

# Health condition estimation of spacecraft key components using belief rule base

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## KEYWORDS

Spacecraft; health condition estimation; belief rule base; Markov Chain Monte Carlo

## ABSTRACT

This paper proposes a method for estimating the health status of spacecraft key components based on the belief rule base (BRB), a semi-quantitative method which uses both human judgmental information and numerical data. It not only allows experts to establish rules to provide useful conclusions, but also allows historical data to train its parameters to obtain more accurate outputs. To balance the parameter training and experts' knowledge, the Markov Chain Monte Carlo (MCMC) technique instead of traditional optimization method is used to adjust the BRB parameter. A practical case of estimating the health condition of space application batteries is studied.

## 1. Introduction

Nowadays, spacecraft becomes more and more complex, and it consists of various mechanical and electronic sub-systems (Rui et al. (2014)). Due to the influence of space environment and the degradation of components themselves (Liu et al. (2015)), the health condition of spacecraft components will gradually deteriorate over time. If the health condition of key components is not monitored, the spacecraft may fail unexpectedly, which will cause the spacecraft fail to complete its operational tasks even bring disastrous consequences (Wang and Xie (2010)). Therefore, the health condition estimation of key components has become increasingly important to spacecraft. With the results of health condition estimation, the maintenance strategy of spacecraft system is supported, which is of great significance to prevent major failures of spacecraft systems. (Mohammad and Hussein (2014); Johnson (2005)). The concept of Integrated Vehicle Health Management (IVHM) is proposed by National Aeronautics and Space Administration (NASA) to meet the requirement of reusable spacecraft, which contains health condition monitoring, estimation, fault diagnosis and fault treatment for key subsystems of spacecraft (Figuroa and Melcher (2013)). The health condition estimation method proposed by this paper is used on a CubeSat project, called 'Kaituo-1B', developed by a research team of Hong Kong Polytechnic University. Compared to the weight of traditional satellites ranging from

a few hundred kilograms to several thousand kilograms, the 'Kaituo-1B' weighs only about 2 kg, which greatly reduces the cost of developing and producing micro-satellites for sending small payloads and instruments into space (POLYU (2018)). It is of great significance to monitor and estimate the health condition of 'Kaituo-1B' during its 3-month life cycle in real time (Xhafa and Andrew (2019)).

The parameters that can reflect the health condition of spacecraft system, its subsystems or components are called health index. The health index can be either an actual physical quantity such as capacity for a lithium battery, or an evaluation indicator constructed by experts such as a comprehensive score. No matter what kind of health index it is, it can not be obtained directly from the telemetry data when satellites are in-orbit (Gentz et al. (2005)). Therefore, the relationship needs to be constructed between health index and observed data to estimate the health condition of spacecraft key components. The solutions mainly include model-based methods, knowledge-based methods and data-driven methods. Model-based methods estimate the health condition by establishing the physical or mathematical model which describing the relationship between the health status and observations. For example, Chen et al. (2004) and Zeng and Skibniewski (2013) designed the fault model to monitor the health of satellite. The advantage of this method is that the result of evaluation is credible, however, for a complex system, it is very difficult to build an accurate physical or mathematical model. The knowledge-based approach estimates the health condition of the system by expert judgement which is based on their experience knowledge. The classical methods which use the experts' judgemental information include Analytic Hierarchy Process (AHP) (Wang, Kai, and Wang (2010); Sasmal and Ramanjaneyulu (2008)), Fuzzy Evaluation (Melhem and Aturaliya (2010)) and ontologies-based knowledge (Du et al. (2016); Bao et al. (2018)). These methods are simple, intuitive and easy to understand (Li et al. (2017)). However, human judgement information may be inaccurate because of subjectivity and preference. Data-driven method regard health estimation system as a black box, which takes observable data as input and health indicators as output, and historical data are used to train the black box. Artificial neural network (ANN) (Gao et al. (2006); Abu-Elanien, Salama, and Ibrahim (2011)) and support vector machine (SVM) (Mohammadi and Gharehpetian (2009)) are the most popular data-driven methods. They are characterised by strong applicability and do not require any understanding and knowledge of the estimated system. Only if the training data set is large enough, a relative high accuracy can be achieved. However, for spacecraft systems, they are usually very expensive, so performing a large number of tests, especially destructive tests, to get sufficiently complete data under different health conditions is not feasible. In addition, we should not ignore the fact that human beings, in most cases, bear the ultimate decision-maker in health estimation, but the incomprehensibility of the data-driven method does not allow this.

In order to solve the problems of the above methods, a health condition estimation method for spacecraft key components based on belief rule base Belief Rule Base (BRB) is proposed. BRB is proposed by Yang et al. (2006), which extends from the traditional rule-based system and uses the evidential reasoning approach as its inference methodology. BRB is a semi-quantitative methods which uses both human judgemental

information and numerical data, that is, it not only allows experts to establish rules to provide useful conclusions, but also allows the use of observational data. It has been applied in many fields, such as risk analysis (Kongabcddd (2012); Qiu et al. (2018)), safety assessment (Li et al. (2017)), fault diagnosis (Feng et al. (2017)), health estimation of engineering system (Yin et al. (2018)), failure prognosis (Zhou et al. (2010); Yin et al. (2017); Chang et al. (2015)), system behaviour prediction (Zhou et al. (2013); Hu et al. (2010)), network security prediction (Hu et al. (2016); Hu and Qiao (2016); Zhang et al. (2017)), medicine and medical assessment (Hossain et al. (2017b,a)).

Because of the preference of experts, there must be strong subjectivity in experts' judgement(Liu et al. (2012)), which may lead to the inaccurate parameters of BRB (Yang et al. (2007)). If there are history data, the parameters of the belief rules can be trained and adjusted to decrease the subjectivity of the experts' judgement. Therefore, BRB combines the advantage of knowledge-based method and data-driven method, thus it can not only overcome the inaccuracy of expert knowledge, but also overcome the overfitting problem of data-driven method. However, the traditional training methods for BRB are based on optimisation method (Zhou et al. (2011); Bishop and Nasrabadi (2006); Bialek, Nemenman, and Tishby (2001)), they seek overly the best fit to the historical data, which may result in the dominant role of training rather than the role of experts. Under such situation, the output is even worse than the BRB determined completely by the experts, then such training fails. Therefore, it is very important to balance the BRB parameter training and the experts' knowledge. To solve the problem, instead of using a traditional optimisation method, the Markov Chain Monte Carlo (MCMC) technique is adopted to train the parameters of BRB in this paper. MCMC techniques are often applied to solve integration and optimisation problems in large dimensional spaces, including machine learning, physics, statistics, econometrics and decision analysis (Andrieu et al. (2003)). Application of MCMC in training field is different from that of traditional optimisation method, it estimates the posterior distribution of parameters in stead of finding single optimised values. This allows the output to be estimated by taking into account all possible parameters (Zhu et al. (2017); Spragins (1965); Ho and Lee (1964)).

