Online Appendix to "Valuable E-Waste: Implications for Extended Producer Responsibility"

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A. Analysis of Welfare Components

In this section, we provide the results concerning the change in welfare components as collection target τ increases along with the corresponding proofs.

Proposition A.1 *Under the RC equilibrium (i.e., when $\bar{v}_s > \tau \geq \bar{\tau}_C$), (i.)* $\frac{\partial \Pi_{s,C}^{R*}}{\partial \tau} < 0$, *(ii.)* $\frac{\partial \Pi_{o,C}^{R*}}{\partial \tau} < 0$ *iff* $\tau > \tilde{\tau}_C = \frac{(k+1)\bar{v}_o - k\bar{v}_s}{2}$, *and (iii.)* $\frac{\partial \Pi_{c,C}^{R*}}{\partial \tau} < 0$ *iff* $\mu < \bar{\mu}$ *and* $\tau > \bar{\bar{\tau}}_C = -\frac{k(\mu k+\mu-k+1)\bar{v}_s}{2(\mu k+\mu-k)}$.

Proof of Proposition A.1: Substituting $q_{o,C}^{R*}$ and $q_{s,C}^{R*}$ (from the RC equilibrium in the proof of Proposition 2) in (1) and (3), we get the OEM and IR profits at the optimality as $\Pi_{o,C}^{R*} = \frac{((k+1)\bar{v}_o - k\bar{v}_s - \tau)\tau}{k+1}$ and $\Pi_{s,C}^{R*} = \frac{(\tau-\bar{v}_s)^2}{\mu(k+1)^2}$, respectively. Furthermore, the acquisition cost is $p_C^{R*} = \frac{kv_s+a+\tau}{k+1}$ and the waste-holder surplus is $\Pi_{c,C}^{R*} = (p_C^{R*} - a)q_{t,C}^{R*} = \frac{(\tau+\bar{v}_s k)((\mu k+\mu-k)\tau + k\bar{v}_s)}{\mu(k+1)^2}$. Taking derivatives with respect to τ we find (i.) $\frac{\partial \Pi_{s,C}^{R*}}{\partial \tau} = \frac{2(\tau-\bar{v}_s)}{\mu(k+1)^2} = -\frac{2q_s^{R*}}{k+1} < 0$, (ii.) $\frac{\partial \Pi_{o,C}^{R*}}{\partial \tau} = \frac{(k+1)\bar{v}_o - k\bar{v}_s - 2\tau}{k+1} < 0$ iff $\tau > \tilde{\tau}_C = \frac{(k+1)\bar{v}_o - k\bar{v}_s}{2}$, and (iii.) $\frac{\partial \Pi_{c,C}^{R*}}{\partial \tau} = \frac{(2(\mu k+\mu-k))\tau + k(\mu k+\mu-k+1)\bar{v}_s}{\mu(k+1)^2} < 0$ iff $\mu < \bar{\mu}$ and $\tau > \bar{\bar{\tau}}_C$ where $\bar{\bar{\tau}}_C = -\frac{k(\mu k+\mu-k+1)\bar{v}_s}{2(\mu k+\mu-k)} = -\frac{k((\mu-\bar{\mu})(k+1)+1)v_s}{2(\mu-\bar{\mu})(k+1)}$. Finally, also note that $\frac{\partial p_C^{R*}}{\partial \tau} = \frac{1}{1+k} > 0$. □

Proposition A.2 *Under the RTC equilibrium (i.e., when $\bar{v}_s > \tau \geq \bar{\tau}_T$ and $\mu > \bar{\mu}$), (i)* $\frac{\partial \Pi_{s,C}^{T*}}{\partial \tau} < 0$, *(ii.)* $\frac{\partial \Pi_{o,C}^{T*}}{\partial \tau} \leq 0$ *iff* $\tau \geq \frac{(k+1)(\mu k+\mu-k)\bar{v}_o - k(\mu k+\mu-k-2)\bar{v}_s}{2\mu(k+1)}$, *and (iii.)* $\frac{\partial \Pi_{c,C}^{T*}}{\partial \tau} > 0$.

