Online Supplement for "Production and Technology Choice under Emissions Regulation: Centralized vs Decentralized Supply Chains" by Jen-Yen Lin, Sean X. Zhou, Fei Gao

In this supplement file, Appendix A provides the supplementary results and their proofs while Appendix B provides the proofs of the results in the paper. For ease of reference, we use (P-Equation Number) to refer to the equations in the paper, e.g., (P-1) is Equation (1) in the paper.

Appendix A: Supplementary Results and Their Proofs

Lemma A1. *$\pi^c(q, q_1^s, q_1^m)$ is concave in (q, q_1^s, q_1^m).*

Proof. Note that

$$\min\{D, q\} = q - (q - D)^+$$

is concave in q. Moreover, as $\rho \geq v$, the combination of the last two terms is concave (q, q_1^s, q_1^m). The other terms in $\pi^c(q, q_1^s, q_1^m)$ are linear. So $\pi^c(q, q_1^s, q_1^m)$ is concave. ■

Lemma A2. *$\pi^m(q, q_1^m)$ is concave in (q, q_1^m).*

Proof. Since $\rho > v$, the combination of the last two terms is concave. The other terms in $\pi^m(q, q_1^m)$ are all linear. So $\pi^m(q, q_1^m)$ is concave in (q, q_1^m). ■

Proposition A1. *The optimal production quantities $(q(w), q_1^m(w))$ of the manufacturer are given as follows.*

(i) If $v < \rho \leq C^m$, then $q_1^m(w) = 0$ and

$$q(w) = \begin{cases} \bar{F}^{-1}(w + c_2^m + \rho\xi_2^m) & \text{if } w + c_2^m \leq \bar{F}(\frac{e^m}{\xi_2^m}) - \rho\xi_2^m, \\ \frac{e^m}{\xi_2^m} & \text{if } \bar{F}(\frac{e^m}{\xi_2^m}) - \rho\xi_2^m < w + c_2^m \leq \bar{F}(\frac{e^m}{\xi_2^m}) - v\xi_2^m, \\ \bar{F}^{-1}(w + c_2^m + v\xi_2^m) & \text{if } w + c_2^m > \bar{F}(\frac{e^m}{\xi_2^m}) - v\xi_2^m. \end{cases} \tag{1}$$

(ii) If $C^m < v < \rho$, then $q_1^m(w) = q(w)$ and

$$q(w) = \begin{cases} \bar{F}^{-1}(w + c_1^m + \rho\xi_1^m) & \text{if } w + c_1^m \le \bar{F}(\frac{e^m}{\xi_1^m}) - \rho\xi_1^m, \\ \frac{e^m}{\xi_1^m} & \text{if } \bar{F}(\frac{e^m}{\xi_1^m}) - \rho\xi_1^m < w + c_1^m \le \bar{F}(\frac{e^m}{\xi_1^m}) - v\xi_1^m, \\ \bar{F}^{-1}(w + c_1^m + v\xi_1^m) & \text{if } w + c_1^m > \bar{F}(\frac{e^m}{\xi_1^m}) - v\xi_1^m. \end{cases} \tag{2}$$

(iii) If $v \le C^m < \rho$, then

$$q(w) = q_1^m(w) = \begin{cases} \bar{F}^{-1}(w + c_1^m + \rho\xi_1^m) & \text{if } w + c_1^m \le \bar{F}(\frac{e^m}{\xi_1^m}) - \rho\xi_1^m, \\ \frac{e^m}{\xi_1^m} & \text{if } \bar{F}(\frac{e^m}{\xi_1^m}) - c_1^m - \rho\xi_1^m < w \le \bar{F}(\frac{e^m}{\xi_1^m}) - c_2^m - C^m\xi_2^m; \end{cases} \tag{3}$$

if $\bar{F}(\frac{e^m}{\xi_1^m}) - C^m\xi_2^m < w + c_2^m \le \bar{F}(\frac{e^m}{\xi_2^m}) - C^m\xi_2^m$,

$$q(w) = \bar{F}^{-1}(w + c_2^m + C^m\xi_2^m), \;\; q_1^m(w) = \frac{\xi_2^m\bar{F}^{-1}(w + c_2^m + C^m\xi_2^m) - e^m}{\xi_2^m - \xi_1^m}; \tag{4}$$

and

$$(q(w), q_1^m(w)) = \begin{cases} \left(\frac{e^m}{\xi_2^m}, 0\right) & \text{if } \bar{F}(\frac{e^m}{\xi_2^m}) - C^m\xi_2^m < w + c_2^m \le \bar{F}(\frac{e^m}{\xi_2^m}) - v\xi_2^m, \\ \left(\bar{F}^{-1}(w + c_2^m + v\xi_2^m), 0\right) & \text{if } w + c_2^m > \bar{F}(\frac{e^m}{\xi_2^m}) - v\xi_2^m. \end{cases} \tag{5}$$

Proof. The manufacturer's profit function $\pi^m(q, q_1^m)$ can be regarded as a special case of the integrated supply chain's profit function $\pi^c(q, q_1^s, q_1^m)$ with $c_1^m = c_2^m = 0$, $\xi_1^m = \xi_2^m = 0$ and c_1^s, c_2^s substituted by $c_1^m + w$ and $c_2^m + w$ respectively. For case (i), (ii), (iii) in this theorem, the manufacturer's problem can be solved similarly to case (i) (ii) in Theorem 2 and (ii) in Theorem 3 respectively. Rewrite the condition in the solution expressions as ranges specified by the wholesale price w, we can get (1), (2) and (3). ■

Before characterizing the detailed optimal solution of the supplier, we further divide the supplier's problem into several cases based on Proposition A1 because the corresponding $w(q)$ is different.

Define

$$\mathcal{J}(q, q_1^s) = \bar{F}(q)q - c_1^s q_1^s - c_2^s(q - q_1^s) - \rho[t^s(q, q_1^s) - e^s]^+ + v[e^s - t^s(q, q_1^s)]^+. \tag{6}$$

For case (i) in Proposition A1, the corresponding $w(q)$ is either $\bar{F}(q) - c_2^m - \rho\xi_2^m$ or $\bar{F}(q) - c_2^m - v\xi_2^m$, depending on $q \le e^m/\xi_2^m$ or $q \ge e^m/\xi_2^m$. Hence, the supplier aims to solve the following:

$$\max_{0 \le q_1^s \le q} \pi^s(q, q_1^s) = \max\left\{ \max_{0 \le q \le \frac{e^m}{\xi_2^m}} \mathcal{F}_1(q), \max_{q \ge \frac{e^m}{\xi_2^m}} \mathcal{F}_2(q) \right\}, \tag{7}$$

where $\mathcal{F}_1(q) = \max_{0 \le q_1^s \le q}\{\mathcal{J}(q, q_1^s) - (c_2^m + v\xi_2^m)q\}$ and $\mathcal{F}_2(q) = \max_{0 \le q_1^s \le q}\{\mathcal{J}(q, q_1^s) - (c_2^m + \rho\xi_2^m)q\}$. Let $q_{\mathcal{F}_i} = \arg\max \mathcal{F}_i(q)$, $i = 1, 2$. As $\mathcal{F}_1(q) \ge \mathcal{F}_2(q)$, the optimal q could be either $q_{\mathcal{F}_1}$, $q_{\mathcal{F}_2}$, or e^m/ξ_2^m.

Similarly, for case (ii) in Proposition A1, the supplier aims to solve the following:

$$\max_{0\le q_1^s\le q}\pi^s(q,q_1^s)=\max\left\{\max_{0\le q\le \frac{e^m}{\xi_1^m}}\mathcal{G}_1(q),\max_{q\ge\frac{e^m}{\xi_1^m}}\mathcal{G}_2(q)\right\},\tag{8}$$

where $\mathcal{G}_1(q)=\max_{0\le q_1^s\le q}\{\mathcal{J}(q,q_1^s)-(c_1^m+v\xi_1^m)q\}$, $\mathcal{G}_2(q)=\max_{0\le q_1^s\le q}\{\mathcal{J}(q,q_1^s)-(c_1^m+\rho\xi_1^m)q\}$ and let $q_{\mathcal{G}_i}=\arg\max\mathcal{G}_i(q)$, $i=1,2$.

And for case (iii) in Proposition A1, the supplier aims to solve the following:

$$\max_{0\le q_1^s\le q}\pi^s(q,q_1^s)=\max\left\{\max_{0\le q\le\frac{e^m}{\xi_2^m}}\mathcal{H}_1(q),\max_{\frac{e^m}{\xi_2^m}\le q\le\frac{e^m}{\xi_1^m}}\mathcal{H}_2(q),\max_{q\ge\frac{e^m}{\xi_1^m}}\mathcal{H}_3(q)\right\},\tag{9}$$

where $\mathcal{H}_1(q)=\max_{0\le q_1^s\le q}\{\mathcal{J}(q,q_1^s)-(c_2^m+v\xi_2^m)q\}$, $\mathcal{H}_2(q)=\max_{0\le q_1^s\le q}\{\mathcal{J}(q,q_1^s)-(c_2^m+C^m\xi_2^m)q\}$, $\mathcal{H}_3(q)=\max_{0\le q_1^s\le q}\{\mathcal{J}(q,q_1^s)-(c_1^m+\rho\xi_1^m)q\}$; and $q_{\mathcal{H}_i}=\arg\max\mathcal{H}_i(q)$, $i=1,2,3$.

The problems (7), (8) and (9) involve two layers of optimization. We need to first solve the optimization problems within the brackets and after that, we compare the resulting maximum values to determine the maximizer for $\pi^s(q,q_1^s)$. Based on the assumption that demand has IGFR, we have the following properties that facilitate the characterization of the equilibrium solutions.

Proposition A2. *$\mathcal{F}_i(q)$, $\mathcal{G}_i(q)$ $(i=1,2)$ and $\mathcal{H}_i(q)$ $(i=1,2,3)$ are quasi-concave in q, and $q_{\mathcal{F}_1}\ge q_{\mathcal{F}_2}$, $q_{\mathcal{G}_1}\ge q_{\mathcal{G}_2}$, $q_{\mathcal{H}_1}\ge q_{\mathcal{H}_2}\ge q_{\mathcal{H}_3}$.*

Proof. In the proof, we only show case (i) because cases (ii) and (iii) can be proved analogously. We focus on solving the optimal q_1^s in different scenarios to derive the resulting $\mathcal{F}_i(q)$. For simplicity, let $M_1=c_2^m+v\xi_2^m$ and $M_2=c_2^m+\rho\xi_2^m$.

First consider the situation when $v\le C^s<\rho$. If q and q_1^s satisfy $\xi_1^s q_1^s+\xi_2^s(q-q_1^s)-e^s>0$,

$$\begin{aligned}
&\max_{0\le q_1^s\le q}\mathcal{J}(q,q_1^s)-M_iq\\
=\;&\max\bar{F}(q)q-(M_i+c_2^s+\rho\xi_2^s)q+[\rho(\xi_2^s-\xi_1^s)-(c_1^s-c_2^s)]q_1^s+\rho e^s,\\
&s.t.,\quad 0\le q_1^s\le q,\ \xi_1^s q_1^s+\xi_2^s(q-q_1^s)-e^s>0,\\
=\;&\max_{q>e^s/\xi_1^s}\bar{F}(q)q-(M_i+c_1^s+\rho\xi_1^s)q+\rho e^s,
\end{aligned}$$

where the equality is because that $\rho>C^s$ implies $q_1^s=q$.

If $\xi_1^s q_1^s+\xi_2^s(q-q_1^s)-e^s=0$, which implies $q_1^s=(\xi_2^s q-e^s)/(\xi_2^s-\xi_1^s)$, the problem becomes

$$\max_{e^s/\xi_2^s\le q\le e^s/\xi_1^s}\bar{F}(q)q-(M_i+c_2^s+C^s\xi_2^s)q+C^se^s,$$

in which the constraint $e^s/\xi_2^s\le q\le e^s/\xi_1^s$ is to ensure that $q_1^s=(\xi_2^s q-e^s)/(\xi_2^s-\xi_1^s)\in[0,q]$.

If $\xi_1^s q_1^s + \xi_2^s(q - q_1^s) - e^s < 0$, the problem is

$$\begin{aligned}
& \max \bar{F}(q)q - (M_i + c_2^s + v\xi_2^s)q + [v(\xi_2^s - \xi_1^s) - (c_1^s - c_2^s)]q_1^s + \rho e^s, \\
& s.t., \quad 0 \leq q_1^s \leq q, \; \xi_1^s q_1^s + \xi_2^s(q - q_1^s) - e^s < 0, \\
= & \max_{q < e^s/\xi_2^s} \bar{F}(q)q - (M_i + c_2^s + v\xi_2^s)q + ve^s,
\end{aligned}$$

where the equality is due to $v \leq C^s$, which results in $q_1^s = 0$. In summary,

$$\mathcal{F}_i(q) = \begin{cases} \bar{F}(q)q - (M_i + c_1^s + \rho\xi_1^s)q + \rho e^s, & q > e^s/\xi_1^s, \\ \bar{F}(q)q - (M_i + c_2^s + C^s\xi_2^s)q + C^s e^s, & e^s/\xi_2^s \leq q \leq e^s/\xi_1^s, \\ \bar{F}(q)q - (M_i + c_2^s + v\xi_2^s)q + ve^s, & q < e^s/\xi_2^s. \end{cases} \tag{10}$$

When $q > e^s/\xi_1^s$, take derivative of $\mathcal{F}_i(q)$,

$$\mathcal{F}_i^{'}(q) \quad = \quad \bar{F}(q)[1 - g(q)] - (M_i + c_1^s + \rho\xi_1^s).$$

Let $\tilde{q}$ satisfy that $g(\tilde{q}) = 1$, then for $q \leq \tilde{q}$, as $g(q)$ is increasing, $1 - g(q) \geq 0$ and $\bar{F}(q)$ is decreasing, then $\mathcal{F}_i^{'}(q)$ is decreasing, and so $\mathcal{F}_i(q)$ is concave for $q \leq \tilde{q}$. For $q > \tilde{q}$, as $1 - g(q) < 0$ and $M_i > 0$, then $\mathcal{F}_i^{'}(q) < 0$ and so $\mathcal{F}_i(q)$ is decreasing for $q > \tilde{q}$. As a result, $\mathcal{F}_i(q)$ is quasi-concave in q when $q > e^s/\xi_1^s$. It can also be shown that $\mathcal{F}_i(q)$ is quasi-concave in the other two regions specified in (10). Moreover, as $\rho > C^s \geq v$ and $c_1^s > c_2^s$, it can be shown that $\mathcal{F}_i'(q)$ is smallest for $q > e^s/\xi_1^s$, followed by region $e^s/\xi_2^s \leq q \leq e^s/\xi_1^s$, and largest for region $q < e^s/\xi_2^s$. Therefore, once $\mathcal{F}_i'(q)$ becomes negative, it will always be negative. Thus, by definition, $\mathcal{F}_i(q)$ is quasi-concave in the whole region.

