# Iterative Demodulation of M-ary Orthogonal Signaling Formats in Coded DS-CDMA Systems with Soft Interference Cancellation and Channel Estimation

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SUMMARY The system under study is a convolutionally coded and orthogonally modulated DS-CDMA system over timevarying frequency-selective Rayleigh fading channels in multiuser environments. Iterative soft demodulation and decoding using the Turbo principle can be applied to such a system to increase the system capacity and performance. To combat multiple access interference (MAI), we incorporate the interference cancellation (IC) and decision-directed channel estimation (CE) in the demodulator. However, both IC and CE are subject to performance degradation due to incorrect decisions. In order to prevent error propagation from the decision feedback, soft interference cancellation and channel estimation assisted demodulation is proposed in this paper. The performance of this strategy is evaluated numerically and proved to be superior to the hard decision-directed approach with a minor increase in complexity.

key words: code division multiple access, multipath channel estimation, interference cancellation, iterative soft demodulation.

# 1. Introduction

PAPER

In this paper, we study a coded CDMA system with orthogonal signalling formats. The signal modulation is accomplished with the Walsh (Hadamard) code, which is used by CDMA users as orthogonal spreading sequences and by the coding community as an error correcting code. The use of Walsh code is widespread in practical CDMA systems. For example, it is used in IS-95 system for orthogonal modulation in the uplink and user separation in the downlink; in 3G systems, it is used for spreading and channelization.

It is well-known that multiuser detection (MUD) [1] is an effective tool to increase the capacity of interference limited CDMA systems and alleviate some technical requirements, such as power control. Several iterative MUD schemes were proposed e.g., in [2]–[4] for uncoded M-ary orthogonal systems with affordable complexity (much less than that of an optimum receiver) and perform much better than the conventional noncoherent receiver, especially in high-capacity networks in which the interference from other users is large. In order to fully explore the potential of multiuser detectors, we need to acquire accurate measurements of the fading channel to do coherent detection or interference cancellation. It was shown in the papers [2], [4] that the use of iterative multiuser detection (interference cancellation) with decision-directed channel estimation provides substantial capacity gains compared to the conventional receiver.

Convolutional coding is employed in this system to improve the error correcting capability and power efficiency of the system. It is believed that CDMA systems exhibit their full potential when combined with forward error correction (FEC) coding [5]. Combined with FEC coding, MUD can overcome its limitations in highly correlated multiuser systems [6]. Therefore, in some proposed systems, MUD is employed in conjunction with FEC coding to obtain greater capacity and throughput.

The problem of joint multiuser detection and decoding was treated, e.g., in [6]–[10]. The multiuser detector and decoder utilize SISO algorithms and operate in an iterative feedback mode, as in turbo decoding. Soft interference cancellation, linear MMSE filtering, trellis based Log-MAP multiuser detection, or blind Bayesian multiuser detection were proposed in those papers to reduce the deteriorative effect of interference before single user decoding is done. An outline of the basic principles of low-complexity turbo multiuser detectors was given in [11]. However, the algorithms developed in the above papers are constrained to nonconcatenated systems with a single convolutional code, and the issue of joint detection/decoding and channel estimation was not investigated.

In mobile data communication systems, the required bit error rate (BER) is well below  $10^{-5}$ . Thus, block codes with a long codeword length or convolutional codes with a very long constraint length and a low code rate are needed to meet the requirement, and the complexity of the decoder significantly increases which makes the decoding physically unrealizable. In addition, the bandwidth efficiency of the systems becomes poor due to the low code rate [12]. Some concatenated codes with interleavers, e.g., parallel concatenated convolutional codes [13] and serially concatenated convolutional codes [14] were proposed in order to achieve large coding gain and good bandwidth ef-

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ficiency with practically acceptable complexity. The moderate decoding complexity for the concatenated codes is achieved by applying the turbo processing principle and alternately decoding the component codes and passing the soft information to the next decoding stage.

The system under study has a structure that is different from the aforementioned systems. It falls into the category of non-concatenated systems if the orthogonal modulation is viewed as part of the spreading process. Alternatively, it can be classified as a serially concatenated system, since the orthogonal modulation is accomplished by the Walsh code, which is essential a block code. Iterative decoding of such kind of a system has not been fully investigated except in very few papers, e.g., in [15], [16], in which a MAP demodulator and SOVA (soft-output Viterbi algorithm) decoder were applied to a similar system using M-ary orthogonal modulation and FEC. However, some important issues, e.g., channel estimation and MAI mitigation were not addressed in these references.

In this paper, we investigate different approaches to iterative demodulation and decoding for this convolutionally coded CDMA system with orthogonal modulation in a multi-user environment over multipath fading channels. It differs from the previous work in that iterative multiuser detection and demodulation are performed jointly in the demodulator and channel estimation is also incorporated in the demodulation process to facilitate the coherent demodulation. Soft decisiondirected interference cancellation and channel estimation are proposed to reduce the effect of error propagation and to improve the reliability of the demodulation process, which is the main contribution of this paper. If soft information is to be used for channel estimation and interference cancellation, a serially concatenated system would be rather different from the nonconcatenated systems in that the soft values are not directly available for all the inner code bits from the outer decoder. In particular, in our case, only the soft information can be extracted for the systematic bits of the Walsh codewords from a SISO channel decoder. In this paper, we design a soft modulator to derive the soft estimates for parity bits so that the demodulator can use the soft decision feedback from the decoder for the IC and CE.

The remainder of this paper is organized as follows. The system model is introduced in Section 2. Several iterative demodulation and decoding schemes are presented in Section 3. The algorithms for deriving soft information for IC and CE either from the demodulator or from the channel decoder are introduced. Section 4 is dedicated to the presentation of the soft demodulator based on the interference cancellation algorithm, as well as how the soft information is used for interference cancellation. Estimation of frequency-selective fading channels using both hard and soft decision feedback is addressed in Section 5. Different algorithms are examined and compared numerically in Section 6, and conclusions are drawn in Section 7.

# 2. System Model

The system model is only briefly described in this section. For a more detailed description, readers are referred to [2]. To facilitate reading, the frequently used acronyms and notations are summarized in Table 1.

The block diagram of the transmitter is shown in the upper part of Fig. 1. The  $k^{th}$  user's  $l^{th}$  information bit is denoted as  $b_k[l] \in \{+1, -1\}$   $(k = 1, \ldots, K, l = 1, \ldots, L_b$ , and  $L_b$  is the block length). The information bits are convolutionally encoded into code bits  $\{u_k[l_l]\} \in \{+1, -1\}$ , where  $u_k[l_l]$  denotes the  $n^{th}$  code bit due to  $b_k[l]$ . For example, in case of a rate  $R_c = 1/3$ code,  $b_k[l]$  is encoded into  $u_k[l_l], u_k[l_l], u_k[l_l]$ .

Code bits are subsequently interleaved and each block of  $\log_2 M$  coded and interleaved bits  $\{u'_k | l^n \} \in$  $\{+1, -1\}$  is mapped into  $\mathbf{w}_{i_k(j)} \in \{\mathbf{w}_0, \dots, \mathbf{w}_{M-1}\},\$ which is one of the M Walsh codewords. The subscript  $i_k(j) \in \{0, 1, \dots, M-1\}$  denotes the  $k^{th}$  user's  $j^{th}$ Walsh symbol index. In the case M = 8, the mapping rule is given in Table 2 below. The three systematic bits are  $\mathbf{w}_{i_k(j)}^1, \mathbf{w}_{i_k(j)}^2,$  and  $\mathbf{w}_{i_k(j)}^4,$  where  $\mathbf{w}_{i_k(j)}^p$  denotes the  $p^{th}$  bit of the codeword. The columns corresponding to the systematic bits are highlighted in the table. To ease understanding, we use M = 8 as an example for the derivation of the soft demodulation and decoding algorithms throughout this paper. However, the extension of the proposed algorithms to other values of M is straightforward. The interleaver and deinterleaver are denoted as  $\Pi$  and  $\Pi^{-1}$ , respectively, in Fig. 1 and in the following figures. The purpose of interleaving is to separate adjacent code bits in time so that, ideally, each code bit will experience independent fading.

