

A Geo-Aware Server Assignment Problem for Mobile Edge Computing

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ABSTRACT

As mobile devices have become the preferred tool for communication, work, and entertainment, traffic at the edge of the network is growing more rapidly than ever. To improve user experience, commodity servers are deployed in the edge to form a decentralized network of mini datacenters each serving a localized region. A challenge is how to place these servers geographically to maximize the offloading benefit and be close to the users they respectively serve. We introduce a formulation for this problem to serve applications that involve pairwise communication between mobile devices at different geolocations. We explore several heuristic solutions and compare them in an evaluation using both real-world and synthetic datasets.

KEYWORDS

Mobile edge computing; optimization; heuristic; server assignment; geo-aware; cloud computing

1. Introduction

Mobile Edge Computing (MEC) [1] has emerged as a viable technology for mobile operators to push computing resources closer to the users so that requests can be served locally without long-haul crossing of the network core, thus improving network efficiency and user experience. In a nutshell, a typical MEC architecture consists of four layers of entities: the mobile users, the base-stations, the edge servers, and the cloud datacenter. The edge servers are introduced in the edge of the network connecting the base-stations to the network core, each server being an aggregation hub, or a mini datacenter to offload processing tasks from the remote datacenter. Because the region, or “cloudlet” [2], served by an edge server is much smaller, a commodity virtualization-enabled computer can be used to run compute and network functions that would otherwise be provided by the datacenter.

MEC can benefit many compute-hungry or latency-critical applications involving video optimization [3], content delivery [4], big data analytics [5], and augmented reality [6], to name a few. Originally initiated for cellular networks to realize the 5G vision, MEC has been generalized for broader wireless and mobile networks [7]. It is becoming more of a phenomenon with the Internet of Things; more than 5 billion IoT devices would be connected to MEC by 2020 according to a January 2017 forecast by BI Intelligence [8].

A challenge with MEC is how to align the edge servers with the base-stations geographically to maximize the edge computing benefits. To address this challenge is application-specific. We focus on applications involving pairwise transactions between devices. Cellphone calls made from one user to another, peer-to-peer video streaming, and multi-player online gaming are examples of this type of communication. If the two devices are served by different edge servers, the datacenter must get involved, thus incurring a backhaul cost. This cost is avoided if the same server serves both devices. However, it makes no practical sense if they are located in far remote geographic locations because a server should be geographically close to where it serves to avoid high installation cost and long latency [9,10]. On the other hand, those devices with many transactions should belong to the same server. Although geographic proximity tends to imply high transactional activity, this relationship is not straightforward. As the number of servers is finite and their capacities limited, it is impossible to equally please all the users.

Therefore, we are motivated to solve the following optimization problem: where to place a set of edge servers of limited capacity and assign them to the base-stations such that (1) offloading benefit is maximized and (2) each server is geographically close to its respective users. We require that the server locations be chosen from a set of predetermined geographic sites; this constraint applies to practical cases involving non-technical factors in the deployment of the servers, for example, due to economics, policies, or management.

The server assignment problem is not new outside MEC. Indeed, it belongs to the body of work on distributed allocation of resources (virtual machines) widely studied in the area of cloud computing [11–14]. The MEC problem is similar, however with unique constraints. Firstly, the servers in MEC need be near the user side, not the datacenter side, and so the communication cost to be minimized is due to the use of the backhaul network (towards the datacenter), not the front-haul (towards the cells). Secondly, the geographic spread of the cells served by a MEC server should be a design factor, which is not a typical priority for a distributed cloud solution.

The MEC server assignment problem has been addressed in some forms, only recently [9,15]. Our key contribution is a new practical formulation for the server assignment problem. We prove its NP-hardness and subsequently explore an approximation solution based on local search heuristics. We evaluate its effectiveness and efficiency using both real-world and synthetic datasets and comparing to intuitive approaches.

The remainder of the paper is organized as follows. Related work is discussed in Section 2. The problem is stated and formulated in Section 3. The algorithm is proposed in Section 4. The results of the evaluation study are analyzed in Section 5. The paper concludes in Section 6 with pointers to our future work.

2. Related Work

Every finite computing system serving a large number of resource-hungry requests faces the challenge of how to assign resources to computing units to optimize hardware consumption and best satisfy application QoS requirements. The MEC server assignment problem shares the same challenge, which can arise in various scenarios.

The assignment problem in [16] applies to a MEC network supporting multiple applications of known request load and latency expectation and the challenge is to determine which edge servers to run the required virtual machines (VMs), constrained by server capacity and inter-server communication delay. In [17,18], where only one

application is being considered and consists of multiple inter-related components organizable into a graph, the challenge is how to place this component graph on top of the physical graph of edge servers to minimize the cost to run the application. In the case that edge servers must be bound to certain geographic locations, a challenge is to decide which among these locations we should place the servers and inter-connect them for optimal routing and installation costs [15].

The above works do not take into account the geographic spread of the region served by a server. The cells served by the same server can be highly scattered geographically, causing long latency and high management cost. This motivates the work in [9] proposing a spatial partitioning of the geographic area such that the cells of the same server are always contiguous. For this partitioning, a graph-based clustering algorithm is proposed that repeatedly merges adjacent cells to form clusters as long as the merger results in better offloading and no cluster exceeds the server capacity. In a similar research [10], where the cells served by each server are also contiguous, the objectives are to minimize the server deployment cost, the front-haul link cost for each server to reach its assigned base-stations, and the cell-to-cell latency via the edge; the proposed algorithm is to repeatedly select the next remaining server of the least deployment cost and assign it to all the nearby base-stations of the least front-haul link cost so long as the server capacity is met.