Proof of Proposition A.2: Plugging $q_{s,C}^{T*}$ and $q_{o,C}^{T*}$ (from region 2 in the proof of Lemma 6) in (1) and (3), we get the profits at the optimality as $\Pi_{o,C}^{T*} = \frac{(\bar{v}_s k - \mu\tau(k+1))((k-\mu k)(\bar{v}_o - \bar{v}_s) - \mu(\bar{v}_o+\tau)}{(\mu k+\mu-k)^2}$ and $\Pi_{s,C}^{T*} = \frac{\mu(\tau-\bar{v}_s)^2}{(\mu k+\mu-k)^2}$, respectively. Furthermore, the acquisition cost at the optimality is $p_C^{T*} = \frac{(\mu-1)k\bar{v}_s + \mu\tau}{\mu k+\mu-k} + a$ and the waste-holder surplus is $\Pi_{c,C}^{T*} = (p_C^{T*} - a)q_{t,C}^{T*} = \frac{\tau(\mu\bar{v}_s k + \mu\tau - \bar{v}_s k)}{\mu k+\mu-k}$.

Taking derivatives we find that (i.) $\frac{\partial \Pi_{s,C}^{T*}}{\partial \tau} = \frac{2\mu(\tau-\bar{v}_s)}{(\mu k+\mu-k)^2} = -\frac{2\mu q_{s,C}^{T*}}{\mu k+\mu-k} < 0$ (recall from the proof of Proposition 6 that this solution arises only when $\bar{\tau}_T < \bar{v}_s$ which can be rewritten as $\mu k + \mu - k \geq 0$), (ii.) $\frac{\partial \Pi_{o,C}^{T*}}{\partial \tau} = \frac{\mu((k+1)(\mu k+\mu-k)\bar{v}_o - k(\mu k+\mu-k-2)\bar{v}_s - 2\mu\tau(k+1))}{(\mu k+\mu-k)^2} > 0$ iff $\tau < \frac{(k+1)(\mu k+\mu-k)\bar{v}_o - k(\mu k+\mu-k-2)\bar{v}_s}{2\mu(k+1)}$, and (iii.) $\frac{\partial \Pi_{c,C}^{T*}}{\partial \tau} = \frac{(\mu k-k)\bar{v}_s + 2\mu\tau}{\mu k+\mu-k} = \frac{\mu\tau + (p_C^{T*}-a)(\mu k+\mu-k)}{\mu k+\mu-k} > 0$. □

B. A Generalized Model: Differentiated Prices and Environmental Benefits

In this section, we generalize our model to allow (i) differentiated acquisition prices and (ii) different environmental benefits from IR versus OEM recovery. The proofs of the results are similar to proofs for the base model and therefore omitted for brevity but available from the authors upon request. Here, we consider the most general form of differentiated acquisition prices:

$$p_o = a + \alpha_1 \sum_{i=1}^{k} q_{s,i} + \alpha_2 q_o \tag{B.1}$$

$$p_{s,i} = a + \mu_1 q_{s,i} + \mu_1 \sum_{j \neq i}^{k} q_{s,j} + \mu_2 q_o \tag{B.2}$$

where q_o denotes the OEM's collection volume and $q_{s,i}$ denotes an IR i's collection volume. In addition, α_1 (α_2) denotes the sensitivity of OEM's acquisition cost to IR (OEM) acquisition volume and μ_1 (μ_2) denotes the sensitivity of IR acquisition cost to IR (OEM) acquisition volume. Hence, this model allows waste holders to charge different prices to each player. (Note that the base model in the paper is a special case of this one where $\alpha_1 = \mu_1 = \mu$ and $\alpha_2 = \mu_2 = 1$.) With this inverse demand characterization it is straightforward to show that given q_o and $q_{s,i}$ values the waste-holder surplus becomes $\Pi_c = (p_o - a)q_o + \sum_{i=1}^{k}(p_{s,i} - a)q_{s,i}$. The OEM maximizes her profit $\Pi_o = (v_o - p_o)q_o$ subject to $q_o \geq \tau$. We consider k identical IRs. Each IR i (denoted by subscript s, i) decides on her collection volume $q_{s,i}$.[1] Then, IR i's problem is $\max_{q_{s,i} \geq 0} \Pi_{s,i} = (v_s - p_{s,i})q_{s,i}$.