Next consider the case $v < \rho \leq C^s$. Note that, in this case, $q_1^s = 0$ at optimum, and it can be easily derived that

$$\mathcal{F}_i(q) = \begin{cases} \bar{F}(q)q - (M_i + c_2^s + \rho\xi_2^s)q + \rho e^s & \text{if } q > \frac{e^s}{\xi_2^s} \\ \bar{F}(q)q - (M_i + c_2^s + v\xi_2^s)q + ve^s & \text{if } q \leq \frac{e^s}{\xi_2^s}. \end{cases} \tag{11}$$

Finally, for the case of $C^s < v < \rho$, $q_1^s = q$ at optimum, and

$$\mathcal{F}_i(q) = \begin{cases} \bar{F}(q)q - (M_i + c_1^s + \rho\xi_1^s)q + \rho e^s & \text{if } q > \frac{e^s}{\xi_1^s} \\ \bar{F}(q)q - (M_i + c_1^s + v\xi_1^s)q + ve^s & \text{if } q \leq \frac{e^s}{\xi_1^s}. \end{cases} \tag{12}$$

For these two cases, it can be similarly proved that $\mathcal{F}_i(q)$ is quasi-concave .

Note $\mathcal{F}_i(q)$ and $(\mathcal{F}_i(q))'$ are decreasing in i because M_i is increasing in i. Also by quasi-concavity of $\mathcal{F}_i(q)$ and the definition of $q_{\mathcal{F}_i}$, it is clear that $q_{\mathcal{F}_i}$ is decreasing in i, $i = 1, 2$. $\blacksquare$

Definitions for Propositions A3 and A4

Define

$$\hat{\mathcal{F}}_1(q) = \bar{F}(q)q - (c^s + c^m + v\xi)q,$$
$$\hat{\mathcal{F}}_2(q) = \bar{F}(q)q - (c^s + c^m + \rho\xi)q.$$

Recall $\mathcal{K}(x) = \bar{F}(x)(1 - g(x))$. Based on the first order condition, the maximizer $q_{\hat{\mathcal{F}}_i}$ of $\hat{\mathcal{F}}_i(q)$, $i = 1, 2$ is

$$q_{\hat{\mathcal{F}}_1} = \mathcal{K}^{-1}(c^s + c^m + v\xi) \geq q_{\hat{\mathcal{F}}_2} = \mathcal{K}^{-1}(c^s + c^m + \rho\xi).$$

In addition, define

$$\hat{\mathcal{G}}_1(q) = \bar{F}(q)q - (c^s + (c^m + c) + v(\xi - \delta))q,$$
$$\hat{\mathcal{G}}_2(q) = \bar{F}(q)q - (c^s + (c^m + c) + \rho(\xi - \delta))q,$$

of which the maximizers are

$$q_{\hat{\mathcal{G}}_1} = \mathcal{K}^{-1}(c^s + (c^m + c) + v(\xi - \delta)) \geq q_{\hat{\mathcal{G}}_2} = \mathcal{K}^{-1}(c^s + (c^m + c) + \rho(\xi - \delta)).$$

Finally, define

$$\hat{\mathcal{H}}(q) = \bar{F}(q)q - (c^s + c^m + c\xi/\delta)q,$$

and its maximizer $q_{\hat{\mathcal{H}}}$ satisfies

$$q_{\hat{\mathcal{F}}_1} \geq q_{\hat{\mathcal{H}}} = \mathcal{K}^{-1}(c^s + c^m + c\xi/\delta) \geq q_{\hat{\mathcal{G}}_2}.$$

Note that ξ in the definitions above shall be replaced by ξ^s in Proposition A3 and ξ^m in Proposition A4.

Proposition A3. *For the decentralized chain, when only the supplier is regulated, the optimal production quantities* $(\bar{q}^d, \bar{q}_1^{ds})$ *are given as*

(i) if $v < \rho \leq c/\delta$, *then* $\bar{q}_1^{ds} = 0$ *and*

$$\bar{q}^d = \min\left\{\max\left\{q_{\hat{\mathcal{F}}_2}, \frac{\bar{e}}{\xi^s}\right\}, q_{\hat{\mathcal{F}}_1}\right\}, \tag{13}$$

(ii) if $c/\delta < v < \rho$, *then*

$$\bar{q}^d = \bar{q}_1^{ds} = \min\left\{\max\left\{q_{\hat{\mathcal{G}}_2}, \frac{\bar{e}}{\xi^s - \delta}\right\}, q_{\hat{\mathcal{F}}_1}\right\} \tag{14}$$

(iii) if $v \leq c/\delta < \rho$, *let* $\bar{q}_\alpha = \mathcal{K}^{-1}(c^s + c^m + c\xi^s/\delta)$, *then*

$$\begin{cases} \bar{q}^d = \bar{q}_1^{ds} = \max\{q_{\hat{\mathcal{G}}_2}, \frac{\bar{e}}{\xi^s - \delta}\} & \text{if } \bar{e} \leq (\xi^s - \delta)\bar{q}_\alpha, \\ \bar{q}^d = \bar{q}_\alpha, \bar{q}_1^{ds} = \frac{\xi^s \bar{q}_\alpha - \bar{e}}{\delta} & \text{if } (\xi^s - \delta)\bar{q}_\alpha < \bar{e} \leq \xi^s \bar{q}_\alpha, \\ \bar{q}^d = \min\{q_{\hat{\mathcal{F}}_1}, \frac{\bar{e}}{\xi^s}\}, \bar{q}_1^{ds} = 0 & \text{if } \bar{e} > \xi^s \bar{q}_\alpha. \end{cases} \tag{15}$$

The optimal wholesale price $\bar{w}^* = \bar{F}(\bar{q}^d) - c^m$.

Proof. The supplier's problem is $\max_{0\le q_1^s\le q}\pi^s(q,q_1^s)$ with the objective given in (P-15). The optimal $\bar{q}_1^{ds}$ is characterized the same as (P-14). Therefore $\max_{0\le q_1^s\le q}\pi^s(q,q_1^s) = \max_{0\le q}\mathcal{F}(q)$, in which is given in (11), (12) and (10) for $\mathcal{F}(q)$ $v<\rho\le C^s$, $C^s<v<\rho$ and $v\le C^s<\rho$, respectively, with $M_i = c^m$. $\mathcal{F}(q)$ is quasi-concave (Proposition A2), then the optimal production $\bar{q}^d$ can be characterized as given in Proposition A3. ■

Proposition A4. *For the decentralized chain, when only the manufacturer is regulated, the optimal production quantities $(\hat{q}^d, \hat{q}_1^{dm})$ are given as follows.*

(i) If $v<\rho\le c/\delta$, then $\hat{q}_1^{dm}=0$ and

$$\hat{q}^d \in \begin{cases} q_{\hat{\mathcal{F}}_2}, & \text{if } \bar{e}/\xi^m \le q_{\hat{\mathcal{F}}_2} \text{ and } \hat{\mathcal{F}}_2(q_{\hat{\mathcal{F}}_2}) \ge \hat{\mathcal{F}}_1(\frac{\bar{e}}{\xi^m}) \\ \frac{\bar{e}}{\xi^m}, & \text{if } \bar{e}/\xi^m \le q_{\hat{\mathcal{F}}_2} \text{ and } \hat{\mathcal{F}}_2(q_{\mathcal{F}_2}) < \hat{\mathcal{F}}_1(\frac{\bar{e}}{\xi^m}) \\ \min\left\{q_{\hat{\mathcal{F}}_1}, \dfrac{\bar{e}}{\xi^m}\right\} & \text{if } q_{\hat{\mathcal{F}}_2} < \bar{e}/\xi^m, \end{cases}$$

(ii) if $c/\delta<v<\rho$, then $\hat{q}_1^{dm}=\hat{q}^d$ and

$$\hat{q}^d = \begin{cases} q_{\hat{\mathcal{G}}_2} & \text{if } \bar{e}/\xi^m < q_{\hat{\mathcal{G}}_2} \text{ and } \hat{\mathcal{G}}_1(\bar{e}/(\xi^m-\delta)) \le \hat{\mathcal{G}}_2(q_{\hat{\mathcal{G}}_2}), \\ \bar{e}/(\xi^m-\delta) & \text{if } \bar{e}/(\xi^m-\delta) < q_{\hat{\mathcal{G}}_2} \text{ and } \hat{\mathcal{G}}_1(\bar{e}/(\xi^m-\delta)) > \hat{\mathcal{G}}_2(q_{\hat{\mathcal{G}}_2}), \\ \min\{q_{\hat{\mathcal{G}}_1}, \bar{e}/(\xi^m-\delta)\} & \text{if } \bar{e}/(\xi^m-\delta) \ge q_{\hat{\mathcal{G}}_2}, \end{cases}$$

(iii) if $v\le c/\delta<\rho$, then

$$\hat{q}^d = \begin{cases} q_{\hat{\mathcal{G}}_2} & \text{if } \bar{e}/(\xi^m-\delta) < q_{\hat{\mathcal{G}}_2} \text{ and } \hat{\mathcal{G}}_2(q_{\hat{\mathcal{G}}_2}) > \max\{\hat{\mathcal{F}}_1(\bar{e}/\xi^m), \hat{\mathcal{H}}(\bar{e}/(\xi^m-\delta))\}, \\ \bar{e}/(\xi^m-\delta) & \text{if } \bar{e}/(\xi^m-\delta) < q_{\hat{\mathcal{G}}_2} \text{ and } \hat{\mathcal{H}}(\bar{e}/(\xi^m-\delta)) > \max\{\hat{\mathcal{F}}_1(\bar{e}/\xi^m), \hat{\mathcal{G}}_2(q_{\hat{\mathcal{G}}_2})\}, \\ \bar{e}/\xi^m & \text{if } \bar{e}/(\xi^m-\delta) < q_{\hat{\mathcal{G}}_2} \text{ and } \hat{\mathcal{F}}_1(\bar{e}/\xi^m) > \max\{\hat{\mathcal{H}}(\bar{e}/(\xi^m-\delta)), \hat{\mathcal{G}}_2(q_{\hat{\mathcal{G}}_2})\}, \\ \min\{q_{\hat{\mathcal{H}}}, \bar{e}/(\xi^m-\delta)\} & \text{if } \bar{e}/\xi^m < q_{\hat{\mathcal{H}}}, \bar{e}/(\xi^m-\delta) \ge q_{\hat{\mathcal{G}}_2}, \hat{\mathcal{F}}_1(\bar{e}/\xi^m) \le \hat{\mathcal{H}}(\min\{q_{\hat{\mathcal{H}}}, \bar{e}/(\xi^m-\delta)\}), \\ \bar{e}/\xi^m & \text{if } \bar{e}/\xi^m < q_{\hat{\mathcal{H}}}, \bar{e}/(\xi^m-\delta) \ge q_{\hat{\mathcal{G}}_2}, \hat{\mathcal{F}}_1(\bar{e}/\xi^m) > \hat{\mathcal{H}}(\min\{q_{\mathcal{H}}, \bar{e}/(\xi^m-\delta)\}), \\ \min\{q_{\hat{\mathcal{F}}_1}, \bar{e}/\xi^m\} & \text{if } \bar{e}/\xi^m \ge q_{\hat{\mathcal{H}}}, \end{cases} \tag{16}$$

and

$$\hat{q}_1^{dm} = \begin{cases} 0 & \text{if } \hat{q}^d \le \bar{e}/\xi^m, \\ \dfrac{\xi^m \hat{q}^d - \bar{e}}{\delta} & \text{if } \bar{e}/\xi^m < \hat{q}^d \le \bar{e}/(\xi^m-\delta), \\ \hat{q}^d & \text{if } \hat{q}^d > \bar{e}/(\xi^m-\delta), \end{cases} . \tag{17}$$

Proof. The result can be obtained from Proposition A1. ■

Appendix B: Proofs

Proof of Proposition 1.