The remainder of this paper is organised as follows. In section 2, the main problems of the health condition estimation of spacecraft needed to be solved are summarised. Section 3 gives a brief introduction of BRB. In Section 4, we describe the BRB parameter adjustment by using MCMC technique. Section 5 presents a case study to explain how the proposed method is used for health estimation of spacecraft and verify its effectiveness. A conclusion is provided in Section 6.

## 2. Problem description for the health condition estimation of spacecraft system

### *2.1. Representation format for input information and output consequent*

The purpose of health estimation is to estimate the health status of the system by observing some signal parameters. Therefore, observations and health status can be considered as input information and outputs consequent of health estimation systems, respectively. Here, we use  $x$  denotes the vector of observations, and

use  $y_{\text{HP}}$  denotes the health status. For a complex spacecraft system, observations as the input information for health estimation can be expressed in two ways:

- (a) Quantitative format. They are often continuous, numerical values obtained through measurements, for example, 'Voltage is 28 V'.
- (b) Qualitative format. They are often linguistic variables from human judgement or domain knowledge, for example, 'the temperature is high'.

There is a similar situation for the expression of health status. The health status can be a continuous score, and also can be expressed as 'good' or 'poor'.

The task of health estimation is essentially to build a mapping between input information (observations) and outputs consequents (health status). From data-driven perspective, a black-box model is built and trained by history data, and achieves the mapping. However, the data-driven method excludes humans judgement, so it can not deal with the qualitative information. Furthermore, for a complex spacecraft system, it's almost impossible to collect complete data to train the black-box model. Therefore, experts' experienced knowledge and their judgement will always play an important role in the task of health estimation. For experts, they will use the IF-THEN logic to express the mapping relationship between inputs and outputs. And because the qualitative format is more natural and expressive than quantitative format, they are always used to describe input information and output consequent by experts. For example, experts may present their experienced knowledge like: 'IF temperature is high, Then the health status is poor'.

A set of subjective linguistic terms which are meaningful evaluation standards is used to describe the input information and output consequent. The input information is represented as  $X = \{x_i; i = 1, \dots, T\}$ , where  $T$  is the number of observations. The value of each observation is obtained from the finite sets  $A = \{A_1, A_2, \dots, A_T\}$ , where  $A_i = \{A_{ij}; j = 1, \dots, J_i\}$  is the reference set for observation  $x_i$ , and  $J_i$  is the number of reference values for the  $i$ th observation. For example, if there are two observations: temperature and voltage,  $X = \{\text{Temperature}; \text{Voltage}\}$ , and for the observation of temperature,  $A_1 = \{\text{Low}; \text{Medium}; \text{High}\}$  is its referential set. The output consequent is represented as  $D = \{D_n; n = 1, \dots, N\}$ , where  $N$  is the number of consequents. For example, a set  $D = \{\text{good}; \text{poor}; \text{fail}\}$  is used to describe the health condition. The experts' experienced knowledge can be express as a rule like: 'IF temperature is high, Then the health status is poor'.

## 2.2. Uncertainties

Uncertainty is a form of imprecision in expert knowledge, and it is caused because experts do not have 100% confidence in their opinions or judgements (Cai et al. (2017)). Therefore, it is very important to deal with the uncertainties of experts' knowledge. Generally, the uncertainties of experts' knowledge can be represented by probability distribution function (PDF) (Li et al. (2017)). However, the PDF can not model ignorance may occur when an expert is unable to know all possible consequents. In other word, the integration of PDF or the accumulation of all

possibilities must be 1, but the sum of possibilities may be less than 1 when ignorance occur. The Dempster-Shafer (D-S) theory of evidence describes and handles uncertainties using the concept of the degrees of belief, which can model ignorance explicitly (Yang et al. (2006)). Therefore, in this paper, the matching degree and the belief degree are used to deal with the uncertainties of input and output, respectively. The matching degree that the  $i$ th observation belongs to the  $j$ th reference is represented by  $\alpha_{ij}$ , where  $\sum_{j=1}^P \alpha_{ij} \leq 1$ . The input information in the form of matching degree can be represented as  $\{\delta x_i; \delta A_{i1}; \alpha_{i1}; \dots; \delta A_{ij}; \alpha_{ij}\}$ .

For example, when temperature is the input information, instead of the statement of 'the temperature is high', it can be stated as 'the temperature is (high,0.7),(medium,0.3),(low,0)'.

For output, the belief degree is represented by  $\beta_n$ , and  $\sum_{n=1}^N \beta_n \leq 1$ . Note that  $\beta_n < 1$  implies there exist ignorance in expert's knowledge,  $\beta_n = 1$  represents that the knowledge is complete and  $\beta_n = 0$  denotes total ignorance about the consequent. Similar to the input, the belief degree with consequent set can be represented as  $\{\delta y; \delta D_1; \beta_1; \dots; \delta D_N; \beta_N\}$ . For example, for output of the health condition estimation, instead of using definitive statements, it will be stated as 'The health condition is (good,0.1),(poor,0.8),(fail,0.1)'.

Therefore, the relationship between premises and conclusions is not precise (Liu et al. (2009)), but only with belief degree. The rule base with such premise and conclusion is called Belief Rule Base (BRB). Traditional logical reasoning engine is not applicable to the belief rule base containing uncertainty, so the rule-base inference methodology using the evidential reasoning (RIMER) proposed by Yang et al. (2006) is used in this paper. From the perspective of function, the RIMER achieves the mapping between input and output, represented as  $\delta y; \delta A; \psi$ , where  $\psi$  is the parameter vector, for example,  $\beta_n$  is one of parameters.

In addition, the observations are usually continuous numerical values obtained by measurements, and a continuous score may be needed to present the health condition. Therefore, a technique is needed to transform continuous numerical values to matching degree for referential values, or to transform belief degree for consequents to continuous numerical values. For example, the temperature is 50°C, which may be transformed into {(high,0.7),(medium,0.3),(low,0)}. The transformation technique will be discussed in more detail below.