The Walsh codeword  $\mathbf{w}_{i_k(j)} \in \{+1, -1\}^M$ , is then repetition encoded into

$$\mathbf{s}_k(j) = \operatorname{rep}(\mathbf{w}_{i_k(j)}, N/M) \in \{+1, -1\}^N$$
 (1)

where rep $(\cdot, \cdot)$  denotes the repetition encoding operation, where its first argument is the input bits and the second one is the repetition factor. Therefore, each bit of the Walsh codeword is spread (repetition coded) into  $N_c = N/M$  chips, and each Walsh symbol is represented by N chips and denoted as  $\mathbf{s}_k(j)$ .

The Walsh sequence  $\mathbf{s}_k(j)$  is then scrambled (randomized) by a scrambling code unique to each user to form the transmitted chip sequence

$$\mathbf{a}_k(j) = \mathbf{C}_k(j)\mathbf{s}_k(j) \in \{+1, -1\}^N$$

where  $\mathbf{C}_k(j) \in \{-1, 0, +1\}^{N \times N}$  is a diagonal matrix whose diagonal elements correspond to the scrambling code for the  $k^{th}$  user's  $j^{th}$  symbol. The purpose of scrambling is to separate users. In this paper, we focus XIAO and STRÖM: ITERATIVE DEMODULATION OF M-ARY ORTHOGONAL SIGNALING FORMATS IN CODED DS-CDMA SYSTEMS WITH SOFT

CMF, Sampling



Fig. 1 Transmitter, channel and receiver front end.

MAP	maximum a posteriori	$i_k(j)$	$k^{th}$ user's $j^{th}$ Walsh symbol index
VA	Viterbi algorithm	$\mathbf{w}_{i_k(j)}$	$k^{th}$ user's $j^{th}$ Walsh codeword
MF	matched filter	$\mathbf{w}_{i_k(j)}^p$	$p^{th}$ bit of $\mathbf{w}_{i_k(j)}$
IC	interference cancellation	$b_k[l]$	$k^{th}$ user's $l^{th}$ information bit
HDIC	hard (decision) interference cancellation	$u_k \begin{bmatrix} n \\ l \end{bmatrix}$	$n^{th}$ code bit due to $b_k[l]$
SDIC	soft (decision) interference cancellation	$u_k^\prime [{}^n_l]$	interleaved version of $u_k \begin{bmatrix} n \\ l \end{bmatrix}$
CE	channel estimation	$\mathbf{C}_k(j)$	$k^{th}$ user's scrambling matrix for the $j^{th}$ symbol
k	user index	$\mathbf{s}_k(j)$	$k^{th}$ user's $j^{th}$ transmitted chip sequence
i	Walsh symbol index	$h_{k,l}$	$l^{th}$ path complex channel gain for $k^{th}$ user's $j^{th}$ symbol

Table 1 Acronyms and notations

 Table 2
 Mapping between input bits, symbol indices, and Walsh codewords

Code bits	Symbol index	Walsh codeword	
$u_k'[{}^0_l] \; u_k'[{}^1_l] \; u_k'[{}^2_l]$	$m = i_k(j)$	$\mathbf{w}_m$	
+1 +1 +1	0	$\mathbf{w}_0: +1 + 1 + 1 + 1 + 1 + 1 + 1 + 1$	
+1 $+1$ $-1$	1	$w_1:+1 +1 +1 +1 -1 -1 -1 -1$	
+1 -1 +1	2	$w_2:+1 +1 -1 -1 +1 +1 -1 -1$	
+1 -1 -1	3	$w_3:+1 +1 -1 -1 -1 -1 +1 +1$	
-1 +1 +1	4	$\mathbf{w}_4:+1$ -1 +1 -1 +1 -1 +1 -1	
-1 +1 -1	5	$\mathbf{w}_5: +1 \ -1 \ +1 \ -1 \ -1 \ +1 \ -1 \ +1$	
-1 $-1$ $+1$	6	$\mathbf{w}_6: +1 \ -1 \ -1 \ +1 \ +1 \ -1 \ -1 \ +1$	
-1 $-1$ $-1$	7	$w_7: +1 -1 -1 +1 -1 +1 -1$	

on the use of long codes, e.g., the scrambling code differs from symbol to symbol. From the above equation, one can see that scrambling is accomplished by chipwise multiplication, and does not introduce any rate increase.

The scrambled sequence  $\mathbf{a}_k(j)$  is pulse amplitude modulated using a unit-energy chip waveform  $\psi(t)$  to form the baseband signal. For simplicity, we assume that  $\psi(t)$  is a rectangular pulse with support  $t \in [0, T_c)$ , where the chip duration is denoted by  $T_c$  (its relation with symbol duration T is  $T = NT_c$ ). However, the proposed methods can be extended for other waveforms, e.g., square-root raised cosine pulses.

The baseband signal is multiplied with a carrier and transmitted over a Rayleigh fading channel with noise power spectral density  $N_0/2$  and with  $L_k$  resolvable paths, having time-varying complex channel gains  $h_{k,1}(t), h_{k,2}(t), \ldots, h_{k,L_k}(t)$  and delays  $\tau_{k,1}, \tau_{k,2}, \ldots, \tau_{k,L_k}$  (see the lower part of Fig. 1). The received signal is the sum of K users' signals plus additive white complex Gaussian noise n(t). After frequency down-conversion and chip matched filtering (CMF), the received signal corresponding to the  $k^{th}$ user's  $j^{th}$  transmitted Walsh sequence  $\mathbf{s}_k(j)$  can be written in vector form as

$$\mathbf{r}(k,j) = \mathbf{A}(k,j)\mathbf{h}(j) + \mathbf{n}(k,j) = \mathbf{X}_k(j)\mathbf{h}_k(j) + \mathrm{ISI}(k,j) + \mathrm{MAI}(k,j) + \mathbf{n}(k,j) \in \mathbb{C}^{N_k}$$
(2)

where the columns of the matrix  $\mathbf{A}(k, j)$  are the delayed version of transmitted chip sequences  $\mathbf{a}_k(j)$  for  $k = 1, 2, \cdots, K$ , one column per path. The length of the processing window  $N_k$ , is larger than the symbol interval N to account for the asynchronous and multipath nature of the channel. The columns are weighted together by  $\mathbf{h}(j)$ , whose elements are the path gains of all users' paths. The received vector  $\mathbf{r}(k,j)$  can be written as the sum of four terms: the signal of interest  $\mathbf{X}_k(j)\mathbf{h}_k(j)$ , the intersymbol interference (ISI), the multiple access interference (MAI), and the noise represented by  $\mathbf{n}(k, j)$ , which is a vector of complex noise samples with zero mean and variance  $N_0$ . The columns of the matrix  $\mathbf{X}_k(j)$  are essentially shifted versions of the chips due to the  $k^{th}$ user's  $j^{th}$  symbol, one column per path (the shift is determined by the path delay). The vector  $\mathbf{h}_k(j) =$  $[h_{k,1}(jT) \ h_{k,2}(jT) \ \cdots \ h_{k,l}(jT) \cdots \ h_{k,L_k}(jT)]^T$  corresponds to the channel gains of the  $k^{th}$  user's paths and is a part of  $\mathbf{h}(j)$ .

# 3. Derivation of Soft Information

The task of the receiver is to detect the information bits transmitted from all users, i.e., detect  $b_k[l]$ for  $l = 1, 2, \ldots, L_b$  and  $k = 1, 2, \cdots, K$  given the observation  $\mathbf{r}(k, j)$  for  $k = 1, 2, \cdots, K$  and  $j = 1, 2, \cdots, L_b/(R_c \log_2 M)$ . To this end, we first need to demodulate the received signal to the code bits  $\{u'_k[^n_l]\}$ , which are subsequently decoded to obtain an estimate of  $\{b_k[l]\}$ . Depending on whether there is feedback from the channel decoder to the demodulator or not, the demodulation and decoding said to be either partitioned or integrated. Decision-directed IC and CE can be used to improve the demodulation performance. Derivation of soft information for IC and CE in both partitioned and integrated schemes is studied in this paper, and will be discussed in detail next. Readers are referred to Table 1 for the acronyms and abbreviations.

