



ing sequences with repetitive structure. On the other hand, quite a bit of research on OFDM is devoted to CFO estimation based on frequency domain training sequence [10–14]. In [10], Ma and Giannakis propose an adaptive gradient descent method for CFO estimation based on null subcarriers. Here, for description convenience, we treat the null subcarriers as a kind of special ‘training sequence’. In [11, 12], Nogami and Zhang introduce fast Fourier-transform (FFT) based CFO estimation methods with proper window functions on the relevant frequency-assignment schemes. Lei proposes a consistent ML CFO estimator in [13], which exploits the relationship between CFO and the periodogram of the frequency domain training sequence. By employing the so-called time-frequency training sequence, a low complexity CFO estimator is introduced in [14].

Concentrating on Lei’s estimator in [13], we find that different from previous *ad hoc* estimators, this kind of CFO estimator is systematically derived. Analysis demonstrates that this kind of ML estimator is asymptotically unbiased and efficient. However, the computational complexity of Lei’s estimator, which exploits large point FFT grid-searching to improve the estimation performance, is rather high. Furthermore, since the residual CFO is approximately calculated by exploiting the magnitude attenuation in the vicinities of those CFO-shifted pilot tones, the estimate precision can’t be guaranteed in radio channels with large multipath spread and small coherence bandwidth. In the above channels, the adjacent pilot tones with frequency separation greater than the coherence bandwidth may undergo different fading. When the current pilot tone suffers from deep fading, the adjacent pilot tones may hardly undergo any fading. Therefore, the right-hand sides of (41) and (42) in [13] should contain the items concerning the adjacent pilot tones. Otherwise the errors originating from the approximations (41), (42) can’t be ignored even if the averaging operation in (44) is performed. Accordingly, the fine CFO estimation of Lei’s estimator can’t be obtained with sufficient precision. Besides, since the side information about the residual CFO in (41) and (42) don’t increase proportionably with the oversize ratio, the performance improvement of Lei’s estimator by increasing the oversize ratio is not obvious.

To obtain a precise estimate of the true CFO with low complexity, we propose a novel frequency domain training sequence and the corresponding CFO estimator. Different from the training sequence comprising distinctively spaced pilot tones in [13], the proposed training sequence is composed of distinctively spaced pilot tones with high energies and uniformly spaced ones with low energies. Especially, the uniformly spaced pilot tones are generated from Chu sequence having constant amplitude and zero auto-correlation (CAZAC) [15]. Moreover, we also propose an approach to reduce the peak-to-average power ratio (PAPR) concerning the proposed training sequence with low computational complexity. With the aid of the proposed training sequence, we then develop the corresponding CFO estimator which overcomes the drawbacks of Lei’s estimator. Utilizing the structure of the training sequence, we propose to construct

a lookup table storing all kinds of pilot-spacing combinations for the distinctively spaced pilot tones. Aided by the distinctively spaced pilot tones and the predefined lookup table, integer CFO estimation can then be obtained without the need of large point FFT grid searching. Accordingly, the complexity issue of Lei’s estimator is resolved. After interference cancellation is done, with the aid of the uniformly spaced pilot tones, fractional CFO estimation based on the best linear unbiased estimation (BLUE) principle can be achieved. Since the uniformly spaced pilot tones are generated from cyclically orthogonal Chu sequence, the fractional CFO estimator aided by them has strong ability to combat multipath effect. Correspondingly, the estimate precision issue of Lei’s estimator in channels with large multipath spread is also resolved. In conclusion, the proposed CFO estimator assisted by the novel frequency domain training sequence has fairly good performance with comparatively little computational complexity.

The rest of the paper is organized as follows. We begin with the OFDM system model in Section II. The novel frequency domain training sequence, which consists of distinctively spaced pilot tones and uniformly spaced ones, is presented in Section III. The approach concerning PAPR reduction is also introduced in this section. And then in Section IV, the corresponding CFO estimator, which is composed of the integer CFO estimator with great complexity reduction and the fractional CFO estimator with strong ability to combat multipath effect, is elaborated. Also included in this section is the complexity analysis. Simulation results are shown in Section V. Finally, Section VI concludes this paper.

Notation: Upper (lower) bold-face letters are used for matrices (column vectors). Superscript  $*$ ,  $T$  and  $H$  denote conjugate, transpose and Hermitian transpose, respectively.  $diag\{\cdot\}$  stands for a diagonal matrix with the elements within the brackets on its diagonal.  $[\mathbf{x}]_m$  denotes the  $m$ -th entry of a column vector  $\mathbf{x}$ .  $[\mathbf{A}]_{m,n}$  denotes the  $(m, n)$ -th entry of a matrix  $\mathbf{A}$ .  $\|\cdot\|^2$  represents the Euclidean norm operation.  $\lfloor \cdot \rfloor$  denotes the nearest integer that the number within the brackets is rounded to.  $((\cdot))_N$  denotes the modulus  $N$  operation.  $E[\cdot]$  denotes the expectation operation.

## 2. System Model

The baseband-equivalent OFDM system model with the proposed CFO estimator is shown in Fig. 1. At transmitter side, the training sequence symbol is inserted before fixed number of data symbols to construct a transmitted frame. The data symbols are generated by passing the information bits through encoder, interleaver,  $M$ -ary modulator ( $M = 2^{M_c}$ ) and inverse fast Fourier-transform (IFFT) module. Let  $\tilde{\mathbf{p}}_N = [\tilde{p}_0, \tilde{p}_1, \dots, \tilde{p}_{N-1}]^T$  denote the frequency domain training sequence. The training sequence symbol is directly generated with the application of the  $N \times N$  normalized IFFT matrix  $\mathbf{F}_N$  to  $\tilde{\mathbf{p}}_N$ ,

$$\mathbf{p}_N = \mathbf{F}_N \tilde{\mathbf{p}}_N \quad (1)$$

In order to prevent possible inter-symbol interference (ISI) between OFDM symbols, a CP with length  $N_g$ , which is longer than the multipath delay spread  $L$ , is inserted before every symbol. Then the baseband samples are transmitted through a frequency-selective fading channel with additive white Gaussian noise (AWGN).

At receiver side, it is assumed that the received samples are affected by the normalized CFO  $\varepsilon$ , which equals the actual CFO  $\Delta F$  divided by the OFDM subcarrier spacing  $\Delta f$ . Firstly, timing synchronization is accomplished, then CPs are removed. Since the proposed CFO estimator is insensitive to timing synchronization error, ideal timing synchronization is assumed in this paper. Let  $\mathbf{h}_L = [h_0, h_1, \dots, h_{L-1}]^T$  denote the normalized finite impulse response of the multipath channel in discrete-time equivalent form, let  $\mathbf{H}$  denote the  $N \times N$  cyclic matrix with first column  $\mathbf{h}_N = [\mathbf{h}_L, \mathbf{0}_{N-L}]^T$ , and define

$$\Psi(\varepsilon) = \text{diag}\{1, e^{j2\pi\varepsilon/N}, \dots, e^{j2\pi(N-1)\varepsilon/N}\},$$

then the received sequence corresponding to the transmitted training sequence can be written as follows,

$$\begin{aligned} \mathbf{r} &= e^{j2\pi\varepsilon N_g/N} \Psi(\varepsilon) \mathbf{H} \mathbf{p}_N + \mathbf{w} \\ &= e^{j2\pi\varepsilon N_g/N} \Psi(\varepsilon) \mathbf{F}_N \tilde{\mathbf{p}}_N \tilde{\mathbf{h}}_N + \mathbf{w} \end{aligned} \quad (2)$$

where

$$\tilde{\mathbf{p}}_N = \text{diag}\{\tilde{p}_0, \tilde{p}_1, \dots, \tilde{p}_{N-1}\}, \tilde{\mathbf{h}}_N = \mathbf{F}_N^H \mathbf{h}_N,$$

$\mathbf{w}$  stands for zero-mean AWGN with covariance matrix equal to  $\sigma^2 \mathbf{I}_N$ ,  $\mathbf{I}_N$  denotes the  $N \times N$  identity matrix. Based on  $\mathbf{r}$  in (2), the proposed integer CFO estimation and fractional CFO estimation are performed in turn. With the estimated CFO, CFO correction is applied to the following data symbols. Finally, the information bits are obtained by passing the corrected data symbols through FFT module, demodulator, de-interleaver and decoder, respectively.