Note that

$$\frac{\partial \pi^c(q, q_1^s, q_1^m)}{\partial q_1^s} = \begin{cases} -c_1^s + c_2^s - v(\xi_1^s - \xi_2^s), & \text{if } t(q, q_1^s, q_1^m) < \bar{e} \\ -c_1^s + c_2^s - \rho(\xi_1^s - \xi_2^s), & \text{if } t(q, q_1^s, q_1^m) > \bar{e}, \end{cases}$$

and if $t(q, q_1^s, q_1^m) = \bar{e}$, then

$$\begin{cases} \bar{e} - t(q, q_1^s + \varepsilon, q_1^m) = -(\xi_1^s - \xi_2^s)\varepsilon > 0, & \text{if } \varepsilon > 0 \\ t(q, q_1^s + \varepsilon, q_1^m) - \bar{e} = (\xi_1^s - \xi_2^s)\varepsilon > 0, & \text{if } \varepsilon < 0 \end{cases}$$

and

$$\pi^c(q, q_1^s + \varepsilon, q_1^m) - \pi^c(q, q_1^s, q_1^m) = \begin{cases} (-c_1^s + c_2^s)\varepsilon - v(\xi_1^s - \xi_2^s)\varepsilon, & \text{if } \varepsilon > 0 \\ (-c_1^s + c_2^s)\varepsilon - \rho(\xi_1^s - \xi_2^s)\varepsilon, & \text{if } \varepsilon < 0 \end{cases}.$$

Similarly,

$$\frac{\partial \pi^c(q, q_1^s, q_1^m)}{\partial q_1^m} = \begin{cases} -c_1^m + c_2^m - v(\xi_1^m - \xi_2^m), & \text{if } t(q, q_1^s, q_1^m) < \bar{e} \\ -c_1^m + c_2^m - \rho(\xi_1^m - \xi_2^m), & \text{if } t(q, q_1^s, q_1^m) > \bar{e} \end{cases}$$

and if $t(q, q_1^s, q_1^m) = \bar{e}$, then

$$\begin{cases} \bar{e} - t(q, q_1^s, q_1^m + \varepsilon) = -(\xi_1^m - \xi_2^m)\varepsilon > 0, & \text{if } \varepsilon > 0 \\ t(q, q_1^s, q_1^m + \varepsilon) - \bar{e} = (\xi_1^m - \xi_2^m)\varepsilon > 0, & \text{if } \varepsilon < 0 \end{cases}$$

and

$$\pi^c(q, q_1^s, q_1^m + \varepsilon) - \pi^c(q, q_1^s, q_1^m) = \begin{cases} (-c_1^m + c_2^m)\varepsilon - v(\xi_1^m - \xi_2^m)\varepsilon, & \text{if } \varepsilon > 0 \\ (-c_1^m + c_2^m)\varepsilon - \rho(\xi_1^m - \xi_2^m)\varepsilon, & \text{if } \varepsilon < 0 \end{cases}$$

(i) If $\rho \leq C^s$, then the values of

$$-c_1^s + c_2^s - v(\xi_1^s - \xi_2^s) \leq -c_1^s + c_2^s - \rho(\xi_1^s - \xi_2^s) \leq 0.$$

Hence $q_1^{cs} = 0$. The case of $\rho \leq C^m$ can be similarly proved.

(ii) If $C^s < v$, then the values of

$$0 \leq -c_1^s + c_2^s - v(\xi_1^s - \xi_2^s) \leq -c_1^s + c_2^s - \rho(\xi_1^s - \xi_2^s).$$

Hence $q_1^{cs} = q^c$. The case $C^m < v$ can be similarly proved.

(iii) If $v \leq C^s < \rho$, then

$$-c_1^s + c_2^s - v(\xi_1^s - \xi_2^s) \leq 0, -c_1^s + c_2^s - \rho(\xi_1^s - \xi_2^s) > 0.$$

Hence, if $t(q^c, q_1^{cs}, q_1^{cm}) < \bar{e}$, then

$$\frac{\partial \pi^c(q, q_1^s, q_1^m)}{\partial q_1^s} = -c_1^s + c_2^s - v(\xi_1^s - \xi_2^s) < 0$$

and $q_1^{cs} = 0$; if $t(q^c, q_1^{cs}, q_1^{cm}) = \bar{e}$, then

$$q_1^{cs} = \frac{(\xi_2^s + \xi_2^m)q^c + (\xi_1^m - \xi_2^m)q_1^{cm} - \bar{e}}{\xi_2^s - \xi_1^s} = \frac{(\xi_2^s + \xi_2^m)q^c - \bar{e}}{\xi_2^s - \xi_1^s};$$

if $t(q^c, q_1^{cs}, q_1^{cm}) > \bar{e}$, then

$$\frac{\partial \pi^c(q, q_1^s, q_1^m)}{\partial q_1^s} = -c_1^s + c_2^s - \rho(\xi_1^s - \xi_2^s) > 0$$

and $q_1^{cs} = q^c$. Therefore,

$$q_1^{cs} = \begin{cases} 0, & \text{if } t(q^c, q_1^{cs}, q_1^{cm}) < \bar{e} \\ \dfrac{(\xi_2^s + \xi_2^m)q^c - \bar{e}}{\xi_2^s - \xi_1^s}, & \text{if } t(q^c, q_1^{cs}, q_1^{cm}) = \bar{e} \\ q^c, & \text{if } t(q^c, q_1^{cs}, q_1^{cm}) > \bar{e} \end{cases}.$$

Therefore, the supplier may use both technologies simultaneously. The case of $v \leq C^m < \rho$ can be similarly argued. ■

Lemma B1. *Let*

$$f(q) = (1 - A)q - \rho\,[Bq - \tilde{e}]^+ + v\,[\tilde{e} - Bq]^+ - \mathrm{E}[q - D]^+.$$

We can characterize the results as follows.

(i) *If* $\dfrac{\tilde{e}}{B} \leq \bar{F}^{-1}(A + Bv)$, *then* $\max_{0 \leq q \leq \frac{\tilde{e}}{B}} f(q) = f\left(\dfrac{\tilde{e}}{B}\right)$

(ii) *If* $\dfrac{\tilde{e}}{B} \geq \bar{F}^{-1}(A + Bv)$, *then* $\max_{0 \leq q \leq \frac{\tilde{e}}{B}} f(q) = f\left(\bar{F}^{-1}(A + Bv)\right)$

(iii) *If* $\dfrac{\tilde{e}}{B} \leq \bar{F}^{-1}(A + B\rho)$, *then* $\max_{q \geq \frac{\tilde{e}}{B}} f(q) = f\left(\bar{F}^{-1}(A + B\rho)\right)$

(iv.) *If* $\dfrac{\tilde{e}}{B} \geq \bar{F}^{-1}(A + B\rho)$, *then* $\max_{q \geq \frac{\tilde{e}}{B}} f(q) = f\left(\dfrac{\tilde{e}}{B}\right)$

(v.) *If* $\dfrac{\tilde{e}}{B} \leq \bar{F}^{-1}(A + Bv)$, *then*

$$\max_{0 \leq q} f(q) = f\left(\max\left\{\bar{F}^{-1}(A + B\rho), \frac{\tilde{e}}{B}\right\}\right)$$

(vi) *If* $\dfrac{\tilde{e}}{B} > \bar{F}^{-1}(A + Bv)$, *then*

$$\max_{0 \leq q} f(q) = f\left(\bar{F}^{-1}(A + Bv)\right)$$

Proof. Firstly, the derivative of $(1-A)q - \mathrm{E}[q-D]^+$ is $1 - A - F(q) = \bar{F}(q) - A$. Secondly, if $0 < q < \dfrac{\tilde{e}}{B}$, then the derivative of $f(q)$ is $\bar{F}(q) - A - Bv$. If $q > \dfrac{\tilde{e}}{B}$, then the derivative of $f(q)$ is $\bar{F}(q) - A - B\rho$. Therefore, we can obtain the result (i)-(vi). ■

Lemma B2. *Suppose $C^s \le C^m$. Then we have the following equalities*

$$c_1^s + c_2^m + C^m(\xi_1^s + \xi_2^m) - [c_2^s + c_2^m + C^s(\xi_2^s + \xi_2^m)] = (\xi_1^s + \xi_2^m)(C^m - C^s) \tag{18}$$

$$c_1^s + c_2^m + \rho(\xi_1^s + \xi_2^m) - [c_2^s + c_2^m + C^s(\xi_2^s + \xi_2^m)] = (\xi_1^s + \xi_2^m)(\rho - C^s) \tag{19}$$

$$c_1^s + c_1^m + \rho(\xi_1^s + \xi_1^m) - [c_1^s + c_2^m + C^m(\xi_1^s + \xi_2^m)] = (\xi_1^s + \xi_2^m)(\rho - C^m) \tag{20}$$

Proof. This can be easily proved by some algebra and the definition of C^s and C^m. Thus we omit the details. ■

Proof of Theorem 2.

(i) By Proposition 1, if $v < \rho \le C^s \le C^m$, then $q_1^{cm} = 0$ and $q_1^{cs} = 0$. Hence the integrated supply chain problem can be reduced as follows:

$$\max_{q\ge 0} \left\{(1 - c_2^s - c_2^m)q - \rho[(\xi_2^s + \xi_2^m)q - \bar{e}]^+ + v[\bar{e} - (\xi_2^s + \xi_2^m)q]^+ - \mathrm{E}[q-D]^+\right\}.$$

Following (v) and (vi) of Lemma **??**, we can obtain the optimal solution.

(ii) By Proposition 1, if $C^s \le C^m < v < \rho$, then $q_1^{cm} = q^c$ and $q_1^{cs} = q^c$. Hence the integrated supply chain problem can be reduced as follows:

$$\max_{q\ge 0} \left\{[1 - c_1^s - c_1^m]q - \rho[(\xi_1^s + \xi_1^m)q - \bar{e}]^+ + v[\bar{e} - (\xi_1^s + \xi_1^m)q]^+ - \mathrm{E}[q-D]^+\right\}.$$

Following (v) and (vi) of Lemma **??**, we can obtain the optimal solution.

(iii) By Proposition 1, if $C^s < v < \rho \le C^m$, then $q_1^{cm} = 0$ and $q_1^{cs} = q^c$. Hence the integrated supply chain problem can be reduced as follows:

$$\max_{q\ge 0} \left\{(1 - c_1^s - c_2^m)q - \rho[(\xi_1^s + \xi_2^m)q - \bar{e}]^+ + v[\bar{e} - (\xi_1^s + \xi_2^m)q]^+ - \mathrm{E}[q-D]^+\right\}.$$

Again the optimal solution can be obtained following (v) and (vi) of Lemma **??**.

■

Proof of Theorem 3.

(i) By Proposition 1, if $v \le C^s < \rho \le C^m$, then the optimal solution $(q^c, q_1^{cs}, q_1^{cm})$ satisfies $q_1^{cm} = 0$ and

$$q_1^{cs} = \begin{cases} 0, & \text{if } t(q^c, q_1^{cs}, q_1^{cm}) < \bar{e} \\ \dfrac{(\xi_2^s + \xi_2^m)q^c - \bar{e}}{\xi_2^s - \xi_1^s}, & \text{if } t(q^c, q_1^{cs}, q_1^{cm}) = \bar{e} \\ q^c, & \text{if } t(q^c, q_1^{cs}, q_1^{cm}) > \bar{e} \end{cases}.$$

Hence, the integrated supply chain's problem can be reduced as follows.

$$\max_{q \ge q_1^s \ge 0} \left\{ (1 - c_2^s - c_2^m)q - (c_1^s - c_2^s)q_1^s - \rho[(\xi_2^s + \xi_2^m)q - (\xi_2^s - \xi_1^s)q_1^s - \bar{e}]^+ \right.$$
$$\left. + v[\bar{e} - (\xi_2^s + \xi_2^m)q + (\xi_2^s - \xi_1^s)q_1^s]^+ - \mathrm{E}[q - D]^+ \right\}. \tag{21}$$

Moreover,

- if $t(q^c, q_1^{cs}, q_1^{cm}) < \bar{e}$(i.e. $(\xi_2^s + \xi_2^m)q^c - (\xi_2^s - \xi_1^s)q_1^{cs} - \bar{e} < 0$), then we have $q_1^{cs} = 0$ and the problem can be further simplified to

$$\begin{aligned} & \max \left\{ [1 - c_2^s - c_2^m - v(\xi_2^s + \xi_2^m)]q - \mathrm{E}[q - D]^+ + v\bar{e} \right\} \\ & s.t., q \ge 0, (\xi_2^s + \xi_2^m)q - \bar{e} < 0. \\ = & \max_{0 \le q < \frac{\bar{e}}{\xi_2^s + \xi_2^m}} \left\{ [1 - c_2^s - c_2^m - v(\xi_2^s + \xi_2^m)]q - \mathrm{E}[q - D]^+ + v\bar{e} \right\}, \end{aligned}$$

  The unconstrained maximizer is $q_\beta(v, v) = \bar{F}^{-1}(c_2^s + c_2^m + v(\xi_2^s + \xi_2^m))$ by applying the first-order condition.

- if $t(q^c, q_1^{cs}, q_1^{cm}) = \bar{e}$, then we have $q_1^{cs} = \dfrac{(\xi_2^s + \xi_2^m)q^c - \bar{e}}{\xi_2^s - \xi_1^s}$ and the problem can be further simplifed to

$$\max_{\frac{\bar{e}}{\xi_2^s + \xi_2^m} \le q \le \frac{\bar{e}}{\xi_1^s + \xi_2^m}} \left\{ [1 - c_2^s - c_2^m - C^s(\xi_2^s + \xi_2^m)]q - \mathrm{E}[q - D]^+ + C^s\bar{e} \right\},$$

  The unconstrained maximizer is $q_\beta(C^s, C^s) = \bar{F}^{-1}(c_2^s + c_2^m + C^s(\xi_2^s + \xi_2^m))$ by applying the first-order condition.

- if $t(q^c, q_1^{cs}, q_1^{cm}) > \bar{e}$(i.e. $(\xi_2^s + \xi_2^m)q^c - (\xi_2^s - \xi_1^s)q_1^{cs} - \bar{e} > 0$), then we have $q_1^{cs} = q^c$ and the problem can be further simplified to

$$\begin{aligned} & \max \left\{ [1 - c_1^s - c_2^m - \rho(\xi_1^s + \xi_2^m)]q - \mathrm{E}[q - D]^+ + \rho\bar{e} \right\} \\ & s.t., q \ge 0, (\xi_1^s + \xi_2^m)q - \bar{e} > 0. \\ = & \max_{\frac{\bar{e}}{\xi_1^s + \xi_2^m} < q} \left\{ [1 - c_1^s - c_2^m - \rho(\xi_1^s + \xi_2^m)]q - \mathrm{E}[q - D]^+ + \rho\bar{e} \right\}, \end{aligned}$$

  The unconstrained maximizer is $q_\gamma(\rho, \rho) = \bar{F}^{-1}(c_1^s + c_2^m + \rho(\xi_1^s + \xi_2^m))$ by applying the first-order condition.

