Supplementary Material

A generalized SEIRD model with implicit quarantine mechanism: a Bayesian approach for the identification of the spread of COVID-19 with applications in Brazilian locations

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SM A. Supplementary Material Information

This Supplementary Material (SM) is organized as follows. Sections SM A.1 and SM A.2 expand the Results section of the main document with additional figures for the BR and RJ scenarios, respectively. The last part of the document shows additional model setting information and the calibration data used in this work.

SM A.1. BR: Additional Results

Fig. A.2a shows the prediction of P, I, A, D, and C in BR. A total of 638.8 thousand (95% CI: 630.6–647.1) active cases are expected on the simulation day 145 (95% CI: 143–146), with D and C expected to be around 149.3 thousand (95% CI: 145.1–153.6) and 4.392 million (95% CI: 4.306–4.482), respectively. The posterior distribution of the peak position is displayed in Fig. A.2b, with the vertical dashed lines corresponding to those displayed in Fig. A.2a. Fig. A.2c depicts the time evolution of $\mathcal{R}(t)$ and displays two vertical lines identifying the credible interval (95% CI: 133–136) of the time above which $\mathcal{R}(t) \leq 1$. The same lines are depicted in Fig. A.2d that shows the uncertainty associated with that time value.
Fig. A.3 provides the model forecasts for the considered hypothetical scenarios of social distancing relaxation. Fig. A.3a shows what to expect in case of a sudden release from social distancing after the simulation day 165 ($t_{1/2} = 0.1$ days as half-life decay). In this case, C reaches 5.656 million (95% CI: 5.426–5.930) and D 192.2 thousand (95% CI: 183.4–202.4) at the end of the simulation (day 397), which corresponds to an increase of approximately 29% in both values when compared to the original scenario with $\omega(t) = \omega^*$ fixed at its MAP estimate. For a gradual release after the simulation day 165 ($t_{1/2} = 15$ days as half-life decay), the numbers of C and D at the end of simulation are 4.790 million (95% CI: 4.660–4.941) and 162.8 thousand (95% CI: 157.3–169.0), respectively, as shown in Fig. A.3b. Figs. A.3c and A.3d compare the variability of C and D at the end of simulation in the form of box plots for all scenarios considered. Outliers appear as individual points and the samples medians are depicted in red.

SM A.2. RJ: Additional Results

Fig. A.5a shows the prediction of P, I, A, D, and C in RJ. A total of 32.2 thousand (95% CI: 31.7–32.7) active cases are expected on the simulation day 115 (95% CI: 114–116), with D and C expected to be around 16.6 thousand (95% CI: 16.3–17.0) and 212.5 thousand (95% CI: 207.6–217.7), respectively. The posterior distribution of the peak position is displayed in Fig. A.5b, with the vertical dashed lines corresponding to those displayed in Fig. A.5a. Fig. A.5c depicts the time evolution of $\mathcal{R}(t)$ and displays two vertical lines identifying the credible interval (95% CI: 103–107) of the time above which $\mathcal{R}(t) \leq 1$. The same lines are depicted in Fig. A.5d that shows the uncertainty associated with that time value.
Fig. A.6 provides the model forecasts for the considered hypothetical scenarios of social distancing relaxation. Fig. A.6a shows what to expect in case of a sudden release from social distancing after the simulation day 165 ($t_{1/2} = 0.1$ days as half-life decay). In this case, C reaches 220.5 thousand (95% CI: 214.4–227.7) and D 17.3 thousand (95% CI: 16.9–17.8) at the end of the simulation (day 392), which corresponds to an increase of approximately 4% in both values when compared to the original scenario with $\omega(t) = \omega^*$ fixed at its MAP estimate. For a gradual release after the simulation day 165 ($t_{1/2} = 15$ days as half-life decay), the numbers of C and D at the end of simulation are 215.2 thousand (95% CI: 210.0–221.1) and 16.8 thousand (95% CI: 16.5–17.2), respectively, as shown in Fig. A.6b. Figs. A.6c and A.6d compare the variability of C and D at the end of simulation in the form of box plots for all scenarios considered. Outliers appear as individual points and the samples medians are depicted in red.