The problem in [9] is aimed to optimize for workloads involving cell-to-cell communication, whereas the problem in [10] is for individual-cell workloads. The latter also requires that all the processing be fulfilled by the edge servers, hence zero backhaul cost. In this aspect, our work is more similar to [9] because we also optimize for cell-to-cell workloads and cannot avoid backhaul use (thus, the objective to minimize its cost). However, there are key differences. First, the number of servers is a constraint in our problem, but not in [9]. Second, we minimize the geographic spread of the cells served by each server, instead of enforcing their geographic contiguity. We argue that these cells should not be too far from their server but do not have to be contiguous; in contrast, the solution in [9] enforces contiguity, but has no control of spread. Third, we require that the servers be bound to predetermined locations (as in [10,15], but not a constraint of [9]).

3. Problem Statement

The geographic area \mathcal{A} is partitioned into a set \mathcal{C} of $N_{\mathcal{C}}$ cells, each cell $i \in \mathcal{C}$ served by a base-station at a known location in \mathcal{A} ; we also refer to this base-station as base-station i . The meanings of “base-station” and “cell” are general, not necessarily understood in conventional meaning as in cellular networks; for example, as a Wi-Fi access router and its coverage area in Wi-Fi networks. The edge layer consists of $N_{\mathcal{S}} < N_{\mathcal{C}}$ edge servers, whose locations are chosen from a set $\mathcal{L} \subset \mathcal{A}$ of $N_{\mathcal{L}} \geq N_{\mathcal{S}}$ candidate locations. For example, \mathcal{L} can be a subset of base-station locations; in this case, an edge server must be co-located with some base-station (as in [10]). In general, we admit any arbitrary candidate location in \mathcal{A} .

The input workload is a symmetric $N_{\mathcal{C}} \times N_{\mathcal{C}}$ matrix of non-negative real values w_{ij} representing the transaction demand between users in cell $i \in \mathcal{C}$ with users in cell $j \in \mathcal{C}$. Note that w_{ii} is the transaction demand between users of the same cell i . Denote by $w_i = \sum_{j \in \mathcal{C}} w_{ij}$ the total workload involving cell i . Without loss of generality, assume that the total workload involving all the cells equals 1; i.e., $\sum_{i \leq j \in \mathcal{C}} w_{ij} = 1$. Each server exclusively manages the workload for a group of base-stations. If base-station

i is assigned to a server at location l , we have a front-haul link, whose usage cost should increase with their distance; denote this cost by d_{il} (assumed given, e.g., equal to distance). Because the backhaul links to reach the remote data center are much more expensive, representing the worst-case scenario, we assume their cost to be a fixed cost d much higher than all front-haul link costs. We want to avoid backhaul links as much as possible.

Our goal is to (1) server location assignment (SLA): assign the best location for each server and (2) cell-server assignment (CSA): assign the best server for each cell. We use 0-1 integer programming to formulate these assignments. Define a binary variable $z = (z_{il})_{\mathcal{C} \times \mathcal{L}}$ such that $z_{il} = 1$ iff there is a server at location l and this server serves cell i . Given z , we can tell exactly the server locations (SLA) and the cell-to-server assignment (CSA). A location l is a server location iff $\sum_{i \in \mathcal{C}} z_{il} \geq 1$, i.e., at least one cell is assigned to location l . Another way to express this condition is

$$\prod_{i \in \mathcal{C}} (1 - z_{il}) = 0.$$

Because N_S different locations must be chosen for the servers, we have the constraint below,

$$\sum_{l \in \mathcal{L}} \left(1 - \prod_{i \in \mathcal{C}} (1 - z_{il}) \right) = N_S.$$

Also, each cell must be assigned to exactly one server, another constraint is

$$\sum_{l \in \mathcal{L}} z_{il} = 1 \quad \forall i \in \mathcal{C}.$$

We assume that there is a capacity $W < 1$ on the compute load a server can process. In the case a server is fully saturated, the residual workload must be serviced by the data center. Our objectives are to minimize the backhaul cost and geo-spread under this assumption.

3.1. Backhaul Cost

A transaction can be one of the following types: between users of the same cell, between users of two different cells assigned to the same server, and between users of two different cells assigned to different servers. The compute demand for the edge comes from transactions of the first two types. Specifically, if a server is placed at location l , we represent its compute (demand) load as

$$W(l) = \sum_{i \leq j \in \mathcal{C}} z_{il} z_{jl} w_{ij}. \quad (1)$$

Of course, $W(l) = 0$ for every non-server location l .

If the server capacity is infinite, all of this load will be fulfilled by the server. However, limited by the server capacity W , if $W(l) > W$, the remaining amount $(W(l) - W)$ of workload must be processed at the datacenter, thus incurring a backhaul cost. Consequently, the total backhaul cost is due to not only the transactions between

cells assigned to different servers, but also those transactions assigned to the same server that exceed its capacity. We represent the backhaul cost as

$$COST = \sum_{i \leq j \in \mathcal{C}} w_{ij} \underbrace{\prod_{l \in \mathcal{L}} (1 - z_{il}z_{jl})}_{i,j \text{ not assigned to same server}} + \sum_{l \in \mathcal{L}} \underbrace{\max\left(0, \sum_{i \leq j \in \mathcal{C}} z_{il}z_{jl}w_{ij} - W\right)}_{\text{overload amount of server at } l}, \quad (2)$$

which can also be expressed as

$$\begin{aligned} COST &= \left(1 - \sum_{s \in \mathcal{S}} \sum_{i \leq j \in \mathcal{C}} w_{ij}x_{is}x_{js}\right) + \sum_{s \in \mathcal{S}} \max\left(0, \sum_{i \leq j \in \mathcal{C}} x_{is}x_{js}w_{ij} - W\right) \\ &= 1 + \sum_{s \in \mathcal{S}} \max\left(-W, -\sum_{i \leq j \in \mathcal{C}} w_{ij}x_{is}x_{js}\right) \\ &= 1 - \sum_{s \in \mathcal{S}} \min\left(W, \sum_{i \leq j \in \mathcal{C}} w_{ij}x_{is}x_{js}\right). \end{aligned} \quad (3)$$

We want to minimize $COST$.

