13011 20 2.5 4255 40517 5390 24478 3366 20.6 h 53 85.98 5 20756 2.4 2.5 10516 22044 7450 6610 2795 15.7 h 54 86.04 5 20840 20 2.5 7153 62231 8139 63273 5577 14.0 h 55 86.13 5 20838 20 2.5 7367 63987 8559 54708 4900 10.	46	85.59	5	20797	20	1.5	7244	63668	8524	61191		
49 85.73 6 26392 2 3 16159 24514 9487 3153 749 13.8 h 50 85.92 6 26363 2 3 16376 24473 9358 7417 631 12.1 h 51 85.93 3 13041 20 1.5 4117 39602 5212 39395 3713 17.4 h 52 85.95 3 13011 20 2.5 4255 40517 5390 24478 3366 20.6 h 53 85.98 5 20756 2.4 2.5 10516 22044 7450 6610 2795 15.7 h 54 86.04 5 20840 20 2.5 7153 62231 8319 63273 5557 14.0 h 55 86.13 5 20838 20 2.5 7009 63089 8220 57513 5620 11.9 h 56 86.18 5 20688 20 2.5 7047 64778 8836 47154 4419 10.7 h<	47	85.70	5	20712	20	1.5	6637	56910	6862	96694	7226	10.3 h
50 85.92 6 26363 2 3 16376 24473 9358 7417 631 12.1 h 51 85.93 3 13041 20 1.5 4117 39602 5212 39395 3713 17.4 h 52 85.95 3 13011 20 2.5 4255 40517 5390 24478 3366 20.6 h 17.5 4255 40517 5390 24478 3366 20.6 h 17.5 18.6 20.6 h 18.6 20.6 h 18.6 20.6 h 18.6 20.6 18.6 20.6 18.6 20.6 18.6 20.6 17.5 18.6 20.6 19.6 20.6 19.6 12.6 11.9 h 19.6 86.16 5 20787 22 2.5 7009 63089 8220 57513 5620 11.9 h 19.6 86.18 5 20688 20 2.5 7447 64778 8836 47154 4419 10.7 h 10.7 h 586.18	48	85.72	5	20911	20	1.5	6311	56349	6710	120384	7895	15.4 h
51 85.93 3 13041 20 1.5 4117 39602 5212 39395 3713 17.4 h 52 85.95 3 13011 20 2.5 4255 40517 5390 24478 3366 20.6 h 53 85.98 5 20756 2.4 2.5 10516 22044 7450 6610 2795 15.7 h 54 86.04 5 20840 20 2.5 7153 62231 8139 63273 5557 14.0 h 55 86.13 5 20838 20 2.5 7009 63089 8220 57513 5620 11.9 h 56 86.16 5 20787 22 2.5 7367 63987 8559 54708 4900 10.8 h 57 86.18 5 20688 20 2.5 7447 64778 8836 47154 4419 10.7 h 58 86.22 5 20852 20 2.5	49	85.73	6	26392	2	3	16159	24514	9487	3153	749	
52 85.95 3 13011 20 2.5 4255 40517 5390 24478 3366 20.6 h 53 85.98 5 20756 2.4 2.5 10516 22044 7450 6610 2795 15.7 h 54 86.04 5 20840 20 2.5 7153 62231 8139 63273 5557 14.0 h 55 86.13 5 20838 20 2.5 7009 63089 8220 57513 5620 11.9 h 56 86.16 5 20787 22 2.5 7367 63987 8559 54708 4900 10.8 h 57 86.18 5 20688 20 2.5 7447 64778 8836 47154 4419 10.7 h 58 86.22 5 20852 20 2.5 6202 54180 6282 144069 8376 11.6 h 59 86.22 5 20947 20 2.5	50	85.92	6	26363	2	3	16376	24473				
53 85.98 5 20756 2.4 2.5 10516 22044 7450 6610 2795 15.7 h 54 86.04 5 20840 20 2.5 7153 62231 8139 63273 5557 14.0 h 55 86.13 5 20838 20 2.5 7009 63089 8220 57513 5620 11.9 h 56 86.16 5 20787 22 2.5 7367 63987 8559 54708 4900 10.8 h 57 86.18 5 20688 20 2.5 7447 64778 8836 47154 4419 10.7 h 58 86.22 5 20852 20 2.5 6202 54180 6282 144069 8376 11.6 h 59 86.22 5 20947 20 2.5 6722 63620 8412 52191 5091 9.35 h 60 86.27 5	51	85.93		13041		1.5						