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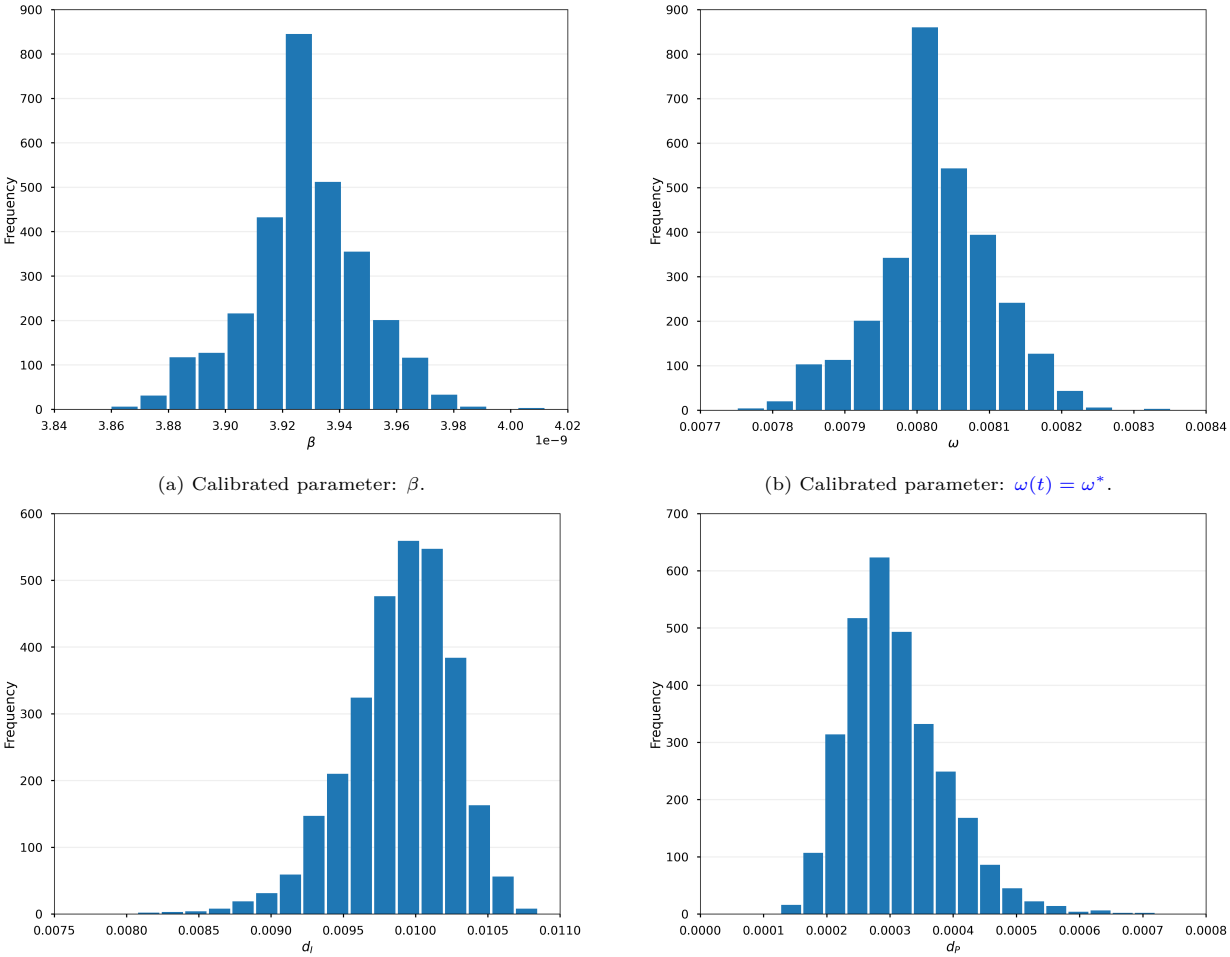
SM A.3. Compartments Predictive Distributions

Considering the posterior distribution $\pi_{\text{post}}(\boldsymbol{\theta}|\boldsymbol{y})$, it is possible to obtain probability distribution for compartments such as $D(t)$. This is obtained by computing:

$$\pi(D(t)|\boldsymbol{y}) = \int_{\Theta} \pi(D(t)|\boldsymbol{\theta})\pi_{\text{post}}(\boldsymbol{\theta}|\boldsymbol{y})d\boldsymbol{\theta}. \tag{A.1}$$

This probability distribution incorporates uncertainties in both model and data, as it marginalizes the posterior parameter distribution $\pi_{\text{post}}(\boldsymbol{\theta}|\boldsymbol{y})$ for every prediction obtained with a parameter $\boldsymbol{\theta}$.
Figures A.2a, A.3a, A.3b, A.5a, A.6a, and A.6b present the compartments predictions MAP estimate curves, as well as their 95% credible interval in the shaded regions.