We develop the environmental benefit measure assuming that the OEM and IRs' recovery efficiencies are different. Therefore, it becomes $\Pi_e = (\epsilon_o q_o + \epsilon_s \sum_{i=1}^{k} q_{s,i})$ where ϵ_o and ϵ_s represent the environmental benefits from OEM and IRs' recovery processes, respectively. Finally, our welfare construct remains the same, i.e., $W = \Pi_o + \sum_{i=1}^{k} \Pi_{s,i} + \Pi_e + \Pi_c$.

All results under the benchmark scenario (i.e., without IRs) continue to hold as are. Next we present the results under competition. All our findings continue to hold, the only differences are in the expressions of thresholds on μ and τ that define regions of findings.

B.1 Implications of EPR on Valuable Waste

We assume that $\frac{\bar{v}_o \mu_2}{2\alpha_2} < \bar{v}_s < \frac{(k+1)\bar{v}_o \mu_1}{k\alpha_1}$ and $\bar{v}_s < \frac{2\alpha_2\mu_1(k+1) - \alpha_1\mu_2 k - (\mu_1(k+1) - \mu_2 k)\bar{v}_o}{k(2\alpha_2 - \alpha_1)}$. Also, we assume $2\alpha_2\mu_1(k+1) - \alpha_1\mu_2 k \geq 0$. The first condition ensures a competitive equilibrium where both the OEM and the IRs profitably coexist in the absence of EPR, and the second condition ensures that

[1] Recall, for ease of exposition, we also define $\bar{v}_o = v_o - a$ and $\bar{v}_s = v_s - a$.

only a fraction of e-waste is acquired by these parties in the absence of EPR. The characterization of the equilibrium is as follows:

Proposition B.3 *In the presence of IRs, there are three possible equilibria for a given collection target τ imposed on the OEM. Those are (i)* ***The NC equilibrium****: when $\tau < \bar{\tau}_C$ the OEM's acquisition volume is $q_{o,C}^{N*} = \frac{(k+1)\mu_1\bar{v}_o - k\alpha_1\bar{v}_s}{2\alpha_2\mu_1(k+1)-\alpha_1\mu_2 k}$ and an IR's acquisition volume is $q_{s,C}^{N*} = \frac{2\alpha 2\bar{v}_s - \mu_2\bar{v}_o}{2\alpha_2\mu_1(k+1)-\alpha_1\mu_2 k}$; (ii)* ***The RC equilibrium****: when $\bar{\tau}_C \leq \tau < \frac{\bar{v}_s}{\mu_2}$, the OEM's acquisition volume is $q_{o,C}^{R*} = \tau$ and an IR's acquisition volume is $q_{s,C}^{R*} = \frac{\bar{v}_s - \tau\mu_2}{(k+1)\mu_1}$; and (iii)* ***The RM equilibrium****: when $\frac{\bar{v}_s}{\mu_2} \leq \tau$, the OEM's acquisition volume is $q_{o,M}^{R*} = \tau$ and the IRs do not acquire any waste, where $\bar{\tau}_C = \frac{(k+1)\mu_1\bar{v}_o - k\alpha_1\bar{v}_s}{2\alpha_2\mu_1(k+1)-\alpha_1\mu_2 k}$.*

The total acquisition volume under NC equilibrium (i.e., the volume diverted from landfills and sent to recycling, $q_{t,C}^{N*} = q_{o,C}^{N*} + kq_{s,C}^{N*} = \frac{\bar{v}_o(\mu_1 k - \mu_2 k + \mu_1) + k\bar{v}_s(2\alpha_2 - \alpha_1)}{2\alpha_2\mu_1(k+1)-\alpha_1\mu_2 k}$) may decrease in the OEM's per unit revenue v_o, as formalized by the next proposition.