### 2.3. Parameter adjustment

Since the subjectivity caused by experts' preference may bring inaccurate outputs for health condition estimation, the BRB parameters need to be adjusted. For example, there is a belief rule like 'If the temperature is high, then the health condition is (good,0.1),(poor,0.8),(fail,0.1)'. The parameters  $\beta_1 = 0.1$ ;  $\beta_2 = 0.8$  and  $\beta_3 = 0.1$  are subjectively determined by the expert. It's hard to say whether the values of these  $\beta$

given by the experts are accurate. If there is historical data, the parameters of the belief rule can be trained and adjusted to decrease the subjectivity. As discussed in the introduction section, the generality of current methods is that the parameter training of BRB is regarded as an optimisation problem. The optimisation goal is to minimise the difference between the estimated output of the BRB system and the expected output. The solution is to find the best model parameters to achieve the best fitting with training data (Zhou et al. (2011); Bishop and Nasrabadi (2006); Bialek, Nemenman, and Tishby (2001)). It is worth noting that these parameters are trained based on a training data set, but they are used to predict the outputs on new input information (Dietterich (1995)). As discussed before, for a complex spacecraft system, it is difficult to collect complete data under different health conditions. Under such circumstances, seeking overly the best fit to the incomplete training data may result in an overfitting problem. That is, while a very small training error is obtained, a very poor prediction is produced (Tang et al. (2019)). And sometimes the prediction error is even larger than the BRB determined by the expert. If this happens, then training is a failure. To avoid this problem, we need to balance the training and the experts' knowledge. In other words, expert's prior knowledge should play the most important role for building the BRB model, and the training just makes the BRB model more accurate. With this idea, the Markov Chain Monte Carlo (MCMC) technique is used to train the parameters of BRB. MCMC is essentially a Bayesian estimation method. Therefore, different from optimisation methods, the principle of the training parameters based on MCMC is to estimate the posterior distribution of parameters instead of finding single values. This allows the output to be estimated by taking into account the parameters of various possibilities (Zhu et al. (2017); Spragins (1965); Ho and Lee (1964)). Therefore, the method proposed in this paper is a combination of subjectivity (experts' knowledge) and objectivity (data training), and its framework is presented in Figure 1.

### 3. An introduction of BRB

#### 3.1. The basic BRB model

A belief rule can be represented as follows (Yang et al. (2006)),

$$\begin{aligned}
 & R_k : \\
 & \text{IF } x_1 \text{ is } A_1^k \wedge x_2 \text{ is } A_2^k \wedge \dots \wedge x_T \text{ is } A_T^k \\
 & \text{THEN } \delta_{1,k} \beta_{1,k} p_{1,k} ; \delta_{2,k} \beta_{2,k} p_{2,k} ; \dots ; \delta_{N,k} \beta_{N,k} p_{N,k} ; \psi \\
 & \text{with rule weight } \theta_k
 \end{aligned} \tag{1}$$

where  $x_i$  is the  $i$ th input information, called antecedent attribute, whose reference in the  $k$ th rule is  $A_i^k$ .  $D_j$  refers to the  $j$ th consequent reference, whose belief degree is  $\beta_{j,k}$ .  $\delta_j \in [0, 1]$ ;  $1 \leq j \leq NP$  in  $k$ th rule;  $\theta_k$  is the relative weight of  $k$ th rule.  $\delta_i$  is the relative weight of  $i$ th attribute.  $T$  is the number of antecedent attributes, and  $L$  is the number of all belief rules. ‘ $\wedge$ ’ represents the logic operator ‘AND’. Here,  $\theta_k$ ,  $\delta_i$ ,  $\beta_{j,k}$  are parameters, and  $\psi$  is used to describe the parameter vector of BRB, which can be represented as

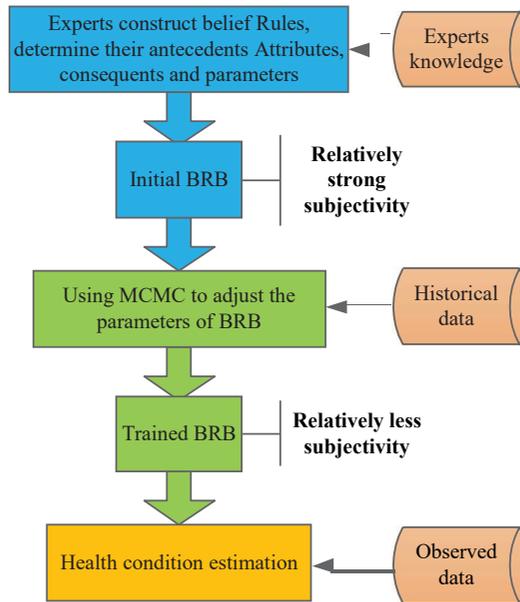


Figure 1. The general methodology to combine subjectivity and objectivity.

$$\psi \sim \theta; \delta; \beta > \quad (2)$$

### 3.2. BRB inference using ER approach

BRB reasoning is based on evidence reasoning (ER) algorithm, which calculates the final conclusion by aggregating activated rules (Yang et al. (2006)). The calculation process consists of two steps: calculating activation weight and calculating belief degree.

The activation weight is represented by  $\omega_k$ , which can be calculated as follow:

$$\omega_k = \frac{\theta_k \prod_{i=1}^{T_k} \delta_i \alpha_{i,j}^k}{\theta_k + \prod_{i=1}^{T_k} \delta_i \alpha_{i,j}^k} \quad \text{and} \quad \delta_i = \frac{\mu_i}{\max_{j=1, \dots, T_k} \mu_j} \quad (3)$$

where  $\alpha_{i,j}^k \in [0, 1]$ ;  $i = 1, 2, \dots, T_k$  refers to the individual matching degree between the  $i$ th attribute input information and the  $j$ th reference value for the  $i$ th attribute in the  $k$ th rule.

The calculation of belief degree  $\beta_j$  is as follow,

$$\beta_j = \frac{\sum_{k=1}^K \omega_k \beta_{j;k}}{\sum_{k=1}^K \omega_k} \quad ; j = 1, 2, \dots, N \quad (4)$$

where,

$$\mu = \frac{1}{N} \sum_{j=1}^N \sum_{k=1}^N \omega_k \beta_{j;k} \quad \beta_{i;k} = \frac{1}{N} \sum_{k=1}^N \omega_k \beta_{i;k} \quad \beta_{i;k} = \frac{1}{N} \sum_{k=1}^N \omega_k \beta_{i;k} \quad (5)$$

### 3.3. Transformation techniques

As discussed in Section 2.2, the input information for BRB shall be represented in the form of matching degree, so the numerical input should be transformed into the matching degree. A transformation technique is proposed by Yang (2001). Several numerical point is selected to quantify the elements of reference set  $A_{ij}; j = 1, \dots, J_i$ , and  $A_{ij}$  is assumed to be smaller than  $A_{i(j+1)}$  without loss of generality. Then an input in the form of matching degree can be represented by

$$I_{\delta x_i} = (A_{ij}; \alpha_{ij}; j = 1, \dots, J_i) \quad (6)$$

where

$$\alpha_{ij} = \frac{A_{i(j+1)} - x_i}{A_{i(j+1)} - A_{ij}} \quad (7)$$

$\alpha_{i(j+1)} = 1 - \alpha_{ij}$  if  $A_{ij} < x_i < A_{i(j+1)}$ , and  $\alpha_{ik} = 0$  for  $k = 1, \dots, J_i; k \neq j; j = 1, \dots, J_i$ .

