Irrevocable Belief Revision and Epistemic Entrenchment

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Abstract

In recent papers [10, 11] Krister Segerberg introduced *Irrevocable Belief Revision*, as closely related to AGM revision [2]. In this paper we present irrevocable belief revision in terms of an epistemic entrenchment relation.[1]

Keywords: Logic of Theory Change, Irrevocable belief revision, Epistemic Entrenchment.

1 Background

The AGM model [2] characterizes changes in the belief of a rational agent. AGM recognizes three different kind of change: expansion (that consists in adding a new belief), contraction (eliminating a belief from the belief corpus of the agent) and revision (adding new beliefs preserving consistency). Krister Segerberg [10, 11] argues that there is a distinction between *actual belief revision* and *merely hypothetical belief revision*, and to capture it, proposes a new model, closely related with AGM: Irrevocable belief revision. [2]

In irrevocable belief revision, just as in the AGM model the beliefs of a rational agent are represented by a belief set $\mathbf{K}$, a set of sentences in a language $\mathcal{L}$. $\mathbf{K}$ is closed under logical consequence Cn, where Cn satisfies: $\mathbf{A} \subseteq Cn(\mathbf{A})$, $Cn(Cn(\mathbf{A})) \subseteq Cn(\mathbf{A})$, and $Cn(\mathbf{A}) \subseteq Cn(\mathbf{B})$ if $\mathbf{A} \subseteq \mathbf{B}$. We assume that Cn includes classical logical consequence, satisfies the rule of introduction of disjunction into premises and is compact. $\vdash \alpha$ is an alternative notation for $\alpha \in \mathrm{Cn}(\emptyset)$ and $\mathbf{K}+\alpha$ for $Cn(\mathbf{K} \cup \{\alpha\})$. $\top$ is an arbitrary tautology, $\perp$ the falsity constant, $\mathbf{K}_\perp$ the inconsistent belief set[3] and $\mathcal{K}$ the set of all belief sets.
Irrevocable belief revision also includes a second belief set $\mathbf{V}$, that represents a set of

[1] Full version of a contributed paper presented at the *6th Workshop on Logic, Language, Information and Computation* (*WoLLIC'99*), `http://www.di.ufpe.br/~wollic/wollic99`, held at the Hotel Simon, National Park of Itatiaia, Rio de Janeiro, Brazil, May 25–28 1999, with scientific sponsorship by IGPL, FoLLI, ASL, SBC, and SBL, and organised by Univ. Federal de Pernambuco (UFPE) and Univ. Estadual de Campinas (UNICAMP). Financial support was given by CNPq (Brazil), CAPES (Brazil), and Facoltà di Scienze della Univ. di Verona (Italy).

[2] "Ordinary theories of belief change do not seem suited to handle the sort of hypothetical belief change that goes on, for example, in debates where the participants agree, ≪for the sake of argument≫, on a certain common ground on which possibilities can be explored and disagreements can be aired. One need not actually believe what one accepts in this way. Nevertheless such acceptance amounts to what may be called a doxastic commitment, one that cannot be given up within the perimeter of the debate. Someone who no longer wishes to honour such a commitment may be described as in effect abandoning the debate, perhaps in order to initiate another debate with a different set of doxastic commitments." [10]

[3] Note that $\mathbf{K}_\perp = \mathcal{L}$.

doxastic commitments, which are treated as irrevocable.

Definition 1.1 *[11] Let* $\mathbf{V}$, $\mathbf{K}$ *be belief sets in* $\mathcal{L}$. *A pair* $(\mathbf{V}, \mathbf{K})$ *is called a* complex *if and only if* $\mathbf{V} \subseteq \mathbf{K}$.

Let $\mathcal{C}$ be the set of all complexes. Segerberg proposed the following axiomatic for irrevocable belief revision:

Definition 1.2 *[11] Let* $(\mathbf{V}, \mathbf{K})$ *be a complex in a language* $\mathcal{L}$. $*$: $\mathcal{C} \times \mathcal{L} \to (\mathcal{K}, \mathcal{K})$[4] *is an* irrevocable revision, *where for all* α, $(\mathbf{V}, \mathbf{K}) * \alpha = (\mathbf{V}_\alpha, \mathbf{K}_\alpha)$ *if and only if it satisfies:* [5]

(∗1) $(\mathbf{V}, \mathbf{K}) * \alpha$ *is always a complex.*

(∗2) $\mathbf{K}_\alpha \vdash \alpha$.

(∗b) $\mathbf{V}_\alpha = Cn(\mathbf{V} \cup \{\alpha\})$.

(∗3) *If* $\mathbf{K} \nvdash \neg\alpha$, *then* $\mathbf{K}_\alpha = Cn(\mathbf{K} \cup \{\alpha\})$.