3.1 Derivation of soft information in partitioned demodulation and decoding

It is a well-known fact that the soft-input channel decoding is between 2 and 3 dB better than the hardinput decoding. In order to do soft decoding, we need derive soft reliability value for each bit  $u'_{k}{n \brack l}$  from the received vector  $\mathbf{r}(k, j)$ . The soft metric for the bit  $u'_{k}{n \brack l}$ is defined according to [17] as

$$\lambda(u_k'[_l^n]; O) = \ln \frac{\sum_{m:u_k'[_l^n] = +1} f(\mathbf{r}|\mathbf{s}_m)}{\sum_{m:u_k'[_l^n] = -1} f(\mathbf{r}|\mathbf{s}_m)}$$
(3)

In the above equation,  $f(\mathbf{r}|\mathbf{w}_m)$  can be approximated as  $f(\mathbf{r}|\mathbf{s}_m) = A \exp\{Bz_k(m)\}$  [17], where  $z_k(m)$  is the decision statistic from the demodulator, based on the hypothesis that the  $m^{th}$  Walsh symbol is transmitted from user k, and A and B are constants. The derivation of  $z_k(m)$  is studied in Section 4. In (3), we denote  $m : u'_k[_l^n] = \pm 1$  as the set of Walsh sequences  $\{\mathbf{s}_m\}$  that correspond to the code bit  $u'_k[_l^n] = \pm 1$ . Typically, one term will dominate each sum in (3), which suggests the "dual-maxima" rule [23]

$$\lambda(u_{k}'[{}_{l}^{n}]; O) \approx \ln \frac{\max_{m:u_{k}'[{}_{l}^{n}]=+1} f(\mathbf{r}|\mathbf{s}_{m})}{\max_{m:u_{k}'[{}_{l}^{n}]=-1} f(\mathbf{r}|\mathbf{s}_{m})}$$
  
= 
$$\max_{m:u_{k}'[{}_{l}^{n}]=+1} \{z_{k}(m)\} - \max_{m:u_{k}'[{}_{l}^{n}]=-1} \{z_{k}(m)\}$$
(4)

In the case M = 8, the  $k^{th}$  user's  $j^{th}$  Walsh codeword  $\mathbf{w}_{i_k(j)}$ , or equivalently, the Walsh sequence  $\mathbf{s}_k(j)$ corresponds to 3 code bits:  $u'_k[_l^0], u'_k[_l^1], u'_k[_l^2]$ . The soft metrics for each 3-bit block can be computed as [17]

$$\lambda(u_k'[{}_l^0]; O) \approx \max\{z_k(0), z_k(1), z_k(2), z_k(3)\} - \max\{z_k(4), z_k(5), z_k(6), z_k(7)\} \\\lambda(u_k'[{}_l^1]; O) \approx \max\{z_k(0), z_k(1), z_k(4), z_k(5)\} - \max\{z_k(2), z_k(3), z_k(6), z_k(7)\} \\\lambda(u_k'[{}_l^2]; O) \approx \max\{z_k(0), z_k(2), z_k(4), z_k(6)\} - \max\{z_k(1), z_k(3), z_k(5), z_k(7)\}$$
(5)

The deinterleaved soft value  $\lambda(u_k[_l^n];I) = \Pi^{-1}\{\lambda(u'_k[_l^n];O)\}$  is delivered as soft input to the decoder using Viterbi algorithm (VA) to estimate the

transmitted information bits  $\{b_k[l]\}$ . We use the notations  $\lambda(\cdot; I)$  and  $\lambda(\cdot; O)$  at the input and output ports of a SISO. They refer to the unconstrained LLRs when the second argument is I, and modified LLRs according to the code constraints when it is O. The second argument I or O is sometimes omitted to simplify notation whenever no ambiguity arises. Other soft values are denoted by  $L(\cdot)$ . They are usually soft input and output of non-SISO devices.

The scheme based on this soft decision rule is shown in Fig. 2. Note that there are two outputs from the demodulator: the soft decision about the transmitted Walsh symbol  $z_k(m)$  and the LLRs for code bits  $\lambda(u'_k[^n_l]; O)$ . The former one is used to make hard decision on the Walsh symbol index  $i_k(j)$  or transmitted Walsh sequence  $\mathbf{s}_k(j)$ 

$$\hat{i}_k(j) = \arg \max_{m \in \{0,\dots,M-1\}} z_k(m), \quad \text{or}$$
$$\hat{\mathbf{s}}_k(j) = \arg \max_{m \in \{0,\dots,M-1\}} f(\mathbf{r}(k,j)|\mathbf{s}_m) \tag{6}$$

The above two equations are equivalent under the assumption stated earlier. The estimated sequence  $\hat{\mathbf{s}}_k(j)$  is needed for estimating the multipath complex channel gains and the IC-based demodulation.

The use of hard decisions of  $\mathbf{s}_k(j)$  for interference cancellation (IC) and channel estimation (CE) makes the system vulnerable to error propagation. The probability of error propagation can be reduced through the feedback of soft information instead of hard decisions. Fig. 3 shows the scenario of demodulation and CE using  $L(\mathbf{s}_k(j))$ , some soft estimate of the Walsh sequence  $\mathbf{s}_k(j)$ . The derivation of  $L(\mathbf{s}_k(j))$  is discussed below. In this case, we need not only the LLRs for code bits  $\{u'_k[_l^n]\}$ , but also for the Walsh codeword  $\mathbf{w}_{i_k(j)}$ . For systematic bits of  $\mathbf{w}_{i_k(j)}$ , the LLRs simply are

$$\lambda(\mathbf{w}_{i_{k}(j)}^{1}; O) = \lambda(u_{k}'[{}^{0}_{l}]; O);$$
  

$$\lambda(\mathbf{w}_{i_{k}(j)}^{2}; O) = \lambda(u_{k}'[{}^{1}_{l}]; O);$$
  

$$\lambda(\mathbf{w}_{i_{k}(j)}^{4}; O) = \lambda(u_{k}'[{}^{2}_{l}]; O)$$

The first parity bit is always +1, therefore, its LLR value  $\lambda(\mathbf{w}_{i_k(j)}^0; O) = \infty$ . For the other parity bits

$$\lambda(\mathbf{w}_{i_{k}(j)}^{3}; O) \approx \max\{z_{k}(0), z_{k}(1), z_{k}(6), z_{k}(7)\} - \max\{z_{k}(2), z_{k}(3), z_{k}(4), z_{k}(5)\} \lambda(\mathbf{w}_{i_{k}(j)}^{5}; O) \approx \max\{z_{k}(0), z_{k}(2), z_{k}(5), z_{k}(7)\} - \max\{z_{k}(1), z_{k}(3), z_{k}(4), z_{k}(6)\} \lambda(\mathbf{w}_{i_{k}(j)}^{6}; O) \approx \max\{z_{k}(0), z_{k}(3), z_{k}(4), z_{k}(7)\} - \max\{z_{k}(1), z_{k}(2), z_{k}(5), z_{k}(6)\} \lambda(\mathbf{w}_{i_{k}(j)}^{7}; O) \approx \max\{z_{k}(0), z_{k}(3), z_{k}(5), z_{k}(6)\} - \max\{z_{k}(1), z_{k}(2), z_{k}(4), z_{k}(7)\}$$
(7)

The soft estimates  $\{L(\mathbf{s}_k(j))\}\$  are derived by spreading

 $\{\lambda(\mathbf{w}_{i_k(j)}; O)\}$  using equation (1). Clearly, the added complexity by deriving soft values rather than making hard decisions is minor if we compare (7) with (6). The use of  $L(\mathbf{s}_k(j))$  in soft demodulation and soft CE will be discussed in Section 4 and 5.