From the above description, we can see that the performance of the CFO estimator depends on the training sequence  $\tilde{\mathbf{p}}_N$ . By designing the training sequence properly, we can obtain the corresponding CFO estimator with good performance and low complexity.

### 3. The Novel Frequency Domain Training Sequence

#### 3.1 Training Sequence Construction

We propose to construct the frequency domain training sequence  $\tilde{\mathbf{p}}_N$  from two types of pilot tones, namely distinctively spaced ones with high energies and uniformly spaced ones with low energies. The structure of the proposed training sequence is illustrated in Fig. 2. The ratio of the total power of the distinctively spaced pilot tones  $\xi_0$  to that of the uniformly spaced ones  $\xi_1$  is set to  $\xi_0 : \xi_1 = \alpha : (1 - \alpha)$  under constant power constraint  $\xi = \xi_0 + \xi_1 = N$ .

Let  $\mathcal{D}$  denote the set of the indices for the  $N_D$  distinctively spaced pilot tones  $\{\tilde{p}_{d_k}\}_0^{N_D-1}$  and let  $\mathcal{U}$  denote the set of the indices for the  $N_U$  uniformly spaced pilot tones  $\{\tilde{p}_{u_k}\}_0^{N_U-1}$  with  $N_U > N_D$ . Then, the set of the indices for the  $N_C$  non-zero pilot tones  $\{\tilde{p}_{c_k}\}_0^{N_C-1}$  denoted by  $\mathcal{C}$  can be defined as  $\mathcal{C} = \mathcal{D} \cup \mathcal{U}$ , where  $\mathcal{D} \cap \mathcal{U} = \phi$ . In order to recover the channel exactly by exploiting the proposed training sequence, the number of the non-zero pilot tones  $N_C$  should be no less than the multipath delay spread of the channel  $L$  [16].

For the distinctively spaced pilot tones, their indices are designed to satisfy the following condition,

$$\begin{aligned} ((d_{((n+1)N_D} - d_n))_N \neq ((d_{((m+1)N_D} - d_m))_N, \\ \text{if } d_n \neq d_m, \text{ and } d_n, d_m \in \mathcal{D} \end{aligned} \quad (3)$$

Suppose  $((N))_{2N_U} = 0$ , let  $X = N/N_U$  denote the subcarrier spacing between the adjacent uniformly spaced pilot tones, let  $d_n$  denote the index of the  $n$ -th distinctively spaced pilot tone and let  $u_{n'}$  denote the index of the left neighboring uniformly spaced pilot tone to the  $n$ -th distinctively spaced pilot tone, then it can be observed from Fig. 2 that the following relationship holds,

$$d_n = u_{n'} + v, \quad \text{for } v \in [1, X - 1] \quad (4)$$

Assume  $((N_U))_2 = 0$  and let  $P_{ICI,d_k}$  denote the average power of the total inter-carrier interference (ICI) that  $\tilde{p}_{d_k}$  imposes on the  $N_U$  uniformly spaced pilot tones at receiver side, then we have the following theorem (whose proof is shown in the appendix),

**Theorem 1:** With the proposed training sequence  $\tilde{\mathbf{p}}_N$ , the optimum value of  $v$  within the range  $[1, X - 1]$  which makes  $P_{ICI,d_k}$  achieve its minimum is  $X/2$ .

According to the above theorem, we set  $v = X/2$  in (4) for the proposed training sequence, which helps to decouple the

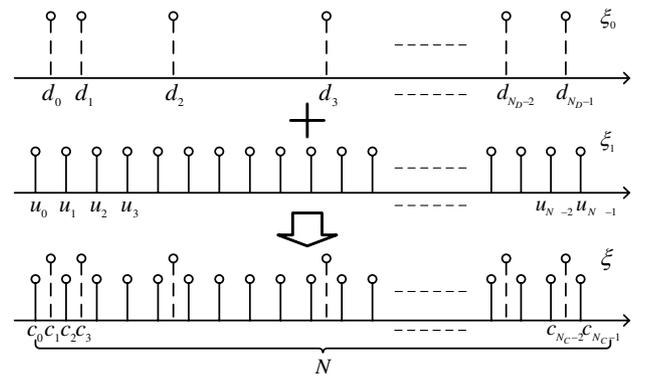


Fig. 2 Structure of the proposed frequency domain training sequence with power constraint  $\xi = \xi_0 + \xi_1$ . Power  $\xi_0$  is allocated to the distinctively spaced pilot tones, while power  $\xi_1$  is allocated to the uniformly spaced ones. X-coordinate and Y-coordinate represent the indices and amplitudes of the pilots tones respectively.

two types of pilot tones with least interference at receiver side. Furthermore, since the distinctively spaced pilot tones with high energies are exploited for the integer CFO estimation as shown in the following section, their amplitudes are made constant without loss of generality.

For the uniformly spaced pilot tones, they are constructed from a length- $N_U$  Chu sequence  $\mathbf{s}_{N_U}$  which has CAZAC. Let  $\tilde{\mathbf{s}}_{N_U} = \mathbf{F}_{N_U}^H \mathbf{s}_{N_U}$ , then the uniformly spaced pilot tones can be expressed as follows,

$$\tilde{p}_{u_k} = \sqrt{(1-\alpha)X} [\tilde{\mathbf{s}}_{N_U}]_k, \text{ for } k = 0, 1, \dots, N_U - 1. \quad (5)$$

Without loss of generality, we set the set of the indices for the uniformly spaced pilot tones to  $\mathcal{U} = \{kX\}_{k=0}^{N_U-1}$ . An appealing feature has been shown in [17] that a sequence has CAZAC if and only if its discrete Fourier-transform (DFT) has CAZAC and vice versa. Therefore, the uniformly spaced pilot tones generated from Chu sequence according to (5) also have constant amplitude.

### 3.2 Peak-to-Average Power Ratio Reduction

Since the PAPR problem often needs to be resolved for low cost linear power amplifier at transmitter side in many systems that utilize frequency domain for data recovery and the amplifier efficiency increases monotonically as the PAPR decreases [18, Fig. 1], the next question considered here is how to make PAPR concerning the proposed training sequence as small as possible.