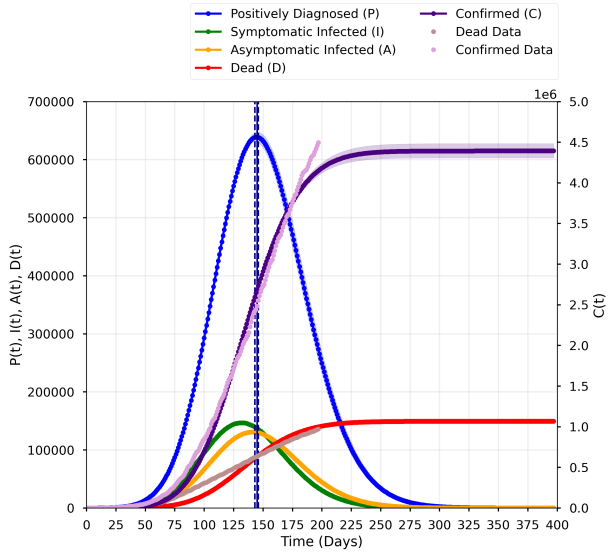
(a) Calibrated parameter: β.

(b) Calibrated parameter: $\omega(t) = \omega^*$.

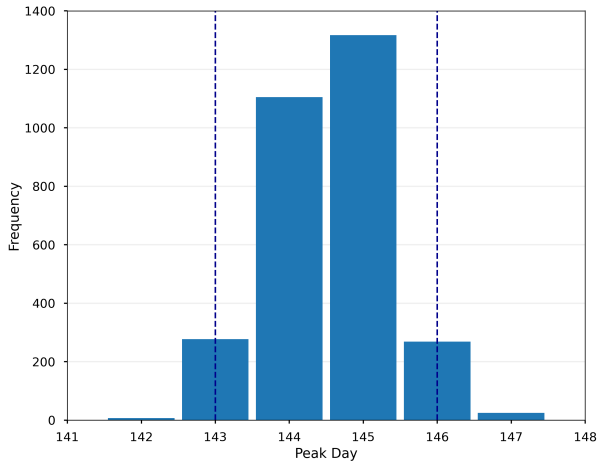
(c) Calibrated parameter: d_I.

(d) Calibrated parameter: d_P.

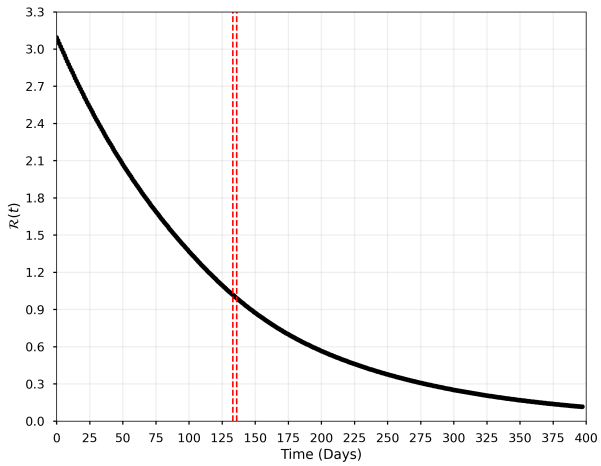
Figure A.1: Frequency histograms for the calibrated parameters (BR).



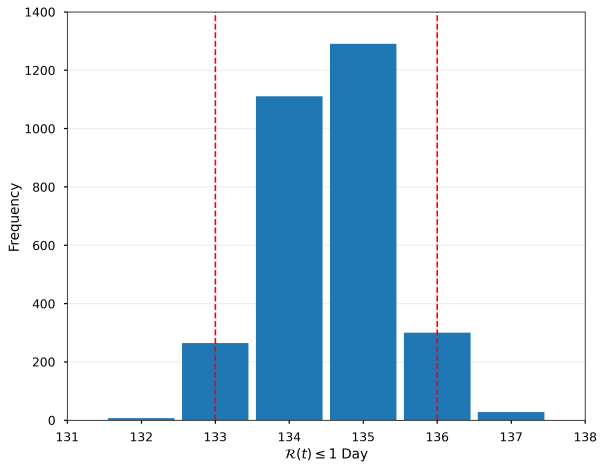
(a) Prediction of P, I, A, D, and C (right y-axis) in BR. The vertical line indicates the curve peak, and the dashed lines display the corresponding 95% CI.