By combining these three cases, under the condition $C^s < v \le C^m < \rho$, the problem becomes

$$\max \left\{ \max_{0 \le q < \frac{\bar{e}}{\xi_2^s + \xi_2^m}} \pi^c(q,0,0), \max_{\frac{\bar{e}}{\xi_2^s+\xi_2^m} \le q \le \frac{\bar{e}}{\xi_1^s+\xi_2^m}} \pi^c\left(q, \frac{(\xi_2^s + \xi_2^m)q - \bar{e}}{\xi_2^s - \xi_1^s}, 0\right), \max_{\frac{\bar{e}}{\xi_1^s+\xi_2^m} < q} \pi^c(q,q,0) \right\}.$$

Since $q_\gamma(\rho,\rho)) < q_\beta(C^s, C^s) < q_\beta(v,v)$, we consider the following cases:

- $\frac{\bar{e}}{\xi_1^s+\xi_2^m} < q_\gamma(\rho,\rho)$
- $q_\gamma(\rho,\rho) \le \frac{\bar{e}}{\xi_1^s+\xi_2^m} \le q_\beta(C^s, C^s)$
- $\frac{\bar{e}}{\xi_2^s+\xi_2^m} \le q_\beta(C^s, C^s) < \frac{\bar{e}}{\xi_1^s+\xi_2^m}$
- $q_\beta(C^s, C^s) < \frac{\bar{e}}{\xi_2^s+\xi_2^m} < q_\beta(v,v)$
- $q_\beta(v,v) \le \frac{\bar{e}}{\xi_2^s+\xi_2^m}$

which can be used to specify the optimal solution of the following functions

$$\begin{cases} \pi^c(q,q,0), & \text{if } q > \frac{\bar{e}}{\xi_1^s+\xi_2^m} \\ \pi^c\left(q, \frac{(\xi_2^s+\xi_2^m)q-\bar{e}}{\xi_2^s-\xi_1^s}, 0\right), & \text{if } 0 \le \frac{(\xi_2^s+\xi_2^m)q-\bar{e}}{\xi_2^s-\xi_1^s} \le q \\ \pi^c(q,0,0) & \text{if } q < \frac{\bar{e}}{\xi_2^s+\xi_2^m} \end{cases}.$$

For example, if $q_\beta(C^s, C^s) \in \left[\frac{\bar{e}}{\xi_2^s+\xi_2^m}, \frac{\bar{e}}{\xi_1^s+\xi_2^m}\right)$, then

- $\pi^c(q,0,0)$ is increasing on $\left(0, \dfrac{\bar{e}}{\xi_2^s + \xi_2^m}\right)$;
- $\pi^c\left(q, \dfrac{(\xi_2^s + \xi_2^m)q - \bar{e}}{\xi_2^s - \xi_1^s}, 0\right)$ is increasing on $\left[\dfrac{\bar{e}}{\xi_2^s + \xi_2^m}, q_\beta(C^s, C^s)\right)$;
- $\pi^c\left(q, \dfrac{(\xi_2^s + \xi_2^m)q - \bar{e}}{\xi_2^s - \xi_1^s}, 0\right)$ is decreasing on $\left(q_\beta(C^s, C^s), \dfrac{\bar{e}}{\xi_1^s + \xi_2^m}\right)$;
- $\pi^c(q,q,0)$ is decreasing on $\left[\dfrac{\bar{e}}{\xi_1^s + \xi_2^m}, \infty\right)$.

Hence the optimal solution of the integrated supply chain is $q_\beta(C^s, C^s)$.

Other cases can be proved using similar arguments. For brevity, we omit the details. The complete proof is available from the authors upon request. ■

Proof of Lemma 1.

We only need to prove that

$$\begin{aligned} &-\rho \left\{[t^s(q,q_1^s) - e^s]^+ + [t^m(q,q_1^m) - e^m]^+\right\} + v\left\{[e^s - t^s(q,q_1^s)]^+ + [e^m - t^m(q,q_1^m)]^+\right\} \\ &- \left\{-\rho\left[t^s(q,q_1^s) + t^m(q,q_1^m) - e^s - e^m\right]^+ + v\left[e^s + e^m - t^s(q,q_1^s) - t^m(q,q_1^m)\right]^+\right\} \ge 0 \end{aligned}$$

Because the triangular inequality of $[x]^+$ holds and $[x]^+ - [-x]^+$ is linear, we can obtain

$$\begin{aligned}
& -\rho\left\{[t^s(q,q_1^s)-e^s]^+ + [t^m(q,q_1^m)-e^m]^+\right\} + v\left\{[e^s - t^s(q,q_1^s)]^+ + [e^m - t^m(q,q_1^m)]^+\right\} \\
= \; & (v-\rho)\left\{[t^s(q,q_1^s)-e^s]^+ + [t^m(q,q_1^m)-e^m]^+\right\} \\
& +v\left\{[t^s(q,q_1^s)-e^s]^+ + [t^m(q,q_1^m)-e^m]^+ - [t^s(q,q_1^s)-e^s]^+ - [t^m(q,q_1^m)-e^m]^+\right\} \\
\leq \; & (v-\rho)\left\{[t^s(q,q_1^s)+t^m(q,q_1^m)-e^s-e^m]^+\right\} \\
& +v\left\{[t^s(q,q_1^s)-e^s]^+ + [t^m(q,q_1^m)-e^m]^+ - [t^s(q,q_1^s)-e^s]^+ - [t^m(q,q_1^m)-e^m]^+\right\} \\
= \; & -\rho\left\{[t^s(q,q_1^s)+t^m(q,q_1^m)-e^s-e^m]^+\right\} + v\left\{[t^s(q,q_1^s)+t^m(q,q_1^m)-e^s-e^m]^+\right\}.
\end{aligned}$$

So the result follows. ∎

Proof of Theorem 4.

(i) By Proposition 1, if $v < \rho \leq C^s \leq C^m$, then $q_1^{scm} = 0$ and $q_1^{scs} = 0$. Assume $\frac{e^s}{\xi_2^s} \leq \frac{e^m}{\xi_2^m} \leq \frac{e^s}{\xi_1^s} \leq \frac{e^m}{\xi_1^m}$. Hence the integrated supply chain problem can be reduced as follows:

$$\max_{q\geq 0}\left\{(1-c_2^s-c_2^m)q - \rho[\xi_2^s q - e^s]^+ - \rho[\xi_2^m q - e^m]^+ + v[e^s - \xi_2^s q]^+ + v[e^m - \xi_2^m q]^+ - \mathrm{E}[q-D]^+\right\}.$$

We divide the domain $[0,\infty)$ into three parts: $\left[0, \frac{e^s}{\xi_2^s}\right)$, $\left[\frac{e^s}{\xi_2^s}, \frac{e^m}{\xi_2^m}\right)$, $\left[\frac{e^m}{\xi_2^m}, \infty\right)$. If $q \in \left[0, \frac{e^s}{\xi_2^s}\right)$, then the objective function can be simplified to

$$(1-c_2^s-c_2^m)q + v(e^s - \xi_2^s q) + v(e^m - \xi_2^m q) - \mathrm{E}[q-D]^+$$

and the corresponding stationary point is $q_\beta(v,v) = \bar{F}^{-1}(c_2^s + c_2^m + v(\xi_2^s + \xi_2^m))$. Also, if $q \in \left[\frac{e^s}{\xi_2^s}, \frac{e^m}{\xi_2^m}\right]$, then the objective function can be simplified to

$$(1-c_2^s-c_2^m)q + \rho(e^s - \xi_2^s q) + v(e^m - \xi_2^m q) - \mathrm{E}[q-D]^+$$

and the corresponding stationary points are $q_\beta(\rho,v) = \bar{F}^{-1}(c_2^s + c_2^m + \rho\xi_2^s + v\xi_2^m)$. If $q \in \left(\frac{e^m}{\xi_2^m}, \infty\right)$, then the objective function can be simplified to

$$(1-c_2^s-c_2^m)q + \rho(e^s - \xi_2^s q) + \rho(e^m - \xi_2^m q) - \mathrm{E}[q-D]^+$$

and the corresponding stationary point is $q_\beta(\rho,\rho) = \bar{F}^{-1}(c_2^s + c_2^m + \rho(\xi_2^s + \xi_2^m))$.

If $q_\beta(v,v) \lneqq \frac{e^s}{\xi_2^s}$, then $q^{sc} = q_\beta(v,v)$; if $q_\beta(v,v) \geq \frac{e^s}{\xi_2^s}$ and $q_\beta(\rho,v) \lneqq \frac{e^m}{\xi_2^m}$, then $q^{sc} = \max\left\{\frac{e^s}{\xi_2^s}, q_\beta(\rho,v)\right\}$; if $q_\beta(\rho,v) \geq \frac{e^m}{\xi_2^m}$, then $q^{sc} = \max\left\{\frac{e^m}{\xi_2^m}, q_\beta(\rho,\rho)\right\}$. Thus, we can obtain the optimal solution.

(ii) By Proposition 1, if $C^s \leq C^m < v < \rho$, then $q_1^{scm} = q^{sc}$ and $q_1^{scs} = q^{sc}$. Hence the integrated supply chain problem can be reduced as follows:

$$\max_{q\geq 0}\left\{[1-c_1^s-c_1^m]q - \rho[\xi_1^s q - e^s]^+ - \rho[\xi_1^m q - e^m]^+ + v[e^s - \xi_1^s q]^+ + v[e^m - \xi_1^m q]^+ - \mathrm{E}[q-D]^+\right\}.$$

We divide the domain $[0, \infty)$ into three parts: $\left[0, \frac{e^s}{\xi_1^s}\right)$, $\left[\frac{e^s}{\xi_1^s}, \frac{e^m}{\xi_1^m}\right)$, $\left[\frac{e^m}{\xi_1^m}, \infty\right)$. If $q \in \left[0, \frac{e^s}{\xi_1^s}\right)$, then the objective function can be simplified to

$$(1 - c_1^s - c_1^m)q + v(e^s - \xi_1^s q) + v(e^m - \xi_1^m q) - \mathrm{E}[q - D]^+$$

and the corresponding stationary point is $q_\alpha(v, v) = \bar{F}^{-1}(c_1^s + c_1^m + v(\xi_1^s + \xi_1^m))$. Also, if $q \in \left[\frac{e^s}{\xi_1^s}, \frac{e^m}{\xi_1^m}\right]$, then the objective function can be simplified to

$$(1 - c_1^s - c_1^m)q + \rho(e^s - \xi_1^s q) + v(e^m - \xi_1^m q) - \mathrm{E}[q - D]^+$$

and the corresponding stationary points are $q_\alpha(\rho, v) = \bar{F}^{-1}(c_1^s + c_1^m + \rho\xi_1^s + v\xi_1^m)$. If $q \in \left(\frac{e^m}{\xi_1^m}, \infty\right)$, then the objective function can be simplified to

$$(1 - c_1^s - c_1^m)q + \rho(e^s - \xi_1^s q) + \rho(e^m - \xi_1^m q) - \mathrm{E}[q - D]^+$$

and the corresponding stationary point is $q_\alpha(\rho, \rho) = \bar{F}^{-1}(c_1^s + c_1^m + \rho(\xi_1^s + \xi_1^m))$.

If $q_\alpha(v, v) \lneqq \frac{e^s}{\xi_1^s}$, then $q^{sc} = q_\alpha(v, v)$; if $q_\alpha(v, v) \geq \frac{e^s}{\xi_1^s}$ and $q_\alpha(\rho, v) \lneqq \frac{e^m}{\xi_1^m}$, then $q^{sc} = \max\left\{\frac{e^s}{\xi_1^s}, q_\alpha(\rho, v)\right\}$; if $q_\alpha(\rho, v) \geq \frac{e^m}{\xi_1^m}$, then $q^{sc} = \max\left\{\frac{e^m}{\xi_1^m}, q_\alpha(\rho, \rho)\right\}$. Thus, we can obtain the optimal solution.

(iii) By Proposition 1, if $C^s < v < \rho \leq C^m$, then $q_1^{cm} = 0$ and $q_1^{cs} = q^c$.

$$\max_{q \geq 0} \left\{[1 - c_1^s - c_2^m]q - \rho[\xi_1^s q - e^s]^+ - \rho[\xi_2^m q - e^m]^+ + v[e^s - \xi_1^s q]^+ + v[e^m - \xi_2^m q]^+ - \mathrm{E}[q - D]^+\right\},$$

We divide the domain $[0, \infty)$ into three parts: $\left[0, \frac{e^s}{\xi_1^s}\right)$, $\left[\frac{e^s}{\xi_1^s}, \frac{e^m}{\xi_2^m}\right)$, $\left[\frac{e^m}{\xi_2^m}, \infty\right)$. If $q \in \left[0, \frac{e^s}{\xi_1^s}\right)$, then the objective function can be simplified to

$$(1 - c_1^s - c_2^m)q + v(e^s - \xi_1^s q) + v(e^m - \xi_2^m q) - \mathrm{E}[q - D]^+$$

and the corresponding stationary point $q_\gamma(v, v) = \bar{F}^{-1}(c_1^s + c_2^m + v(\xi_1^s + \xi_2^m))$. Also, if $q \in \left[\frac{e^s}{\xi_1^s}, \frac{e^m}{\xi_2^m}\right]$, then the objective function can be simplified to

$$(1 - c_1^s - c_2^m)q + \rho(e^s - \xi_1^s q) + v(e^m - \xi_2^m q) - \mathrm{E}[q - D]^+$$

and the corresponding stationary point $q_\gamma(\rho, v) = \bar{F}^{-1}(c_1^s + c_2^m + \rho\xi_1^s + v\xi_2^m)$. If $q \in \left(\frac{e^m}{\xi_2^m}, \infty\right)$, then the objective function can be simplified to

$$(1 - c_1^s - c_2^m)q + \rho(e^s - \xi_1^s q) + \rho(e^m - \xi_2^m q) - \mathrm{E}[q - D]^+$$

and the corresponding stationary point $q_\gamma(\rho, \rho) = \bar{F}^{-1}(c_1^s + c_2^m + \rho(\xi_1^s + \xi_2^m))$.

If $q_\gamma(v, v) \lneqq \frac{e^s}{\xi_1^s}$, then $q^{sc} = q_\gamma(v, v)$; if $q_\gamma(v, v) \geq \frac{e^s}{\xi_1^s}$ and $q_\gamma(\rho, v) \lneqq \frac{e^m}{\xi_2^m}$, then $q^{sc} = \max\left\{\frac{e^s}{\xi_1^s}, q_\gamma(\rho, v)\right\}$; if $q_\gamma(\rho, v) \geq \frac{e^m}{\xi_2^m}$, then $q^{sc} = \max\left\{\frac{e^m}{\xi_2^m}, q_\gamma(\rho, \rho)\right\}$. Thus, we can obtain the optimal solution.