Note that in the schemes illustrated in Fig. 2 and Fig. 3, the iteration is only inside the demodulation block. Channel decoding is carried out only once with Viterbi algorithm. The performance is improved in an iterative manner due to the fact that the channel is more accurately measured and interference is better detected (meaning better interference cancellation), as the iteration goes on.

3.2 Derivation of soft information in integrated demodulation and decoding

The above two schemes are not optimized in the sense that the demodulator can not benefit from information derived from the channel decoder. Demodulation and decoding are strictly partitioned into two blocks. In the following, we present how the soft information can be derived in the integrated scheme, where the problem of joint soft demodulation and decoding is approached by expanding the iteration loop over the concatenation of demodulation and decoding blocks. In this case, the demodulator and decoder are each implemented with a SISO algorithm and operate in an iterative feedback mode.

To provide a reference for comparison, we first introduce an integrated scheme with hard IC and CE, which is illustrated in Fig. 4. It differs from the aforementioned algorithms in that the decisions from the channel decoder are fed back to the demodulator. The estimate of  $\hat{\mathbf{s}}_k(j)$  (hard decision) needed for the IC and CE is not delivered from the demodulator itself, but from the output of the channel decoder. As will be clear later on, spanning the iteration loop over the two blocks is really crucial in improving the system performance. The price to pay is the added complexity mainly due to the channel decoding at every iteration instead of doing it once for all. This is especially true when we use the Log-MAP based decoder, which needs forward and backforward recursions at each iteration. Going through convolutional encoding, interleaving and modulation processes every time also slightly increase the complexity.

To derive soft output from the decoder for soft IC and CE, we replace the VA decoder with some SISO decoder which produces soft output for both information bits  $\{b_k[l]\}$  and code bits  $\{u_k[_l^n]\}$ . Based on the soft input  $\lambda(u_k[_l^n]; I)$  and the trellis structure of the convolutional code, the  $k^{th}$  user's SISO channel decoder computes the a posteriori LLR of each information bit  $\lambda(b_k[l]; O)$  and each code bit  $\lambda(u_k[_l^n]; O)$ , where  $\lambda(b_k[l]; O)$  is used to make decision on the transmitted information bit at the final iteration, while  $\lambda(u_k[_l^n]; O)$ 



Fig. 2 Partitioned demodulation and decoding with hard IC and CE.

PSfrag replacements

Walsh

Encoder





zero



Fig. 4 Integrated demodulation and decoding with hard IC and CE.

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is used for the IC and CE in the demodulator at the next iteration. Several SISO algorithms can be used to compute the channel decoder outputs. To be consistent with the "dual-maxima" rule adopted in the demodulator introduced in Section 3.1, we consider the use of Max-Log-MAP for the purpose of this study. Refer to [18] for a thorough discussion on the complexity and performance of different optimal and sub-optimal decoding algorithms.

The improved iterative scheme is shown in Fig. 5. It differs from the first one shown by Fig. 4 in that it uses soft information  $L(\mathbf{s}_k(j))$  for the IC and CE in an attempt to reduce the likelihood of error propagation. Instead of estimating  $\mathbf{s}_k(j)$ , we compute the soft estimate of  $\mathbf{s}_k(j)$  by feeding  $\lambda(u'_k[_l^n]) = \prod\{\lambda(u_k[_l^n]; O)\}$  into a soft modulator which computes  $\lambda(\mathbf{w}_{i_k(j)})$ , the LLRs of the codeword  $\mathbf{w}_{i_k(j)}$ , then derive  $L(\mathbf{s}_k(j))$  by repetition encoding (spreading)  $\lambda(\mathbf{w}_{i_k(j)})$ . Next, we shall explain how the soft modulator is implemented.

From Table 2, we can see that the parity bits are formed by systematic bits  $\mathbf{w}_{i_k(j)}^1, \mathbf{w}_{i_k(j)}^2, \mathbf{w}_{i_k(j)}^4$  as

The LLRs for systematic bits are

$$\begin{aligned} \lambda(\mathbf{w}_{i_k(j)}^1) &= \lambda(u_k'[{}^0_l]); \quad \lambda(\mathbf{w}_{i_k(j)}^2) = \lambda(u_k'[{}^1_l]); \\ \lambda(\mathbf{w}_{i_k(j)}^4) &= \lambda(u_k'[{}^2_l]) \end{aligned}$$

Considering the fact that the interleaver breaks the memory of the convolutional encoding process, the bits  $u'_k[^0_l], u'_k[^1_l], u'_k[^2_l]$  can be modeled as statistically independent random variables. Also assume that they are independent conditioned on the received signal, then the LLRs for parity bits can thus be computed according to [20] by

$$\begin{split} \lambda(\mathbf{w}_{i_{k}(j)}^{3}) &= \lambda(\mathbf{w}_{i_{k}(j)}^{1} \oplus \mathbf{w}_{i_{k}(j)}^{2}) \\ &= 2 \arctan\left\{ \tanh(\lambda(u_{k}'[l_{l}^{0}])/2) \cdot \tanh(\lambda(u_{k}'[l_{l}^{1}])/2) \right\} \\ &\approx \operatorname{sgn}\{\lambda(u_{k}'[l_{l}^{0}])\} \cdot \operatorname{sgn}\{\lambda(u_{k}'[l_{l}^{1}])\} \cdot \min\{|\lambda(u_{k}'[l_{l}^{0}])|, |\lambda(u_{k}'[l_{l}^{1}])|\} \\ \lambda(\mathbf{w}_{i_{k}(j)}^{5}) &= \lambda(\mathbf{w}_{i_{k}(j)}^{1} \oplus \mathbf{w}_{i_{k}(j)}^{4}) \\ &= 2 \arctan\left\{ \tanh(\lambda(u_{k}'[l_{l}^{0}])/2) \cdot \tanh(\lambda(u_{k}'[l_{l}^{0}])|, |\lambda(u_{k}'[l_{l}^{0}])|\} \\ &\approx \operatorname{sgn}\{\lambda(u_{k}'[l_{l}^{0}])\} \cdot \operatorname{sgn}\{\lambda(u_{k}'[l_{l}^{1}])/2) \cdot \tanh(\lambda(u_{k}'[l_{l}^{0}])|, |\lambda(u_{k}'[l_{l}^{0}])|\} \\ \lambda(\mathbf{w}_{i_{k}(j)}^{6}) &= \lambda(\mathbf{w}_{i_{k}(j)}^{2} \oplus \mathbf{w}_{i_{k}(j)}^{4}) \\ &= 2 \arctan\left\{ \tanh(\lambda(u_{k}'[l_{l}^{1}])/2) \cdot \tanh(\lambda(u_{k}'[l_{l}^{1}])|, |\lambda(u_{k}'[l_{l}^{0}])|\} \\ &\approx \operatorname{sgn}\{\lambda(u_{k}'[l_{l}^{1}])\} \cdot \operatorname{sgn}\{\lambda(u_{k}'[l_{l}^{0}])\} \cdot \min\{|\lambda(u_{k}'[l_{l}^{1}])|, |\lambda(u_{k}'[l_{l}^{0}])|\} \\ &\lambda(\mathbf{w}_{i_{k}(j)}^{7}) &= \lambda(\mathbf{w}_{i_{k}(j)}^{1} \oplus \mathbf{w}_{i_{k}(j)}^{2} \oplus \mathbf{w}_{i_{k}(j)}^{4}) \\ &= 2 \operatorname{arctanh}\left\{ \prod_{n=0}^{2} \tanh(\lambda(u_{k}'[l_{l}^{n}])/2) \right\} \end{aligned}$$

$$(8)$$

The approximation in (8) can be further approximated by omitting the  $\min\{\cdot\}$  operations, which yields