54 86.04 5 20840 20 2.5 7153 62231 8139 63273 5557 14.0 h 55 86.13 5 20838 20 2.5 7009 63089 8220 57513 5620 11.9 h 56 86.16 5 20787 22 2.5 7367 63987 8559 54708 4900 10.8 h 57 86.18 5 20688 20 2.5 7447 64778 8836 47154 4419 10.7 h 58 86.22 5 20852 20 2.5 6202 54180 6282 144069 8376 11.6 h 59 86.22 5 20947 20 2.5 7522 63620 8412 52191 5091 9.35 h 60 86.27 5 20978 40 2.5 6387 72868 6909 116000 7520 11.1 h 62 86.38 5 20754 40 2.5 6881 76861 7638 86915 6253 <t< td=""><td>52</td><td>85.95</td><td>3</td><td>13011</td><td>20</td><td>2.5</td><td>4255</td><td>40517</td><td>5390</td><td>24478</td><td>3366</td><td></td></t<>	52	85.95	3	13011	20	2.5	4255	40517	5390	24478	3366	
55 86.13 5 20838 20 2.5 7009 63089 8220 57513 5620 11.9 h 56 86.16 5 20787 22 2.5 7367 63987 8559 54708 4900 10.8 h 57 86.18 5 20688 20 2.5 7447 64778 8836 47154 4419 10.7 h 58 86.22 5 20852 20 2.5 6202 54180 6282 144069 8376 11.6 h 59 86.22 5 20947 20 2.5 7522 63620 8412 52191 5091 9.35 h 60 86.27 5 20978 40 2.5 6773 79489 8184 75416 6035 14.2 h 61 86.29 5 20797 40 2.5 6387 72868 6909 116000 7520 11.1 h 62 86.38 5 20754 40 2.5 6881 76861 7638 86915 6253 <t< td=""><td>53</td><td>85.98</td><td>5</td><td>20756</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></t<>	53	85.98	5	20756								
56 86.16 5 20787 22 2.5 7367 63987 8559 54708 4900 10.8 h 57 86.18 5 20688 20 2.5 7447 64778 8836 47154 4419 10.7 h 58 86.22 5 20852 20 2.5 6202 54180 6282 144069 8376 11.6 h 59 86.22 5 20947 20 2.5 7522 63620 8412 52191 5091 9.35 h 60 86.27 5 20978 40 2.5 6773 79489 8184 75416 6035 14.2 h 61 86.29 5 20797 40 2.5 6387 72868 6909 116000 7520 11.1 h 62 86.38 5 20754 40 2.5 6881 76861 7638 86915 6253 6.72 h 63 86.43 5 20920 40 2.5 7177 76854 7706 82085 6054 <t< td=""><td>54</td><td>86.04</td><td>5</td><td>20840</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></t<>	54	86.04	5	20840								
57 86.18 5 20688 20 2.5 7447 64778 8836 47154 4419 10.7 h 58 86.22 5 20852 20 2.5 6202 54180 6282 144069 8376 11.6 h 59 86.22 5 20947 20 2.5 7522 63620 8412 52191 5091 9.35 h 60 86.27 5 20978 40 2.5 6773 79489 8184 75416 6035 14.2 h 61 86.29 5 20797 40 2.5 6387 72868 6909 116000 7520 11.1 h 62 86.38 5 20754 40 2.5 6881 76861 7638 86915 6253 6.72h 63 86.43 5 20920 40 2.5 7177 76854 7706 82085 6054 8.81h 64 86.45 5 20631 40 2.5 5806 85167 6249 148784 8876 <td< td=""><td>55</td><td>86.13</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></td<>	55	86.13										
58 86.22 5 20852 20 2.5 6202 54180 6282 144069 8376 11.6 h 59 86.22 5 20947 20 2.5 7522 63620 8412 52191 5091 9.35 h 60 86.27 5 20978 40 2.5 6773 79489 8184 75416 6035 14.2 h 61 86.29 5 20797 40 2.5 6387 72868 6909 116000 7520 11.1 h 62 86.38 5 20754 40 2.5 6881 76861 7638 86915 6253 6.72 h 63 86.43 5 20920 40 2.5 7177 76854 7706 82085 6054 8.81 h 64 86.45 5 20631 40 2.5 6329 74485 7262 92362 7046 14.6 h 65 86.63 5 20902 80 2.5 5806 85167 6249 148784 88	56	86.16	5	20787	22	2.5	7367	63987	8559	54708	4900	