3.2. Geographic Spread

For better latency and easier management, we should keep the geographic region served by an edge server from spreading too far, especially for cells with many transactions. We quantify the geo-spread of a server as the sum of its distance to each assigned base-station, weighted by transaction demand. If this server is placed at location l , its geo-spread is quantified as

$$S(l) = \sum_{i \in \mathcal{C}} z_{il}d_{il}w_i. \quad (4)$$

We want to minimize the total geo-spread for all the servers

$$SPREAD = \sum_{l \in \mathcal{L}} S(l) = \sum_{l \in \mathcal{L}} \sum_{i \in \mathcal{C}} z_{il}d_{il}w_i. \quad (5)$$

3.3. Optimization Problem and NP-Hardness

In summary, our problem is the following two-objective optimization problem.

Problem 3.1 (Min Cost-Spread Assignment (MCSA)).

$$\min \{COST, SPREAD\}$$

s.t.

$$1) \sum_{l \in \mathcal{L}} \left(1 - \prod_{i \in \mathcal{C}} (1 - z_{il}) \right) = N_S. \quad (6)$$

$$2) \sum_{l \in \mathcal{L}} z_{il} = 1 \quad \forall i \in \mathcal{C} \quad (7)$$

$$3) z_{il} \in \{0, 1\} \quad \forall i \in \mathcal{C}, l \in \mathcal{L}. \quad (8)$$

Theorem 3.2. *MCSA is NP-hard.*

Proof. Consider a simple configuration of our problem: $w_{ij} = 0$ for all $(i, j) \in \mathcal{C} \times \mathcal{C}$ except for $i = j$, and $d_{il} = 1$ for all $(i, l) \in \mathcal{C} \times \mathcal{L}$. Then, it is easy to see that

$$COST = \sum_{l \in \mathcal{L}} \max(0, \sum_{i \in \mathcal{C}} z_{il} w_{ii} - W)$$

$$SPREAD = \sum_{l \in \mathcal{L}} \sum_{i \in \mathcal{C}} z_{il} w_{ii} = \sum_{i \in \mathcal{C}} w_{ii} \sum_{l \in \mathcal{L}} z_{il} = \sum_{i \in \mathcal{C}} w_{ii}.$$

Because *SPREAD* is a constant, we can choose any N_S locations to place the servers. Given these server locations, what remains is to minimize *COST*,

$$\min \left\{ COST = \sum_{l \in \mathcal{L}} \max(0, \sum_{i \in \mathcal{C}} z_{il} w_{ii} - W) \right\}$$

This minimization is NP-hard because we show below that an algorithm for it can be used to solve the optimization version of the partition problem, which is known to be NP-hard: partition a given set of positive integers, $\{x_1, x_2, \dots, x_n\}$, into two subsets such that the respective subset sums differ the least. To reduce to our problem, consider $N_S = 2$ servers and $N_C = n$ cells $\{1, 2, \dots, n\}$ with $w_{ii} = x_i$, and let $W = \frac{1}{2} \sum_{i=1}^n w_{ii}$. Suppose that an optimal *COST* solution, (A, B) , assigns a cellset $A \subset [n]$ to server 1 and a cellset $B \subset [n]$ to server 2. Without loss of generality, let $\sum_{i \in A} w_{ii} \leq \sum_{i \in B} w_{ii}$. The corresponding *COST* is

$$COST(A, B) = \max(0, \sum_{i \in A} w_{ii} - W) + \max(0, \sum_{i \in B} w_{ii} - W) = \sum_{i \in B} w_{ii} - W.$$

There are two cases. First, if $\sum_{i \in A} w_{ii} = \sum_{i \in B} w_{ii} = W$, then partition $A \cup B$ is optimal for the partition problem because the subset sums are identical. Second, in the otherwise case, $\sum_{i \in A} w_{ii} < W < \sum_{i \in B} w_{ii}$, partition $A \cup B$ is optimal for the partition problem because no partition $A' \cup B'$ can offer a smaller subset sum difference,

$$\sum_{i \in B'} w_{ii} - \sum_{i \in A'} w_{ii} < \sum_{i \in B} w_{ii} - \sum_{i \in A} w_{ii}.$$

Indeed, suppose by contradiction that this partition (A', B') exists. Without loss of generality, let $\sum_{i \in B'} w_{ii} \geq \sum_{i \in A'} w_{ii}$; hence, we must have $\sum_{i \in B} w_{ii} > \sum_{i \in B'} w_{ii} \geq W \geq \sum_{i \in A'} w_{ii}$. Then, if we assign A' to server 1 and B' to server 2, $COST$ will be

$$\begin{aligned} COST(A', B') &= \max(0, \sum_{i \in A'} w_{ii} - W) + \max(0, \sum_{i \in B'} w_{ii} - W) \\ &= 0 + \sum_{i \in B'} w_{ii} - W < \sum_{i \in B} w_{ii} - W \\ &= COST(A, B). \end{aligned}$$

This is contradictory to the assumption that (A, B) is the optimal $COST$ solution. \square

4. Heuristic Approach

We propose a three-phase algorithm approach: focus on spread optimization first, then refine the solution based on the cost, which as a side effect may worsen the spread, and, finally, improve the solution again, this time for a better spread. As local search is widely used for hard combinatorial optimization problems, we present below the local search methods to optimize each individual objective and how to apply them in the three-phase approach.