$$\hat{\mathbf{w}}_{i_{k}(j)}^{0} = +1; \quad \hat{\mathbf{w}}_{i_{k}(j)}^{1} = \operatorname{sgn}\{\lambda(u_{k}'[{}_{l}^{0}])\}; \\
\hat{\mathbf{w}}_{i_{k}(j)}^{2} = \operatorname{sgn}\{\lambda(u_{k}'[{}_{l}^{1}])\}; \quad \hat{\mathbf{w}}_{i_{k}(j)}^{4} = \operatorname{sgn}\{\lambda(u_{k}'[{}_{l}^{2}])\}; \\
\hat{\mathbf{w}}_{i_{k}(j)}^{3} = \hat{\mathbf{w}}_{i_{k}(j)}^{1} \oplus \hat{\mathbf{w}}_{i_{k}(j)}^{2} = \operatorname{sgn}\{\lambda(u_{k}'[{}_{l}^{0}])\} \cdot \operatorname{sgn}\{\lambda(u_{k}'[{}_{l}^{1}])\}; \\
\hat{\mathbf{w}}_{i_{k}(j)}^{5} = \hat{\mathbf{w}}_{i_{k}(j)}^{1} \oplus \hat{\mathbf{w}}_{i_{k}(j)}^{4} = \operatorname{sgn}\{\lambda(u_{k}'[{}_{l}^{0}])\} \cdot \operatorname{sgn}\{\lambda(u_{k}'[{}_{l}^{2}])\}; \\
\hat{\mathbf{w}}_{i_{k}(j)}^{6} = \hat{\mathbf{w}}_{i_{k}(j)}^{2} \oplus \hat{\mathbf{w}}_{i_{k}(j)}^{4} = \operatorname{sgn}\{\lambda(u_{k}'[{}_{l}^{1}])\} \cdot \operatorname{sgn}\{\lambda(u_{k}'[{}_{l}^{2}])\}; \\
\hat{\mathbf{w}}_{i_{k}(j)}^{7} = \hat{\mathbf{w}}_{i_{k}(j)}^{1} \oplus \hat{\mathbf{w}}_{i_{k}(j)}^{2} \oplus \hat{\mathbf{w}}_{i_{k}(j)}^{4} = \prod_{n=0}^{2} \operatorname{sgn}\{\lambda(u_{k}'[{}_{l}^{n}])\}; \\$$
(9)

which are the hard decisions. Compared to hard decisions expressed by (9), the soft derivation only slightly increases the computational complexity by introducing  $\min\{\cdot\}$  operations as shown in (8).

Another approach to compute  $\lambda(\mathbf{w}_{i_k(j)})$  is given in the Appendix. In Section 4 and 5, we shall see how the soft estimate of  $\mathbf{s}_k(j)$  can be used for the IC and CE.

As a common practice in iterative decoding, the soft outputs  $\{\lambda(u_k[_l^n]; O)\}$  from the Max-Log-MAP decoder can also be used as extrinsic information for the inner soft demodulation. This issue is thoroughly treated in [19]. It is shown in this paper that extrinsic information really helps improve the quality of demodulator, if handled properly. Several enhancement ideas were proposed to make more efficient use of extrinsic information. However, the main focus of this paper is the soft demodulation without extrinsic feedback. The soft outputs from the Max-Log-MAP decoder are used by the demodulator only for the purposes of channel estimation and interference cancellation.

![](_page_7_Figure_2.jpeg)

Fig. 5 Integrated demodulation and decoding with soft IC and CE.

# 4. Demodulation with HDIC/SDIC

The algorithms discussed above require the design of a demodulator that can produce soft outputs to enable soft input channel decoding. In a serially concatenated system, the quality of the inner demodulation or decoding is decisive for the system performance. The noncoherent matched filter (MF) based soft demodulation algorithms were presented in [21], [22]. They are useful in the beginning of the detection process when the estimates of the fading channel are lacking, we must therefore carry out the detection in a noncoherent manner. However, they have poor performance in multiuser environments since multiple access interference (MAI) is considered as additive noise and the knowledge about MAI is not exploited in any way. An effective tool to increase the capacity of interference-limited CDMA systems is multiuser detection (MUD), a method of jointly detecting all the users in the system. Among different MUD techniques, the multistage interference cancellation (IC) schemes are known to be simple and effective for mitigation of MAI in the DS-CDMA systems employing long scrambling codes. Here, we adopt the IC technique to improve the demodulation performance. The derivation of the demodulation algorithm using the IC technique with hard/soft decision feedback (HDIC/SDIC) is given below. For simplicity of notation we will suppress the index k and/or j from  $\mathbf{s}_k(j), \mathbf{C}_k(j), \mathbf{r}(k, j), \mathbf{A}(k, j), \mathbf{n}(k, j), \mathbf{X}_k(j) \text{ and } \mathbf{h}_k(j),$ etc., whenever no ambiguity arises.

In the beginning of the process, we use a noncoherent MF based demodulator to make a rough estimate of the transmitted signal in the absence of any knowledge of the underlying channel. Once the transmitted signals are estimated for all the users at the previous iteration, the channel can be estimated using the decision feedback and the interference can be removed by subtracting the estimated signals of the interfering users from the received signal  $\mathbf{r}$  to form a new signal vector  $\mathbf{r}'$  for demodulating the signal transmitted from user k, i.e.,

$$\mathbf{r}_{\text{hard}}' = \mathbf{r} - \hat{\mathbf{y}} + \mathbf{X}_k \mathbf{h}_k \tag{10}$$

where  $\mathbf{r} \in \mathbb{C}^{N_k}$  denote the received signal vector due to the transmission of the  $j^{th}$  symbol from the  $k^{th}$  user and  $\mathbf{r}'_{hard} \in \mathbb{C}^{N_k}$  is its interference canceled version after subtracting the contributions from all the other users using hard decision feedback. The vector  $\hat{\mathbf{y}} = \hat{\mathbf{A}}\hat{\mathbf{h}}$ represents the estimated contribution from all the users calculated using the data matrix  $\hat{\mathbf{A}}$  and channel vector  $\hat{\mathbf{h}}$  estimated at the previous iteration. The vector  $\hat{\mathbf{X}}_k \hat{\mathbf{h}}_k$ is the estimated contribution from all the paths of user k.

When  $L(\mathbf{s}_k)$ , the soft estimate of the transmitted sequence, is available either from the soft demodulator (see section 3.1) or from the soft decoder (see section 3.2), we can carry out soft IC. The rationale is that the hard IC tends to propagate errors and increase the interference with incorrect decision feedback; while with soft cancellation, an erroneously estimated symbol usually has small LLR, and hence the soft estimate of this symbol is small and does not make much contribution to the feedback, therefore error propagation is avoided.

If  $\lambda(\mathbf{w}_{i_k(j)}^p)$  is derived from the soft demodulator expressed by (7), the soft estimate (expected value given the received observation) for each bit of the Walsh codeword is computed by

$$\begin{split} \mathbf{E}[\mathbf{w}_{i_{k}(j)}^{p}|\mathbf{r}] &= (+1) \times P\{\mathbf{w}_{i_{k}(j)}^{p} = +1|\mathbf{r}\} + \\ &(-1) \times P\{\mathbf{w}_{i_{k}(j)}^{p} = -1|\mathbf{r}\} \\ &= (+1)\frac{e^{\lambda(\mathbf{w}_{i_{k}(j)}^{p})}}{1 + e^{\lambda(\mathbf{w}_{i_{k}(j)}^{p})}} + (-1)\frac{e^{-\lambda(\mathbf{w}_{i_{k}(j)}^{p})}}{1 + e^{-\lambda(\mathbf{w}_{i_{k}(j)}^{p})}} \\ &= \tanh\{\lambda(\mathbf{w}_{i_{k}(j)}^{p})/2\} \end{split}$$