■

Proof of Theorem 5.

(i) By Proposition 1, if $v \le C^s < \rho \le C^m$, then the optimal solution $(q^{sc}, q_1^{scs}, q_1^{scm})$ satisfies $q_1^{scm} = 0$ and

$$q_1^{scs} = \begin{cases} 0, & \text{if } t^s(q^{sc}, q_1^{scs}) < e^s \\ \dfrac{\xi_2^s q^{sc} - e^s}{\xi_2^s - \xi_1^s}, & \text{if } t^s(q^{sc}, q_1^{scs}) = e^s \\ q^{sc}, & \text{if } t^s(q^{sc}, q_1^{scs}) > e^s \end{cases} .$$

Hence, the integrated supply chain problem can be reduced as follows.

$$\max_{q \ge q_1^s \ge 0} \pi^{sc}(q, q_1^s, 0) = \big\{(1 - c_2^s - c_2^m)q - (c_1^s - c_2^s)q_1^s - \rho[\xi_2^s q - (\xi_2^s - \xi_1^s)q_1^s - e^s]^+ - \rho[\xi_2^m q - e^m]^+ \\ + v[e^s - \xi_2^s q + (\xi_2^s - \xi_1^s)q_1^s]^+ + v[e^m - \xi_2^m q]^+ - \mathrm{E}[q - D]^+\big\} . \tag{22}$$

Since we assume that $\frac{e^s}{\xi_2^s} \le \frac{e^m}{\xi_2^m} \le \frac{e^s}{\xi_1^s} \le \frac{e^m}{\xi_1^m}$, we will discuss the following *four* subproblems in order to obtain the optimal solutions of problem (22):

$$\max\left\{ \max_{0 \le q < \frac{e^s}{\xi_2^s}} \pi^{sc}(q,0,0), \max_{\frac{e^s}{\xi_2^s} \le q < \frac{e^m}{\xi_2^m}} \pi^{sc}\left(q, \frac{\xi_2^s q - e^s}{\xi_2^s - \xi_1^s}, 0\right), \max_{\frac{e^m}{\xi_2^m} \le q < \frac{e^s}{\xi_1^s}} \pi^{sc}\left(q, \frac{\xi_2^s q - e^s}{\xi_2^s - \xi_1^s}, 0\right), \max_{\frac{e^s}{\xi_1^s} \le q} \pi^{sc}(q,q,0) \right\}.$$

This is because

- if $t^s(q, q_1^s) < e^s$, i.e. $\xi_2^s q - (\xi_2^s - \xi_1^s)q_1^s - e^s < 0$, then we have $q_1^{scs} = 0$ (because the resulting objective function is decreasing in q_1^s), and $\xi_2^s q^{sc} - e^s < 0$. Meanwhile, $\xi_2^m q^{sc} - e^m < 0$. Hence the first considered subproblem is

$$\max_{0 \le q < \frac{e^s}{\xi_2^s}} \pi^{sc}(q,0,0) = \big\{[1 - c_2^s - c_2^m - v(\xi_2^s + \xi_2^m)]q - \mathrm{E}[q - D]^+ + v(e^s + e^m)\big\},$$

  whose unconstrained maximizer $q_\beta(v, v) = \bar{F}^{-1}(c_2^s + c_2^m + v(\xi_2^s + \xi_2^m))$ by applying the first-order condition.
- if $t^s(q, q_1^s) = e^s$ and $\xi_2^m q - e^m < 0$, then we have $q_1^{scs} = \dfrac{\xi_2^s q^{sc} - e^s}{\xi_2^s - \xi_1^s}$, and $\dfrac{e^s}{\xi_2^s} \le q^{sc} \le \dfrac{e^m}{\xi_2^m}$. Hence the second considered subproblem is

$$\max_{\frac{e^s}{\xi_2^s} \le q < \frac{e^m}{\xi_2^m}} \pi^{sc}\left(q, \frac{\xi_2^s q - e^s}{\xi_2^s - \xi_1^s}, 0\right) = \big\{[1 - c_2^s - c_2^m - C^s \xi_2^s - v\xi_2^m]q - \mathrm{E}[q - D]^+ + C^s e^s + v e^m\big\},$$

  whose unconstrained maximizer $q_\beta(C^s, v) = \bar{F}^{-1}(c_2^s + c_2^m + C^s \xi_2^s + v \xi_2^m)$ by applying the first-order condition.
- if $t^s(q, q_1^s) = e^s$ and $\xi_2^m q - e^m \ge 0$, then we have $q_1^{scs} = \dfrac{\xi_2^s q^{sc} - e^s}{\xi_2^s - \xi_1^s}$, $\dfrac{e^m}{\xi_2^m} \le q^{sc} \le \dfrac{e^s}{\xi_1^s}$. Hence the third considered subproblem is

$$\max_{\frac{e^m}{\xi_2^m} \le q < \frac{e^s}{\xi_1^s}} \pi^{sc}\left(q, \frac{\xi_2^s q - e^s}{\xi_2^s - \xi_1^s}, 0\right) = \big\{[1 - c_2^s - c_2^m - C^s \xi_2^s - \rho\xi_2^m]q - \mathrm{E}[q - D]^+ + C^s e^s + \rho e^m\big\},$$

whose unconstrained maximizer $q_\beta(C^s,\rho) = \bar{F}^{-1}(c_2^s + c_2^m + C^s\xi_2^s + \rho\xi_2^m)$ by applying the first-order condition.

– if $t^s(q,q_1^s) > e^s$, i.e. $\xi_2^s q - (\xi_2^s - \xi_1^s)q_1^s - e^s > 0$, then we have $q_1^{scs} = q^{sc}$, $\xi_1^s q^{sc} - e^s > 0$; and the fourth considered subproblem is

$$\max_{\frac{e^s}{\xi_1^s}<q} \pi^{sc}(q,q,0) = \left\{[1 - c_1^s - c_2^m - \rho\xi_1^s + \rho\xi_2^m)]q - \mathrm{E}[q-D]^+ + \rho e^s + \rho e^m\right\},$$

whose unconstrained maximizer $q_\gamma(\rho,\rho) = \bar{F}^{-1}(c_1^s + c_2^m + \rho\xi_1^s + \rho\xi_2^m)$ by applying the first-order condition.

Now, if $q_\beta(v,v) < \frac{e^s}{\xi_2^s}$, then because $q_\beta(v,v) \geq q_\beta(C^s,v) \geq q_\beta(C^s,\rho) \geq q_\gamma(\rho,\rho)$, the optimal solution $q^{sc} = q_\beta(v,v)$. Similarly, if $q_\beta(v,v) \geq \frac{e^s}{\xi_2^s}$ and $q_\beta(C^s,v) < \frac{e^m}{\xi_2^m}$, then

$$q^{sc} = \max\left\{\frac{e^s}{\xi_2^s}, q_\beta(C^s,v)\right\}.$$

If $q_\beta(C^s,v) \geq \frac{e^m}{\xi_2^m}$ and $q_\beta(C^s,\rho) < \frac{e^s}{\xi_1^s}$, then

$$q^{sc} = \max\left\{\frac{e^m}{\xi_2^m}, q_\beta(C^s,\rho)\right\}.$$

If $q_\beta(C^s,\rho) \geq \frac{e^s}{\xi_1^s}$, then

$$q^{sc} = \max\left\{\frac{e^s}{\xi_1^s}, q_\gamma(\rho,\rho)\right\}.$$

Cases (ii) and (iii) can be similarly proven and so we omit the details. ■

Lemma 1. *The inverse function $w(q)$ can be specified as follows.*

(i) If $v < \rho \leq C^m$

$$w(q) = \begin{cases} \bar{F}(q) - c_2^m - v\xi_2^m & \text{if } q \leq \frac{e^m}{\xi_2^m}, \\ \bar{F}(q) - c_2^m - \rho\xi_2^m & \text{if } q > \frac{e^m}{\xi_2^m}. \end{cases} \tag{23}$$

(ii) if $C^m < v < \rho$

$$w(q) = \begin{cases} \bar{F}(q) - c_1^m - v\xi_1^m & \text{if } q \leq \frac{e^m}{\xi_1^m}, \\ \bar{F}(q) - c_1^m - \rho\xi_1^m & \text{if } q > \frac{e^m}{\xi_1^m}. \end{cases} \tag{24}$$

(iii) if $v \leq C^m < \rho$

$$w(q) = \begin{cases} \bar{F}(q) - c_2^m - v\xi_2^m & \text{if } q \leq \frac{e^m}{\xi_2^m}, \\ \bar{F}(q) - c_2^m - C^m\xi_2^m & \text{if } \frac{e^m}{\xi_2^m} < q \leq \frac{e^m}{\xi_1^m}, \\ \bar{F}(q) - c_1^m - \rho\xi_1^m & \text{if } q > \frac{e^m}{\xi_1^m}. \end{cases} \tag{25}$$

Proof. In the proof, we only show case (iii) because cases (i) and (ii) can be proved analogously.

(iii) $v \le C^m < \rho$: The function $q(w)$ in Proposition A1 is decreasing on the intervals $\left(0, \bar{F}(\frac{e^m}{\xi_1^m}) - c_1^m - \rho\xi_1^m\right)$, $\left(\bar{F}(\frac{e^m}{\xi_1^m}) - c_2^m - C^m\xi_2^m, \bar{F}(\frac{e^m}{\xi_2^m}) - c_2^m - C^m\xi_2^m\right)$, and $\left(\bar{F}(\frac{e^m}{\xi_2^m}) - c_2^m - v\xi_2^m, \infty\right)$. Hence $q(w)$ has the inverse $w(q)$ on these three intervals. Then we can obtain the result (25) except at $\frac{e^m}{\xi_1^m}$ and $\frac{e^m}{\xi_2^m}$. Moreover, since

$$\left\{w \middle| q(w) = \frac{e^m}{\xi_1^m}\right\} = \left(\bar{F}(\frac{e^m}{\xi_1^m}) - c_1^m - \rho\xi_1^m, \bar{F}(\frac{e^m}{\xi_1^m}) - c_2^m - C^m\xi_2^m\right)$$

and

$$\left\{w \middle| q(w) = \frac{e^m}{\xi_2^m}\right\} = \left(\bar{F}(\frac{e^m}{\xi_2^m}) - c_2^m - C^m\xi_2^m, \bar{F}(\frac{e^m}{\xi_2^m}) - c_2^m - v\xi_2^m\right),$$

the supplier will choose the highest possible wholesale price for a given ordering q, we can get the whole result of (25). ■

Proof of Theorem 7.

From Proposition A1 and Corollary 1 we can derive the expression of q_1^{dm}. From the proof of Proposition A1, it can be shown that the expression of q_1^{dm} is valid to characterize q_1^{ds}. We illustrate how to solve q^d using the most complicated case (iii). Note that the optimal q^d is the solution of

$$\max\left\{\max_{q\le\frac{e^m}{\xi_2^m}} \mathcal{H}_1(q), \max_{\frac{e^m}{\xi_2^m}<q\le\frac{e^m}{\xi_1^m}} \mathcal{H}_2(q), \max_{q>\frac{e^m}{\xi_1^m}} \mathcal{H}_3(q)\right\},$$

where $\mathcal{H}_i(q)$ is defined in (9) and $\mathcal{H}_1(q) \ge \mathcal{H}_2(q) \ge \mathcal{H}_3(q)$, $q_{\mathcal{H}_1} \ge q_{\mathcal{H}_2} \ge q_{\mathcal{H}_3}$. If $e^m/\xi_2^m \ge q_{\mathcal{H}_1}$, then

$$\max_{q\le e^m/\xi_2^m} \mathcal{H}_1(q) = \mathcal{H}_1(q_{\mathcal{H}_1}) \ge \mathcal{H}_1(e^m/\xi_2^m) \ge \mathcal{H}_2(e^m/\xi_2^m) \ge \max_{e^m/\xi_2^m<q\le e^m/\xi_1^m} \mathcal{H}_2(q).$$

$$\max_{q\le e^m/\xi_2^m} \mathcal{H}_1(q) = \mathcal{H}_1(q_{\mathcal{H}_1}) \ge \mathcal{H}_1(e^m/\xi_2^m) \ge \mathcal{H}_3(e^m/\xi_2^m) \ge \max_{q>e^m/\xi_1^m} \mathcal{H}_3(q).$$

If $q_{\mathcal{H}_2} \le e^m/\xi_2^m < q_{\mathcal{H}_1}$, then

$$\max_{q\le e^m/\xi_2^m} \mathcal{H}_1(q) = \mathcal{H}_1(e^m/\xi_2^m) \ge \mathcal{H}_2(e^m/\xi_2^m) \ge \max_{e^m/\xi_2^m<q\le e^m/\xi_1^m} \mathcal{H}_2(q).$$

$$\max_{q\le e^m/\xi_2^m} \mathcal{H}_1(q) = \mathcal{H}_1(e^m/\xi_2^m) \ge \mathcal{H}_3(e^m/\xi_2^m) \ge \max_{q>e^m/\xi_1^m} \mathcal{H}_3(q).$$

If $e^m/\xi_2^m < q_{\mathcal{H}_2} \le e^m/\xi_1^m$, then there are two cases. If $\mathcal{H}_1(e^m/\xi_2^m) > \mathcal{H}_2(q_{\mathcal{H}_2})$, then

$$\max_{q\le e^m/\xi_2^m} \mathcal{H}_1(q) = \mathcal{H}_1(e^m/\xi_2^m) > \mathcal{H}_2(q_{\mathcal{H}_2}) \ge \mathcal{H}_3(e^m/\xi_1^m) \ge \max_{q>e^m/\xi_1^m} \mathcal{H}_3(q).$$