(b) Frequency histogram for the peak day of P.

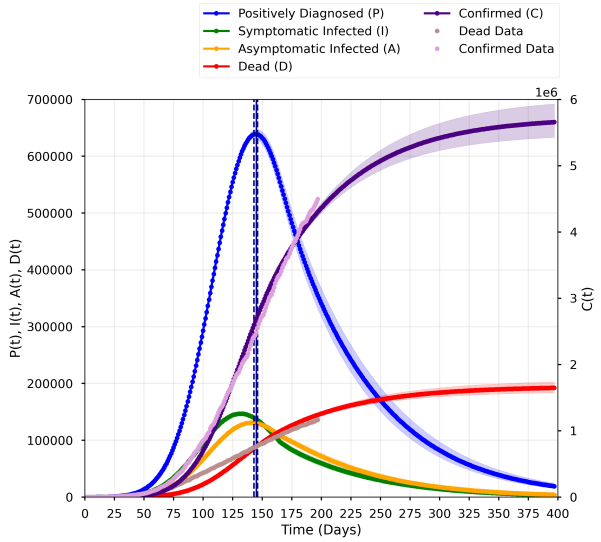


(c) $\mathcal{R}(t)$ of the COVID-19 in BR.

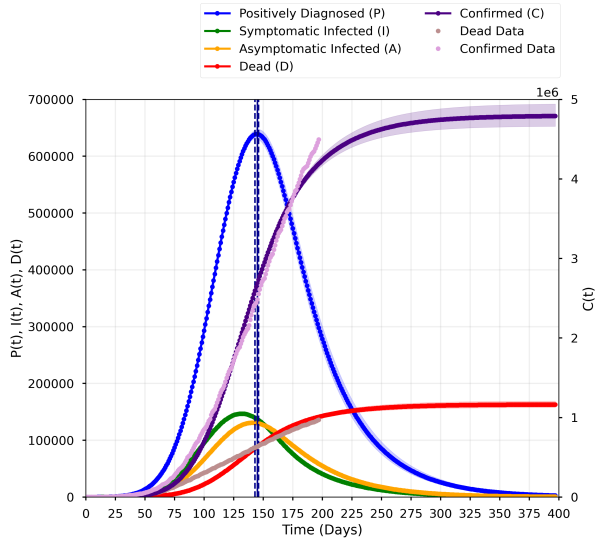


(d) Frequency histogram for the day at which $\mathcal{R}(t) \leq 1$.

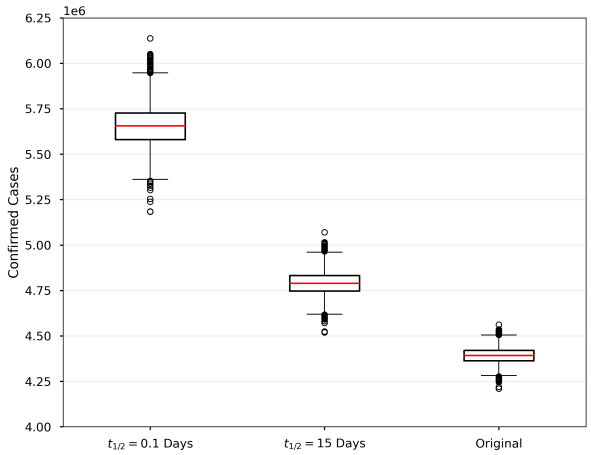
Figure A.2: Dynamics of the COVID-19 in BR modeled with the available data (Mathematical Modeling and Methods).



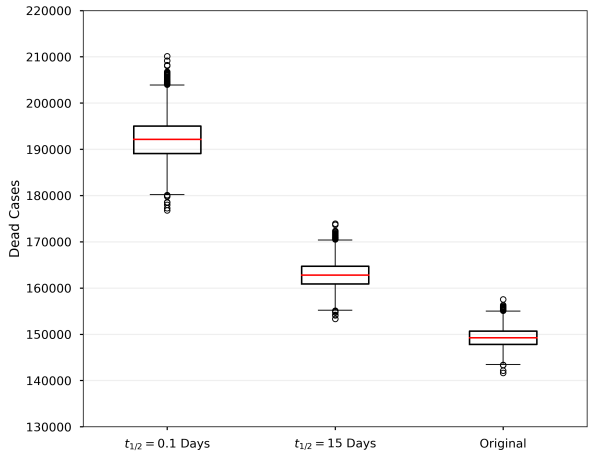
(a) Prediction of P, I, A, D, and C (right y-axis) in BR considering a sudden release from social distancing ($t_d = 165$ days and $t_{1/2} = 0.1$ days). The vertical line indicates the curve peak, and the dashed lines display the corresponding 95% CI.