Define

$$\begin{aligned} \tilde{\mathbf{p}}_{N_D} &= [\tilde{p}_{d_0}, \tilde{p}_{d_1}, \dots, \tilde{p}_{d_{N_D-1}}]^T, \\ \tilde{\mathbf{p}}_{N_U} &= [\tilde{p}_{u_0}, \tilde{p}_{u_1}, \dots, \tilde{p}_{u_{N_U-1}}]^T, \\ \mathbf{F}_{\beta N \times N_D} &= [\mathbf{f}_{d_0}^{\beta N}, \mathbf{f}_{d_1}^{\beta N}, \dots, \mathbf{f}_{d_{N_D-1}}^{\beta N}], \\ \mathbf{F}_{\beta N \times N_U} &= [\mathbf{f}_{u_0}^{\beta N}, \mathbf{f}_{u_1}^{\beta N}, \dots, \mathbf{f}_{u_{N_U-1}}^{\beta N}], \end{aligned}$$

where  $\mathbf{f}_k^{\beta N}$  denotes the  $k$ -th column of the  $\beta N \times \beta N$  IFFT matrix  $\mathbf{F}_{\beta N}$ , then the  $\beta$ -times oversampled time domain sequence  $\mathbf{p}_{\beta N}$  corresponding to the proposed training sequence can be written as follows,

$$\mathbf{p}_{\beta N} = \sqrt{\beta} \mathbf{F}_{\beta N \times N_D} \tilde{\mathbf{p}}_{N_D} + \sqrt{\beta} \mathbf{F}_{\beta N \times N_U} \tilde{\mathbf{p}}_{N_U} \quad (6)$$

It is well known that the PAPR of the continuous-time signal can't be obtained precisely by the use of Nyquist rate sampling, which corresponds to the case of  $\beta = 1$ . It is shown in [19] that  $\beta = 4$  can provide sufficiently accurate PAPR results. The PAPR computed from the  $\beta$ -times oversampled time domain sequence  $\mathbf{p}_{\beta N}$  is given by

$$PAPR = \max_{n \in [0, \beta N - 1]} \{ |[\mathbf{p}_{\beta N}]_n|^2 \} \quad (7)$$

After  $N_D, N_U, \alpha, \mathcal{U}$  are all decided, we need to design  $\mathcal{D}$  and  $\tilde{\mathbf{p}}_{N_D}$  to obtain a small PAPR concerning the proposed training sequence. Under the constraints that the elements of  $\mathcal{D}$  satisfy the conditions (3), (4) with  $v = X/2$  and that

the elements of  $\tilde{\mathbf{p}}_{N_D}$  have constant amplitude, the optimal  $\mathcal{D}$  and  $\tilde{\mathbf{p}}_{N_D}$  can be obtained by solving the following *minimax* problem,

$$\{\mathcal{D}, \tilde{\mathbf{p}}_{N_D}\} = \arg \min_{\mathcal{D}, \tilde{\mathbf{p}}_{N_D}} \left\{ \max_{n \in [0, \beta N - 1]} \{ |[\mathbf{p}_{\beta N}]_n|^2 \} \right\} \quad (8)$$

In principle, the elements of  $\mathcal{D}$  should be distributed in the range  $[0, N - 1]$  as 'evenly' as possible in order to track the variety of the fading channel accurately. After  $\mathcal{D}$  is determined according to the above principle, the  $2N_D$ -dimensional optimization problem concerning PAPR in (8) is reduced to a  $N_D$ -dimensional optimization one. Furthermore, since the integer CFO estimation as shown in the following section is only relevant to the amplitudes of the distinctively spaced pilot tones, in order to further simply the  $N_D$ -dimensional optimization problem, we put the following constraint on the distinctively spaced pilot tones,

$$\tilde{p}_{d_k} = \sqrt{\frac{\alpha N}{N_D}} (-1)^{i_k}, \text{ for } i_k \in \{0, 1\}, k = 0, 1, \dots, N_D - 1. \quad (9)$$

Therefore,  $\tilde{\mathbf{p}}_{N_D}$  is determined solely by  $[i_0, i_1, \dots, i_{N_D-1}]$  with definite  $\alpha, N_D$  and  $N$ . Correspondingly, we can express the above  $N_D$ -dimensional optimization problem concerning PAPR as follows,

$$\begin{aligned} [i_0, i_1, \dots, i_{N_D-1}] &= \\ \arg \min_{[i_0, i_1, \dots, i_{N_D-1}] \in \{0, 1\}^{N_D}} &\left\{ \max_{n \in [0, \beta N - 1]} \{ |[\mathbf{p}_{\beta N}]_n|^2 \} \right\} \quad (10) \end{aligned}$$

Compared with the approach as shown in (8), the above approach has rather less computational complexity. Effectiveness of the above approach will be shown through simulation results in Section V.

## 4. Low Complexity OFDM CFO Estimator

The CFO estimation based on the proposed training sequence is separated into two phases: the integer and fractional CFO estimation. In this section, the integer CFO estimator based on the distinctively spaced pilot tones with great complexity reduction and the fractional CFO estimator based on the uniformly spaced ones with strong ability to combat multipath effect will be described in detail separately. The corresponding complexity analysis will also be presented subsequently.

### 4.1 Integer CFO Estimator Based on the Distinctively Spaced Pilot Tones

Define

$$\begin{aligned} \mathbf{F}_{N \times N_C} &= [\mathbf{f}_{c_0}^N, \mathbf{f}_{c_1}^N, \dots, \mathbf{f}_{c_{N_C-1}}^N], \\ \tilde{\mathbf{P}}_{N_C} &= \text{diag}\{\tilde{p}_{c_0}, \tilde{p}_{c_1}, \dots, \tilde{p}_{c_{N_C-1}}\}, \\ \tilde{\mathbf{h}}_{N_C} &= [[\tilde{\mathbf{h}}_N]_{c_0}, [\tilde{\mathbf{h}}_N]_{c_1}, \dots, [\tilde{\mathbf{h}}_N]_{c_{N_C-1}}]^T, \end{aligned}$$

then the received sequence corresponding to the proposed training sequence can be written as follows,

$$\mathbf{r} = \mathbf{A}(\varepsilon)\tilde{\mathbf{h}}_{N_C} + \mathbf{w} \quad (11)$$

where

$$\mathbf{A}(\varepsilon) = e^{j2\pi\varepsilon N_g/N} \Psi(\varepsilon) \mathbf{F}_{N \times N_C} \tilde{\mathbf{P}}_{N_C}.$$

With the assumption that  $\mathbf{w}$  is zero-mean AWGN with covariance matrix equal to  $\sigma^2 \mathbf{I}_N$ , the log-likelihood function of  $\mathbf{r}$  conditioned on  $\varepsilon$  and  $\tilde{\mathbf{h}}_{N_C}$  can be obtained as (ignoring the constant items),

$$\ln p(\mathbf{r}|\varepsilon, \tilde{\mathbf{h}}_{N_C}) = -\sigma^{-2} \|\mathbf{r} - \mathbf{A}(\varepsilon)\tilde{\mathbf{h}}_{N_C}\|^2 \quad (12)$$

Fixing  $\varepsilon$  and maximizing (12) with respect to  $\tilde{\mathbf{h}}_{N_C}$ , we can get the ML estimate of  $\tilde{\mathbf{h}}_{N_C}$  [20],

$$\begin{aligned} \hat{\mathbf{h}}_{N_C} &= [\mathbf{A}^H(\varepsilon)\mathbf{A}(\varepsilon)]^{-1} \mathbf{A}^H(\varepsilon)\mathbf{r} \\ &= e^{-j2\pi\varepsilon N_g/N} \tilde{\mathbf{P}}_{N_C}^{-1} \mathbf{F}_{N \times N_C}^H \Psi^H(\varepsilon)\mathbf{r} \end{aligned} \quad (13)$$

Substituting  $\hat{\mathbf{h}}_{N_C}$  into (12), after some straightforward manipulations and dropping the irrelevant items, we can obtain the reformulated log-likelihood function conditioned on  $\varepsilon$  as follows,

$$\ln p(\mathbf{r}|\varepsilon) = \|\mathbf{F}_{N \times N_C}^H \Psi^H(\varepsilon)\mathbf{r}\|^2 \quad (14)$$

Therefore, the ML estimate of  $\varepsilon$  is given by

$$\begin{aligned} \hat{\varepsilon} &= \arg \max_{\hat{\varepsilon}} \{ \|\mathbf{F}_{N \times N_C}^H \Psi^H(\hat{\varepsilon})\mathbf{r}\|^2 \} \\ &= \arg \max_{\hat{\varepsilon}} \left\{ \sum_{k=0}^{N_C-1} \Xi(\hat{\varepsilon}_k) \right\} \end{aligned} \quad (15)$$

where

$$\hat{\varepsilon}_k = \hat{\varepsilon} + c_k, \Xi(\hat{\varepsilon}_k) = \frac{1}{N} \left| \sum_{n=0}^{N-1} [\mathbf{r}]_n e^{-j2\pi n \hat{\varepsilon}_k / N} \right|^2.$$