4.1. Cost-Only Optimization

Because $COST$ does not involve geography, we need compute only the best cell-server assignment based on the workload demand; any location choice for the servers would work. In a nutshell, our algorithm starts with a random assignment (random cell-server assignment and random server-location assignment), and repeatedly apply a local operation such that the new assignment improves $COST$. A local operation, denoted by `move_cells`(l_0, l_1), runs an algorithm to migrate cells between a pair of servers at locations l_0 and l_1 ; the servers are referred to as the l_0 -server and l_1 -server, respectively. As long as we can find a local operation that improves $COST$,

$$COST_{new} < COST, \quad (9)$$

we make the new assignment permanent and repeat the same process until no such local operation is found.

Let $A_0(A'_0)$ and $A_1(A'_1)$ denote the cellsets of the l_0 -server and l_1 -server before (after) the location, respectively. According to Eq. (2), $COST$ will decrease if the quantity

$$\Delta_{COST}(A_0, A_1) = \sum_{i \in A_0} \sum_{j \in A_1} w_{ij} + \max\left(0, \sum_{i \leq j \in A_0} w_{ij} - W\right) + \max\left(0, \sum_{i \leq j \in A_1} w_{ij} - W\right) \quad (10)$$

decreases as a result of replacing (A_0, A_1) with (A'_0, A'_1) . Consequently, we should design the cell-moving algorithm such that its objective is to minimize Δ_{COST} .

This challenge can be translated into a graph bipartitioning problem. Let H be a weighted graph (self-loop possible) where each cell in C is a vertex and an edge

Algorithm 1: `move_cells(l_0, l_1)`

```
1  $A_0, A_1$ : cellsets of  $l_0$ -server and  $l_1$ -server, respectively;
2 Compute  $W(A_0), W(A_1)$ : edge weight sum (compute load) of  $A_0$  and  $A_1$ ,
   respectively;
3 while true do
4    $k^* = 0, GAIN_{max} = 0, GAIN = 0$ ;
5   Unlock all vertices;
6   Compute gain for every vertex;
7   Save the current partition  $C = A_0 \cup A_1$ ;
8   for  $k = 1, 2, \dots, |C|$  do
9     if  $\exists$  an unlocked vertex  $i_k \in C$  of highest gain satisfying InEq. (11) then
10       $GAIN += gain(i_k)$ ;
11      if  $GAIN > GAIN_{max}$  then
12         $GAIN_{max} = GAIN$ ;
13         $k^* = k$ ;
14      end
15      Move vertex  $i_k$  to the other component;
16      Update gain for every unlocked vertex;
17      Update  $W(A_0)$  and  $W(A_1)$ ;
18      Lock vertex  $i_k$ ;
19    end
20    else break ;
21  end
22  if  $GAIN_{max} > 0$  then
23    Retrieve the original current partition  $C = A_0 \cup A_1$ ;
24    Move vertices  $i_1, i_2, \dots, i_{k^*}$  each from its respective original component to
      the other component;
25    Update gain for every unlocked vertex;
26    Update  $W(A_0)$  and  $W(A_1)$ ;
27  end
28  else break ;
29 end
30 return;
```

connects cell i and cell j if they have transactions; the weight of edge (i, j) is w_{ij} . A feasible solution is a partition of H into two components. The first additive term of Eq. (10) is the cut weight of this partition and the second and third additive terms represent a capacity-constrained quality for the partition. We derive an algorithm to compute the best partition based on the Fiduccia-Mattheyses (FM) heuristic [19]. FM is effective for solving the classic graph min-cut bipartitioning problem whose objective is to minimize the cut weight while balancing the vertex weight. FM is fast (linear time in terms of the number of vertices) and simple (each local operation involves moving only one vertex across the cut). Because our objective is different (minimizing Δ_{COST}), we need to modify FM.

The cell-moving algorithm works as follows (see Algorithm 1). Resembling FM, the algorithm runs in passes and in each pass we compute a sequence of cell migrations each moving a vertex from A_0 to A_1 or from A_1 to A_0 such that $\Delta_{COST}(A_0, A_1)$ after this series is maximally improved. The algorithm stops when no improvement can be

made. To determine which vertex to move, let us define for each vertex a quantity called “gain”, which is the cost reduction if the vertex were moved from its component to the other component. Consider a vertex i and, without loss of generality, suppose that $i \in A_0$. If vertex i were moved to A_1 , its gain in the cut weight is

$$gain_{cut}(i) = \sum_{j \in A_1} w_{ij} - \sum_{j \in A_0, j \neq i} w_{ij}.$$

The edge weight sum of A_0 and that of A_1 would be changed to

$$\begin{aligned} W'(A_0) &= W(A_0) - \sum_{j \in A_0} w_{ij} \\ W'(A_1) &= W(A_1) + w_{ii} + \sum_{j \in A_1} w_{ij} \end{aligned}$$

and so the gain in the capacity-constrained quality is

$$\begin{aligned} gain_{cap}(i) &= \max(0, W(A_0) - W) + \max(0, W(A_1) - W) \\ &\quad - \max(0, W'(A_0) - W) - \max(0, W'(A_1) - W). \end{aligned}$$

The gain of vertex i is

$$gain(i) = gain_{cut}(i) + gain_{cap}(i).$$

Intuitively, a positive (negative) gain would result in a smaller (larger) Δ_{COST} if the vertex switched its component.