If  $\lambda(\mathbf{w}_{i_k(j)}^p)$  is derived from the soft modulator expressed by (8), the soft estimate of each bit can be computed as

$$\begin{split} \mathbf{E}[\mathbf{w}_{i_{k}(j)}^{0}|\mathbf{r}] &= +1; \quad \mathbf{E}[\mathbf{w}_{i_{k}(j)}^{1}|\mathbf{r}] = \tanh\{\lambda(u_{k}'[_{l}^{0}])/2\};\\ \mathbf{E}[\mathbf{w}_{i_{k}(j)}^{2}|\mathbf{r}] &= \tanh\{\lambda(u_{k}'[_{l}^{1}])/2\};\\ \mathbf{E}[\mathbf{w}_{i_{k}(j)}^{3}|\mathbf{r}] &= \tanh\{\lambda(u_{k}'[_{l}^{0}])/2\};\\ \mathbf{E}[\mathbf{w}_{i_{k}(j)}^{3}|\mathbf{r}] &= \tanh\{\lambda(u_{k}'[_{l}^{0}])/2\}\cdot\tanh\{\lambda(u_{k}'[_{l}^{1}])/2\};\\ \mathbf{E}[\mathbf{w}_{i_{k}(j)}^{5}|\mathbf{r}] &= \tanh\{\lambda(u_{k}'[_{l}^{0}])/2\}\cdot\tanh\{\lambda(u_{k}'[_{l}^{2}])/2\};\\ \mathbf{E}[\mathbf{w}_{i_{k}(j)}^{6}|\mathbf{r}] &= \tanh\{\lambda(u_{k}'[_{l}^{1}])/2\}\cdot\tanh\{\lambda(u_{k}'[_{l}^{2}])/2\};\\ \mathbf{E}[\mathbf{w}_{i_{k}(j)}^{6}|\mathbf{r}] &= \tanh\{\lambda(u_{k}'[_{l}^{1}])/2\}\cdot\tanh\{\lambda(u_{k}'[_{l}^{2}])/2\};\\ \mathbf{E}[\mathbf{w}_{i_{k}(j)}^{7}|\mathbf{r}] &= \prod_{n=0}^{2}\tanh\{\lambda(u_{k}'[_{l}^{n}])/2\} \end{split}$$

Comparing with equation (9), it is evident that the additional complexity by computing  $E[\mathbf{w}_{i_k(j)}^p|\mathbf{r}]$  instead of making hard decision on  $\mathbf{w}_{i_k(j)}^p$  is small. The complexity increase is due to the replacement of  $\operatorname{sgn}(\cdot)$ operation with  $\tanh(\cdot)$ .

The soft estimate  $\operatorname{E}[\mathbf{s}_{k}^{q}|\mathbf{r}]$  for each Walsh chip  $\mathbf{s}_{k}^{q}, q = 1, \cdots, N$  is derived by spreading (repetition encoding) the soft bit of Walsh codeword  $\operatorname{E}[\mathbf{w}_{i_{k}(j)}^{p}|\mathbf{r}], p = 1, \cdots, M$ . The repetition factor is N/M. The cancellation residual after soft cancellation becomes

$$\mathbf{r}_{ ext{soft}}' = \mathbf{r} - \mathrm{E}[\mathbf{y}|\mathbf{r}] + \mathrm{E}[\mathbf{X}_k|\mathbf{r}]\mathbf{h}_k$$

where  $\mathbf{E}[\mathbf{y}|\mathbf{r}] = \mathbf{E}[\mathbf{A}|\mathbf{r}]\mathbf{h}$ , and the columns of  $\mathbf{E}[\mathbf{A}|\mathbf{r}]$  and  $\mathbf{E}[\mathbf{X}_k|\mathbf{r}]$  are derived by scrambling  $\mathbf{E}[\mathbf{s}_k|\mathbf{r}]$  with  $\mathbf{C}_k$  and compensating with the path delays. The received vector after HDIC/SDIC contains the contribution from the  $k^{th}$  user plus the cancellation residual and original additive Gaussian noise, i.e.,

$$\mathbf{r}' = \mathbf{X}_k \mathbf{h}_k + \mathbf{r}_c + \mathbf{n} = \mathbf{X}_k \mathbf{h}_k + \mathbf{w}$$
(11)

where  $\mathbf{r}_c$  stands for the cancellation residual and  $\mathbf{w}$  is defined as  $\mathbf{w} = \mathbf{r}_c + \mathbf{n}$ . Based on the fact that the long scrambling code sequence can be modeled as a random binary sequence [24], it was proven in [25] that both MAI and ISI terms can be accurately approximated as independent zero mean complex Gaussian random vectors which are uncorrelated with the noise vector. Therefore, the combined noise and residual interference  $\mathbf{w} \in \mathbb{C}^{N_k}$  is a complex random vector with PDF  $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \sigma_w^2 \mathbf{I}_{N_k})$ . Thus we have

$$f(\mathbf{r}'|\mathbf{s}_m) = \frac{1}{(\pi \sigma_w^2)^{N_k}} \exp\left(-\frac{\|\mathbf{r}' - \mathbf{X}_{k,m} \mathbf{h}_k\|^2}{\sigma_w^2}\right)$$
$$\lambda_{\mathrm{IC}}(u_k'[{}_l^n]; O) \approx \ln \frac{\max_{m:u_k'[{}_l^n]=+1} f(\mathbf{r}'|\mathbf{s}_m)}{\max_{m:u_k'[{}_l^n]=-1} f(\mathbf{r}'|\mathbf{s}_m)}$$
$$= \ln \frac{\exp(-\|\mathbf{r}' - \mathbf{X}^+ \mathbf{h}_k\|^2 / \sigma_w^2)}{\exp(-\|\mathbf{r}' - \mathbf{X}^- \mathbf{h}_k\|^2 / \sigma_w^2)}$$
$$= \frac{1}{\sigma_w^2} \left\{ \|\mathbf{r}' - \mathbf{X}^- \mathbf{h}_k\|^2 - \|\mathbf{r}' - \mathbf{X}^+ \mathbf{h}_k\|^2 \right\}$$
$$= \frac{2}{\sigma_w^2} \operatorname{Re} \left\{ \mathbf{h}_k^* \mathbf{X}^{+*} \mathbf{r}' - \mathbf{h}_k^* \mathbf{X}^{-*} \mathbf{r}' \right\}$$
(12)

where  $\mathbf{X}_{k,m} = \begin{bmatrix} \mathbf{x}_{k,1,m} & \mathbf{x}_{k,2,m} & \cdots & \mathbf{x}_{k,L_k,m} \end{bmatrix}$ , and

 $\mathbf{x}_{k,l,m}$  denotes the transmitted chip sequence due to the  $k^{th}$  user's  $j^{th}$  symbol from the  $l^{th}$  path based on the hypothesis that the  $m^{th}$  Walsh symbol is transmitted. It is formed by  $\mathbf{s}_m$  scrambled with  $\mathbf{C}_k$  and compensated with the path delay  $\tau_{k,l}$ . In (12),  $\mathbf{X}^+$  denotes the  $\mathbf{X}_{k,m}$ that corresponds to  $\max_{m:u'_k[l^n]=+1} f(\mathbf{r}'|\mathbf{s}_m)$ , and  $\mathbf{X}^-$  is defined similarly. Referring to equation (4),  $z_k(m) = \frac{2}{\sigma_w^2} \operatorname{Re} \left\{ \mathbf{h}_k^* \mathbf{X}_{k,m}^* \mathbf{r}' \right\}$  for this IC-based demodulator. From (11), we know that

$$\begin{split} \mathbf{E}[\|\mathbf{r}'\|^2] &= \mathbf{E}[(\mathbf{X}_k \mathbf{h}_k + \mathbf{w})(\mathbf{X}_k \mathbf{h}_k + \mathbf{w})^*] \\ &= \mathbf{E}[\|\mathbf{X}_k \mathbf{h}_k\|^2] + \mathbf{E}[\|\mathbf{w}\|^2] \approx N \, \mathbf{E}[\|\mathbf{h}_k\|^2] + N_k \sigma_w^2 \\ &= N P_k + N_k \sigma_w^2 \end{split}$$

Therefore, the variance of  ${\bf w}$  can be derived statistically as

$$\sigma_w^2 = \frac{\mathrm{E}[\|\mathbf{r}'\|^2] - NP_k}{N_k} \approx \frac{\|\overline{\mathbf{r}'}\|^2 - NP_k}{N_k} \qquad (13)$$

where  $P_k = \sum_{l=1}^{L_k} |h_{k,l}|^2 = \sum_{l=1}^{L_k} P_{k,l}$  denotes the total average received power from user k's different paths, and  $\|\mathbf{r}'\|^2$  is the energy of the vector  $\mathbf{r}'$  averaged over the whole block of symbols.