We point out here that  $\Xi(f)$  denotes the periodogram of the received sequence  $\mathbf{r}$  with period  $N$ . Suppose that at least two neighboring distinctively-spaced pilot tones and two uniformly-spaced pilot tones with indices  $u_n$  and  $u_m$  do not coincide with channel nulls of the frequency-selective fading channels, where  $u_n$  and  $u_m$  satisfy the following condition,

$$(u_n - u_m) \notin \{d | d = d_k - d_l, \text{ for } d_k \neq d_l \text{ and } d_k, d_l \in \mathcal{D}\}. \quad (16)$$

Then exploiting the similar approach as in [13], we can prove that  $\hat{\varepsilon} = \varepsilon$  is the unique value to maximize  $\ln p(\mathbf{r}|\varepsilon)$  for  $\hat{\varepsilon}, \varepsilon \in (-N/2, N/2]$  with the proposed training sequence  $\tilde{\mathbf{p}}_N$ . Accordingly, the estimation range of the CFO estimator in (15) is  $(-N/2, N/2]$ .

Furthermore, it can be seen that  $\Xi(f)$  can be computed through FFT technique. In order to simply the computation, we only invoke  $N$ -point FFT over  $\mathbf{r}$ . Define

$$\tilde{r}_{((k'))_N} = \mathbf{f}_{((k'))_N}^{N,H} \mathbf{r}, \text{ for } k' = -\frac{N}{2} + 1, -\frac{N}{2} + 2, \dots, \frac{N}{2}. \quad (17)$$

then we can obtain the integer CFO estimate of  $\varepsilon$  as follows,

$$\hat{\varepsilon}_I = \arg \max_{k'} \left\{ \sum_{k=0}^{N_C-1} |\tilde{r}_{((k'+c_k))_N}|^2 \right\} \quad (18)$$

Notice that the computational complexity of the integer CFO estimation in (18) is still very high. In order to further simplify the computation, we take the received sequence transformed from time domain to frequency domain into consideration. With the assumption that  $N$ ,  $X$  and the signal-to-noise ratio (SNR) are all large enough, the ICI resulting from adjacent non-zero pilot tones can be ignored and the following approximation can be achieved,

$$\begin{aligned} |\tilde{r}_{((c_k + \lfloor \varepsilon \rfloor))_N}| &\doteq |\tilde{p}_{c_k}| |[\tilde{\mathbf{h}}_N]_{c_k} \text{sinc}(\varepsilon - \lfloor \varepsilon \rfloor)| \\ &\times \left| 1 + \frac{[\tilde{\mathbf{w}}]_{((c_k + \lfloor \varepsilon \rfloor))_N}}{\tilde{p}_{c_k} [\tilde{\mathbf{h}}_N]_{c_k} \text{sinc}(\varepsilon - \lfloor \varepsilon \rfloor)} e^{-j \frac{\pi(\varepsilon - \lfloor \varepsilon \rfloor)(N-1) + 2\pi\varepsilon N_g}{N}} \right| \\ &\doteq |\tilde{p}_{c_k}| |[\tilde{\mathbf{h}}_N]_{c_k} \text{sinc}(\varepsilon - \lfloor \varepsilon \rfloor)| \end{aligned} \quad (19)$$

where

$$\tilde{\mathbf{w}} = \mathbf{F}_N^H \mathbf{w}, \text{sinc}(x) = \sin(\pi x) / (\pi x).$$

Taking the above approximation into consideration, we can get the suboptimal integer CFO estimation as follows,

$$\begin{aligned} \hat{\varepsilon}_I &= \\ &\arg \max_{k'} \left\{ \sum_{k=0}^{N_D-1} |\tilde{r}_{((k'+d_k))_N}|^2 + \sum_{k=0}^{N_U-1} |\tilde{r}_{((k'+u_k))_N}|^2 \right\} \\ &\doteq \arg \max_{k'} \left\{ \sum_{k=0}^{N_D-1} |\tilde{r}_{((k'+d_k))_N}|^2 \right\} \end{aligned} \quad (20)$$

Let

$$d_{k_0} = \arg \max_{d_k \in \mathcal{D}} \{ |\tilde{r}_{((d_k + \lfloor \varepsilon \rfloor))_N}| \},$$

$$u_{k_1} = \arg \max_{u_k \in \mathcal{U}} \{ |\tilde{r}_{((u_k + \lfloor \varepsilon \rfloor))_N}| \}.$$

Taking (19) into consideration again, if and only if the following condition is satisfied,

$$|[\tilde{\mathbf{h}}_N]_{d_{k_0}} / [\tilde{\mathbf{h}}_N]_{u_{k_1}}| > \sqrt{(1-\alpha)N_D / (\alpha N_U)}, \quad (21)$$

we can obtain

$$\arg \max_{c_k \in \mathcal{C}} \{ |\tilde{r}_{((c_k + \lfloor \varepsilon \rfloor))_N}| \} \in \mathcal{D} \quad (22)$$

Let

$$P_{\text{correct}} = P(\arg \max_{c_k \in \mathcal{C}} \{ |\tilde{r}_{((c_k + \lfloor \varepsilon \rfloor))_N}| \} \in \mathcal{D}) \quad (23)$$

denote the probability that (22) holds. Using extensive simulations in the next section, we find that by increasing the value of  $N_D$  with  $\alpha N_U / [(1-\alpha)N_D]$  kept invariable, the

Table 1 Contents of the Predefined Lookup Table <sup>1</sup>

$d_1 - d_0$	$d_2 - d_0$	$\dots$	$d_{N_D-1} - d_0$
$d_2 - d_1$	$d_3 - d_1$	$\dots$	$d_0 - d_1 + N$
$\dots$	$\dots$	$\dots$	$\dots$
$d_0 - d_{N_D-1} + N$	$d_1 - d_{N_D-1} + N$	$\dots$	$d_{N_D-2} - d_{N_D-1} + N$

<sup>1</sup>Each row stores a possible pilot-spacing combination.

probability  $P_{correct}$  approaches 1.

With condition (22), we propose a low complexity integer CFO estimator as illustrated in Fig. 3. Firstly, the peak pilot tone which is affected by the fading channel least is found,

$$\zeta = \arg \max_{k \in [0, N-1]} \{ |\tilde{r}_k| \} \quad (24)$$

Since  $((\zeta - \lfloor \varepsilon \rfloor))_N$  is a member of the set  $\mathcal{D}$  and the spacings between the adjacent members of  $\mathcal{D}$  are distinctive, we then exploit a lookup table with size  $N_D \times (N_D - 1)$ , which stores every possible pilot-spacing combination for the distinctively spaced pilot tones as shown in Table 1, to locate the distinctively spaced pilot tone corresponding to the peak pilot tone in  $\mathcal{D}$ . The corresponding operation is,

$$\kappa = \arg \max_{k \in [0, N_D-1]} \left\{ \sum_{g=0}^{N_D-2} |\tilde{r}_{((\Pi_{k,g} + \zeta))_N}|^2 \right\} \quad (25)$$

where  $\Pi_{k,g}$  denotes the content stored in the  $k$ -th row  $g$ -th column of the lookup table. Then, the shift of the searched peak pilot tone is calculated,

$$\delta = \zeta - d_\kappa \quad (26)$$

Finally, the integer CFO estimate of  $\varepsilon$  can be readily obtained by normalizing the shift to  $N$ ,

$$\hat{\varepsilon}_I = \begin{cases} -N + \delta, & \text{if } \delta > N/2, \\ N + \delta, & \text{else if } \delta \leq -N/2, \\ \delta, & \text{else.} \end{cases} \quad (27)$$