At the beginning of each pass, we construct a priority queue of vertices based on their gain. This queue includes only those vertices whose migration would result in

$$\max(W'(A_0), W'(A_1)) \leq \max(W, W(A_0), W(A_1)). \quad (11)$$

In other words, we consider moving a vertex only if the moving improves the load balancing between the two servers or keeps them under the server capacity.

During the current pass, we repeatedly select the vertex of highest gain from the priority queue, move it, and update the queue. After this vertex is moved, it is “locked” so that it cannot be moved again in the current pass. Then, we repeat this process until the queue is empty. We keep track of the gain accumulation after each k^{th} step:

$$GAIN_k = \sum_{t=1}^k gain(i_t),$$

where i_t is the vertex chosen in step $1 \leq t \leq k$. The best move decision would be to move vertices i_1, i_2, \dots, i_{k^*} such that $k^* = \arg \max_{k \leq N_c} GAIN_k$.

If $GAIN_{k^*} > 0$, these moves would result in better Δ_{COST} because the new Δ_{COST} is $\Delta_{COST} - GAIN_{k^*} < \Delta_{COST}$; we make these moves permanent and go on to the next pass which repeats the same procedure. Else, the algorithm makes no change (i.e., keep the same partition as that before the pass starts) and stops.

When a local operation `move_cells(l_0, l_1)` finishes, we will use the final assignment resulted from this operation. The algorithm continues repeatedly with finding the next

`move_cells()` local operation that can further improve *COST* and stops when no such local operation is found.

4.2. Spread-Only Optimization

To minimize *SPREAD* can be reduced to solving a k-median problem [20]. In k-median, given a set of n clients and a set of m facilities, the goal is to choose k facilities to open and assign an open facility to each client such that the total assignment cost is minimum, assuming that the assignment cost to service client i by facility l is γ_{il} (by default, a metric). We can consider each server a facility to open ($k = N_S$, $m = N_L$) and each cell a client ($n = N_C$). The cost to assign client i to facility l (if open) is $\gamma_{il} = w_i d_{il}$ (alternatively, we can think of d_{il} as the assignment cost per unit of service and w_i as the service demand). Then the total service cost of the corresponding k-median problem is

$$\sum_{l \in \mathcal{L}: \text{open}} \sum_{i \in \mathcal{C}} z_{il} \gamma_{il} = \sum_{l \in \mathcal{L}} \sum_{i \in \mathcal{C}} z_{il} d_{il} w_i = \text{SPREAD}.$$

Therefore, any k-median solution z_{il} ($z_{il} = 1$ iff facility l is open and client i is served by this facility) is a solution that minimizes *SPREAD* for our problem.

K-median is NP-hard [20] and the best approximation factor known to date is $2.675 + \epsilon$, achieved by Byrka et al. [21]. Using the local search approach, one can obtain an approximation factor of $(3 + 2/p)$, for example by Arya et al.'s polynomial-time algorithm [22]. This algorithm starts with a feasible assignment and repeatedly perform a p -facility swap until no further cost reduction.

Similarly, our algorithm starts with a random assignment and then repeatedly applies a series of local operations. Let `assign_cells(L)` denote an algorithm that assigns the cells in \mathcal{C} to the servers located at a given subset of locations, $L \subset \mathcal{L}$, such that a cell is always assigned to the nearest server; i.e.,

$$\text{assign_cells}(L) : i \mapsto \arg \min_{l \in L} d_{il}.$$

A local operation, denoted by `swap_locations(l_0, l_1)`, involves a pair of a server location l_0 in the current server location set L and a non-server location $l_1 \notin L$, and does the following:

- Remove location l_0 from the server set
- Add location l_1 as a new server location set
- Run `assign_cells(L')` to obtain a new cell-server assignment where $L' = L - \{l_0\} + \{l_1\}$ is the new server set.

A local operation is chosen to take place permanently if *SPREAD* of the resultant assignment is improved by at least a constant factor $\kappa \in (0, 1)$; i.e.,

$$\text{SPREAD}_{\text{new}} < (1 - \kappa) \times \text{SPREAD}. \tag{12}$$

Subsequently, the algorithm goes on repeatedly with finding another local operation satisfying this inequality until none is found.

4.3. Three-Phase Algorithm

The above algorithms are designed for only one objective, cost or spread. We propose the following three-phase algorithm; a summary is given in Algorithm 2.

In Phase 1 (lines 1-4 of Algorithm 2), we run the spread-only algorithm presented above to obtain an assignment with the (approximately) best spread.

In Phase 2 (lines 5-7 of Algorithm 2), we start with this assignment and adjust the cell-server assignment to improve cost. During the process, the server-location assignment is intact. For the adjustment, we apply the same local search algorithm (same local operation) as in the cost-only algorithm except for one small modification. Specifically, a local operation, `move_cells(l_0, l_1)`, is made permanent not only if the resultant cost is less (Eq. (9)), but also the resulted spread remains below a threshold,

$$SPREAD_{new} \leq (1 + \epsilon) \times SPREAD_0, \quad (13)$$

Because a local operation, while lessening the cost may worsen the spread, the threshold is introduced to keep the spread within a reasonable factor of $SPREAD_0$ that is the spread at the start of Phase 2. Here, $\epsilon \in (0, \infty)$; we can set $\epsilon = \infty$ if the goal is to bring down the cost aggressively.