(b) Prediction of P, I, A, D, and C (right y-axis) in BR considering a gradual release from social distancing ($t_d = 165$ days and $t_{1/2} = 15$ days). The vertical line indicates the curve peak, and the dashed lines display the corresponding 95% CI.

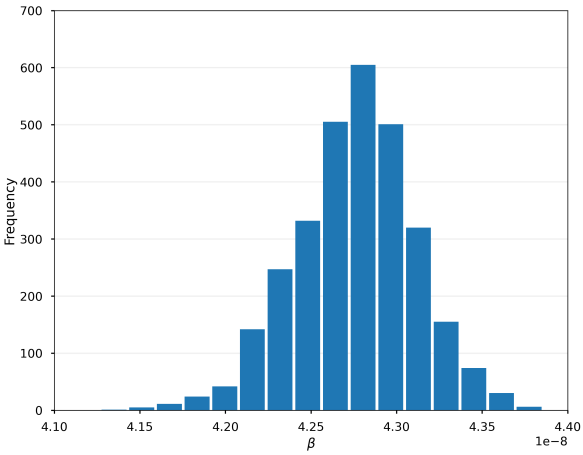


(c) Box plot for C in BR at the end of the simulation.

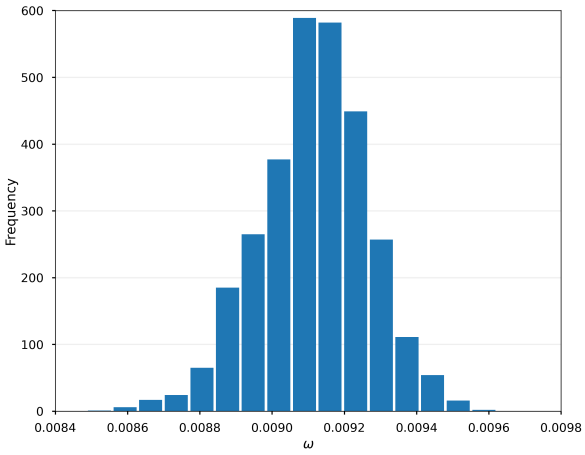


(d) Box plot for D in BR at the end of the simulation.

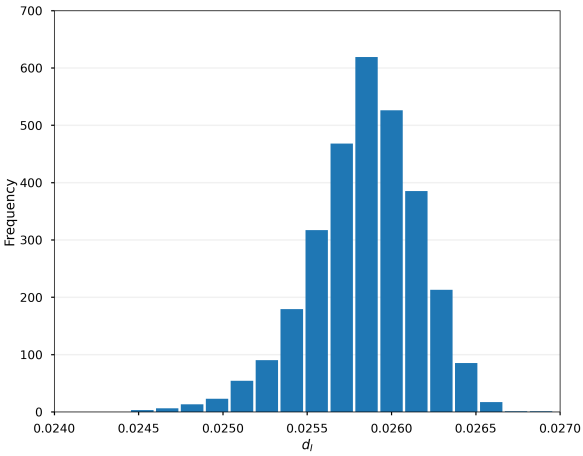
Figure A.3: Model forecasts in BR considering sudden and gradual releases from social distancing ($t_d = 165$ days and $t_{1/2} = 0.1$ and 15 days). The original (baseline) scenario of the main text is also presented to ease comparison.



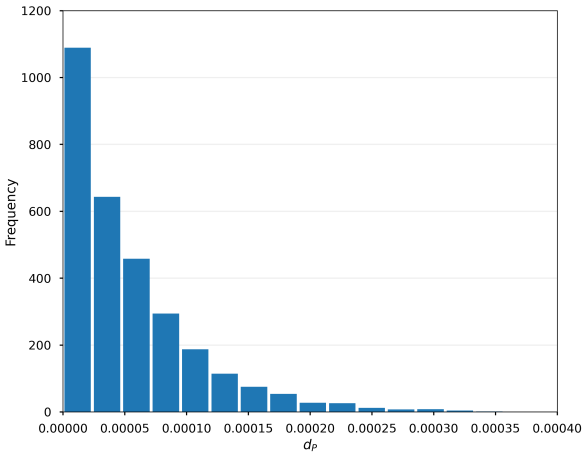
(a) Calibrated parameter: β.