#### 4.2 Fractional CFO Estimator Based on the Uniformly Spaced Pilot Tones

After the integer CFO estimation is accomplished, the distinctively spaced pilot tones and the neighboring ones are nulled for the sake of cancelling the interference they impose on the uniformly spaced pilot tones. Let  $\tilde{\mathbf{r}}^{ic}$  denote the interference-cancelled frequency domain sequence, then we can express  $\tilde{\mathbf{r}}^{ic}$  according to the above description as follows,

$$[\tilde{\mathbf{r}}^{ic}]_k = \begin{cases} 0, & \text{if } k = ((d_{\tilde{k}} + \hat{\varepsilon}_I))_N \text{ for } d_{\tilde{k}} \in \mathcal{D}, \\ & \text{or } k = ((d_{\tilde{k}} + \hat{\varepsilon}_I - 1))_N \text{ for } d_{\tilde{k}} \in \mathcal{D} \\ & \text{and } \frac{\sum_{d_{\tilde{k}} \in \mathcal{D}} |\tilde{r}_{((d_{\tilde{k}} + \hat{\varepsilon}_I + 1))_N}|^2}{\sum_{d_{\tilde{k}} \in \mathcal{D}} |\tilde{r}_{((d_{\tilde{k}} + \hat{\varepsilon}_I - 1))_N}|^2} < 1, \\ & \text{or } k = ((d_{\tilde{k}} + \hat{\varepsilon}_I + 1))_N \text{ for } d_{\tilde{k}} \in \mathcal{D} \\ & \text{and } \frac{\sum_{d_{\tilde{k}} \in \mathcal{D}} |\tilde{r}_{((d_{\tilde{k}} + \hat{\varepsilon}_I + 1))_N}|^2}{\sum_{d_{\tilde{k}} \in \mathcal{D}} |\tilde{r}_{((d_{\tilde{k}} + \hat{\varepsilon}_I - 1))_N}|^2} > 1, \\ \tilde{r}_k, & \text{else.} \end{cases} \quad (28)$$

After that,  $\tilde{\mathbf{r}}^{ic}$  is transformed from frequency domain to time domain as follows,

$$\mathbf{r}^{ic} = \mathbf{F}_N \tilde{\mathbf{r}}^{ic} \quad (29)$$

Then integer CFO correction is carried out on  $\mathbf{r}^{ic}$ ,

$$\mathbf{r}^{cc} = e^{-j \frac{2\pi \hat{\varepsilon}_I N q}{N}} \Psi(-\hat{\varepsilon}_I) \mathbf{r}^{ic} \quad (30)$$

Taking the property of periodical repetition for  $\mathbf{r}^{cc}$  into account, we have

$$\begin{aligned} R_m &= \frac{1}{N - mN_U} \sum_{n=mN_U}^{N-1} [\mathbf{r}^{cc}]_n [\mathbf{r}^{cc}]_{n-mN_U}^* \\ &= \sigma_r^2 e^{j2\pi m N_U \varepsilon_F / N} (1 + \varrho_m), \text{ for } m \in [0, X/2] \end{aligned} \quad (31)$$

where

$$\begin{aligned} \varepsilon_F &= \varepsilon - \hat{\varepsilon}_I, \\ \sigma_r^2 &\doteq \frac{1}{N - mN_U} \sum_{n=mN_U}^{N-1} \left\{ \left\{ \sum_{l=0}^{L-1} \{ [\mathbf{s}_{N_U}]_{((n-l))_{N_U}} h_l \} \right\} \right. \\ &\quad \times \left. \left\{ \sum_{l=0}^{L-1} \{ [\mathbf{s}_{N_U}]_{((n-mN_U-l))_{N_U}} h_l \} \right\}^* \right\}, \end{aligned}$$

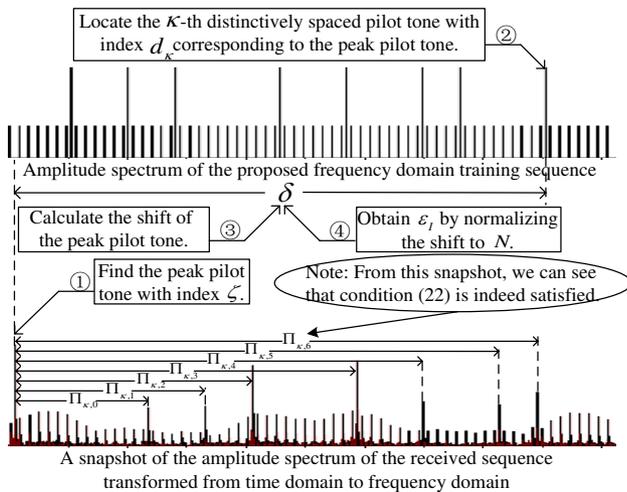


Fig. 3 Schematic diagram of the proposed integer CFO estimator with equations (24)-(27).

Table 2 Computational complexity of FBE and LNE

	Number of Real Additions	Number of Real Multiplications
FBE	$4N \log_2 N + N(1.5X + 1) + N_D(N_D + 1)$	$4N \log_2 N + N(1.5X + 4) + N_D(N_D - 2)$
LNE <sup>1</sup>	$2L'N \log_2 N + 2N_D + N_D M'$	$2L'N \log_2 N + 6N_D + N_D M'$

<sup>1</sup>For LNE,  $L'$  denotes the oversize ratio of the corresponding FFT interpolation,  $M'$  denotes the number of the interpolated signals in frequency domain whose amplitudes are larger than the predefined threshold.

$$\varrho_m \doteq \frac{1}{\sigma_r^2(N - mN_U)} \left\{ \sum_{l=0}^{L-1} \{[\mathbf{s}_{N_U}]_{((n-l))_{N_U}} h_l\} \check{w}_{n-mN_U}^* \right. \\ \left. + \left\{ \sum_{l=0}^{L-1} \{[\mathbf{s}_{N_U}]_{((n-mN_U-l))_{N_U}} h_l\} \check{w}_n + \check{w}_n \check{w}_{n-mN_U}^* \right\} \right\},$$

$\check{w}_n$  denotes zero mean AWGN. With the assumption that the multipath channel is time-invariant over each OFDM symbol, the following result can be obtained by exploiting cyclically orthogonal property of Chu sequence,

$$\sigma_r^2 \doteq \frac{1}{N - mN_U} \sum_{n=mN_U}^{N-1} \left\{ \sum_{l=0}^{L-1} \{ |[\mathbf{s}_{N_U}]_{((n-l))_{N_U}}|^2 |h_l|^2 \} \right. \\ \left. + \sum_{l,l'=0, l \neq l'}^{L-1} \{ [\mathbf{s}_{N_U}]_{((n-l))_{N_U}} [\mathbf{s}_{N_U}]_{((n-l'))_{N_U}}^* h_l h_{l'}^* \} \right\} \\ = \frac{1}{N - mN_U} \sum_{n=mN_U}^{N-1} \sum_{l=0}^{L-1} \{ |[\mathbf{s}_{N_U}]_{((n-l))_{N_U}}|^2 |h_l|^2 \} \quad (32)$$

We can see from (31) and (32) that Chu sequence with cyclically orthogonal property can enhance the ability of the estimator to combat multipath effect. Actually, with the concept of zero-correlation zone (ZCZ) in [21], the uniformly spaced pilot tones generated from a sequence with ZCZ width greater than  $L$ , i.e., the multipath delay spread of the channel, is enough.

