

Supplementary material 1: the construction process of DGGS

To construct a hexagonal DGGS (Fig. 1), we first recursively divided the surface of the polyhedron to form multi-resolution hexagonal grids, and then mapped them to the surface of the earth.

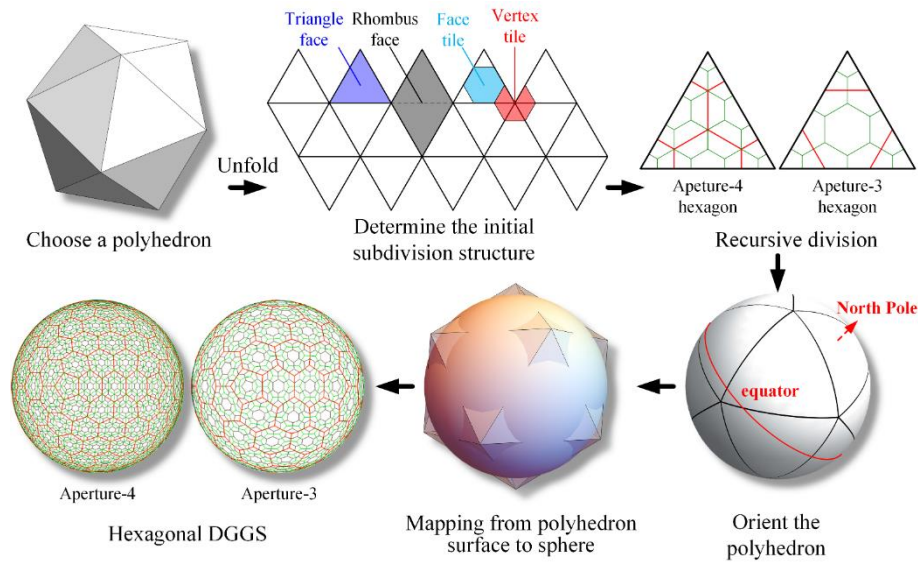


Fig. 1. Complete process of constructing a hexagonal DGGS.

Supplementary material 2: HHUT hierarchical structure on the spherical surface (from the perspective of tiles)

We map the icosahedral HHUT grids of the first four levels to the earth according to the Snyder equal area projection (Snyder et al., 1992). The results are shown in Fig. 2 (different colors represent cells of different tiles). According to HHUT hierarchical structure and equal area conditions, the number and resolution of the cell in each level can be calculated as shown in Table 1.

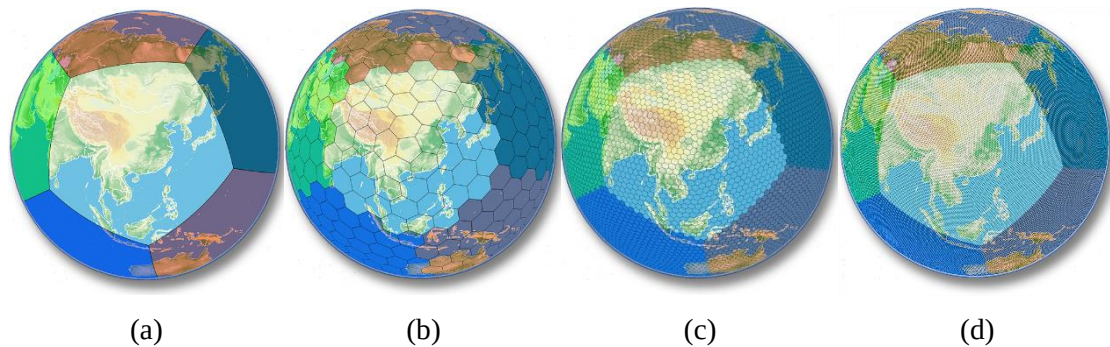


Fig. 2. HHUT hierarchical structure on the spherical surface (from the perspective of tiles).

Level (a) 0, (b) 3, (c) 5, and (d) 7.

Supplementary material 3: main parameters of HHUT cells in each level

Table 1 Main parameters of HHUT cells in each level

Level	Total cell number	Average area of a cell (km^2)	Distance between adjacent cell centers (km)
0	12	51006562.17248	7529.84000
1	42	12751640.54310	3764.92000
2	162	3187910.13578	1913.88000
3	642	796977.53394	961.97800
4	2562	199244.38349	481.77100
5	10242	49811.09587	241.04700
6	40962	12452.77397	120.56000
7	163842	3113.19349	60.28930
8	655362	778.29837	30.14700
9	2621442	194.57459	15.07410
10	10485762	48.64365	7.53719
11	41943042	12.16091	3.76863
12	167772162	3.04023	1.88432
13	671088642	0.76006	0.94217
14	2684354562	0.19001	0.47108
15	10737418242	0.04750	0.23554

Supplementary material 4: the multiplication operation for HHUT

The multiplication operation (denoted by $\otimes$) of the hierarchy-based code is defined as follows. Taking the direction of ω as a reference, the hexadecimal digital set $\{1, 2, 3, 4, 5, 6\}$ can be obtained from the position of the set $\mathbb{D}$, which corresponds to a rotation of 0° , counterclockwise rotation of 60° , counterclockwise rotation of 120° , rotation of 180° , clockwise rotation of 120° , and clockwise rotation of 60° , respectively, in the multiplication operation. When multiplying, the hexadecimal digital set is calculated bit by bit, and the result is obtained by looking up the multiplication table. The lookup table of the multiplication operation is presented in Table 2. Taking $210 \otimes 2$ in Figure 12 as an example, according to the multiplication table,

multiplication operations are performed on symbols other than tile bits, $0 \otimes 2 = 0$ and $1 \otimes 2 = 2$; thus, the result is 20, and because it belongs to the same tile, the result 220 is obtained, which is same as the result of counterclockwise rotation of 60° .

Table 2 Lookup table of the multiplication operation

$\otimes$	0	1	2	3	4	5	6	a	b	c	d	e	f
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	a	b	c	d	e	f
2	0	2	3	4	5	6	1	b	c	d	e	f	a
3	0	3	4	5	6	1	2	c	d	e	f	a	b
4	0	4	5	6	1	2	3	d	e	f	a	b	c
5	0	5	6	1	2	3	4	e	f	a	b	c	d
6	0	6	1	2	3	4	5	f	a	b	c	d	e

Supplementary material 5: main parameters of H3 cells in each level

Table 3 Main parameters of H3 cells in each level

Level	Total cell number	Average area of a cell (km^2)	Distance between adjacent cell centers (km)
0	122	4,250,546.8477000	2503.44599
1	842	607,220.9782429	715.27028
2	5882	86,745.8540347	283.07334
3	41162	12,392.2648621	109.84913
4	288122	1,770.3235517	40.44423
5	2016842	252.9033645	15.71654
6	14117882	36.1290521	5.76536
7	98825162	5.1612932	2.24498
8	691776122	0.7373276	0.82337
9	4842432842	0.1053325	0.32071
10	33897029882	0.0150475	0.11762
11	237279209162	0.0021496	0.04581
12	1,660,954,464,122	0.0003071	0.01680
13	11,626,681,248,842	0.0000439	0.00566
14	81,386,768,741,882	0.0000063	0.00240
15	569,707,381,193,162	0.0000009	0.00093