Conversely, outputs in the form of belief degree can be converted to numerical value  $y$  as shown in Equation (6). From Yang et al. (2006)

$$y = \sum_{j=1}^N u_{\delta D_j} \beta_j \quad (8)$$

here  $u_{\delta D_j}$  refers to the reference value for individual consequent  $D_j$ .

## 4. BRB parameter adjustment with MCMC technique

### 4.1. Metropolis-Hastings (MH) algorithm

In this paper, the MCMC technique is used to estimate the posterior distribution of BRB parameters. And the MH algorithm (Andrieu et al. (2003)), which is the most popular MCMC method, will be used. Suppose that  $v = (v_1; v_2; \dots; v_m)^T$  is a  $m$ -dimensional vector, and its probability density is  $p(v)$ . A proposal distribution  $q(v^* | v)$  samples a candidate value  $v^*$  given the current value  $v$ . An MH step then moves towards  $v^*$  with acceptance probability  $A(v^* | v) = \min\{1, \frac{p(v^*)q(v | v^*)}{p(v)q(v^* | v)}\}$ , otherwise it remains at  $v$ . In this paper, Gaussian distribution is selected as a symmetric random walk proposal,

thus  $q(x^* | x) = q(x | x^*)$ . Hence, the acceptance probability is

$$A(v^* | v) = \min\left\{1, \frac{p(v^*)}{p(v)}\right\} \quad (9)$$

The algorithm of MH is shown in algorithm 1.

#### 4.2. BRB parameter adjustment with MH algorithm

With the historical data  $x; y \quad f_{\delta x \delta 1 p}; y_{\delta 1 p p}; \dots; \delta x_{\delta t p}; y_{\delta t p p}$ , the posterior distribution of BRB parameters, which is symbolised as  $p_{\delta y j x}; y_p$ , needs to be estimated. Here, we assume that the inputs  $f_{\delta x \delta 1 p}; x_{\delta 2 p}; \dots; x_{\delta t p}$  are independent, so its outputs  $f_{y_{\delta 1 p}}; y_{\delta 2 p}; \dots; y_{\delta t p}$  are also independent. Then, there is

$$p_{\delta y_{\delta 1 p}; \dots; y_{\delta t p} | \psi; x_{\delta 1 p}; \dots; x_{\delta t p}} \propto \prod_{t=1}^T p_{\delta y_{\delta t p} | \psi; x_{\delta t p}} \quad (10)$$

#### Algorithm 1 MH algorithm

```

Init( $v^{\delta 0 p}$ )
for  $i \quad 0$  to  $N - 1$  do
  Sample  $u \sim U_{[0,1]}$ 

  Sample  $v^{\delta} \sim q_{\delta v^{\delta}}(v^{\delta | p})$ 
  if  $u < \frac{p_{\delta v^{\delta} | p}(v^{\delta})}{q_{\delta v^{\delta}}(v^{\delta | p})} \min\{1, \frac{p_{\delta v^{\delta} | p}(v^{\delta | p})}{p_{\delta v^{\delta} | p}(v^{\delta})}\}$  then
     $v^{\delta | p} \quad v^{\delta}$ 
  else
     $v^{\delta | p} \quad v^{\delta | p}$ 
  end if
end for

```

According to Bayes' principle, when the training data  $f_x; y_g$  is given, the posterior distribution of the parameters vector  $\psi$  is

$$p_{\delta \psi | x; y_p} \propto p_{\delta \psi} p_{\delta y | \psi; x_p} \propto p_{\delta \psi} \prod_{t=1}^T p_{\delta y_{\delta t p} | \psi; x_{\delta t p}} \quad (11)$$

where  $p_{\delta \psi}$  is the prior distribution of BRB parameters given by experts.

Therefore,  $p_{\delta \mu p} \propto p_{\delta \psi | x; y_p}$  is substituted into the Equation (9), then the acceptance probability is

$$A(\psi^{\delta | p} | \delta x; y_p; \psi^{\delta | p} | \delta x; y_p) \propto \min\left\{1; \frac{p_{\delta \psi^{\delta | p}}(Q_t)}{p_{\delta \psi^{\delta | p}}(Q_{t-1})} \frac{p_{\delta y_{\delta t p} | \psi^{\delta | p}; x_{\delta t p}}}{p_{\delta y_{\delta t p} | \psi^{\delta | p}; x_{\delta t p}}}\right\} \quad (12)$$

For a BRB model, several constraints for the parameters vector  $\psi$  should be satisfied as follows,

$$\begin{aligned}
& 0 < \beta_{j;k} < 1; j = 1, \dots, N; k = 1, \dots, L \\
& \beta_{j;k} = 1 \\
& j \leq 1 \\
& 0 < \theta_k < 1; k = 1, \dots, L \\
& 0 < \delta_i < 1; i = 1, \dots, T \\
& A_{ij} < A_{i;j+1}; i = 1, \dots, T; j = 1, \dots, J_i \\
& u \delta D_i < u \delta D_j \text{ if } i < j; i, j = 1, \dots, N
\end{aligned} \tag{13}$$

When sampling with MCMC technique, a simple method to handle the constraints is to reject the sample which deviates the constraints, and to resample until the constraint is satisfied.

When sampling with MCMC technique, a simple approach to deal with constraints is to eliminate the samples deviating from the constraints, and then re-sample until the constraints are met.

The algorithm of BRB parameters training based on MH is shown in algorithm 2.