Proposition B.4 *Under the NC equilibrium, i.e., when $\tau < \bar{\tau}_C$, there exists a threshold $\bar{\mu} = \frac{k\mu_2}{k+1}$ such that the total acquisition volume $q_{t,C}^{N*}$ decreases with OEM's per unit revenue v_o iff $\mu_1 < \bar{\mu}$.*

The overall effect of more stringent targets on the total environmental benefit is similar to that under the base model:

Proposition B.5 *Under the RC equilibrium, i.e., when $\bar{\tau}_C \leq \tau < \bar{v}_s$, the total acquisition volume $q_{t,C}^{R*}$ decreases with the collection target τ iff $\mu_1 < \bar{\mu} = \frac{k\mu_2}{k+1}$.*

Finally, we identify the effect of higher collection targets on total welfare as follows:

Proposition B.6 *In a regulated competitive market (i.e., RC equilibrium with $\bar{\tau}_C \leq \tau < \frac{\bar{v}_s}{\mu_2}$), there exists a threshold $\tilde{\mu}$ such that the total welfare $W_C^{R*}(\tau)$ decreases with collection target τ iff $\mu < \tilde{\mu}_1 = \frac{k\mu_2(\bar{v}_s + \epsilon_s)}{(k+1)(\bar{v}_o + \epsilon_o)}$. Furthermore, $\tilde{\mu}$ is increasing in k.*

Comparing Propositions B.3-B.6 with their reciprocals (i.e., Propositions 2-5), we see that naturally the thresholds on μ and τ are more intricate under the generalized model. Nevertheless, observe that all our findings continue to hold.

B.2 IR Activity Towards OEM Obligations

First, we characterize the equilibrium as follows:

Proposition B.7 *When both OEM and IR volumes count towards the collection target, there are three possible equilibria for a given collection target* τ*. Those are (i)* ***The NC equilibrium****: Non-binding regulation, where the OEM acquires* $q_{o,C}^{N*} = \frac{(k+1)\mu_1\bar{v}_o - k\alpha_1\bar{v}_s}{2\alpha_2\mu_1(k+1)-\alpha_1\mu_2 k}$ *and each IR acquires* $q_{s,C}^{N*} = \frac{2\alpha 2\bar{v}_s-\mu_2\bar{v}_o}{2\alpha_2\mu_1(k+1)-\alpha_1\mu_2 k}$*; (ii)* ***The RTC equilibrium****: Binding regulation with IR activity, where the OEM acquires* $q_{o,C}^{T*} = \frac{\mu_1\tau(k+1)-\bar{v}_s k}{\mu_1 k+\mu_1-k\mu_2}$ *and each IR acquires* $q_{s,C}^{T*} = \frac{\bar{v}_s-\tau\mu_2}{\mu_1 k+\mu_1-k\mu_2}$*; and (iii)* ***The RM equilibrium****: Binding regulation without IR activity, where the OEM acquires* $q_{o,M}^{R*} = \tau$ *and the IRs do not acquire any waste. Table 1 characterizes when these equilibria realize with respect to thresholds* $\bar{\mu}$ *and* $\bar{\tau}_T = \frac{\bar{v}_o(\mu_1 k-\mu_2 k+\mu_1)+k\bar{v}_s(2\alpha_2-\alpha_1)}{2\alpha_2\mu_1(k+1)-\alpha_1\mu_2 k} > \bar{\tau}_C$*.*

<table>
<tr><th></th><th>$\tau < \bar{\tau}_T$</th><th colspan="2">$\tau \geq \bar{\tau}_T$</th></tr>
<tr><td>$\mu_1 \leq \bar{\mu}$</td><td>NC</td><td colspan="2">RM</td></tr>
<tr><td rowspan="2">$\mu_1 > \bar{\mu}$</td><td rowspan="2">NC</td><td>$\tau < \frac{\bar{v}_s}{\mu_2}$</td><td>$\frac{\bar{v}_s}{\mu_2} \leq \tau$</td></tr>
<tr><td>RTC</td><td>RM</td></tr>
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Table 1: Equilibrium characterization when IR activity counts towards the OEM obligations under generalized model

Our results regarding the positive impact of higher collection target on total acquisition volume continues to hold:

Proposition B.8 *When both OEM and IR volumes count towards the collection target* τ *and the regulation is binding, the total acquisition volume* q_t^{T*} *is increasing in* τ*.*

As before, we find that the negative impact of higher collection target on total welfare is not mitigated when IR activity counts towards meeting the targets.