(b) Calibrated parameter: $\omega(t) = \omega^*$.

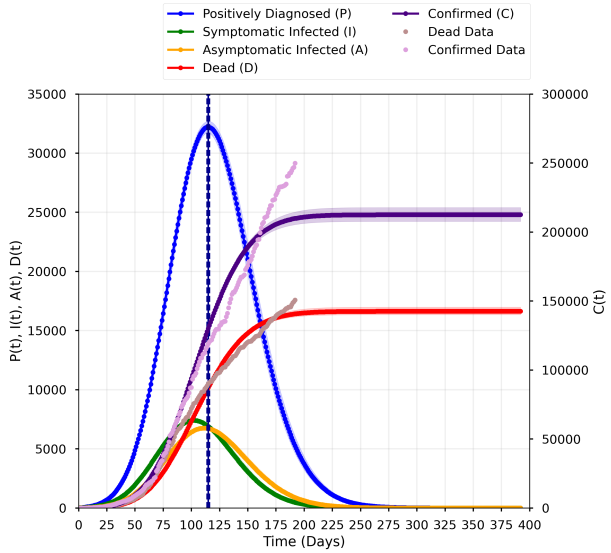


(c) Calibrated parameter: d_I.

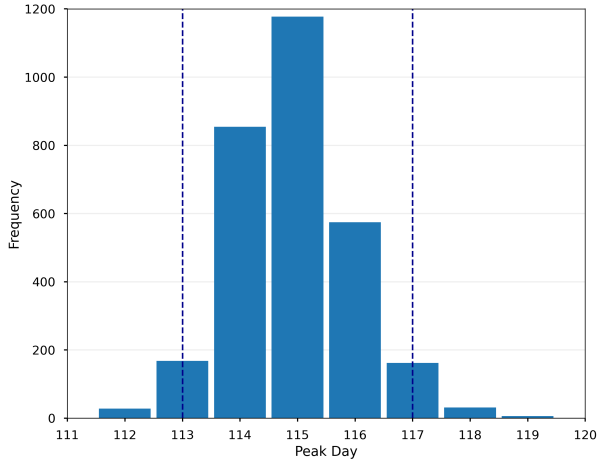


(d) Calibrated parameter: d_P.

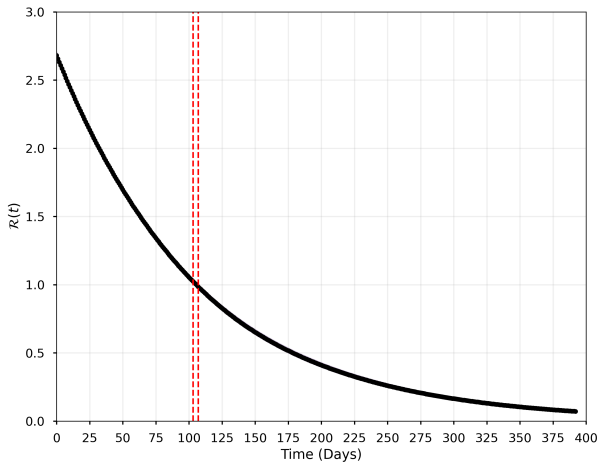
Figure A.4: Frequency histograms for the calibrated parameters (RJ).



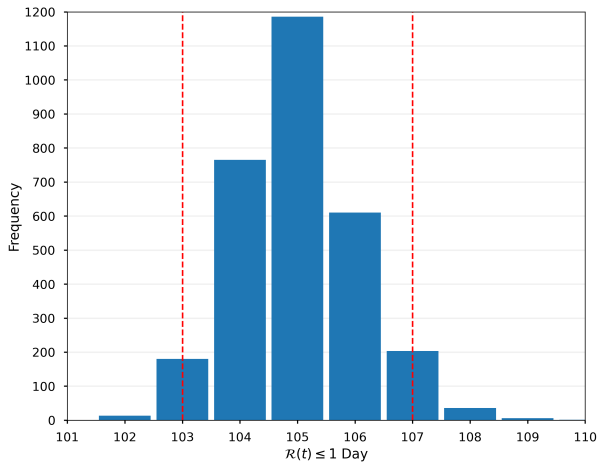
(a) Prediction of P, I, A, D, and C (right y-axis) in RJ. The vertical line indicates the curve peak, and the dashed lines display the corresponding 95% CI.