In Phase 3 (lines 8-10 of Algorithm 2), we start with the assignment of Phase 2 and recompute the server locations for better spread (which has been worsen as tradeoff during Phase 2 compared to that in Phase 1). During the process, the cell-server assignment is intact. Denote the server set by \mathcal{S} and the cellset of each server $s \in \mathcal{S}$ by $cellset(s)$. The unknown to compute is the binary variable y_{sl} , set to 1 iff server s is placed at location l . We have

$$SPREAD = \sum_{l \in \mathcal{L}} \sum_{i \in \mathcal{C}} z_{il} d_{il} w_i = \sum_{s \in \mathcal{S}} \sum_{l \in \mathcal{L}} y_{sl} \underbrace{\sum_{i \in cellset(s)} d_{il} w_i}_{A_{sl}}. \quad (14)$$

Because a server must be assigned to an exclusive location, to minimize $\sum_{s \in \mathcal{S}} \sum_{l \in \mathcal{L}} y_{sl} A_{sl}$ is equivalent to finding a min-cost maximal matching in a complete bipartite graph $(\mathcal{S}, \mathcal{L})$ where an edge connects a vertex $s \in \mathcal{S}$ to a vertex $l \in \mathcal{L}$ with cost A_{sl} (which is known from the intact cell-server assignment). Therefore, we apply the Hungarian Algorithm [23] to compute this matching (y_{sl}), which runs in polynomial time (cubic in the number of vertices).

5. Evaluation

We conducted an evaluation in two scenarios: using a synthetic dataset (Synthetic500) to represent a workload that has no relationship with geography and a real-world dataset (Milano625) to represent a workload in which demand is higher between cells of increasing proximity.

- Synthetic500: The service area is a 2D square area $\mathcal{A} = [0, 1]^2$ where $N_C = 500$ random locations are chosen for the base-stations and their corresponding cells are the Voronoi cells of \mathcal{A} . The workload demand w_{ij} between cell i and cell j is generated uniformly at random: $w_{ij} \sim uniform(0, 1)$.
- Milano625: We constructed this dataset from the collection of geo-referenced Call Detail Records over the city of Milan during Nov 1st, 2013 - Jan 1st, 2014

Algorithm 2: Three-Phase Algorithm

```
/* Phase 1: K-median to minimize SPREAD */
1 Initial assignment: Choose a set  $L$  of  $N_S$  random locations for the servers and
  assign cells to these servers randomly;
2 while  $\exists l_0 \in L$  and  $\exists l_1 \notin L$  such that swap_locations( $l_0, l_1$ ) would result in an
  assignment satisfying InEq. (12) do
3 |   Permanently apply the assignment resulted from swap_locations( $l_0, l_1$ );
4 end
/* Phase 2: FM to improve COST */
5 while  $\exists l_0, l_1 \in L$  such that move_cells( $l_0, l_1$ ) would result in an assignment
  satisfying InEq. (9) and InEq. (13) do
6 |   Permanently apply the assignment resulted from move_cells( $l_0, l_1$ ) ;
7 end
/* Phase 3: Hungarian to improve SPREAD */
8 Let  $\mathcal{S}$  be the set of servers (i.e., those serving cells according to the above
  assignment);
9 Compute matrix  $[A_{sl}]_{\mathcal{S} \times \mathcal{L}}$  as defined in Eq. (14) ;
10 Run Hungarian Algorithm on the cost matrix  $[A_{sl}]_{\mathcal{S} \times \mathcal{L}}$  ;
11 return;
```

(<https://dandelion.eu/>). Specifically, we partition the area into a grid of $25 \times 25 = 625$ cells of size $0.94km \times 0.94km$, and count the calls between these cells made during the Monday of Nov 4th, 2013.

In both studies, the transaction demand values are normalized such that they sum to 1. The number of MEC servers is set to $N_S = 10$ whose location is chosen from $N_{\mathcal{L}} = 50$ random locations. These quantities are reasonable given the number of cells. The server capacity is set to $W \in \{0.03, 0.04, \dots, 0.08\}$, meaning $\{3\%, 4\%, \dots, 8\%\}$ of the total workload. Figure 5 shows the heat map of the workload demand for the Milano625 dataset.

For convenience, we refer to our three-phase algorithm as KMED/FM/HUNG, as it applies k-median, Fiduccia-Mattheyses (FM), and Hungarian algorithms in the three phases, respectively. Serving as benchmark for comparison are: RAND (the random assignment algorithm), KMED (the spread-only algorithm using k-median), and FM/HUNG (the cost-only algorithm using FM with another step using Hungarian to improve spread). Note that the only difference between KMED/FM/HUNG and FM/HUNG is that the former starts with a KMED assignment while the latter starts with a random assignment. The metrics for comparison are cost (*COST*) and spread (*SPREAD*). RAND offers a good upper-bound for both cost and spread, while KMED represents a good lower bound (supposedly best) for spread and FM/HUNG a good lower-bound (supposedly best) for cost.

The κ parameter in Eq. (12) is set to 0.0001 for k-median and ϵ in Eq. (13) is set to ∞ (no spread constraint in the second phase). The simulation runs on 10 random sets of candidate server locations and, for each set, 5 random choices for the initial assignment. The results are averaged over these 50 runs and plotted with 100% confidence interval.