59 86.22 5 20947 20 2.5 7522 63620 8412 52191 5091 9.35 h 60 86.27 5 20978 40 2.5 6773 79489 8184 75416 6035 14.2 h 61 86.29 5 20797 40 2.5 6387 72868 6909 116000 7520 11.1 h 62 86.38 5 20754 40 2.5 6881 76861 7638 86915 6253 6.72 h 63 86.43 5 20920 40 2.5 7177 76854 7706 82085 6054 8.81 h 64 86.45 5 20631 40 2.5 6329 74485 7262 92362 7046 14.6 h 65 86.63 5 20902 80 2.5 5806 85167 6249 148784 8876 13.8 h 66 86.64 6 24404 100 3 7564 124510 9691 89594 7355 <t< td=""><td>57</td><td>86.18</td><td>5</td><td>20688</td><td>20</td><td>2.5</td><td>7447</td><td></td><td></td><td></td><td></td><td></td></t<>	57	86.18	5	20688	20	2.5	7447					
60 86.27 5 20978 40 2.5 6773 79489 8184 75416 6035 14.2 h 61 86.29 5 20797 40 2.5 6387 72868 6909 116000 7520 11.1 h 62 86.38 5 20754 40 2.5 6881 76861 7638 86915 6253 6.72 h 63 86.43 5 20920 40 2.5 7177 76854 7706 82085 6054 8.81 h 64 86.45 5 20631 40 2.5 6329 74485 7262 92362 7046 14.6 h 65 86.63 5 20902 80 2.5 5806 85167 6249 148784 8876 13.8 h 66 86.64 6 24404 100 3 7564 124510 9691 89594 7355 13.2 h 67 86.69 6 24573 100 3 7803 122935 9614 89375 7369 <t< td=""><td>58</td><td>86.22</td><td>5</td><td>20852</td><td>20</td><td>2.5</td><td></td><td></td><td></td><td></td><td></td><td></td></t<>	58	86.22	5	20852	20	2.5						
61 86.29 5 20797 40 2.5 6387 72868 6909 116000 7520 11.1 h 62 86.38 5 20754 40 2.5 6881 76861 7638 86915 6253 6.72 h 63 86.43 5 20920 40 2.5 7177 76854 7706 82085 6054 8.81 h 64 86.45 5 20631 40 2.5 6329 74485 7262 92362 7046 14.6 h 65 86.63 5 20902 80 2.5 5806 85167 6249 148784 8876 13.8 h 66 86.64 6 24404 100 3 7564 124510 9691 89594 7355 13.2 h 67 86.69 6 24573 100 3 7803 122935 9614 89375 7369 14.1 h 68 86.70 6 24495 100 3 6571 105037 7010 149698 11272 12.1 h 69 86.73 6 24538 100 3 7635 120888 9389 99930 7811 11.5 h 70 86.75 6 24374 100 3 7827 126178 9899 80444 6862 14.7 h 71 86.82 6 24615 100 3 6532 121187 9864 167590 8507 11.8 h 72 86.96 6 24658 100 3 7762 116023 8546 126334 8798 7.66 h	59	86.22	5	20947								
62 86.38 5 20754 40 2.5 6881 76861 7638 86915 6253 6.72 h 63 86.43 5 20920 40 2.5 7177 76854 7706 82085 6054 8.81 h 64 86.45 5 20631 40 2.5 6329 74485 7262 92362 7046 14.6 h 65 86.63 5 20902 80 2.5 5806 85167 6249 148784 8876 13.8 h 66 86.64 6 24404 100 3 7564 124510 9691 89594 7355 13.2 h 67 86.69 6 24573 100 3 7803 122935 9614 89375 7369 14.1 h 68 86.70 6 24495 100 3 6571 105037 7010 149698 11272 12.1 h 69 86.73 6 24538 100 3 7635 120888 9389 99930 7811 11.5 h 70 86.75 6 24374 100 3 7827 126178 9899 80444 6862 14.7 h 71 86.82 6 24615 100 3 6532 121187 9864 167590 8507 11.8 h 72 86.96 6 24658 100 3 7762 116023 8546 126334 8798 7.66 h	60	86.27	5	20978	40	2.5	6773	79489	8184			