And finally, if $\mathcal{H}_1(e^m/\xi_2^m) \le \mathcal{H}_2(q_{\mathcal{H}_2})$, then

$$\max_{q\le e^m/\xi_2^m} \mathcal{H}_1(q) = \mathcal{H}_1(e^m/\xi_2^m) \le \mathcal{H}_2(q_{\mathcal{H}_2}).$$

$$\max_{q>e^m/\xi_1^m} \mathcal{H}_3(q) \leq \mathcal{H}_3(e^m/\xi_1^m) \leq \mathcal{H}_2(q_{\mathcal{H}_2}).$$

If $q_{\mathcal{H}_3} \leq e^m/\xi_1^m < q_{\mathcal{H}_2}$, there are also two cases. If $\mathcal{H}_1(e^m/\xi_2^m) > \mathcal{H}_2(e^m/\xi_1^m)$, then

$$\max_{q\leq e^m/\xi_2^m} \mathcal{H}_1(q) = \mathcal{H}_1(e^m/\xi_2^m) > \mathcal{H}_2(e^m/\xi_1^m) \geq \mathcal{H}_3(e^m/\xi_1^m) \geq \max_{q>e^m/\xi_1^m} \mathcal{H}_3(q).$$

And finally, if $\mathcal{H}_1(e^m/\xi_2^m) \leq \mathcal{H}_2(e^m/\xi_1^m)$, then

$$\max_{q\leq e^m/\xi_2^m} \mathcal{H}_1(q) = \mathcal{H}_1(e^m/\xi_2^m) \leq \mathcal{H}_2(e^m/\xi_1^m).$$

$$\max_{q>e^m/\xi_1^m} \mathcal{H}_3(q) \leq \mathcal{H}_3(e^m/\xi_1^m) \leq \mathcal{H}_2(e^m/\xi_1^m).$$

If $e^m/\xi_1^m < q_{\mathcal{H}_3}$, $\max_{q\leq e^m/\xi_2^m} \mathcal{H}_1(q) = \mathcal{H}_1(e^m/\xi_2^m)$, $\max_{e^m/\xi_2^m<q\leq e^m/\xi_1^m} \mathcal{H}_2(q) = \mathcal{H}_2(e^m/\xi_1^m)$, and $\max_{q>e^m/\xi_1^m} \mathcal{H}_3(q) = \mathcal{H}_3(q_{\mathcal{H}_3})$. So Theorem 7 follows. ■

Proof of Proposition 8.

When q and q_1^s are fixed, $\pi^s(q, q_1^s)$ is increasing with e^s. Hence $\pi^s(q^d, q_1^{ds})$ is increasing with e^s.

When q and q_1^s are fixed, the wholesale price $w(q)$ is increasing with e^m. The profit $\pi^s(q, q_1^s)$ is increasing with e^m, so $\pi^s(q^d, q_1^{ds})$ is.

When q and q_1^m are fixed, the wholesale price $w(q)$ and $-\rho\,[t^m(q, q_1^m) - e^m]^+ + v\,[e^m - t^m(q, q_1^m)]^+$ increase with e^m. The optimal profit $\pi^m(q^d, q_1^{dm})$ may be increasing or decreasing with e^m, since the optimal quantity q^d balances the influence on $-w(q)q$ and $-\rho\,[t^m(q, q_1^m) - e^m]^+ + v\,[e^m - t^m(q, q_1^m)]^+$.

We now prove that optimal π^m is increasing in e^s. Note that with (q, q_1^m), π^m is decreasing in w. So if we can show that $w(q)$ is decreasing in e^s, then the result is proved. Note that $w(q)$ is decreasing in q and independent of e^s. So it suffices to show that q^d is increasing in e^s. Consider three cases.

Case 1 $C^s \geq \rho$. Then at optimum, $q_1^s = 0$. Hence, the supplier's problem becomes

$$w(q)q - c_1^s q - c_2^s q - \rho[\xi_2^s q - e^s]^+ + v[e^s - \xi_2^s q]^+$$

which is supermodular in (q, e^s) and so the optimal q^d is increasing in e^s.

Case 2 $C^s \leq v$. Then at optimum $q_1^s = q$. Hence, the supplier's problem becomes

$$w(q)q - c_1^s q - \rho[\xi_1^s q - e^s]^+ + v[e^s - \xi_1^s q]^+$$

which is again supermodular in (q, e^s) and so the optimal q^d is increasing in e^s.

Case 3 $v < C^s < \rho$. Then at optimum, $q_1^s = \min\{\frac{(\xi_2^s q - e^s)^+}{\xi_2^s - \xi_1^s}, q\}$. Note that

$$\xi_1^s q_1^s + \xi_2^s (q - q_1^s) - e^s$$

$$
\begin{aligned}
&= (\xi_1^s - \xi_2^s)\min\{\frac{(\xi_2^s q - e^s)^+}{\xi_2^s - \xi_1^s}, q\} + \xi_2^s q - e^s \\
&= \xi_2^s q - e^s - \min\{(\xi_2^s q - e^s)^+, (\xi_2^s - \xi_1^s)q\} \\
&= -(\xi_2^s q - e^s)^- + [(\xi_2^s q - e^s)^+ - (\xi_2^s - \xi_1^s)q]^+ \\
&= -(\xi_2^s q - e^s)^- + [\xi_1^s q - e^s]^+.
\end{aligned}
$$

Hence, the supplier's problem becomes

$$
\begin{aligned}
& w(q)q - c_1^s \min\{\frac{(\xi_2^s q - e^s)^+}{\xi_2^s - \xi_1^s}, q\} - c_2^s(q - \min\{\frac{(\xi_2^s q - e^s)^+}{\xi_2^s - \xi_1^s}, q\}) \\
& -\rho[-(\xi_2^s q - e^s)^- + [\xi_1^s q - e^s]^+]^+ + v[-(\xi_2^s q - e^s)^- + [\xi_1^s q - e^s]^+]^- \\
= \; & (w(q) - c_2^s)q - \frac{c_1^s - c_2^s}{\xi_2^s - \xi_1^s}[(\xi_2^s q - e^s)^+ - (\xi_1^s q - e^s)^+] \\
& -\rho[-(\xi_2^s q - e^s)^- + [\xi_1^s q - e^s]^+]^+ + v[-(\xi_2^s q - e^s)^- + [\xi_1^s q - e^s]^+]^-
\end{aligned}
$$

which can be shown supermodular because its partial derivative with respect to q is increasing in e^s, i.e.,

$$
\begin{aligned}
& [(w(q) - c_2^s)q]' - \frac{c_1^s - c_2^s}{\xi_2^s - \xi_1^s}[\xi_2^s \mathbf{1}(\xi_2^s q > e^s) - \xi_1^s \mathbf{1}(\xi_1^s q > e^s)] \\
& -\rho \xi_1^s \mathbf{1}(\xi_1^s q > e^s) - v \xi_2^s \mathbf{1}(\xi_2^s q < e^s)
\end{aligned}
$$

is increasing in e^s as $v < C^s < \rho$.

Therefore, q^d is increasing in e^s, then $w(q^d)$ is decreasing in e^s, and the optimal profit of the manufacturer is increasing in e^s. ■

Proof of Proposition 9.

Parts (a) and (b) are proved simultaneously. We first show that for any number $y \in [0,1]$, $\mathcal{K}^{-1}(y) \leq \bar{F}^{-1}(y)$. To prove this, denote $\mathcal{K}^{-1}(y) = z_1$ and $\bar{F}^{-1}(y) = z_2$, then $\bar{F}(z_1)(1 - g(z_1)) = \bar{F}(z_2)$. Because $\bar{F}(\cdot)$ is decreasing and $g(\cdot) \geq 0$, $z_1 \leq z_2$.

(i) If $v < \rho \leq c/\delta$, based on Theorem 2 (i) and Proposition A3 (i) with $\xi_1^s = \xi^s - \delta$, $\xi_2^s = \xi^s$, $c_1^s = c^s + c$, $c_2^s = c^s$, $c_1^m = c_2^m = c^m$, $\xi_1^m = 0$ and $\xi_2^m = 0$, we consider the following cases.

- $\bar{e} \leq \xi^s q_{\hat{\mathcal{F}}_1}$: $\bar{q}^c = \max\{q_\beta(\rho, 0), \bar{e}/\xi^s\} \geq \max\{q_{\hat{\mathcal{F}}_2}, \bar{e}/\xi^s\} = \bar{q}^d$.
- $\xi^s q_{\hat{\mathcal{F}}_1} < \bar{e} \leq \xi^s q_\beta(v, 0)$: $\bar{q}^c = \max\{q_\beta(\rho, 0), \bar{e}/\xi^s\} \geq \bar{e}/\xi^s > q_{\hat{\mathcal{F}}_1} = \bar{q}^d$.
- $\bar{e} > \xi^s q_\beta(v, 0)$: $\bar{q}^c = q_\beta(v, 0) \geq q_{\hat{\mathcal{F}}_1} = \bar{q}^d$.

$\bar{q_1}^{cs} = \bar{q}_1^{ds} = 0$, $\theta^{cs} = \theta^{sd} = 0$. The total emissions of the integrated system is $\xi^s \bar{q}^c$ which is no less than $\xi^s \bar{q}^d$ of the decentralized system.

(ii) If $c/\delta < v < \rho$, based on Theorem 2 (ii) and Proposition A3 (ii) with $\xi_1^s = \xi^s - \delta$, $\xi_2^s = \xi^s$, $c_1^s = c^s + c$, $c_2^s = c^s$, $c_1^m = c_2^m = c^m$, $\xi_1^m = 0$ and $\xi_2^m = 0$, the proof of $q^c \geq \bar{q}^d$ is the same as that in (i) with c^s and ξ^s substituted by $(c^s + c)$ and $(\xi^s - \delta)$ respectively. $\bar{q_1}^{cs} = \bar{q}^c$ and $\bar{q}_1^{ds} = \bar{q}^d$ thus $\bar{q_1}^{cs} \geq \bar{q}_1^{ds}$ and $\theta^{cs} = \theta^{sd} = 1$. The total emissions of the integrated system is $(\xi^s - \delta)\bar{q}^c$ which is no less than $(\xi^s - \delta)\bar{q}^d$ of the decentralized system.

(iii) For brevity, we define $\bar{q}_\alpha^c$ by $\bar{q}_\alpha^c = q_\beta(c/\delta, 0)$. If $v \leq c/\delta < \rho$, based on Theorem 3 (i) and Proposition A3 (iii) with $\xi_1^s = \xi^s - \delta$, $\xi_2^s = \xi^s$, $c_1^s = c^s + c$, $c_2^s = c^s$, $c_1^m = c_2^m = c^m$, $\xi_1^m = 0$ and $\xi_2^m = 0$, we have that

- $\bar{e} \leq (\xi^s - \delta)\bar{q}_\alpha$: $\bar{q}^c = \bar{q}_1^{cs} = \max\{q_\gamma(\rho, 0), \bar{e}/(\xi^s - \delta)\} \geq \bar{q}^d = \bar{q}_1^{ds} = \max\{\hat{\mathcal{G}}_2, \bar{e}/(\xi^s - \delta)\}$. $\theta^{cs} = \theta^{ds} = 1$. The quantity of total emissions of the integrated system is $(\xi^s - \delta)\bar{q}^c$ which is no less than $(\xi^s - \delta)\bar{q}^d$ of the decentralized system.
- $(\xi^s - \delta)\bar{q}_\alpha^c < \xi^s \bar{q}_\alpha$
  - $(\xi^s - \delta)\bar{q}_\alpha < \bar{e} \leq (\xi^s - \delta)\bar{q}_\alpha^c$: $\bar{q}^c = \bar{q}_1^{cs} = \max\{q_\gamma(\rho, 0), \bar{e}/(\xi^s - \delta)\} > \bar{q}_\alpha = \bar{q}^d$ as $\bar{q}_\alpha < \bar{e}/(\xi^s - \delta)$. And $\bar{q}_1^{ds} = \frac{\xi^s \bar{q}_\alpha - \bar{e}}{\delta} < \bar{q}_\alpha < \bar{e}/(\xi^s - \delta) \leq \bar{q}_1^{cs}$. $\theta^{cs} = 1 > \theta^{ds} = \frac{\xi^s - \bar{e}/\bar{q}_\alpha}{\delta}$.
  - $(\xi^s - \delta)\bar{q}_\alpha^c < \bar{e} \leq \xi^s \bar{q}_\alpha$: $\bar{q}^c = \bar{q}_\alpha^c \geq \bar{q}^d = \bar{q}_\alpha$, $\bar{q}_1^{cs} = \frac{\xi^s \bar{q}_\alpha^c - \bar{e}}{\delta} \geq \bar{q}_1^{ds}$. $\theta^{cs} = \frac{\xi^s - \bar{e}/\bar{q}_\alpha^c}{\delta} \geq \frac{\xi^s - \bar{e}/\bar{q}_\alpha}{\delta} = \theta^{ds}$.
  - $\xi^s \bar{q}_\alpha < \bar{e} \leq \xi^s \bar{q}_\alpha^c$: $\bar{q}^c = \bar{q}_\alpha^c \geq \bar{q}^d = \min\{\bar{q}_\alpha, \bar{e}/\xi^s\}$ since $\bar{e}/\xi^s \leq \bar{q}_\alpha^c$. $\bar{q}_1^{cs} = \frac{\xi^s \bar{q}_\alpha^c - \bar{e}}{\delta} > \bar{q}_1^{ds} = 0$. And it is clear that $\theta^{cs} \geq \theta^{ds} = 0$.
- $(\xi^s - \delta)\bar{q}_\alpha^c > \xi^s \bar{q}_\alpha$
  - $(\xi^s - \delta)\bar{q}_\alpha < \bar{e} \leq \xi^s \bar{q}_\alpha$: $\bar{q}^c = \bar{q}_1^{cs} = \max\{q_\gamma(\rho, 0), \bar{e}/(\xi^s - \delta)\} > \bar{q}^d = \bar{q}_\alpha$ as $\bar{q}_\alpha < \bar{e}/(\xi^s - \delta)$. And $\bar{q}_1^{ds} = \frac{\xi^s \bar{q}_\alpha - \bar{e}}{\delta} < q_\alpha < \bar{e}/(\xi^s - \delta) \leq \bar{q}_1^{cs}$. $\theta^{cs} = 1 \geq \theta^{ds}$.
  - $\xi^s \bar{q}_\alpha < \bar{e} \leq (\xi^s - \delta)\bar{q}_\alpha^c$: $\bar{q}^c = \bar{q}_1^{cs} = \max\{\bar{F}^{-1}((c^s + c) + c^m + \rho(\xi^s - \delta)), \bar{e}/(\xi^s - \delta)\} > \bar{q}^d = \min\{\bar{q}_\alpha, \bar{e}/\xi^s\}$ as $\bar{e}/(\xi^s - \delta) > \bar{e}/\xi^s$. And $\bar{q}_1^{ds} = 0 < \bar{q}_1^{cs}$. $\theta^{cs} = 1 \geq \theta^{ds}$.
  - $(\xi^s - \delta)\bar{q}_\alpha^c < \bar{e} \leq \xi^s \bar{q}_\alpha^c$: $\bar{q}^c = \bar{q}_\alpha^c \geq \bar{q}^d = \min\{\bar{q}_\alpha, \bar{e}/\xi^s\}$ as $\bar{q}_\alpha^c \geq \bar{e}/\xi^s$. And $\bar{q}_1^{ds} = 0 < \bar{q}_1^{cs}$. $\theta^{cs} \geq \theta^{ds} = 0$.