(b) Frequency histogram for the peak day of P.

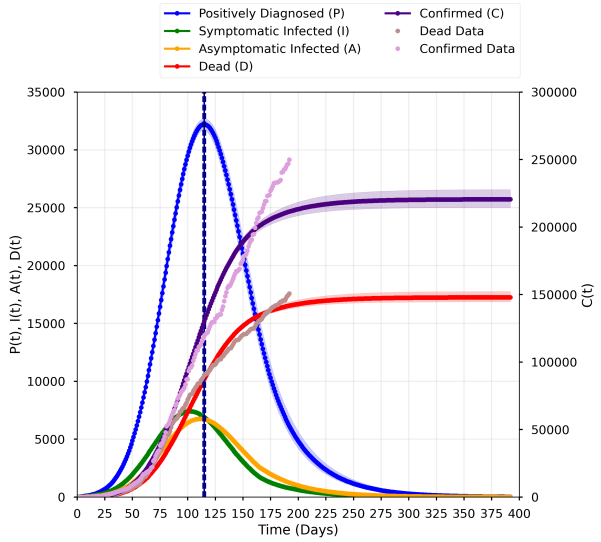


(c) $\mathcal{R}(t)$ of the COVID-19 in RJ.

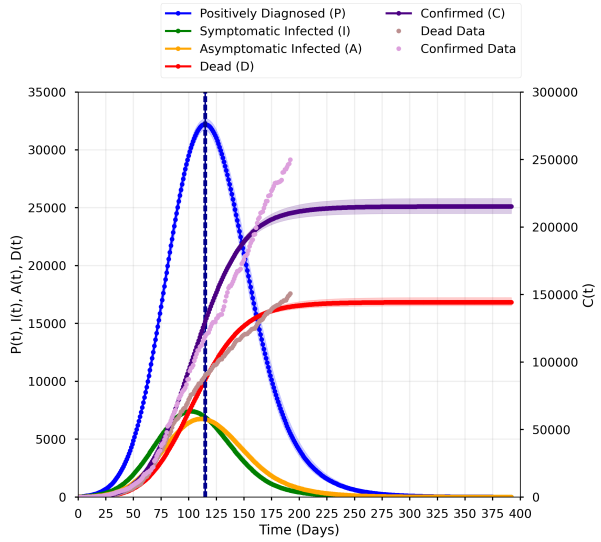


(d) Frequency histogram for the day at which $\mathcal{R}(t) \leq 1$.

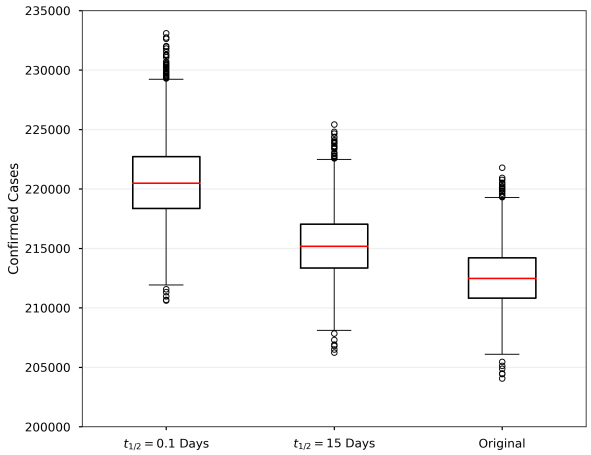
Figure A.5: Dynamics of the COVID-19 in RJ modeled with the available data (Mathematical Modeling and Methods).



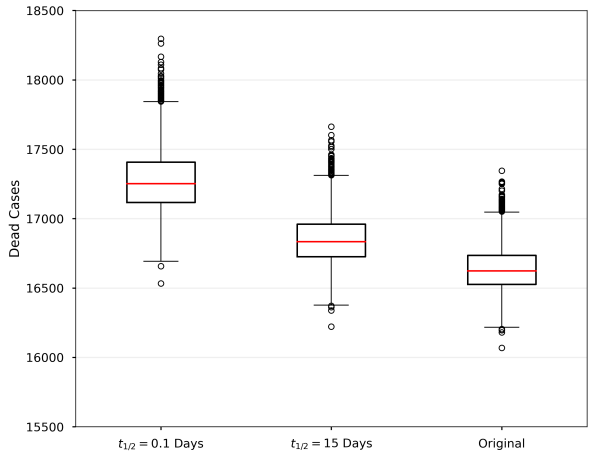
(a) Prediction of P, I, A, D, and C (right y-axis) in RJ considering a sudden release from social distancing ($t_d = 165$ days and $t_{1/2} = 0.1$ days). The vertical line indicates the curve peak, and the dashed lines display the corresponding 95% CI.