63 86.43 5 20920 40 2.5 7177 76854 7706 82085 6054 8.81 h 64 86.45 5 20631 40 2.5 6329 74485 7262 92362 7046 14.6 h 65 86.63 5 20902 80 2.5 5806 85167 6249 148784 8876 13.8 h 66 86.64 6 24404 100 3 7564 124510 9691 89594 7355 13.2 h 67 86.69 6 24573 100 3 7803 122935 9614 89375 7369 14.1 h 68 86.70 6 24495 100 3 6571 105037 7010 149698 11272 12.1 h 69 86.73 6 24538 100 3 7635 120888 9389 99930 7811 11.5 h 70 86.75 6 24374 100 3 7827 126178 9899 80444 6862 14.7 h 71 86.82 6 24615 100 3 6532 121187 9864 167590 8507 11.8 h 72 86.96 6 24658 100 3 7762 116023 8546 126334 8798 7.66 h	61	86.29	5	20797	40							
63 86.43 5 20920 40 2.5 7177 76854 7706 82085 6054 8.81 h 64 86.45 5 20631 40 2.5 6329 74485 7262 92362 7046 14.6 h 65 86.63 5 20902 80 2.5 5806 85167 6249 148784 8876 13.8 h 66 86.64 6 24404 100 3 7564 124510 9691 89594 7355 13.2 h 67 86.69 6 24573 100 3 7803 122935 9614 89375 7369 14.1 h 68 86.70 6 24495 100 3 6571 105037 7010 149698 11272 12.1 h 69 86.73 6 24538 100 3 7635 120888 9389 99930 7811 11.5 h 70 86.75 6 24374 100 3 7827 126178 9899 80444 6862 <		86.38	5	20754								
65 86.63 5 20902 80 2.5 5806 85167 6249 148784 8876 13.8 h 66 86.64 6 24404 100 3 7564 124510 9691 89594 7355 13.2 h 67 86.69 6 24573 100 3 7803 122935 9614 89375 7369 14.1 h 68 86.70 6 24495 100 3 6571 105037 7010 149698 11272 12.1 h 69 86.73 6 24538 100 3 7635 120888 9389 99930 7811 11.5 h 70 86.75 6 24374 100 3 7827 126178 9899 80444 6862 14.7 h 71 86.82 6 24615 100 3 6532 121187 9864 167590 8507 11.8 h 72 86.96 6 24658 100 3 7762 116023 8546 126334 8798 7.66 h		86.43	5	20920	40							
66 86.64 6 24404 100 3 7564 124510 9691 89594 7355 13.2 h 67 86.69 6 24573 100 3 7803 122935 9614 89375 7369 14.1 h 68 86.70 6 24495 100 3 6571 105037 7010 149698 11272 12.1 h 69 86.73 6 24538 100 3 7635 120888 9389 99930 7811 11.5 h 70 86.75 6 24374 100 3 7827 126178 9899 80444 6862 14.7 h 71 86.82 6 24615 100 3 6532 121187 9864 167590 8507 11.8 h 72 86.96 6 24658 100 3 7762 116023 8546 126334 8798 7.66 h	64	86.45	5	20631	40	2.5	6329	74485	7262	92362	7046	
66 86.64 6 24404 100 3 7564 124510 9691 89594 7355 13.2 h 67 86.69 6 24573 100 3 7803 122935 9614 89375 7369 14.1 h 68 86.70 6 24495 100 3 6571 105037 7010 149698 11272 12.1 h 69 86.73 6 24538 100 3 7635 120888 9389 99930 7811 11.5 h 70 86.75 6 24374 100 3 7827 126178 9899 80444 6862 14.7 h 71 86.82 6 24615 100 3 6532 121187 9864 167590 8507 11.8 h 72 86.96 6 24658 100 3 7762 116023 8546 126334 8798 7.66 h	65	86.63	5	20902	80	2.5	5806					