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#### Algorithm 2 BRB parameters training based on MH

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Init( $\psi^{(0)}$ ) and its prior distribution  $p(\delta|\psi)$ , and the observation distribution  $p(\delta|y, \psi)$ 
for  $i = 0$  to  $N$  do
  Sample  $u \sim U_{[0,1]}$ 
  while 1 do
    Sample  $\psi^? \sim q(\delta|\psi^?)j\psi^{(i)}$ 
    if  $\psi^?$  satisfy constraints listed in Equation (12) then
      break
    end if
  end while
  if  $u < A \left( \frac{j\delta x; y; \psi^?; j\delta x; y}{\psi^{(i)} \psi^?} \right) \wedge \min 1;$   $\frac{p(\delta|\psi^?) Q_t}{p(\delta|\psi^{(i)}) Q_{t-1}}$   $\frac{p(\delta|y, \psi^?; x\delta t)}{p(\delta|y, \psi^{(i)}; x\delta t)}$  then
     $\psi^{(i+1)} = \frac{1}{4} \psi^?$ 
  else
     $\psi^{(i+1)} = \frac{1}{4} \psi^{(i)}$ 
  end if
end for

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## 5. Case study

There are thousands of belief rules for a complex spacecraft system or subsystems, and it is impossible to list a large number of belief rules in the article due to the limited space of the paper. Therefore, a relative simple example, which is to estimate the health condition of space application lithium-ion battery, is taken to demonstrate the application of proposed method. Although this example is relatively simple, but for a larger space craft system or subsystems, the application of confidence rule in health condition estimation and its training method are consistent with this example. Besides, lithium-ion batteries have been used in the new satellites widely (Hyder et al. (2000)). Since the power systems have caused a lot of fatal failures of spacecraft, especially the battery subsystems (Zhang and Lee (2011)), the health condition monitoring of lithium-ion

batteries has attracted more attention. Residual capacity can directly indicate the health status of lithium battery (Olivares et al. (2013)). However, it is impossible to measure the capacity when spacecraft is on-line or in-orbit, and only voltage, current and temperature can be obtained from telemetry (Rufus, Lee, and Thakker (2008)). Therefore, the health condition of the spacecraft's batteries can only be estimated indirectly from these observations.

### 5.1. Experiment data

From an intuitive point of view, with the increase of service time, the working time (discharging time) of a full charged battery is shorter and shorter, so the discharging time has a certain relationship with the capacity of lithium-ion batteries. Accordingly, an indicator called equal discharge voltage difference time interval (TIEDVD) (Liu et al. (2013)) is constructed as Figure 2 shows.

Besides, temperature is another important observation for health estimation of battery. When temperature is higher, the ionic conductivity increases and the reaction kinetics tends to be faster. Therefore, although the apparent performance and capacity seem to be higher, the actual available capacity is much lesser (Rufus, Lee, and Thakker (2008)). Accordingly, the mean temperature during the TIEDVD, simplified as MT, is as another input attribute. The lithium-ion batteries experiment data set from NASA is used in this paper. In the data set, each battery is run through three different operational profiles (charge, discharge and impedance) at room temperature, and only the discharge

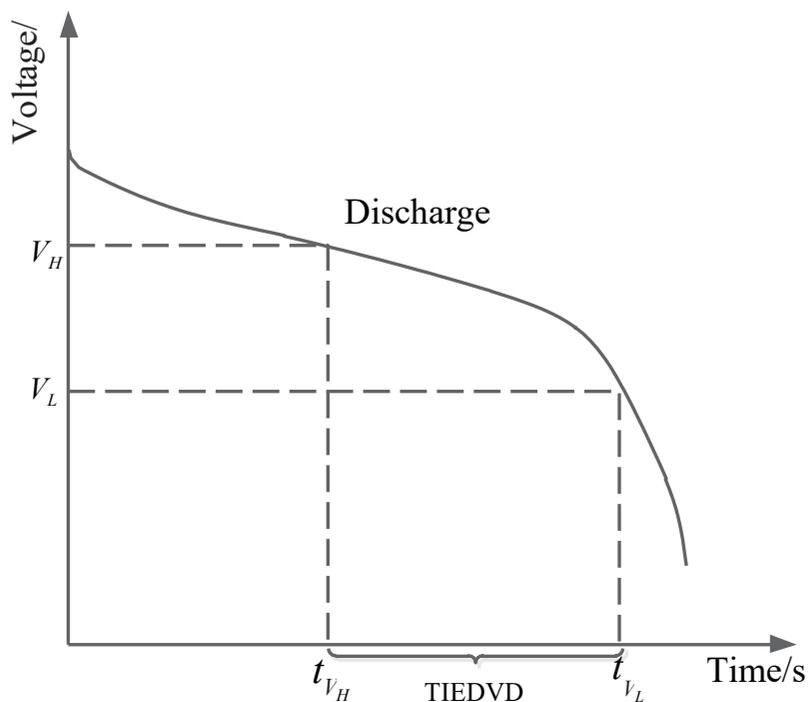


Figure 2. Illustration of TIEDVD.

data is used in this paper. For discharge, there are fields including battery terminal voltage, battery temperature current measured at load, time vector for the cycle and capacity.

To achieve the objective mentioned above, the TIEDVD and MT are extracted from the experiment data set as the inputs of the BRB system, and the capacity in the experiment which can indicates the health condition of batteries directly is taken as the output. When extracting the TIEDVD, the high voltage is set as 3.6 V, and the low voltage is set as 3.2 V, that is  $V_H \approx 3.6V$ ;  $V_L \approx 3.2V$ . Figure 3 shows the TIEDVD, MT and capacity extracted from the experiment data set of battery #0006.

## 5.2. Referential points of the antecedents and consequent

The TIEDVD and MT are numerical values, which needs to be transformed to the matching degree of linguistic terms. In this paper, we use three linguistic terms for TIEDVD and they are short time (s), medium time (M), long time (L). And the referential values for these three linguistic terms are 1200, 1700, 2200 s. That is

$$A_1 \approx \{s \approx 1200s; M \approx 1700s; L \approx 2200s\} \quad (14)$$

Similarly we use two linguistic terms for MT and they are low temperature (L), and high temperature (H). And the referential values for these two linguistic terms are 30°C, 36°C.

$$A_2 \approx \{L \approx 30^\circ C; H \approx 36^\circ C\} \quad (15)$$

With the referential values for linguistic terms, the numerical values can be transformed into the belief degrees with the transformation technique mentioned in Section 3.3. For

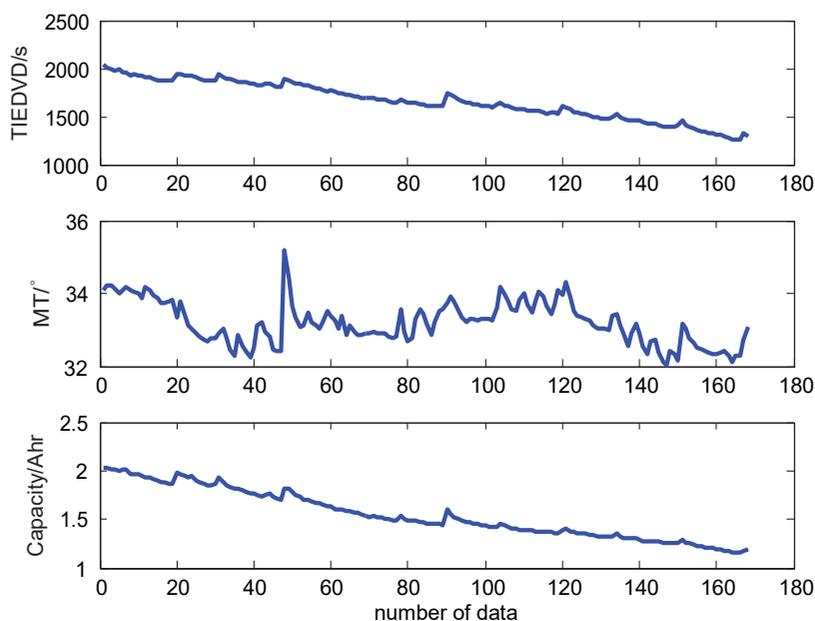


Figure 3. TIEDVD, MT and capacity for #6 battery.

example, if the value of TIEDVD is 1951 s, the matching degree of the referential terms is  $f_{\delta S}; 0:1P; \delta M; 0:9P; \delta L; 0Pg$ .