(b) Prediction of P, I, A, D, and C (right y-axis) in RJ considering a gradual release from social distancing ($t_d = 165$ days and $t_{1/2} = 15$ days). The vertical line indicates the curve peak, and the dashed lines display the corresponding 95% CI.



(c) Box plot for C in RJ at the end of simulation.



(d) Box plot for D in RJ at the end of simulation.

Figure A.6: Model forecasts in RJ considering sudden and gradual releases from social distancing ($t_d = 165$ days and $t_{1/2} = 0.1$ and 15 days). The original (baseline) scenario of the main text is also presented to ease comparison.

Table A.1: Model parameters.

Parameter	Meaning
β	Rate of transmission per contact with symptomatic infected individual
μ	Rate of transmission per contact with asymptomatic infected individual
σ	Transition rate from exposed to infected individuals
ρ	Proportion of infected individuals who have symptoms
ε_I	Diagnosis rate of hospitalized individuals
γ_A	Recovery rate of asymptomatic infected individuals
γ_I	Recovery rate of symptomatic infected individuals
γ_P	Recovery rate of hospitalized individuals
d_I	Death rate of hospitalized individuals
d_P	Death rate of symptomatic infected individuals
ω	The rate at which susceptible, exposed, and infected individuals are removed due to social distancing measures

Table A.2: Fixed values for estimated parameters and ICs.

Parameter	Average Value	Hypothesis
ρ	0.85	The estimated asymptomatic proportion was 15% [2]
σ	1/5 day^{-1}	Estimated incubation (or latent) period based on [1]
ε_I	1/3 day^{-1}	We adopt that only severely ill individuals are diagnosed in BR, which takes about 3 days
γ_A	1/14 day^{-1}	It takes around 14 days for recovering from COVID-19 [3]
γ_I	1/14 day^{-1}	
γ_P	1/14 day^{-1}	
Class	**Initial Population (Ind.)**	**Hypothesis/Source**
N(0)	210147125 (BR), 17264943 (RJ)	IBGE 2019
P(0)	7 (BR), 8 (RJ)	Data at day 2020-03-05 (BR), 2020-03-10 (RJ)
E(0)	$10 \cdot$ P(0)	The number of exposed individuals is about 10 times the positively confirmed ones
I(0)	$5 \cdot$ P(0)	The number of symptomatic infected individuals is about 5 times the positively confirmed ones
A(0)	P(0)	The number of asymptomatic infected individuals is about the same of the positively confirmed ones
D(0)	0	Data at day 2020-03-05 (BR), 2020-03-10 (RJ)
R(0)	0	Data at day 2020-03-05 (BR), 2020-03-10 (RJ)
S(0)	210147006 (BR), 17264837 (RJ)	S(0) = N(0) - {E(0) + A(0) + I(0) + P(0) + R(0) + D(0)}

References

[1] Lauer, S. A., Grantz, K. H., Bi, Q., Jones, F. K., Zheng, Q., Meredith, H. R., Azman, A. S., Reich, N. G., and Lessler, J. (2020). The incubation period of coronavirus disease 2019 (COVID-19) from publicly reported confirmed cases: estimation and application. *Annals of Internal Medicine*, 172(9):577–582.

[2] Mizumoto, K., Kagaya, K., Zarebski, A., and Chowell, G. (2020). Estimating the asymptomatic proportion of coronavirus disease 2019 (COVID-19) cases on board the Diamond Princess cruise ship, Yokohama, Japan, 2020. *Eurosurveillance*, 25(10):2000180.

[3] Pan, F., Ye, T., Sun, P., Gui, S., Liang, B., Li, L., Zheng, D., Wang, J., Hesketh, R. L., Yang, L., and Zheng, C. (2020). Time course of lung changes on chest ct during recovery from 2019 novel coronavirus (COVID-19) pneumonia. *Radiology*, page 200370.