Proposition B.9 *Consider the RTC equilibirum, i.e., let* $\mu_1 > \bar{\mu}$ *and* $\bar{\tau}_T \leq \tau < \frac{\bar{v}_s}{\mu_2}$*, when IR activity counts towards OEM obligations. A binding collection target decreases welfare when* $\bar{v}_s + \epsilon_s > \bar{v}_o + \epsilon_o$ *and* $\mu < \tilde{\mu}$*.*

To sum up, under this generalized model the summary of findings in Table 8 continue to hold.

C. Extension: E-waste Trading

So far we assume that there is no transaction between the IRs and OEM. In this section we relax this assumption and allow the OEM to buy items from IRs to meet the collection target τ. For the sake of simplicity, we assume that $k = 1$, i.e., there is one IR. If the OEM buys q_e units at unit wholesale price w from the IR, her problem becomes:

$$\text{(P3)} \max_{q_o, q_e \geq 0} \Pi_o = (v_o - p)q_o + (v_o - w)q_e$$
$$s.t. \quad q_o + q_e \geq \tau$$
$$q_s \geq q_e \geq 0$$

whereas the IR's problem becomes:

$$\text{(P4)} \max_{q_s \geq 0} \Pi_s = (v_s - p)(q_s - q_e) + (w - p)q_e$$

Selling q_e units to the OEM at price w per unit, the IR makes $w - p$ per unit. As before, he also receives $v_s - p$ from the recovery of remaning $q_s - q_e$ units. With the below analysis we show that there are three possible equilibrium solutions: The first one corresponds to the case with no trading; and therefore our results under Section 5 hold here. The second one captures the case where the IR sells all of her collection to the OEM and is equivalent to the case analyzed under Section 6. Finally, the third one captures the case where the IR sells only a fraction of her collection to the OEM. In this case we show that (i) if μ is smaller than a threshold then the solution again converges to that under one of the two regulatory scenarios analyzed in the paper; (ii) if μ is larger than a threshold (but not too large, see below for the conditions) then we find that the negative implications of higher collection target on the total landfill diversion and welfare are still observed. Next we provide the details of the analysis:

We write the Lagrangian of the OEM's problem as $L_o = \Pi_o + \gamma_1(q_o + q_e - \tau) + \gamma_2 q_e + \gamma_3(q_s - q_e)$. Then we calculate the optimality conditions as $\frac{\partial L_o}{q_o} = -\mu q_s - a + \gamma_1 - 2q_o + v_o = 0, \frac{\partial L_o}{q_e} = v_o - w + \gamma_1 + \gamma_2 - \gamma_3 = 0$, $\frac{\partial \Pi_s}{q_s} = -2\mu q_s - a - q_o + v_s = 0$, $\gamma_1(q_o + q_e - \tau) = 0$, $\gamma_2 q_e = 0$, and $\gamma_3(q_s - q_e) = 0$. Let $\bar{w} = w - a$. Solving these for optimal quantities q_o^*, q_s^*, q_e^* and the Lagrangian multipliers γ_i^* for $i = 1, 2, 3$, we find three possible equilibrium solutions.