In reverse, the belief degrees of different health condition generated by the BRB can be transformed into the numerical values of capacity. For the consequent, the health condition (capacity), 5 linguistic terms are used: very poor (VP), poor (P), medium (M), good (G) and very good (VG). And the referential capacity values for these five linguistic terms are 1.84, 1.89, 1.94, 1.99, 2.04 Ahr. That is

$$D \frac{1}{4} fVP \frac{1}{4} 1:1Ahr; P \frac{1}{4} 1:35Ahr; M \frac{1}{4} 1:6Ahr; G \frac{1}{4} 1:85Ahr; VG \frac{1}{4} 2:1Ahr \quad (16)$$

If the belief degree of different health condition calculated by BRB is  $f_{\delta VP}; 0:1P; \delta P; 0:1P; \delta M; 0:2P; \delta G; 0:2P; \delta M; 0:4Pg$ ; then capacity  $\frac{1}{4} VP \times 0:1P \times 0:1P \times M \times 0:2P \times G \times 0:2P \times VG \times 0:4 \frac{1}{4} 1:84 \times 0:1P \times 1:89 \times 0:1P \times 1:94 \times 0:2P \times 1:99 \times 0:2P \times 2:04 \times 0:4 \frac{1}{4} 1:975$

### 5.3. Rules

As mentioned above, different from traditional rule base, the BRB is based on the belief concept. When building the belief rule base, the experts not only need to give the premise and conclusion for rules, but also need to give a series of parameters, including rule weight  $\theta_k$ , attributes weight  $\delta_{ik}$ , and belief degrees  $\beta_{nk}$ . And the number of rules is decided by antecedent attributes, for example, there are two antecedent attributes, TIEDVD and MT in this case, where the TIEDVD is divided into three linguistic terms, and the MT is divided into two linguistic terms. Therefore, there are  $3 \times 2 \frac{1}{4} 6$  combinations of the two attributes resulting in six rules in the rule base. The experts can give the rules and corresponding parameters with their experienced knowledge or by observing the historical data.

For this case, Table 1 lists the six belief rules decided by the experts.

In the Table 1, the first rule means that:

$$R_1 : \text{IF TIEDVD is S AND MT is L;} \\ \text{THEN capacity is } f_{\delta VP}; 0:7P; \delta P; 0:2P; \delta M; 0:1P; \delta G; 0:0P; \delta VG; 0:0Pg; \quad (17) \\ \text{with rule weight } \theta_1 \frac{1}{4} 1; \text{ and attributes weight } \delta_1 \frac{1}{4} 1; \delta_2 \frac{1}{4} 1$$

Figure 4 shows the results of capacity generated by experts given BRB and also the real capacity for battery #0006. From the comparison, it can be seen that the estimated outcomes of BRB decided by experts basically reflect the variation trend of real capacity. However, the estimated outcomes cannot match the real ones very closely because of the inaccuracy of human knowledge.

Table 1. Belief rule base given by experts.

$R_k$	$\theta_k$	$x_{\delta A_1 P}$	$x_{\delta A_2 P}$	Consequents $D \frac{1}{4} fVP; P; M; G; VPg; \delta_1 \frac{1}{4} 1; \delta_2 \frac{1}{4} 1$
1	1.0	S	L	$f_{\delta VP}; 0:7P; \delta P; 0:2P; \delta M; 0:1P; \delta G; 0:0P; \delta VG; 0:0Pg$
2	1.0	S	H	$f_{\delta VP}; 0:9P; \delta P; 0:1P; \delta M; 0:0P; \delta G; 0:0P; \delta VG; 0:0Pg$
3	1.0	M	L	$f_{\delta VP}; 0:0P; \delta P; 0:1P; \delta M; 0:6P; \delta G; 0:2P; \delta VG; 0:1Pg$
4	1.0	M	H	$f_{\delta VP}; 0:1P; \delta P; 0:2P; \delta M; 0:5P; \delta G; 0:2P; \delta VG; 0:0Pg$
5	1.0	L	L	$f_{\delta VP}; 0:0P; \delta P; 0:0P; \delta M; 0:0P; \delta G; 0:1P; \delta VG; 0:9Pg$
6	1.0	L	H	$f_{\delta VP}; 0:0P; \delta P; 0:0P; \delta M; 0:1P; \delta G; 0:1P; \delta VG; 0:8Pg$

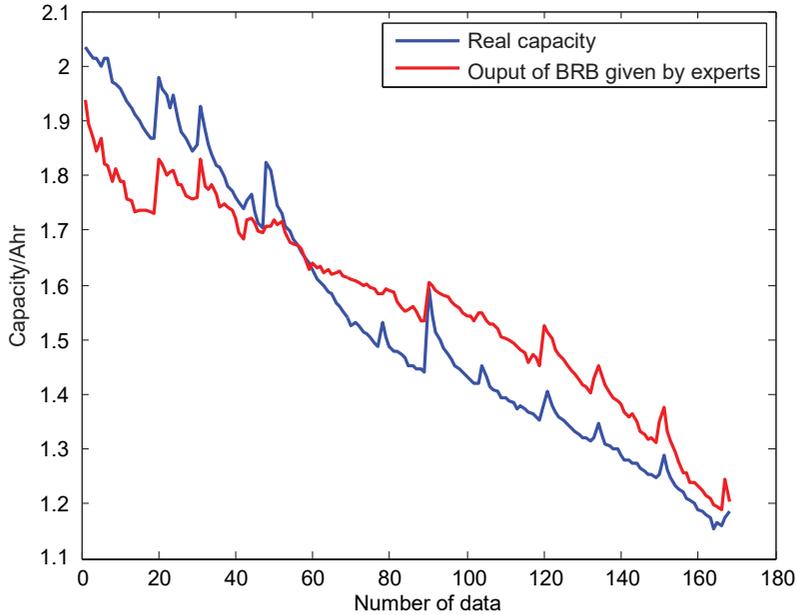


Figure 4. Comparison of capacity generated by experts given BRB with real capacity.