For either case $(\xi^s - \delta)\bar{q}_\alpha^c < \xi^s \bar{q}_\alpha$ or case $(\xi^s - \delta)\bar{q}_\alpha^c > \xi^s \bar{q}_\alpha$, it is not hard to show that the total quantity of emissions of the integrated system is no less than $\bar{e}$ while no more than $\bar{e}$ in the decentralized system.

- $\bar{e} > \xi^s \bar{q}^c_\alpha$: $\bar{q}^c = \min\{q_\beta(v,0), \bar{e}/\xi^s\} \geq \bar{q}^d = \min\{\bar{q}_\alpha, \bar{e}/\xi^s\}$ Moreover, $\bar{q}^{cs}_1 = \bar{q}^{ds}_1 = 0$ and $\theta^{cs} = \theta^{ds} = 0$. The total emissions of the integrated system is $\xi^s \bar{q}^c$ which is no less than $\xi^s \bar{q}^d$ of the decentralized system.

So for all scenarios (i)-(iii), we see that $\bar{q}^d \leq \bar{q}^c, \bar{q}^{ds}_1 \leq \bar{q}^{cs}_1$, and $\theta^{ds} \leq \theta^{cs}$. Meanwhile, the integrated system generates more emissions as well. ■

Proof of Proposition 10.

Parts (a) and (b) are proved simultaneously. We first show that for any number $y \in [0,1]$, $\mathcal{K}^{-1}(y) \leq \bar{F}^{-1}(y)$. To prove this, denote $\mathcal{K}^{-1}(y) = z_1$ and $\bar{F}^{-1}(y) = z_2$, then $\bar{F}(z_1)(1 - g(z_1)) = \bar{F}(z_2)$. Because $\bar{F}(\cdot)$ is decreasing and $g(\cdot) \geq 0$, $z_1 \leq z_2$.

(i) If $v < \rho \leq c/\delta$, based on Theorem 2 (i) and Proposition A4 (i) with $\xi^s_1 = 0$, $\xi^s_2 = 0$, $c^s_1 = c^s_2 = c^s$, $c^m_1 = c^m + c$, $c^m_2 = c^m$, $\xi^m_1 = \xi^m - \delta$ and $\xi^m_2 = \xi^m$, then we use the inequalities $q_{\hat{\mathcal{F}}_2} \leq q_{\hat{\mathcal{F}}_1} \leq q_\beta(0,v)$.

- $\dfrac{\bar{e}}{\xi^m} \leq q_{\hat{\mathcal{F}}_2}$:

$$\hat{q}^c = \max\left\{q_\beta(0,\rho), \frac{\bar{e}}{\xi^m}\right\} \geq \max\left\{q_{\hat{\mathcal{F}}_2}, \frac{\bar{e}}{\xi^m}\right\} \geq \hat{q}^d$$

- $q_{\hat{\mathcal{F}}_2} < \dfrac{\bar{e}}{\xi^m} \leq q_{\hat{\mathcal{F}}_1}$:

$$\hat{q}^c = \max\left\{q_\beta(0,\rho), \frac{\bar{e}}{\xi^m}\right\} \geq \frac{\bar{e}}{\xi^m} \geq \min\left\{q_{\hat{\mathcal{F}}_1}, \frac{\bar{e}}{\xi^m}\right\} = \hat{q}^d$$

- $q_{\hat{\mathcal{F}}_1} < \dfrac{\bar{e}}{\xi^m} \leq q_\beta(0,v)$:

$$\hat{q}^c = \max\left\{q_\beta(0,\rho), \frac{\bar{e}}{\xi^m}\right\} \geq \frac{\bar{e}}{\xi^m} \geq \min\left\{q_{\hat{\mathcal{F}}_1}, \frac{\bar{e}}{\xi^m}\right\} = \hat{q}^d$$

- $q_\beta(0,v) < \dfrac{\bar{e}}{\xi^m}$:

$$\hat{q}^c = q_\beta(0,v) \geq q_{\hat{\mathcal{F}}_1} \geq \min\left\{q_{\hat{\mathcal{F}}_1}, \frac{\bar{e}}{\xi^m}\right\} = \hat{q}^d$$

(ii) If $c/\delta < v < \rho$, based on Theorem 2 (ii) and Proposition A4 (ii) with $\xi^s_1 = 0$, $\xi^s_2 = 0$, $c^s_1 = c^s_2 = c^s$, $c^m_1 = c^m + c$, $c^m_2 = c^m$, $\xi^m_1 = \xi^m - \delta$ and $\xi^m_2 = \xi^m$, then we use the inequalities $q_{\hat{\mathcal{G}}_2} \leq q_{\hat{\mathcal{G}}_1} \leq q_\alpha(0,v)$.

- $\dfrac{\bar{e}}{\xi^m - \delta} \leq q_{\hat{\mathcal{G}}_2}$:

$$\hat{q}^c = \max\left\{q_\alpha(0,\rho), \frac{\bar{e}}{\xi^m - \delta}\right\} \geq \max\left\{q_{\hat{\mathcal{G}}_2}, \frac{\bar{e}}{\xi^m - \delta}\right\} \geq \hat{q}^d$$

– $q_{\hat{\mathcal{G}}_2} < \dfrac{\bar{e}}{\xi^m - \delta} \le q_{\hat{\mathcal{G}}_1}$:

$$\hat{q}^c = \max\left\{q_\alpha(0,\rho), \frac{\bar{e}}{\xi^m - \delta}\right\} \ge \frac{\bar{e}}{\xi^m - \delta} \ge \min\left\{q_{\hat{\mathcal{G}}_1}, \frac{\bar{e}}{\xi^m - \delta}\right\} = \hat{q}^d$$

– $q_{\hat{\mathcal{G}}_1} < \dfrac{\bar{e}}{\xi^m - \delta} \le q_\alpha(0,v)$:

$$\hat{q}^c = \max\left\{q_\alpha(0,\rho), \frac{\bar{e}}{\xi^m - \delta}\right\} \ge \frac{\bar{e}}{\xi^m - \delta} \ge \min\left\{q_{\hat{\mathcal{G}}_1}, \frac{\bar{e}}{\xi^m - \delta}\right\} = \hat{q}^d$$

– $q_\alpha(0,v) < \dfrac{\bar{e}}{\xi^m - \delta}$:

$$\hat{q}^c = q_\alpha(0,v) \ge q_{\hat{\mathcal{G}}_1} \ge \min\left\{q_{\hat{\mathcal{G}}_1}, \frac{\bar{e}}{\xi^m - \delta}\right\} = \hat{q}^d$$

(iii) Suppose $v \le c/\delta < \rho$. Based on Theorem 3 (iii) and Proposition A4 (iii) with $\xi_1^s = 0$, $\xi_2^s = 0$, $c_1^s = c_2^s = c^s$, $c_1^m = c^m + c$, $c_2^m = c^m$, $\xi_1^m = \xi^m - \delta$ and $\xi_2^m = \xi^m$, we have the following.

– $\bar{e} \le (\xi^m - \delta) q_{\hat{\mathcal{G}}_2}$: It implies that $\bar{e} \le (\xi^m - \delta) q_\gamma(0, c/\delta)$

$$\hat{q}_1^{cm} = \hat{q}^c = \max\left\{q_\alpha(0,\rho), \frac{\bar{e}}{\xi^m - \delta}\right\} \ge \max\left\{q_{\hat{\mathcal{G}}_2}, \frac{\bar{e}}{\xi^m - \delta}\right\} = q_{\hat{\mathcal{G}}_2} \ge \hat{q}^d$$
$$\left(\text{since } \hat{q}^d \in \left\{\frac{\bar{e}}{\xi^m}, \frac{\bar{e}}{\xi^m - \delta}, q_{\hat{\mathcal{G}}_2}\right\}\right)$$

Since $\hat{q}_1^{cm} = \hat{q}^c$, $\theta^{cm} = 1$. Moreover, the total quantity of emissions of the integrated system is $(\xi^m - \delta)\hat{q}^c$ which is no less than $\bar{e}$ by

$$(\xi^m - \delta)\hat{q}_1^{cm} + \xi^m(\hat{q}^c - \hat{q}_1^{cm}) = (\xi^m - \delta)\hat{q}^c \ge (\xi^m - \delta)\frac{\bar{e}}{\xi^m - \delta} = \bar{e}.$$

If $\hat{q}^d = \frac{\bar{e}}{\xi^m}$ or $\hat{q}^d = \frac{\bar{e}}{\xi^m - \delta}$, then both the total quantity of emissions of the decentralized system are $\bar{e}$. If $\hat{q}^d = q_{\hat{\mathcal{G}}_2}$, then $\hat{q}^d \ge \frac{\bar{e}}{\xi^m - \delta}$ which implies that $\hat{q}_1^{dm} = \hat{q}^d$ and the total quantity of emissions of the decentralized system is $(\xi^m - \delta)\hat{q}^d$. Therefore, the total quantity of emissions of the integrated system is no less than the total quantity of emissions of the decentralized system.

– $(\xi^m - \delta) q_{\hat{\mathcal{G}}_2} < \bar{e} \le (\xi^m - \delta) q_{\hat{\mathcal{H}}}$

$$\hat{q}_1^{cm} = \hat{q}^c = \max\left\{q_\alpha(0,\rho), \frac{\bar{e}}{\xi^m - \delta}\right\} \ge \max\left\{q_{\hat{\mathcal{G}}_2}, \frac{\bar{e}}{(\xi^m - \delta)}\right\} = \frac{\bar{e}}{(\xi^m - \delta)} \ge \hat{q}^d$$
$$\left(\text{since } \hat{q}^d \in \left\{\frac{\bar{e}}{\xi^m}, \frac{\bar{e}}{\xi^m - \delta}\right\}\right).$$

Since $\hat{q}_1^{cm} = \hat{q}^c$, $\theta^{cm} = 1$. Moreover, the total quantity of emissions of the integrated system is $(\xi^m - \delta)\hat{q}^c$ which is no less than $\bar{e}$ by

$$(\xi^m - \delta)\hat{q}_1^{cm} + \xi^m(\hat{q}^c - \hat{q}_1^{cm}) = (\xi^m - \delta)\hat{q}^c \ge (\xi^m - \delta)\frac{\bar{e}}{(\xi^m - \delta)} = \bar{e}.$$

If $\hat{q}^d = \frac{\bar{e}}{\xi^m}$ or $\hat{q}^d = \frac{\bar{e}}{\xi^m - \delta}$, then both the total quantity of emissions of the decentralized system are $\bar{e}$. Therefore, the total quantity of emissions of the integrated system is no less than the total quantity of emissions of the decentralized system.

- $\xi^m q_{\hat{\mathcal{H}}} < (\xi^m - \delta) q_\gamma(0, c/\delta)$
  * $(\xi^m - \delta) q_{\hat{\mathcal{H}}} < \bar{e} \leq \xi^m q_{\hat{\mathcal{H}}} < (\xi^m - \delta) q_\gamma(0, c/\delta)$

$$\hat{q}_1^{cm} = \hat{q}^c = \max\left\{q_\alpha(0,\rho), \frac{\bar{e}}{\xi^m - \delta}\right\} \geq \max\left\{q_{\hat{\mathcal{G}}_2}, \frac{\bar{e}}{\xi^m - \delta}\right\} = \frac{\bar{e}}{\xi^m - \delta} \geq \hat{q}^d$$
$$\left(\text{since } \hat{q}^d \in \left\{\frac{\bar{e}}{\xi^m}, q_{\hat{\mathcal{H}}}\right\}\right)$$

Since $\hat{q}_1^{cm} = \hat{q}^c$, $\theta^{cm} = 1$. Moreover, the total quantity of emissions of the integrated system is $(\xi^m - \delta)\hat{q}^c$ which is no less than $\bar{e}$ by

$$(\xi^m - \delta)\hat{q}_1^{cm} + \xi^m(\hat{q}^c - \hat{q}_1^{cm}) = (\xi^m - \delta)\hat{q}^c \geq (\xi^m - \delta)\frac{\bar{e}}{\xi^m - \delta} = \bar{e}.$$

If $\hat{q}^d = \frac{\bar{e}}{\xi^m}$, then the total quantity of emissions of the decentralized system is $\bar{e}$. If $\hat{q}^d = q_{\hat{\mathcal{H}}}$, then $(\xi^m - \delta) q_{\hat{\mathcal{H}}} < \bar{e}$. Therefore, the total quantity of emissions of the integrated system is no less than the total quantity of emissions of the decentralized system.