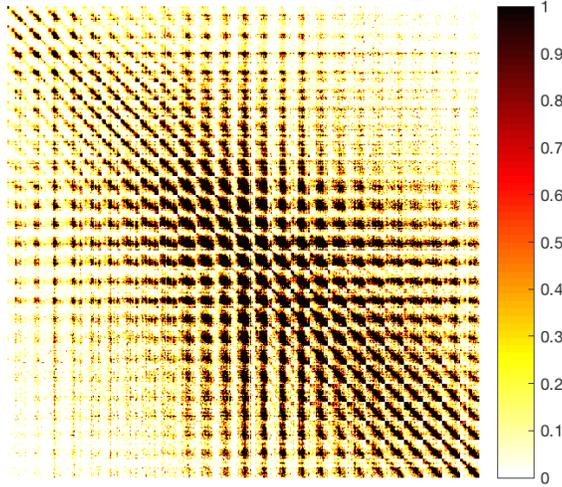


Figure 1. Heat map of workload w_{ij} for Milano625, where i is the x-axis, j the y-axis, and $(0, 0)$ at top-left.

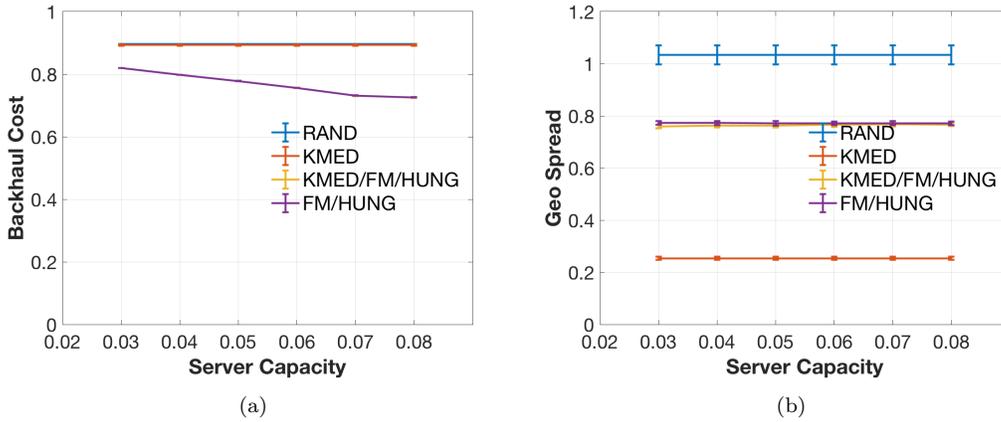


Figure 2. Synthetic500: synthetic workload, no relationship with geography

5.1. Workload Without Geography Correlation

Figure 2 shows the results for the synthetic case in which geography is no factor in workload demand. In terms of cost (Figure 2(a)), KMED is almost identical to RAND, both incurring a high backhaul cost of almost 0.9 (i.e., 90% of the total workload), even when the server capacity increases. This is not surprising because KMED is cost-blind and so when workload has no relationship with geography, minimizing spread results in a cost as bad as that of a random assignment. Perhaps for the same reason, the other two methods, KMED/FM/HUNG and FM/HUNG, also incur almost the same cost. In other words, whether we start with a RAND assignment or a KMED assignment, an application of FM+HUNG would result in similar costs. It is important to note that the FM step is effective, especially as the server capacity increases. With a server capacity of 0.08, applying FM reduces the backhaul cost to 0.73, a 20% improvement from the initial assignment.

In terms of spread (Figure 2(b)), KMED is the best (expected) and RAND the worst (understandable because it is spread-blind). Between the other two, more in-

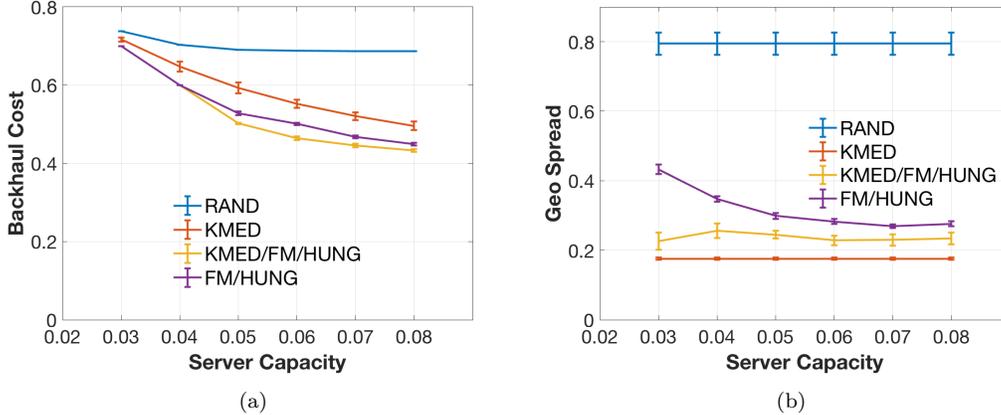


Figure 3. Milano625: real-world workload, strongly correlated to geography.

terestingly, FM/HUNG has a similar spread (only slightly larger) compared to that of KMED/FM/HUNG. This study suggests that, for a workload input that has no relationship with geography, (1) we can do better than a random assignment, (2) the 3-phase algorithm can run without Phase 1 (KMED) which has almost zero benefit, and (3) there is no clear winner between FM/HUNG and KMED; which one should be chosen depends on whether we prefer minimizing cost or spread.

5.2. Workload With Geography Correlation

Figure 3 shows the results for the real-world dataset (Milano625), in which the workload has a strong correlation with geography; specifically, higher between cells of shorter distance [9]. Similar to the above study, and, expectedly, RAND is worse than all the other algorithms in both objectives and KMED has the best spread. There are, however, key differences.