Assume that the SNR is sufficiently high and that  $|\varepsilon_F| \leq X/2$ , then the following approximation holds,

$$\varphi_m = \text{angle}(R_m R_{m-1}^*) \\ \doteq \text{Imag}(\varrho_m) - \text{Imag}(\varrho_{m-1}) \\ + 2\pi N_U \varepsilon_F / N, \quad \text{for } m \in [1, X/2] \quad (33)$$

where  $\text{Imag}(\varrho_m)$  denotes the imaginary component of  $\varrho_m$ . Accordingly, based on the best linear unbiased estimation (BLUE) principle [8, 20], fractional CFO estimation can be obtained as follows,

$$\hat{\varepsilon}_F = \frac{N}{2\pi N_U} \sum_{m=1}^{X/2} \lambda_m \varphi_m \quad (34)$$

where

$$\lambda_m = \frac{6(X-m)(X-m+1) - 0.25X^2}{X(X^2-1)}, \quad \text{for } m \in [1, X/2].$$

After the integer and fractional CFO estimation are both accomplished, the whole CFO is readily obtained as  $\hat{\varepsilon} = \hat{\varepsilon}_I + \hat{\varepsilon}_F$ .

### 4.3 Complexity Analysis

For description convenience, we refer to the proposed estimator as frequency domain training sequence based estimator (FBE). In Table 2, the computational complexity of FBE is evaluated in comparison with that of Lei's estimator (LNE) [13]. We assume that LNE employs a length- $N$  frequency domain training sequence comprising  $N_D$  pilots tones. The comparison is visualized in Fig. 4, where the total numbers of real additions and multiplications involved in the two types of estimators are illustrated as a function of the subcarrier number  $N$ . For the considered subcarrier numbers, we set the number of the distinctively spaced pilot tones to  $N_D = N/128$ , which guarantees accurate running of the integer CFO estimator, and set the subcarrier spacing between the adjacent uniformly spaced pilot tones to  $X = 16$ , which guarantees little ICI between the two types of pilot tones in the proposed training sequence at receiver side and good complexity-performance tradeoff of the fractional CFO estimator. It can be observed from Fig. 4 that the complexity of FBE is obviously lower than that of LNE. Furthermore, since the time domain sequence corresponding to the uniformly spaced pilot tones can be separated into two identical halves for even  $X$ , the correlation-based fractional CFO estimator in [7], whose complexity is lower than that of the fractional CFO estimator in Section IV.B, can be exploited to further reduce the complexity of FBE but with a slightly worse performance.

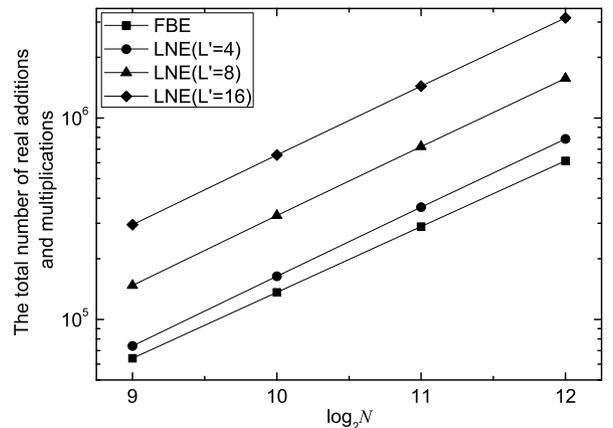


Fig. 4 Complexity comparison between FBE and LNE with  $L' = 4, 8, 16$ .

As far as resource consumption is concerned, we point out here that the IFFT operation included in the above fractional CFO estimation can be implemented through FFT technique by making a conjugate operation on its input and output respectively. Moreover, since  $N$ -point FFT operation is also a prerequisite for an OFDM receiver, the resource that FBE consumes can thus be reduced considerably.

## 5. Simulation Results

This section presents the results of Monte Carlo simulations which illustrate the performance of FBE. Throughout the simulations, we employ a turbo encoder with code rate  $R_c = 1/2$ , an S-random interleaver [22] with size 20480 and a 16-QAM modulator for the OFDM system. Other parameters are set as follows: carrier frequency  $f_c = 5\text{GHz}$ , bandwidth  $B = 10\text{MHz}$ , subcarrier number  $N = 1024$ , CP length  $N_g = 64$ , sampling interval  $T_s = 0.1\mu\text{s}$ , and symbol duration  $T = 108.8\mu\text{s}$ . Two types of multipath Rayleigh fading channels are employed for the simulations: a 4-path slow fading channel with small delay spread (Channel 1), a 6-path fast fading channel with large delay spread (Channel 2). Channel 1 has a classical Doppler spectrum with maximum Doppler shift  $f_d = 50\text{Hz}$ , while Channel 2 has a classical Doppler spectrum with  $f_d = 200\text{Hz}$ . For Channel 1, the relative average-powers and propagation delays of the four paths are  $\{0, -9.7, -19.2, -22.8\}$  dB and  $\{0, 0.2, 0.4, 0.8\}$   $\mu\text{s}$ , respectively. While for Channel 2, all the six paths have identical average-powers and their relative propagation delays are  $\{0, 0.3, 0.7, 1.1, 1.3, 2.4\}$   $\mu\text{s}$ . For the training sequence, we design  $N_D = 8$ ,  $N_U = 64$  and  $\mathcal{D} = \{104, 200, 280, 456, 568, 696, 760, 904\}$ . Besides, according to (11), the SNR of the received sequence is defined as  $E_s/N_0 = E[|\mathbf{A}(\varepsilon)\tilde{\mathbf{h}}_{N_C}|^2]/\sigma^2$ .

With the definition

$$i = \sum_{k=0}^{N_D-1} i_k 2^{N_D-1-k}, \quad (35)$$

Fig. 5 presents simulation results of the PAPR corresponding to each  $i$  for the proposed training sequence with  $\alpha = 0.3, 0.5$ . It can be observed from the figure that the values of PAPR vary with  $i$  in the similar law with different  $\alpha$  and that the PAPR increases with  $\alpha$  at the same  $i$  for almost all cases. When  $\alpha = 0.3$ , the minimum PAPR is achieved at  $i = 16$ , which correspond to  $[i_0, i_1, \dots, i_{N_D-1}] = [00010000]$ . When  $\alpha = 0.5$ , the minimum PAPR is achieved at  $i = 241$ , which corresponds to  $[i_0, i_1, \dots, i_{N_D-1}] = [11110001]$ . We can also observe from the figure that 2.88dB and 3.26dB reductions are obtained when the minimum PAPR is compared with the maximum PAPR for different  $\alpha$ .

Fig. 6 shows the probability  $P_{correct}$  defined in (23) versus  $N_D$  for the proposed training sequence with  $\alpha N_U / [(1-\alpha)N_D] = 24/7, 8$  in Channel 1 and Channel 2, respectively. We can see from the figure that by increasing the value of  $N_D$  with  $\alpha N_U / [(1-\alpha)N_D]$  kept invariable, the probability  $P_{correct}$  can always approaches 1. This verifies the correctness of condition (22).