  * $\xi^m q_{\hat{\mathcal{H}}} < \bar{e} \leq (\xi^m - \delta) q_\gamma(0, c/\delta)$

$$\hat{q}_1^{cm} = \hat{q}^c = \max\left\{q_\alpha(0,\rho), \frac{\bar{e}}{\xi^m - \delta}\right\} \geq \frac{\bar{e}}{\xi^m - \delta} \geq \min\left\{q_{\hat{\mathcal{F}}_1}, \frac{\bar{e}}{\xi^m}\right\} = \hat{q}^d$$

Since $\hat{q}_1^{cm} = \hat{q}^c$, $\theta^{cm} = 1$. Moreover, the total quantity of emissions of the integrated system is $(\xi^m - \delta)\hat{q}^c$ which is no less than $\bar{e}$ by

$$(\xi^m - \delta)\hat{q}_1^{cm} + \xi^m(\hat{q}^c - \hat{q}_1^{cm}) = (\xi^m - \delta)\hat{q}^c \geq (\xi^m - \delta)\frac{\bar{e}}{\xi^m - \delta} = \bar{e}.$$

Since $\hat{q}^d = \min\{\frac{\bar{e}}{\xi^m}, q_{\hat{\mathcal{F}}_1}\} \leq \frac{\bar{e}}{\xi^m}$, $\hat{q}_1^{dm} = 0$ and the total quantity of emissions of the decentralized system is $\bar{e}$. Therefore, the total quantity of emissions of the integrated system is no less than the total quantity of emissions of the decentralized system.

  * $\xi^m q_{\mathcal{H}} < (\xi^m - \delta) q_\gamma(0, c/\delta) < \bar{e} \leq \xi^m q_\gamma(0, c/\delta)$

$$\hat{q}^c = q_\gamma(0, c/\delta) \geq \frac{\bar{e}}{\xi^m} \geq \min\left\{q_{\hat{\mathcal{F}}_1}, \frac{\bar{e}}{\xi^m}\right\} = \hat{q}^d$$

Since $\hat{q}_1^{cm} = \dfrac{\xi^m \hat{q}^c - \bar{e}}{\delta}$, the total quantity of emissions of the integrated system is $\bar{e}$. Since $\hat{q}^d = \min\{\frac{\bar{e}}{\xi^m}, q_{\hat{\mathcal{F}}_1}\} \leq \frac{\bar{e}}{\xi^m}$, $\hat{q}_1^{dm} = 0$ and the total quantity of emissions of the decentralized system is $\bar{e}$. Therefore, the total quantity of emissions of the integrated system is no less than the total quantity of emissions of the decentralized system.

– $\xi^m q_{\hat{\mathcal{H}}} > (\xi^m - \delta) q_\gamma(0, c/\delta)$
  * $(\xi^m - \delta) q_{\hat{\mathcal{H}}} < \bar{e} \le (\xi^m - \delta) q_\gamma(0, c/\delta)$

$$\hat{q}^c = \max\left\{q_\alpha(0,\rho), \frac{\bar{e}}{\xi^m - \delta}\right\} \ge \frac{\bar{e}}{\xi^m - \delta} \ge \hat{q}^d \left(\text{since } \hat{q}^d \in \left\{\frac{\bar{e}}{\xi^m}, q_{\hat{\mathcal{H}}}\right\}\right)$$

  * $(\xi^m - \delta) q_\gamma(0, c/\delta) < \bar{e} \le \xi^m q_{\hat{\mathcal{H}}}$

$$\hat{q}^c = q_\gamma(0, c/\delta) \ge \frac{\bar{e}}{\xi^m} \ge q_{\hat{\mathcal{H}}} \ge \frac{\bar{e}}{\xi^m} \ge \hat{q}^d \left(\text{since } \hat{q}^d \in \left\{\frac{\bar{e}}{\xi^m}, q_{\hat{\mathcal{H}}}\right\}\right)$$

  * $\xi^m q_{\hat{\mathcal{H}}} < \bar{e} \le \xi^m q_\gamma(0, c/\delta)$

$$\hat{q}^c = q_\gamma(0, c/\delta) \ge \frac{\bar{e}}{\xi^m} \ge \min\left\{q_{\hat{\mathcal{F}}_1}, \frac{\bar{e}}{\xi^m}\right\} = \hat{q}^d$$

– $\xi^m q_\gamma(0, c/\delta) < \bar{e}$

$$\hat{q}^c = \min\left\{q_\gamma(0, v), \frac{\bar{e}}{\xi^m}\right\} \ge \min\left\{q_{\hat{\mathcal{F}}_1}, \frac{\bar{e}}{\xi^m}\right\} = \hat{q}^d$$

Since $\hat{q}_1^{cm} = 0$, the total quantity of emissions of the integrated system is $\xi^m \hat{q}^c$ which is no less than $\bar{e}$ by

$$(\xi^m - \delta)\hat{q}_1^{cm} + \xi^m(\hat{q}^c - \hat{q}_1^{cm}) = \xi^m \hat{q}^c \ge \xi^m \frac{\bar{e}}{\xi^m} = \bar{e}.$$

Since $\hat{q}^{dm} = \min\left\{q_{\hat{\mathcal{F}}_1}, \frac{\bar{e}}{\xi^m}\right\} \le \frac{\bar{e}}{\xi^m}$, $\hat{q}_1^{dm} = 0$. Hence the total quantity of emissions of the decentralized system are $\xi^m \hat{q}^d$. Therefore, the total quantity of emissions of the integrated system is no less than the total quantity of emissions of the decentralized system. ■

Proof of Theorem 11.

For part (a), we only show case $v \le c/\delta < \rho$ because cases $v < \rho \le c/\delta$ and $c/\delta < \rho < v$ can be proved analogously. Let $\xi^s = \xi^m = \xi$.

(iii) $v \le c/\delta < \rho$:

- Suppose $\bar{e}/(\xi - \delta) < q_{\hat{\mathcal{G}}_2}$. Then $\hat{q}^d \in \{q_{\hat{\mathcal{G}}_2}, \bar{e}/(\xi - \delta), \bar{e}/\xi\}$ which implies that $\hat{q}^d \le q_{\hat{\mathcal{G}}_2}$. At the same time, the conditions $\bar{e}/\xi < \bar{e}/(\xi - \delta) < q_{\hat{\mathcal{G}}_2} < q_{\hat{\mathcal{H}}}$ hold and imply

$$\bar{q}_1^{ds} = \bar{q}^d = \max\{q_{\hat{\mathcal{G}}_2}, \bar{e}/(\xi - \delta)\} = q_{\hat{\mathcal{G}}_2} \ge \hat{q}^d \ge \hat{q}_1^{dm}$$

  which follows from Proposition A3.

- Suppose $\bar{e}/(\xi-\delta) \geq q_{\hat{\mathcal{G}}_2}$ and $\bar{e}/\xi < q_{\hat{\mathcal{H}}}$.
  - $q_{\hat{\mathcal{H}}} < \bar{e}/(\xi-\delta)$ implies $\hat{q}^d \in \{q_{\hat{\mathcal{H}}}, \bar{e}/\xi\} \leq q_{\hat{\mathcal{H}}} = \bar{q}^d$ and $\hat{q}_1^{dm} = \frac{\xi\hat{q}^d-\bar{e}}{\xi}$, $\bar{q}_1^{ds} = \frac{\xi\bar{q}^d-\bar{e}}{\xi}$. Hence $\hat{q}_1^{dm} \leq \bar{q}_1^{ds}$.
  - $q_{\hat{\mathcal{H}}} \geq \bar{e}/(\xi-\delta)$ implies $\hat{q}^d \in \{\bar{e}/(\xi-\delta), \bar{e}/\xi\} \leq \bar{e}/(\xi-\delta) = \bar{q}^d$ and $\hat{q}_1^{dm} = \frac{\xi\hat{q}^d-\bar{e}}{\xi}$, $\bar{q}_1^{ds} = \frac{\xi\bar{q}^d-\bar{e}}{\xi}$. Hence $\hat{q}_1^{dm} \leq \bar{q}_1^{ds}$.
- Suppose $\bar{e}/\xi \geq q_{\hat{\mathcal{H}}}$. Then $\hat{q}^d = \min\{q_{\mathcal{F}_1}, \bar{e}/\xi\} = \bar{q}^d$, $\hat{q}_1^{dm} = 0$ and $\bar{q}_1^{ds} = 0$.

We obtain part (b) from the following:

$$
\begin{aligned}
& \text{the total cost of the case: regulated supplier} \\
= & \Big[\mathrm{E}[\min\{D, \bar{q}^d\}] - w(\bar{q}^d)\bar{q}^d - c^m\bar{q}^d\Big] + \Big[w(\bar{q}^d)\bar{q}^d - (c^s+c)\bar{q}_1^{ds} - c^s(\bar{q}^d - \bar{q}_1^{ds}) \\
& -\rho[(\xi-\delta)\bar{q}_1^{ds} + \xi(\bar{q}^d - \bar{q}_1^{ds}) - \bar{e}]^+ + v[\bar{e} - (\xi-\delta)\bar{q}_1^{ds} - \xi(\bar{q}^d - \bar{q}_1^{ds})]^+\Big] \\
= & \Big[\mathrm{E}[\min\{D, \bar{q}^d\}] - (\bar{F}(\bar{q}^d) - c^m)\bar{q}^d - c^m\bar{q}^d\Big] + \Big[(\bar{F}(\bar{q}^d) - c^m)\bar{q}^d - (c^s+c)\bar{q}_1^{ds} - c^s(\bar{q}^d - \bar{q}_1^{ds}) \\
& -\rho[(\xi-\delta)\bar{q}_1^{ds} + \xi(\bar{q}^d - \bar{q}_1^{ds}) - \bar{e}]^+ + v[\bar{e} - (\xi-\delta)\bar{q}_1^{ds} - \xi(\bar{q}^d - \bar{q}_1^{ds})]^+\Big] \\
\geq & \Big[\mathrm{E}[\min\{D, \hat{q}^d\}] - (\bar{F}(\bar{q}^d) - c^m)\hat{q}^d - c^m\hat{q}^d\Big] + \Big[(\bar{F}(\bar{q}^d) - c^m)\hat{q}^d - (c^s+c)\hat{q}_1^{dm} - c^s(\hat{q}^d - \hat{q}_1^{dm}) \\
& -\rho[(\xi-\delta)\hat{q}_1^{dm} + \xi(\hat{q}^d - \hat{q}_1^{dm}) - \bar{e}]^+ + v[\bar{e} - (\xi-\delta)\hat{q}_1^{dm} - \xi(\hat{q}^d - \hat{q}_1^{dm})]^+\Big] \\
\geq & \Big[\mathrm{E}[\min\{D, \hat{q}^d\}] - (\bar{F}(\bar{q}^d) - c^m)\hat{q}^d - c^m\hat{q}^d\Big] + \Big[(\bar{F}(\hat{q}^d) - c^m)\hat{q}^d - (c^s+c)\hat{q}_1^{dm} - c^s(\hat{q}^d - \hat{q}_1^{dm}) \\
& -\rho[(\xi-\delta)\hat{q}_1^{dm} + \xi(\hat{q}^d - \hat{q}_1^{dm}) - \bar{e}]^+ + v[\bar{e} - (\xi-\delta)\hat{q}_1^{dm} - \xi(\hat{q}^d - \hat{q}_1^{dm})]^+\Big] \\
= & \mathrm{E}[\min\{D, \hat{q}^d\}] - (\bar{F}(\bar{q}^d) - \bar{F}(\hat{q}^d))\hat{q}^d - c^m\hat{q}^d - (c^s+c)\hat{q}_1^{dm} - c^s(\hat{q}^d - \hat{q}_1^{dm}) \\
& -\rho[(\xi-\delta)\hat{q}_1^{dm} + \xi(\hat{q}^d - \hat{q}_1^{dm}) - \bar{e}]^+ + v[\bar{e} - (\xi-\delta)\hat{q}_1^{dm} - \xi(\hat{q}^d - \hat{q}_1^{dm})]^+ \\
\geq & \mathrm{E}[\min\{D, \hat{q}^d\}] - c^m\hat{q}^d - (c^s+c)\hat{q}_1^{dm} - c^s(\hat{q}^d - \hat{q}_1^{dm}) \\
& -\rho[(\xi-\delta)\hat{q}_1^{dm} + \xi(\hat{q}^d - \hat{q}_1^{dm}) - \bar{e}]^+ + v[\bar{e} - (\xi-\delta)\hat{q}_1^{dm} - \xi(\hat{q}^d - \hat{q}_1^{dm})]^+ \\
\geq & \mathrm{E}[\min\{D, \hat{q}^d\}] - c^m\hat{q}^d - c^s\hat{q}^d - c\hat{q}_1^{dm} \\
& -\rho[(\xi-\delta)\hat{q}_1^{dm} + \xi(\hat{q}^d - \hat{q}_1^{dm}) - \bar{e}]^+ + v[\bar{e} - (\xi-\delta)\hat{q}_1^{dm} - \xi(\hat{q}^d - \hat{q}_1^{dm})]^+ \\
= & \mathrm{E}[\min\{D, \hat{q}^d\}] - w(\hat{q}^d)\hat{q}^d - c^m\hat{q}^d - c\hat{q}_1^{dm} + w(\hat{q}^d)\hat{q}^d - c^s\hat{q}^d \\
& -\rho[(\xi-\delta)\hat{q}_1^{dm} + \xi(\hat{q}^d - \hat{q}_1^{dm}) - \bar{e}]^+ + v[\bar{e} - (\xi-\delta)\hat{q}_1^{dm} - \xi(\hat{q}^d - \hat{q}_1^{dm})]^+ \\
= & \Big[\mathrm{E}[\min\{D, \hat{q}^d\}] - w(\hat{q}^d)\hat{q}^d - (c^m+c)\hat{q}_1^{dm} - c^m(\hat{q}^d - \hat{q}_1^{dm}) \\
& -\rho[(\xi-\delta)\hat{q}_1^{dm} + \xi(\hat{q}^d - \hat{q}_1^{dm}) - \bar{e}]^+ + v[\bar{e} - (\xi-\delta)\hat{q}_1^{dm} - \xi(\hat{q}^d - \hat{q}_1^{dm})]^+\Big] \\
& +w(\hat{q}^d)\hat{q}^d - c^s\hat{q}^d \\
= & \text{the total cost of the case: regulated manufacturer}
\end{aligned}
$$

■