- Solution 1 ($q_e^* = 0$): Here we find $\gamma_1^* = \frac{3\tau + \bar{v}_s - 2\bar{v}_o}{3}, \gamma_2^* = \frac{2\bar{w} - 3\tau - \bar{v}_s}{2}, \gamma_3^* = 0, q_e^* = 0, q_o^* = \tau, q_s^* = \frac{\bar{v}_s - \tau}{2\mu}$. Note that this solution is the same as the RC equilibrium in Proposition 2 (with $k = 1$), i.e., when there is no trading and only OEM counts towards the collection targets. The optimality conditions $\gamma_1^* \geq 0$ can be written as $\tau \geq \bar{\tau}_C = \frac{2\bar{v}_o - \bar{v}_s}{3}$ and $\gamma_2^* \geq 0$ can be written as $\bar{w} \geq \frac{3\tau + \bar{v}_s}{2} = \bar{w}_1$. Also note that $\frac{\partial \Pi_s^*}{\partial w} = 0$. Therefore, if $\bar{w} \geq \bar{w}_1$ then there is no trading and our results under Section 5 regarding the decrease in total landfill diversion and welfare as collection target increases (i.e., Proposition 4 and 5, respectively) hold here.

- Solution 2 ($q_e^* = q_s^*$): Here we have $\gamma_1^* = \frac{3\mu\tau - 2\mu\bar{v}_o + \mu\bar{v}_s + \bar{v}_o - 2\bar{v}_s}{2\mu - 1}$, $\gamma_2^* = 0$, $\gamma_3^* = \frac{-2\mu\bar{w} + 3\mu\tau + \mu\bar{v}_s + \bar{w} - 2\bar{v}_s}{2\mu - 1}$, $q_o^* = \frac{2\mu\tau - \bar{v}_s}{2\mu - 1}$, $q_s^* = q_e^* = \frac{\bar{v}_s - \tau}{2\mu - 1}$. Note that this solution is the same as the RTC equilibrium in Proposition 6, i.e., when the IR collection counts towards collection targets. The optimality condition $\gamma_1^* > 0$ can be written as $\tau \geq \bar{\tau}_T$ and $\gamma_3^* \geq 0$ can be written as $\bar{w} \leq \frac{(3\mu\tau+\mu\bar{v}_s-2\bar{v}_s)}{(2\mu-1)} = \bar{w}_2$. Therefore, if $\bar{w} < \bar{w}_2$ then the OEM buys all of IR's e-waste. In this case, our results under Section 6 regarding the decrease in total welfare as collection target increases (i.e., Proposition 8) hold here. Also note that $\frac{\partial \Pi_s^*}{w} = \frac{\bar{v}_s-\tau}{2\mu-1} = q_s^* > 0$. Therefore, the IR would want to pick a wholesale price as high as possible.

- Solution 3 ($0 < q_e^* < q_s^*$): Here we find $\gamma_1^* = -\bar{v}_o + \bar{w}, \gamma_2^* = 0, \gamma_3^* = 0, q_e^* = \frac{\bar{v}_s-2\bar{w}+3\tau}{3}, q_o^* = \frac{2\bar{w}-\bar{v}_s}{3}, q_s^* = \frac{2\bar{v}_s-\bar{w}}{3\mu}$. Optimality condition $q_s^* - q_e^* > 0$ can be written as $\bar{w} < \frac{3\mu\tau+\mu\bar{v}_s-2\bar{v}_s}{2\mu-1} = \bar{w}_2$ and $q_e^* > 0$ can be written as $w < \frac{\bar{v}_s+3\tau}{2} = \bar{w}_1$. Here, plugging in optimal values we find total acquisition volume as $q_t^* = q_o^* + q_s^* = \frac{(2\mu-1)\bar{w}+(-\mu+2)\bar{v}_s}{3\mu}$. furthermore, profits are $\Pi_o^* = \frac{4\bar{w}^2-9\bar{w}\tau-4\bar{w}\bar{v}_s+9\tau\bar{v}_o+\bar{v}_s^2}{9}$ and $\Pi_s^* = \frac{(-3\mu+4)\bar{v}_s^2+(9\mu\bar{w}-9\mu\tau-4\bar{w})\bar{v}_s-\bar{w}(6\mu\bar{w}-9\mu\tau-\bar{w})}{9\mu}$. Also $p^* = \frac{\bar{v}_s+\bar{w}+3a}{3}$ and $\Pi_c^* = \frac{(\bar{v}_s+\bar{w})(2\mu\bar{w}-\mu\bar{v}_s-\bar{w}+2\bar{v}_s)}{9\mu}$. Finally $W^* = \frac{(2\mu\epsilon+2\mu\bar{v}_s-\epsilon-\bar{v}_s)\bar{w}+3\mu(\bar{v}_o-\bar{v}_s)\tau-\bar{v}_s(\epsilon+\bar{v}_s)(\mu-2)}{3\mu}$.