Define the average bias as follows,

$$b_I = \frac{1}{N_S} \sum_{k=0}^{N_S-1} |(\hat{\varepsilon}_I)_k - \lfloor \varepsilon \rfloor|, \quad (36)$$

$$b_F = \frac{1}{N_S} \sum_{k=0}^{N_S-1} |(\hat{\varepsilon}_F)_k - (\varepsilon - \lfloor \varepsilon \rfloor)|,$$

where  $N_S$  denotes the total number of simulations and  $(\hat{\varepsilon}_I)_k, (\hat{\varepsilon}_F)_k$  denote the estimated integer and fractional CFOs for the  $k$ -th run, respectively. For the properly-operating CFO estimator,  $b_I$  and  $b_F$  should be around zero. In order to show how the ratio  $\alpha$  impacts the estimate per-

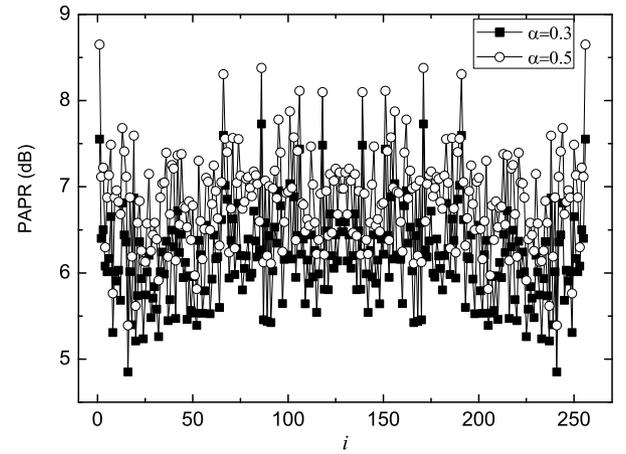


Fig. 5 PAPR versus  $i$  for the proposed training sequence with  $\alpha = 0.3, 0.5$ .

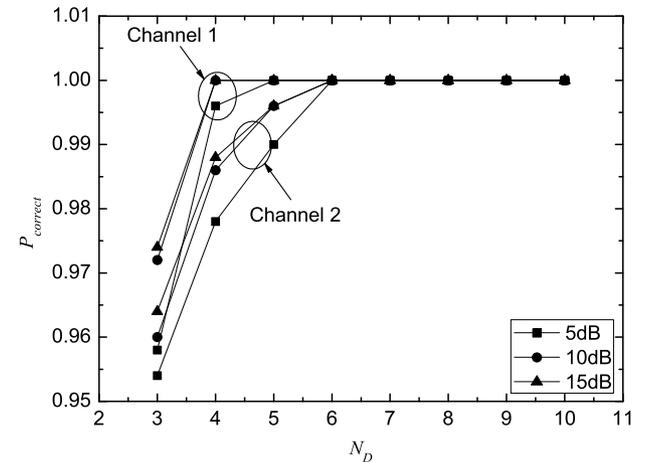


Fig. 6 The probability  $P_{correct}$  versus  $N_D$  for the proposed training sequence with  $\alpha N_U / [(1-\alpha)N_D] = 24/7, 8$ ,  $E_s/N_0 = 5, 10, 15\text{dB}$  in Channel 1 and Channel 2, respectively.

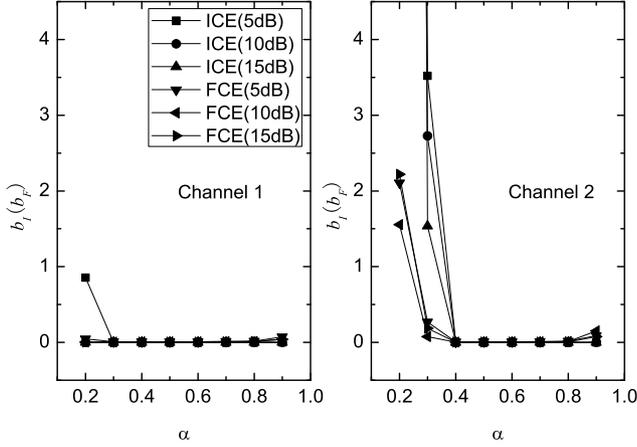


Fig. 7 Average bias versus  $\alpha$  for ICE and FCE with  $E_s/N_0=5,10,15$ dB in Channel 1 and Channel 2, respectively.

formance, we present in Fig. 7 the simulation results of the average bias versus  $\alpha$  for the proposed integer CFO estimator (ICE) and fractional CFO estimator (FCE) with  $E_s/N_0=5,10,15$ dB in Channel 1 and Channel 2, respectively. It can be seen that ICE works reliably with large  $\alpha$  and that FCE works reliably with small  $\alpha$  under the condition that ICE runs accurately. There is a safe zone for both of them to operate well. The safe zone is  $[0.3, 0.8]$  in Channel 1, while it is  $[0.4, 0.8]$  in Channel 2.

To evaluate the performance of FBE, we introduce a variable: the mean square error (MSE)  $\chi$ , which is defined as follows,

$$\chi = \frac{1}{N_S} \sum_{k=0}^{N_S-1} [(\hat{\varepsilon})_k - \varepsilon]^2 \quad (37)$$

where  $(\hat{\varepsilon})_k$  denote the estimated CFO for the  $k$ -th run. Fig. 8 illustrates the MSE performance of FBE with  $\alpha = 0.3, \varepsilon = 9.279$  in Channel 1 and  $\alpha = 0.5, \varepsilon = -8.835$  in Channel 2, respectively. As a benchmark, the performances of LNE with  $L' = 4, 8, 16$ , where  $L'$  denotes the oversize ratio of the corresponding FFT interpolation, are also shown in the figure. For LNE, we assume that a frequency domain training sequence with length  $N = 1024$  consisting of  $N_D = 8$  pilot tones is employed. Also included for comparison is the Cramer-Rao bound (CRB) [8] defined as follows,

$$\text{CRB} = \frac{1.5}{\pi^2 N (1 - N^{-2}) (1 - \alpha) 10^{\text{SNR}/10}} \quad (38)$$

It can be seen from the figure that the estimate performance of FBE is similar to that of LNE in Channel 1 and superior to that of LNE in Channel 2. Thanks to the cyclically orthogonal property of Chu sequence, FBE is comparatively robust against multipath channels and its MSE is quite close to the CRB for both of the considered channels. Whereas the MSE performance of LNE deteriorates in Channel 2 and

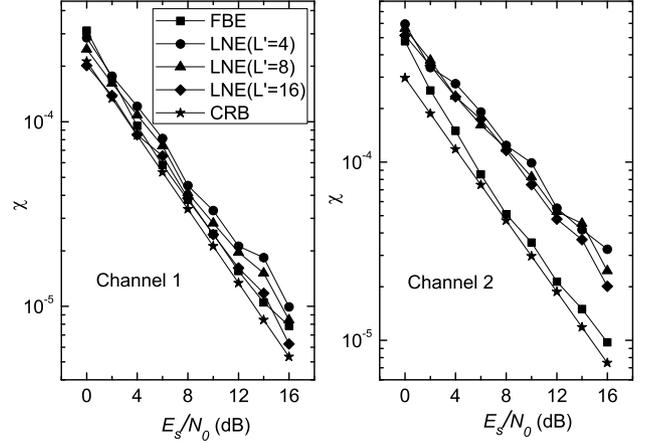


Fig. 8 MSE versus  $E_s/N_0$  for FBE and LNE with  $\alpha = 0.3, \varepsilon = 9.279$  in Channel 1 and  $\alpha = 0.5, \varepsilon = -8.835$  in Channel 2. Also included for comparison is the CRB.

there exists a large performance gap between the MSE of LNE and that of FBE. Recall the complexity comparison in last section, we can find certain advantages of FBE.

In Fig. 9, we plot the bit error rate (BER) curves of FBE and LNE as a function of  $E_b/N_0$  with  $E_b/N_0 = \frac{E_s}{R_c M_c N_0}$ . Also included for comparison is the ideal case when no CFO exists. For these simulations, we set the number of inner iterations in the turbo decoder to 6, and we also assume ideal channel estimation at receiver side. It can be observed from the figure that the BER performance of FBE is similar to that of LNE in Channel 1 and better than that of LNE in Channel 2, which should be owed to the better MSE performance of FBE. Therefore, with relatively low complexity and good performance, FBE is more suitable for practical OFDM systems in comparison with LNE.