67 86.69 6 24573 100 3 7803 122935 9614 89375 7369 14.1 h 68 86.70 6 24495 100 3 6571 105037 7010 149698 11272 12.1 h 69 86.73 6 24538 100 3 7635 120888 9389 99930 7811 11.5 h 70 86.75 6 24374 100 3 7827 126178 9899 80444 6862 14.7 h 71 86.82 6 24615 100 3 6532 121187 9864 167590 8507 11.8 h 72 86.96 6 24658 100 3 7762 116023 8546 126334 8798 7.66h				24404	100	3						
69 86.73 6 24538 100 3 7635 120888 9389 99930 7811 11.5 h 70 86.75 6 24374 100 3 7827 126178 9899 80444 6862 14.7 h 71 86.82 6 24615 100 3 6532 121187 9864 167590 8507 11.8 h 72 86.96 6 24658 100 3 7762 116023 8546 126334 8798 7.66 h		86.69	6	24573	100							
70 86.75 6 24374 100 3 7827 126178 9899 80444 6862 14.7 h 71 86.82 6 24615 100 3 6532 121187 9864 167590 8507 11.8 h 72 86.96 6 24658 100 3 7762 116023 8546 126334 8798 7.66 h	68	86.70	6	24495	100	3	6571	105037	7010	149698	11272	
70 86.75 6 24374 100 3 7827 126178 9899 80444 6862 14.7 h 71 86.82 6 24615 100 3 6532 121187 9864 167590 8507 11.8 h 72 86.96 6 24658 100 3 7762 116023 8546 126334 8798 7.66 h	69	86.73	6	24538	100	3	7635	120888				
71 86.82 6 24615 100 3 6532 121187 9864 167590 8507 11.8 h 72 86.96 6 24658 100 3 7762 116023 8546 126334 8798 7.66 h				24374	100							
72 86.96 6 24658 100 3 7762 116023 8546 126334 8798 7.66 h				24615	100							
			6	24658								
	73	87.54	5	20604	100	1	6101	94651	6605	108893	8048	12.1 h

- [Bollobás 1985] B. Bollobás, Random graphs, Academic Press, London and Orlando, FL, 1985.
- [Brent and te Riele 1992] R. P. Brent and H. J. J. te Riele, "Factorizations of $a^n \pm 1$, $13 \le a < 100$ ", Technical Report NM-R9212, Centrum voor Wiskunde en Informatica, Kruislaan 413, 1098 SJ Amsterdam, The Netherlands, June 1992. Updates to this report have appeared as CWI Report NM-R9419, September 1994 and CWI Report NM-R9609, March 1996, with P. L. Montgomery as a coauthor. The complete tables, incorporating these updates, are available at ftp://nimbus.anu.edu.au/pub/Brent/factors or ftp://ftp.cwi.nl/pub/herman/factors.
- [Bressoud 1989] D. M. Bressoud, Factorization and primality testing, Undergraduate Texts in Math., Springer, New York, 1989.
- [Davis and Holdridge 1983] J. A. Davis and D. B. Holdridge, "Factorization using the quadratic sieve algorithm", Technical Report Sand 83-1346, Sandia National Laboratories, Albuquerque, NM, 1983.
- [Denny 1993] T. F. Denny, "Faktorisieren mit dem quadratischen Sieb", Master's thesis, Universität des Saarlandes, Saarbrücken, 1993.
- [Denny et al. 1994] T. F. Denny, B. Dodson, A. K. Lenstra, and M. S. Manasse, "On the factorization of RSA-120", pp. 166-174 in Advances in Cryptology (CRYPTO '93: Santa Barbara, CA, 1993), edited by D. R. Stinson, Lecture Notes in Comp. Sci. 773, Springer, Berlin, 1994.