  Now assume the IR can choose $\bar{w}^*$ in the first stage maximizing her profit. We find that $\frac{\partial^2 \Pi_s^*}{\partial \bar{w}^2} = -\frac{2(6\mu-1)}{9\mu}$. Therefore, the concavity of Π_s^* depends on μ as we analyze next:

  - If $6\mu \leq 1$ then Π_s^* is convex in $\bar{w}$. In that case, $\bar{w}^*$ will be a corner solution (i.e., at $\bar{w}_1$ or $\bar{w}_2$). We find that $\Pi_s^*(\bar{w}=\bar{w}_1)-\Pi_s^*(\bar{w}=\bar{w}_2) = \frac{(\bar{v}_s-\tau)((4\mu-12\mu^2-1)\tau+(4\mu^2+1)\bar{v}_s)}{4(2\mu-1)^2\mu} > 0$ iff $\tau < \frac{(4\mu^2+1)\bar{v}_s}{12\mu^2-4\mu+1} = \hat{\tau}$. Therefore, if $\tau < \hat{\tau}$ then $\bar{w}^* = \bar{w}_1^*$ and this solution becomes the same as solution 1 above. Otherwise (i.e., if $\tau > \hat{\tau}$), then $\bar{w}^* = \bar{w}_2^*$ and this solution becomes the same as solution 2 above.
  - If $6\mu > 1$ then Π_s^* is concave in $\bar{w}$ and there is a profit maximizing interior $\bar{w}^*$. Solving $\frac{\partial \Pi_s^*}{\partial \bar{w}} = \frac{(-12\mu+2)\bar{w}+9\mu\tau+9\mu\bar{v}_s-4\bar{v}_s}{9\mu} = 0$ for $\bar{w}$, we find that $\bar{w}^* = \frac{9\mu\tau+9\mu\bar{v}_s-4\bar{v}_s}{2(6\mu-1)}$. We also find the total acquisition volume as $q_t(\bar{w} = \bar{w}^*) = \frac{6\mu\tau+2\mu\bar{v}_s-3\tau+3\bar{v}_s}{2(6\mu-1)}$ and $\frac{\partial q_t(\bar{w}=\bar{w}^*)}{\partial \tau} = \frac{3(2\mu-1)}{2(6\mu-1)} \leq 0$ iff $\mu \leq 1/2$. Therefore, similar to our results under Section 5, higher collection target may result in lower waste recovery when the IR demand elasticity μ is small. Finally, we find the total welfare $W^*(\bar{w} = \bar{w}^*) = \frac{6\mu\epsilon\tau+2\mu\epsilon\bar{v}_s+12\mu\tau\bar{v}_o-6\mu\tau\bar{v}_s+2\mu\bar{v}_s^2-3\epsilon\tau+3\epsilon\bar{v}_s-2\tau\bar{v}_o-\tau\bar{v}_s+3\bar{v}_s^2}{2(6\mu-1)}$ and $\frac{\partial W^*}{\partial \tau} = \frac{6\mu\epsilon+12\mu\bar{v}_o-6\mu\bar{v}_s-3\epsilon-2\bar{v}_o-\bar{v}_s}{2(6\mu-1)}$. Therefore $\frac{\partial W^*}{\partial \tau} \leq 0$ iff $\mu \leq \frac{2(\bar{v}_o+\epsilon)+(\bar{v}_s+\epsilon)}{6(2(\bar{v}_o+\epsilon)-(\bar{v}_s+\epsilon))}$. Again, this is similar to our earlier findings, i.e., the total welfare decreases with collection target iff μ is below a threshold.