## 6. Conclusions

In this paper, a novel frequency domain training sequence composed of distinctively and uniformly spaced pilot tones with different energies and the corresponding CFO estimator have been proposed for OFDM systems over frequency-selective fading channels. The integer CFO estimation is achieved based on the distinctively spaced pilot tones, while the fractional CFO estimation is accomplished based on the uniformly spaced ones. The computational complexity of the proposed CFO estimator has been decreased considerably by exploiting a predefined lookup table making the best of the distinctively spaced pilot tones. The ability of the proposed CFO estimator to combat multipath effect has also been enhanced greatly with the aid of the uniformly spaced pilot tones generated from Chu sequence with cyclically orthogonal property. Moreover, the good performance of the proposed CFO estimator has been verified through simulations. Although the CFO estimation discussed here is for

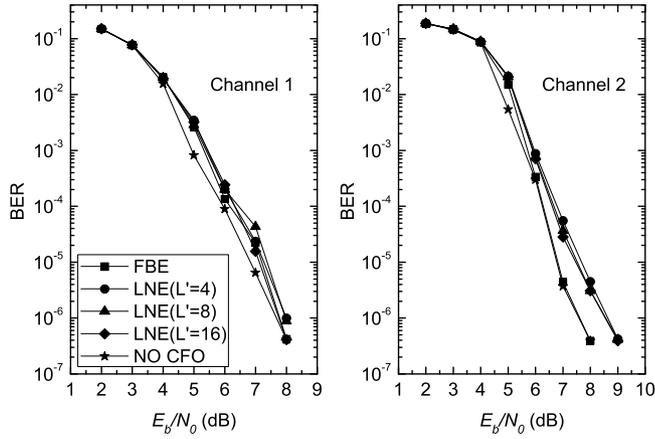


Fig. 9 BER versus  $E_b/N_0$  for FBE and LNE with  $\alpha = 0.3, \varepsilon = 9.279$  in Channel 1 and  $\alpha = 0.5, \varepsilon = -8.835$  in Channel 2. Also included for comparison is the ideal case when no CFO exists.

OFDM systems, the proposed estimator is also well-suited for other block transmission systems such as SC systems.

### Appendix: Proof of Theorem 1

Since integer frequency offset won't introduce ICI and it only has the effect of shifting the pilot tones, we assume  $\varepsilon$  to be in the range  $[-0.5, 0.5]$  in this appendix.

With the  $N \times N$  FFT matrix  $\mathbf{F}_N^H$  applied to  $\mathbf{r}$  in (2), we have

$$\tilde{\mathbf{r}} = \mathbf{F}_N^H \mathbf{r} \quad (\text{A.1})$$

Exploiting the proposed frequency domain training sequence  $\tilde{\mathbf{p}}_N$ , we can express the non-zero pilot tone affected by  $\varepsilon$  as follows,

$$\begin{aligned} [\tilde{\mathbf{r}}]_{c_k} &= \tilde{p}_{c_k} [\tilde{\mathbf{h}}_N]_{c_k} \frac{\sin(\pi\varepsilon)}{N \sin(\pi\varepsilon/N)} e^{j \frac{\pi\varepsilon(2Nq+N-1)}{N}} \\ &+ \sum_{k'=0, k' \neq k}^{N_C-1} \left\{ \tilde{p}_{c_{k'}} [\tilde{\mathbf{h}}_N]_{c_{k'}} \frac{\sin(\pi\varepsilon)}{N \sin[\pi(c_{k'} - c_k + \varepsilon)/N]} \right. \\ &\left. \times e^{j \frac{-\pi(c_{k'} - c_k) + \pi\varepsilon(2Nq+N-1)}{N}} \right\} + [\tilde{\mathbf{w}}]_{c_k}, \quad (\text{A.2}) \end{aligned}$$

where

$$\tilde{\mathbf{w}} = \mathbf{F}_N^H \mathbf{w}.$$

From (A.2), we can see that ICI indeed exists between the two types of pilot tones. In order to obtain the optimum value of  $v$ , we express  $P_{ICI,d_k}$ , i.e., the average power of the total ICI that  $\tilde{p}_{d_k}$  imposes on the  $N_U$  uniformly spaced pilot tones, as a function of  $v$  as follows,

$$P_{ICI,d_k}(v)$$

$$= N^{-2} E[|\tilde{\mathbf{h}}_N]_{d_k}|^2] |\tilde{p}_{d_k}|^2 \sin^2(\pi\varepsilon) \sum_{m=0}^{N_U/2-1} \Upsilon_m(v), \quad (\text{A.3})$$

where

$$\begin{aligned} \Upsilon_m(v) &= \sin^{-2}[\pi(v + mX + \varepsilon)/N] \\ &+ \sin^{-2}[\pi(v - X - mX + \varepsilon)/N]. \quad (\text{A.4}) \end{aligned}$$

For brevity, we define  $\theta_0 = \pi(v + mX + \varepsilon)/N$ ,  $\theta_1 = \pi(v - X - mX + \varepsilon)/N$ . Based on the definition of  $\Upsilon_m(v)$ , we can obtain the following result,

$$\begin{aligned} \frac{\partial \Upsilon_m(v)}{\partial v} &= -\frac{2\pi(\cos \theta_0 \sin^3 \theta_1 + \cos \theta_1 \sin^3 \theta_0)}{N \sin^3 \theta_0 \sin^3 \theta_1} \\ &= -[\sin \theta_0 (\sin \theta_0 - \sin \theta_1 \cos \theta_0 \cos \theta_1) + \cos^2 \theta_0 \sin^2 \theta_1] \\ &\times \frac{2\pi \sin(\theta_0 + \theta_1)}{N \sin^3 \theta_0 \sin^3 \theta_1} \quad (\text{A.5}) \end{aligned}$$

With  $\varepsilon \in [-0.5, 0.5]$ ,  $m \in [0, N_U/2 - 1]$ ,  $v \in [1, X - 1]$  and  $N = XN_U$ , we have

$$\begin{aligned} \theta_0 &\in \left[ \frac{\pi}{2N}, \frac{\pi}{2} - \frac{\pi}{2N} \right], \\ \theta_1 &\in \left[ -\frac{\pi}{2} + \frac{\pi}{2N}, -\frac{\pi}{2N} \right], \\ \theta_0 + \theta_1 &\in \left[ \frac{\pi}{N} - \frac{\pi}{N_U}, \frac{\pi}{N_U} - \frac{\pi}{N} \right]. \end{aligned}$$

Then the following inequality holds,

$$\sin \theta_0 (\sin \theta_0 - \sin \theta_1 \cos \theta_0 \cos \theta_1) + \cos^2 \theta_0 \sin^2 \theta_1 > 0 \quad (\text{A.6})$$

Let  $\frac{\partial \Upsilon_m(v)}{\partial v} = 0$ , we have  $\sin(\theta_0 + \theta_1) = 0$ , which leads to  $v = X/2 - \varepsilon$ . Besides, we also have

$$\begin{aligned} \frac{\partial^2 \Upsilon_m(v)}{\partial v^2} &= \frac{2\pi^2}{N^2 \sin^4 \theta_0 \sin^4 \theta_1} \\ &\times (\sin^4 \theta_0 + \sin^4 \theta_1 + 2 \cos^2 \theta_0 \sin^4 \theta_1 + 2 \cos^2 \theta_1 \sin^4 \theta_0) \\ &> 0 \quad (\text{A.7}) \end{aligned}$$

Furthermore, we can see from (A.4) that  $\Upsilon_m(v)$  is symmetrical about the line  $v = X/2 - \varepsilon$ . Since  $v$  can only be an integer, the value of  $v$  within the range  $[1, X - 1]$  that makes  $\Upsilon_m(v)$  achieves its minimum for any  $\varepsilon$  within the range  $[-0.5, 0.5]$  is  $X/2$ .

Correspondingly, the following result can be obtained,

$$\arg \min_{v \in [1, X-1]} \{P_{ICI,d_k}(v)\} = X/2 \quad (\text{A.8})$$

This completes the proof.

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