#### 5.4. Training with historical data

If there are historical data, the BRB parameters shown in Table 1 can be adjusted to obtain more accurate outcomes. For this case, in order to train the BRB, the extracted TIEDVD, MT and capacity of battery #0005 and battery #0006 are used as the training data set. And the extracted TIEDVD, MT and capacity of battery #0007 is used as the testing data set to verify the effectiveness of training. In order to illustrate the advantages of the proposed method in spacecraft health condition estimation, it will be compared with BRB trained by optimisation method and BP neural network.

##### (1) BRB parameters adjustment with MCMC technique

The prior distribution of parameters follows uniform distribution under constraints described in Eq.12, the transition distribution is set as follows,  $\theta_{i|p_1} \sim N(\theta_i; 0:03p, \beta_{i|p_1})$ ,  $\beta_{i|p_1} \sim N(\beta_i; 0:01p, \delta_{i|p_1})$ ,  $\delta_{i|p_1} \sim N(\delta_i; 0:03p)$ . The observation distribution  $p(y_i | \psi; x_i) \sim N(y_i; 6e-5p)$ , where  $y_i$  is the real output of training data set. And the number of samples  $N \approx 10000$

##### (2) BRB parameters adjustment with optimisation method

Figure 5 shows the framework of training BRB parameter based on optimisation method, where  $x$  is the input vector,  $y$  is the real output,  $\hat{y}$  denotes the predicted output generated by BRB system, and the parameters of BRB is represented as  $\psi = \langle \theta; \delta; \beta \rangle$ . And the target of the optimisation is minimising the error between real output  $y$  and predicted output  $\hat{y}$ , which is denoted by  $\|\hat{y} - y\|$ . There are several methods to represent  $\|\hat{y} - y\|$ ; such as mean square error (MSE); mean average error (MAP), etc. In this paper, MSE is chosen as the optimisation. Therefore,  $\|\hat{y} - y\|$  can be calculated as follow,

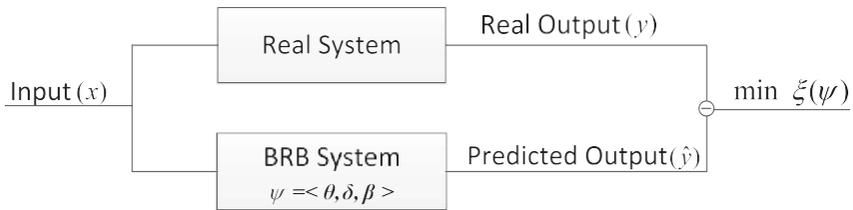


Figure 5. Illustration of BRB parameter training based on optimisation method.

Table 2. Belief rule base trained by MCMC.

$R_k$	$\theta_k$	$x\delta A_{1P}$	$x\delta A_{2P}$	Consequents $D \frac{1}{4} fVP; P; M; G; VPg; \delta_1 \frac{1}{4} 0:3108; \delta_2 \frac{1}{4} 0:4876$
1	0.6477	S	L	$f\delta VP; 0:7011P; \delta P; 0:2232P; \delta M; 0:0757P; \delta G; 0:0P; \delta VG; 0:0Pg$
2	0.2568	S	H	$f\delta VP; 0:8802P; \delta P; 0:1198P; \delta M; 0:0P; \delta G; 0:0P; \delta VG; 0:0Pg$
3	0.3079	M	L	$f\delta VP; 0:0P; \delta P; 0:3951P; \delta M; 0:3073P; \delta G; 0:2028P; \delta VG; 0:0948Pg$
4	0.3085	M	H	$f\delta VP; 0:0277P; \delta P; 0:0402P; \delta M; 0:3373P; \delta G; 0:5948P; \delta VG; 0:0Pg$
5	0.2498	L	L	$f\delta VP; 0:0P; \delta P; 0:0P; \delta M; 0:0P; \delta G; 0:3093P; \delta VG; 0:6907Pg$
6	0.7545	L	H	$f\delta VP; 0:0P; \delta P; 0:0P; \delta M; 0:0873P; \delta G; 0:2218P; \delta VG; 0:6909Pg$

$$y = \sum_{k=1}^m \frac{\theta_k \cdot \min(\delta A_{1P}, \delta A_{2P})}{\sum_{k=1}^m \min(\delta A_{1P}, \delta A_{2P})} \cdot \text{Consequents}_k$$

The BRB trained by this method is called MMSE (minimum MSE) trained BRB below. The above optimisation problem can be solved by using ‘fmincon’ in the optimisation toolbox of MATLAB.

### (3) BP Neural Network

The parameters of BP neural network are: Two hidden layers, six nodes which uses ‘logsig’ activation function for the first hidden layer, one node which uses ‘purelin’ activation function for the second hidden layer. net.trainParam.lr = 0.05; net.trainParam.goal = 1e-5; net.trainParam.epochs = 500;

### (4) Results

After being trained by MCMC method, the adjusted BRB is shown in Table 2. Figure 6 shows the capacity estimated by initial BRB given by experts, MCMC trained BRB, MMSE trained BRB, and the BP network. As can be seen from the figure, the output of the initial BRB deviates largely from the test data. But after the training of MCMC technique and MMSE method, the output results are closer to the real output. The accumulative errors of different method are presented in Figure 7, and the detailed MSE is shown in Table 3, which indicates that the MCMC trained BRB has a higher accuracy than MMSE trained BRB, but MMS-trained BRB has a higher accuracy than BP network.

Figure 8 shows the belief degree of different health conditions generated by MCMC trained BRB. It can be seen that the results of belief degree is accordance with the tendency of real health condition of battery, and also accordance with the understanding our human being. For example, the probability that the battery’s health status is VG is decreasing as time goes, and drop to almost zero in half way.