The original  $\mathbf{h}_k$  is unknown and has to be estimated. In (12), we should replace it with its estimates  $\hat{\mathbf{h}}_k$  instead. An estimate of the channel vector  $\hat{\mathbf{h}}_k$  can be obtained using detected data from previous iteration. Channel estimation will be treated next.

# 5. Hard/Soft Decision-Directed CE

We need estimates of the complex channel gains to do coherent demodulation as discussed above. The maximum likelihood (ML) channel estimator is described in this section to estimate frequency-selective multipath channel gains (technically, the algorithm is truly ML only if  $\hat{\mathbf{A}}$  is the ML estimate of  $\mathbf{A}$ ). Both hard and soft versions of the estimator are presented. It is decisiondirected method (the estimation procedure at the  $n^{th}$ stage uses the data estimates from the  $(n-1)^{th}$  stage) and can be inserted into the iteration loops in the previous section.

According to [2], the channel estimation algorithm using hard decision of the matrices  $\hat{\mathbf{A}}(k, j)$  and  $\hat{\mathbf{A}}(k, j+1)$  can be reformulated as

$$\hat{\mathbf{h}}_{\text{hard}}^{\text{ML}}(k,j) = \begin{bmatrix} \hat{\mathbf{A}}(k,j) \\ \hat{\mathbf{A}}(k,j+1) \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} \mathbf{r}(k,j) \\ \mathbf{r}(k,j+1) \end{bmatrix}$$
(14)

where the matrix  $\hat{\mathbf{A}}^{\dagger} = (\hat{\mathbf{A}}^* \hat{\mathbf{A}})^{-1} \hat{\mathbf{A}}^*$  denotes the left pseudo-inverse of  $\hat{\mathbf{A}}$  (assuming that  $\hat{\mathbf{A}}$  has full column rank). Note that we stack the received vectors and data matrices in (14) to solve the dimensionality problem as addressed in [2].

When  $E[\mathbf{A}|\mathbf{r}]$  is available, the soft version of the

ML channel estimator can be formed as

$$\hat{\mathbf{h}}_{\text{soft}}^{\text{ML}}(k,j) = \begin{bmatrix} \mathbf{E}[\mathbf{A}(k,j)|\mathbf{r}(k,j)]\\ \mathbf{E}[\mathbf{A}(k,j+1)|\mathbf{r}(k,j+1)] \end{bmatrix}^{\dagger} \begin{bmatrix} \mathbf{r}(k,j)\\ \mathbf{r}(k,j+1) \end{bmatrix}$$
(15)

Since the channel gains are correlated in time, the estimation results can be further enhanced by applying smoothing operation on the original channel estimates. In our simulations, channel smoothing is accomplished by an FIR filter derived from the Hamming window of length 19. Comparing (14) vs. (15), one can see the use of soft information in channel estimation itself does not introduce any additional complexity. It is the derivation of soft values that is little more complicated than making hard decisions as discussed in Section 3.1 and Section 3.2. It is also worth noticing that both hard and soft channel estimators introduced above involve matrix inversion at a symbol rate, which is computationally intensive. For a  $N \times N$  matrix, its matrix inversion requires on the order of  $N^3$  operations. Alternatively, the matrix inversion can be solved iteratively by the Jacobi algorithm [26] or Gauss-Seidel method [27]. For the purpose of this work, we directly calculate the matrix inverse and leave other practical and simplified solutions as future tasks.

# 6. Numerical results

The simulation parameters are summarized in Table 3. Channels are independent multipath Rayleigh fading channels with the classical "bath tub" power spectrum [28]. That is, the channel gain  $h_{k,l}(t)$  is a complex circular Gaussian process with autocorrelation function  $E[h_{k,l}^*(t)h_{k,l}(t+\tau)] = P_{k,l}J_0(2\pi f_D\tau)$  where  $f_D$  is the maximum Doppler frequency,  $J_0(x)$  is the zeroth order Bessel function of the first kind, and  $P_{k,l}$  is the power of  $h_{k,l}(t)$ . The Doppler shifts on each of the multipath components are due to the relative motion between the base station and mobile units. Perfect slow power control is assumed in the sense that  $P_k = \sum_{l=1}^{L_k} P_{k,l}$ , the average received power, is equal for all users, and normalized to unity. Channel estimation is conducted with the ML algorithm presented in Section 5.

The long scrambling codes  $\mathbf{C}_k$  are randomly assigned. The path delays  $\tau_{k,1}, \tau_{k,2}, \ldots, \tau_{k,L_k}$  and  $\mathbf{C}_k$  are assumed to be known to the receiver. One simplifying assumption is made such that the delays of all the

 Table 3
 Simulation parameters setting.

convolutional code	$\begin{array}{c} \text{code rate} = 1/3\\ \text{constraint length} = 5 \end{array}$
	polynomials = 25, 33, 37
number of Walsh codewords	M = 8
number of chips per symbol	N = 64
normalized Doppler frequency	$f_D T = 0.01$
number of channel taps	$L_k = 3$ for all $k$
block length	4620 code bits
block interleaver	$66 \times 70$

users' paths are multiples of the chip duration. However, the algorithms presented in this paper are general and can be extended to include arbitrary delays. The simulation results are averaged over random distributions of fading, noise, delay, and scrambling code with minimum of 10 blocks of data transmitted and at least 100 errors generated. To study the behavior of each algorithm, the number of iterations is usually set to 6 or 7, since it is observed that almost all the algorithms would converge after 5 or 6 iterations.

In the beginning of the iterative process, the noncoherent MF based soft demodulation algorithm presented in [22] is used to obtained an initial estimate of the transmitted data for the IC and CE in the subsequent stages. For the SDIC demodulation algorithm, the noise plus residual interference variance  $\sigma_w^2$  is statistically estimated by equation (13), i.e., by averaging  $(\|\mathbf{r}'\|^2 - NP_k)/N_k$  over the whole block of symbols.

The performance of the hard/soft IC and CE is examined and compared in a 15-user system for both the partitioned and the integrated schemes. The results are shown in Fig. 6 and 7. With target BER= $10^{-5}$ , the gain by applying soft information rather than hard decisions for the IC and CE is 0.2 dB for the partitioned approach, and 0.6 dB for the integrated approach after the system reaches convergence. The improvement tends to increase as SNR increases. Apparently, the integrated approach performs much better the partitioned approach. The average received energy of per information bit is  $E_b = \mathbb{E}[\|\mathbf{X}_k(j)\mathbf{h}_k(j)\|^2] / \log_2 M$ , and to achieve a BER=  $10^{-5}$ ,  $E_b/N_0 = 9$  dB is required by the partitioned scheme with soft IC and CE; while only  $E_b/N_0 = 7.2$  dB is required by the integrated scheme with soft IC and CE. From the experiments, we noticed that the LLR computation expressed in (8) without approximation leads to exactly the same soft values as the one expressed in (16) and (17) in the Appendix. The former approach has lower complexity, is thus preferred. Also, the performance degradation introduced by the approximation in (8) is negligible.

To find out their theoretical performance potential, we plot in Fig. 6 and 7 a lower bound on the BER that can be achieved by the proposed schemes. The bound is obtained based on the assumption that the cancellation and channel estimation are perfect, which would be the ideal situation leading to best achievable performance. Fig. 6 shows that the system performance is far from the lower BER bound with the partitioned approach; the gap is over 2 dB. The reason is that errors in the decision feedback significantly degrade performance and prevent the algorithm from reaching its theoretical potential. Fig. 7 shows that the performance achieved by the integrated approach is closer to the BER lower bound; the gap is between 0.3 and 0.7 dB. Clearly, the integrated demodulation and decoding scheme effectively reduces the feedback error probability and approaches the best achievable performance.