- [Erdős and Rényi 1959] P. Erdős and A. Rényi, "On random graphs I", Publ. Math. Debrecen 6 (1959), 290-297.
- [Erdős and Rényi 1960] P. Erdős and A. Rényi, "On the evolution of random graphs", *Publ. Math. Inst. Hungar. Acad. Sci.* 5 (1960), 17–61.
- [Erdős and Rényi 1961] P. Erdős and A. Rényi, "On the evolution of random graphs", Bull. Inst. Int. Statist. Tokyo 38 (1961), 343–347.
- [Kraitchik 1929] M. Kraitchik, Recherches sur la théorie des nombres, II, Gauthier-Villars, Paris, 1929.
- [Lehmer and Powers 1931] D. H. Lehmer and R. E. Powers, "On factoring large numbers", Bull. Amer. Math. Soc. 37 (1931), 770-776.

- [Lenstra and Lenstra 1993] A. K. Lenstra and H. W. Lenstra, Jr. (editors), The development of the number field sieve, Lecture Notes in Math. 1554, Springer, Berlin, 1993.
- [Lenstra and Manasse 1994] A. K. Lenstra and M. S. Manasse, "Factoring with two large primes", Math. Comp. 63 (1994), 785-798.
- [Morrison and Brillhart 1975] M. A. Morrison and J. Brillhart, "A method of factoring and the factorization of F_7 ", Math. Comp. 29 (1975), 183–205.
- [Parkinson and Wunderlich 1984] D. Parkinson and W. Wunderlich, "A compact algorithm for Gaussian elimination over GF(2) implemented on highly parallel computers", Parallel Comput. 1 (1984), 65–73.
- [Paton 1969] K. Paton, "An algorithm for finding a fundamental set of cycles of a graph", Comm. ACM 12 (1969), 514-518.
- [Peralta \geq 1997] R. Peralta, "Implementation of the hypercube variation of the multiple polynomial quadratic sieve". To appear.
- [Pomerance 1982] C. Pomerance, "Analysis and comparison of some integer factoring algorithms", pp. 89-139 in Computational methods in number theory, I, edited by H. W. Lenstra, Jr. and R. Tijdeman, Mathematisch Centrum, Amsterdam, 1982.
- [Pomerance 1985] C. Pomerance, "The quadratic sieve factoring algorithm", pp. 169–182 in Advances in cryptology (EUROCRYPT '84: Paris, 1984), edited by T. Beth et al., Lecture Notes in Comp. Sci. 209, Springer-Verlag, New York, 1985.
- [Pomerance et al. 1988] C. Pomerance, J. W. Smith, and R. Tuler, "A pipeline architecture for factoring large integers with the quadratic sieve algorithm", SIAM J. Comput. 17 (1988), 387-403.
- [te Riele et al. 1989] H. J. J. te Riele, W. M. Lioen, and D. T. Winter, "Factoring with the quadratic sieve on large vector computers", J. Comp. Appl. Math. 27 (1989), 267-278.
- [te Riele et al. 1991] H. te Riele, W. Lioen, and D. Winter, "Factorization beyond the googol with MPQS on a single computer", CWI Quarterly 4 (March 1991), 69-72.

- [Riesel 1985] H. Riesel, Prime numbers and computer methods for factorization, Progress in Math. 126, Birkhäuser, Boston, 1985.
- [Silverman 1987] R. D. Silverman, "The multiple polynomial quadratic sieve", Math. Comp. 48 (1987), 329-339.
- [Wagstaff 1993] S. S. Wagstaff, Jr., December 13, 1993.Regular letter to subscribers describing the state of the Cunningham factorization project.
- Henk Boender, Department of Mathematics and Computer Science, University of Leiden, P.O. Box 9512, 2300 RA Leiden, The Netherlands (henkb@wi.leidenuniv.nl), and Centrum voor Wiskunde en Informatica, P.O. Box 94079, 1090 GB Amsterdam, The Netherlands (henkb@cwi.nl)
- Herman J. J. te Riele, Centrum voor Wiskunde en Informatica, P.O. Box 94079, 1090 GB Amsterdam, The Netherlands (herman@cwi.nl)

Received October 10, 1995; accepted September 5, 1996