First, KMED has a substantially lower cost than RAND’s; this implies that, due to the correlation between workload and geography, by minimizing spread there is, to some extent, a benefit in reducing the cost. Indeed, because KMED tends to cluster cells near each other and workload demand is high between close cells, heavy workloads tend to be served by the edge, hence less workload going backhaul (compared to a random assignment). Second, KMED/FM/HUNG is clearly better than FM/HUNG in both objectives; this substantiates the effectiveness of having Phase 1 (KMED) in our algorithm, leading to not only better cost but also better spread. Third, KMED/FM/HUNG has a spread only slightly worse than KMED; this shows the effectiveness of Phase 3 (HUNG) in improving spread. In short, all the three phases in the proposed algorithm (KMED/FM/HUNG) are important to achieving both objectives.

5.3. Other Observations

Figure 4 gives a visual representation of the assignment map according to KMED and KMED/FM/HUNG. Both methods are consistent with the physical map of Milan (Figure 4(f)); that is, because most transactions involve the inner neighborhoods, a server closer to the the central area covers fewer cells (which have high activity)

than those in the outskirts (which have low activity). While KMED places the servers spatially nicely (Figure 4(a), Figure 4(b)), KMED/FM/HUNG allows for some discontinuity in the cells that belong to the same server (Figure 4(c), Figure 4(d)); the latter does so to reduce the backhaul cost. For example, when the capacity $W = 0.05$, $(COST, SPREAD)$ is $(0.43, 0.21)$ for KMED/FM/HUNG and $(0.49, 0.18)$ for KMED. This is a tradeoff between cost versus spread. Although we cannot avoid this tradeoff, it is important to point out that KMED/FM/HUNG offers a better workload balance, as clearly illustrated in Figure 4(e). The ratio of the maximum to the minimum workload demand is at least two times less with KMED/FM/HUNG than with KMED.

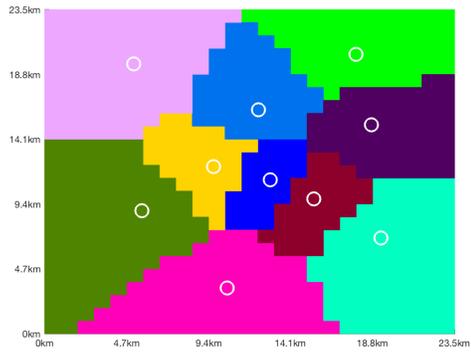
6. Conclusions

We have addressed a new server assignment problem for MEC, whose decision making needs to be made for where geographically to place the servers and how to assign them to the user cells based on transactional workloads. The formulation of the two objectives with respect to the backhaul cost and geographic spread has not appeared in the MEC literature. We have proposed and evaluated a heuristic solution leveraging k-median, Fiduccia-Mattheyses, and Hungarian methods. The solution is not optimal (due to the NP-hardness of the problem), but an effective approximation. For the future work, our next step is to consider the case where the workload demand is not static. In practice, the workload demand varies over the time, but usually follows a pattern. Knowing this pattern, for example, in the form of a probability distribution, an interesting goal is to compute an assignment offering the best expected optimization.

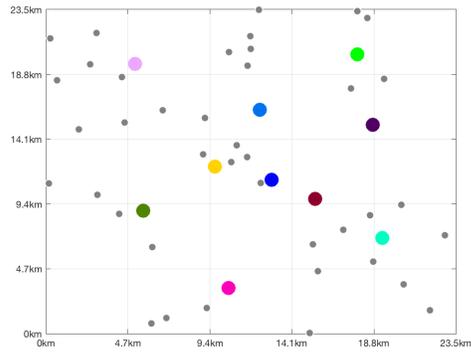
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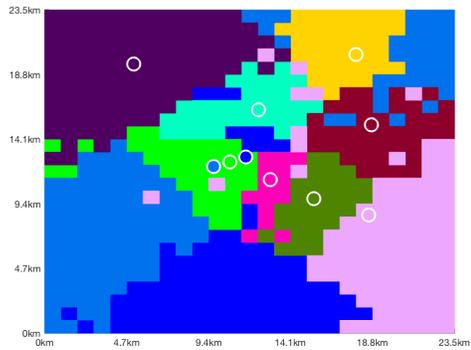
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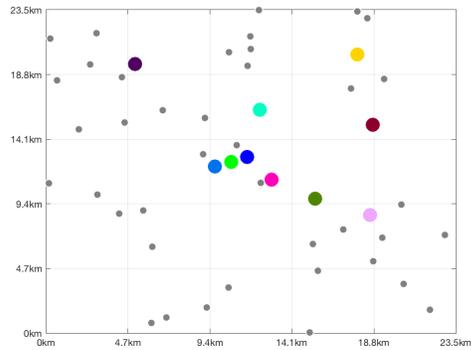
(a) KMED



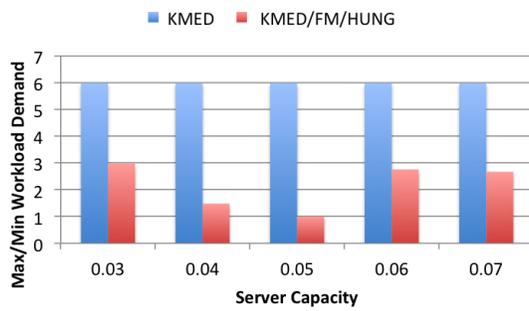
(b) KMED



(c) KMED/FM/HUNG



(d) KMED/FM/HUNG



(e) KMED/FM/HUNG



(f) Milano area

Figure 4. Assignment map of (a,b) KMED and (c,d) KMED/FM/HUNG for a single random run on Milano625 with capacity $W = 5\%$, where each colored-circle represents a server (out of 50 possible locations) and each server and its cells share the same color. Also shown is (e) the ratio of maximum to minimum workload and (f) the physical map of Milan.