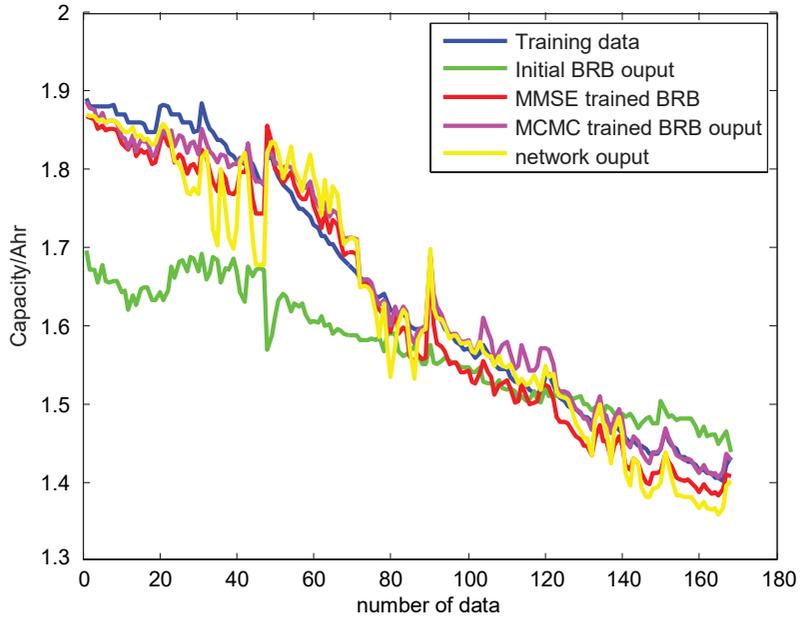


Figure 6. Comparing the results.

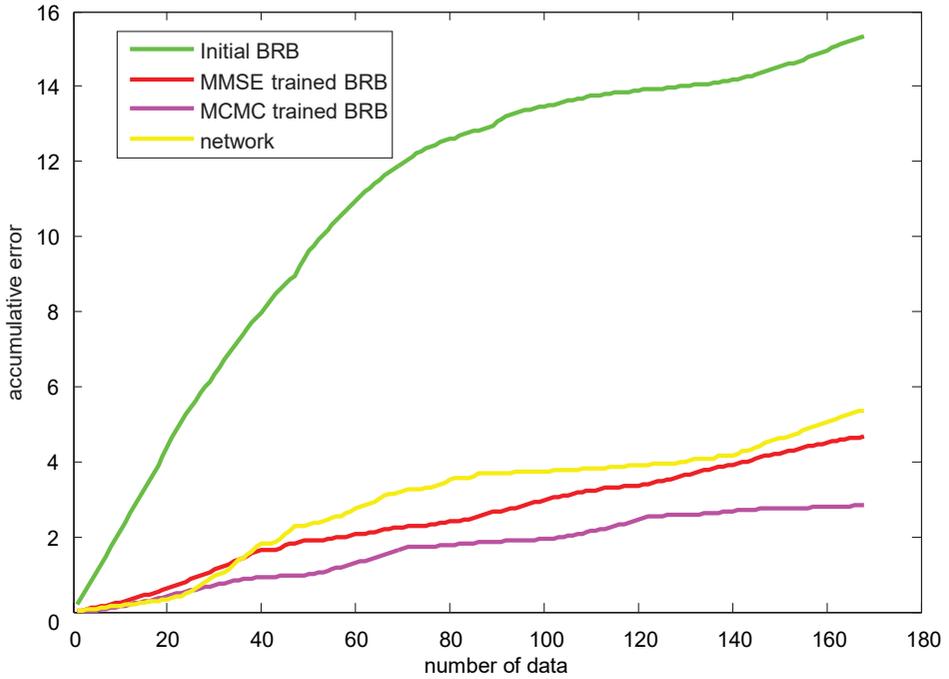


Figure 7. Comparing the accumulative errors.

Table 3. The comparative errors by using different methods.

Method	BRB decided by experts	BRB trained by MCMC	BRB trained by MMSE	BP network
Error (MSE)	1.4e-2	4.6e-4	9.6e-4	1.9e-3

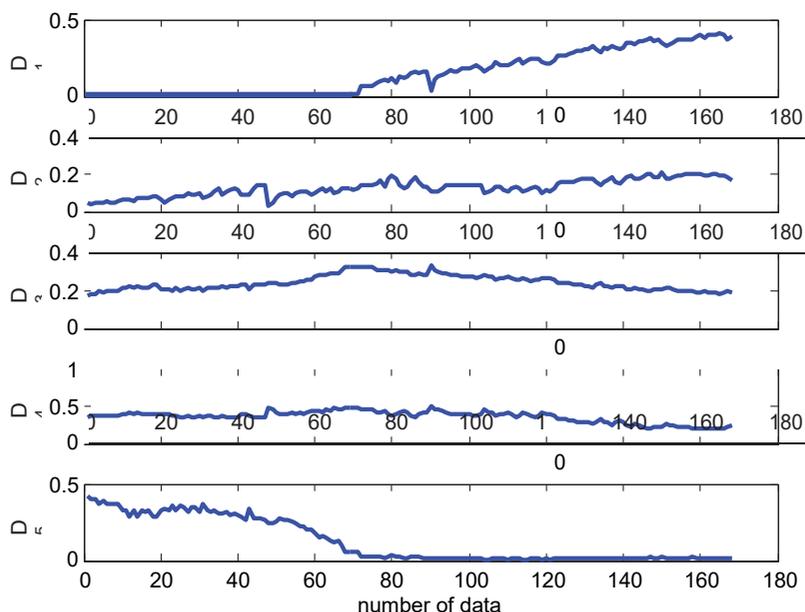


Figure 8. The belief degree of different health conditions generated by MCMC-trained BRB.

## 6. Conclusions

This paper focuses on a study of health condition estimation of spacecraft key components by using BRB. The study demonstrates that the precision of the proposed method for estimating the health condition is superior to the traditional optimised BRB and also the BP network due to the superior capability of overcoming the problem of overfitting. Comparing with the traditional optimisation method for BRB parameters training, the MCMC method estimates the posterior probability of parameters instead of finding optimal single values. Therefore, MCMC method adjusts BRB parameters in a more conservative way, which makes experts play major roles in determining BRB parameters. Comparing with the pure data-driven method (BP network), the proposed method can take full advantage of the prior knowledge of experts to compensate for the insufficient training data or the interference of noise in training data, thus overcoming the over-fitting problem. In addition, BRB is a 'white box' system, and the results can be explained clearly because the reasons can be traced back. Furthermore, the proposed method can generate output of belief degree for different linguistic terms, which can deal with the uncertainties, and such kind of conclusions is more natural and acceptable to human users.

It is worth noting that the battery case is much more simple than real application in order to demonstrate the proposed method clearly. In practice, the spacecraft systems have a large number of subsystems and observations, so the BRB may be a large-scale rule base with hundreds or thousands of rules. Besides, a bottom-up approach is often used to estimate the health condition of a complex system, and the BRB can be of a hierarchical structure exactly Yang et al. (2006). For multi-levels BRB, the lower level attributes can be aggregated as evidence for a higher level attributes. Therefore, using multi-levels BRB to model the hierarchical structure of a complex spacecraft system needs to be further researched.

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