The system capacity (the number of users that can be supported by the system with acceptable bit error rate) achievable by the hard/soft IC and CE with integrated scheme is shown in Fig. 8. Apparently, more users can be supported by the soft IC and CE algorithm than the hard IC and CE algorithm, and the gap in capacity increases as SNR increases.

We observe from the above plots that all the algo-

![](_page_10_Figure_4.jpeg)

**Fig. 6** Hard vs. soft IC & CE with partitioned approach. Six stages of iterative demodulation are performed. The topmost curve represents the noncoherent MF demodulation. The second curve from top represents the first stage IC demodulation. The bottommost curve represents the 5th stage IC demodulation.

![](_page_10_Figure_6.jpeg)

**Fig. 7** Hard vs. soft IC & CE with integrated approach. Six stages of joint demodulation and decoding are performed. The topmost curve represents the noncoherent MF demodulation and decoding. The second curve from top represents the first stage IC demodulation and decoding. The bottommost curve represents the 5th stage IC demodulation and decoding.

![](_page_10_Figure_8.jpeg)

Fig. 8 Measurement of system capacity with integrated approach.

rithms converge to maximum achievable performance after 4 to 6 iterations, beyond which improvement through the iteration process becomes insignificant. Compared with the topmost curve which represents the first noncoherent stage using MF demodulator, the subsequent coherent IC demodulation stages greatly improve the demodulation performance. Clearly, interference cancellation and channel estimation, especially IC and CE using soft decision feedback are needed to reduce error probability and increase the system capacity.

## 7. Conclusions

In this paper, we developed some algorithms for IC and CE assisted demodulation based on soft decision feedback for demodulating and decoding orthogonally modulated and convolutionally coded signals in frequencyselective channels. The IC and CE are needed in order to remove deteriorative effect of interference. They can be implemented using the decisions either from the output of the demodulator or from the output of the channel decoder. In the former case, the demodulation and decoding blocks are partitioned as in the conventional system; in latter case, the two blocks are integrated in an iterative fashion so that the output of the channel decoder can be fed back to the demodulator. The integrated approach outperforms the partitioned approach, at cost of higher complexity.

The use of hard decision for the IC and CE makes the system vulnerable to error propagation. The probability of error propagation can be reduced through the feedback of soft information instead of hard decisions. With integrated approach, the Log-MAP decoding should be employed to enables soft IC and CE, which are impossible with VA decoding. In order to utilize the soft outputs from the Log-MAP decoder, a soft modulator is introduced in this paper to derive a soft estimate of transmitted chip sequence for soft IC and CE. Based on the numerical results and analysis, we conclude that the use of soft information for the IC and CE can improve the demodulation performance and the system capacity with minor increase in complexity compared to the IC and CE using hard decisions. The gain is more obvious for the integrated demodulation and decoding than for the partitioned approach.

## 8. Appendix

The other approach to derive  $\lambda(\mathbf{w}_{i_k(j)}^p)$  starts with the calculation of the conditional probability for each Walsh codeword  $\mathbf{w}_m$ 

$$\begin{split} P(\mathbf{w}_{0}|\mathbf{r}) &= P(u_{k}'[_{l}^{0}] = +1, u_{k}'[_{l}^{1}] = +1, u_{k}'[_{l}^{2}] = +1|\mathbf{r}) \\ &= P(u_{k}'[_{l}^{0}] = +1|\mathbf{r}) \cdot P(u_{k}'[_{l}^{1}] = +1|\mathbf{r}) \cdot P(u_{k}'[_{l}^{2}] = +1|\mathbf{r}) \\ P(\mathbf{w}_{1}|\mathbf{r}) &= P(u_{k}'[_{l}^{0}] = +1, u_{k}'[_{l}^{1}] = +1, u_{k}'[_{l}^{2}] = -1|\mathbf{r}) \\ &= P(u_{k}'[_{l}^{0}] = +1|\mathbf{r}) \cdot P(u_{k}'[_{l}^{1}] = +1|\mathbf{r}) \cdot P(u_{k}'[_{l}^{2}] = -1|\mathbf{r}) \\ &\vdots \end{split}$$

$$P(\mathbf{w}_{7}|\mathbf{r}) = P(u_{k}'[_{l}^{0}] = -1, u_{k}'[_{l}^{1}] = -1, u_{k}'[_{l}^{2}] = -1|\mathbf{r})$$

$$= P(u_{k}'[_{l}^{0}] = -1|\mathbf{r}) \cdot P(u_{k}'[_{l}^{1}] = -1|\mathbf{r}) \cdot P(u_{k}'[_{l}^{2}] = -1|\mathbf{r})$$

$$P(u_{k}'[_{l}^{n}] = \pm 1|\mathbf{r}) = \frac{\exp[\pm\lambda(u_{k}'[_{l}^{n}])]}{1 + \exp[\pm\lambda(u_{k}'[_{l}^{n}])]}$$

$$= \frac{1}{2} \left[ 1 \pm \tanh\left(\frac{1}{2}\lambda(u_{k}'[_{l}^{n}])\right) \right]; \quad n = 0, 1, 2 \quad (16)$$

The last equation in (16) holds according to the definition  $\lambda(u'_k[l]) = \ln \frac{P(u'_k[l]) = +1|\mathbf{r})}{P(u'_k[l]) = -1|\mathbf{r})}$ . The LLR of each bit  $\mathbf{w}_{i_k(j)}^p, p = 0, 1, \cdots, 7$  can then be obtained as

$$P(\mathbf{w}_{i_{k}(j)}^{p} = +1|\mathbf{r}) = \sum_{m:\mathbf{w}_{m}^{p}=+1} P(\mathbf{w}_{m}|\mathbf{r});$$

$$P(\mathbf{w}_{i_{k}(j)}^{p} = -1|\mathbf{r}) = \sum_{m:\mathbf{w}_{m}^{p}=-1} P(\mathbf{w}_{m}|\mathbf{r})$$

$$\lambda(\mathbf{w}_{i_{k}(j)}^{p}) = \ln \frac{P(\mathbf{w}_{i_{k}(j)}^{p} = +1|\mathbf{r})}{P(\mathbf{w}_{i_{k}(j)}^{p} = -1|\mathbf{r})} = \ln \frac{\sum_{m:\mathbf{w}_{m}^{p}=+1} P(\mathbf{w}_{m}|\mathbf{r})}{\sum_{m:\mathbf{w}_{m}^{p}=-1} P(\mathbf{w}_{m}|\mathbf{r})}$$
(17)

where  $m : \mathbf{w}_m^p = \pm 1$  is denoted as the set of Walsh codewords  $\{\mathbf{w}_m\}$  whose  $p^{th}$  bit is  $\pm 1$ . After deriving LLRs by (16) and (17),  $\mathrm{E}[\mathbf{w}_{i_k(j)}^p|\mathbf{r}]$  is computed as

$$E[\mathbf{w}_{i_{k}(j)}^{p}|\mathbf{r}] = (+1) \times P\{\mathbf{w}_{i_{k}(j)}^{p} = +1|\mathbf{r}\} + (-1) \times P\{\mathbf{w}_{i_{k}(j)}^{p} = -1|\mathbf{r}\} = P(\mathbf{w}_{i_{k}(j)}^{p} = +1|\mathbf{r}) - P(\mathbf{w}_{i_{k}(j)}^{p} = -1|\mathbf{r}) = \sum_{m:\mathbf{w}_{m}^{p}=+1} P(\mathbf{w}_{m}|\mathbf{r}) - \sum_{m:\mathbf{w}_{m}^{p}=-1} P(\mathbf{w}_{m}|\mathbf{r})$$
(18)

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