(∗4) *If* $\mathbf{V} \nvdash \neg\alpha$ *then* $\mathbf{K}_\alpha \neq \mathbf{K}_\perp$.

(∗5) *If* $\vdash \alpha \leftrightarrow \beta$, *then* $\mathbf{K}_\alpha = \mathbf{K}_\beta$.

(∗df) $\mathbf{K}_{(\alpha\vee\beta)} = \mathbf{K}_\alpha$ *or* $\mathbf{K}_{(\alpha\vee\beta)} = \mathbf{K}_\beta$ *or* $\mathbf{K}_{(\alpha\vee\beta)} = \mathbf{K}_\alpha \cap \mathbf{K}_\beta$.

(∗c) *For all* α, $*' : \mathcal{C} \times \mathcal{L} \to (\mathcal{K}, \mathcal{K})$ *satisfies that* [6] $((\mathbf{V}, \mathbf{K}) * \alpha) *' \beta = (\mathbf{V}, \mathbf{K}) * (\alpha \wedge \beta)$.

(*2) is redundant, since it follows from (*b) and (*1)[7]. We reproduce the original axiomatic. The following lemma follows trivially from the definition:

Lemma 1.3 $*'$ *satisfies (∗1),(∗2), (∗b), (∗3), (∗4), (∗5), and (∗df).*

2 Entrenchment-Based Irrevocable Revision

The notion of epistemic entrenchment was introduced in [3] by Gärdenfors to represent formally a preference ordering among formulae in a theory. The *standard* postulates of epistemic entrenchment are:

(**EE1**) If $\alpha \leq \beta$ and $\beta \leq \delta$, then $\alpha \leq \delta$. (transitivity)

(**EE2**) If $\alpha \vdash \beta$, then $\alpha \leq \beta$. (dominance)

(**EE3**) $\alpha \leq (\alpha \wedge \beta)$ or $\beta \leq (\alpha \wedge \beta)$. (conjunctiveness)

(**EE4**) If $\mathbf{K} \neq \mathbf{K}_\perp$, then $\alpha \notin \mathbf{K}$ if and only if $\alpha \leq \beta$ for all β. (minimality)

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[4] "Note that the irrevocable belief revision functions are regarded as binary functions, but they are just partially defined on the set $\mathcal{C} \times \mathcal{L}$. Once the first argument has been fixed to be a given complex $(\mathbf{V}, \mathbf{K})$, the function is well defined for every language formula. Consequently, we can take irrevocable belief revision functions as a unary function $*$: $\mathcal{L} \to (\mathcal{K}, \mathcal{K})$" [V. Becher, personal communication]. We conserve the binary notation for clarity of the exposition.

[5] In a first approach, we tried to suppress $\mathbf{V}$ from the axiomatic, defining $\mathbf{V} = \{\beta : \mathbf{K} * \neg\beta = \mathbf{K}_\perp\}$. However, as John Cantwell points out, irrevocable belief revision possess *historic memory*, i.e., it does not satisfy: *If* $\mathbf{K} = (\ldots(\mathbf{H} * \alpha_1)\ldots) * \alpha_n$, *then* $\mathbf{K} * \alpha = ((\ldots(\mathbf{H} * \alpha_1)\ldots) * \alpha_n) * \alpha$. For deep details of iterable functions see [1].
For example, suppose an agent that always prefers the newest beliefs rather than the oldest. Consequently,
$(Cn(\emptyset) + \alpha + \beta) * (\neg\alpha \vee \neg\beta) = Cn(\{\neg\alpha, \beta\})$,
$(Cn(\emptyset) + \beta + \alpha) * (\neg\alpha \vee \neg\beta) = Cn(\{\alpha, \neg\beta\})$,
but $Cn(\emptyset) + \alpha + \beta = Cn(\emptyset) + \beta + \alpha$.

[6] idem footnote 4, $*'$ is defined for $(\mathbf{V}_\alpha, \mathbf{K}_\alpha)$.

[7] This fact was pointed out by an anonymous referee of the Wollic 99, where this papers was previously presented.

(**EE5**) If $\beta \leq_{\mathbf{K}} \alpha$ for all β, then $\vdash \alpha$. (maximality)

To construct an entrenchment-based irrevocable revision, we can make use of (**EE1**)–(**EE4**), but replacing (**EE5**) with a modified version of *maximality*:

(**EEi5**) $\alpha \in \mathbf{V}$ if and only if $\top \leq \alpha$. (i-maximality)

In the standard entrenchment ordering, the maximally entrenched beliefs are exactly the tautologies. I-maximality extends this property to the whole set $\mathbf{V}$.

Lemma 2.1 *Let* $\leq_{\mathbf{K}}$ *be an entrenchment ordering on a belief set* $\mathbf{K}$ *that satisfies* (**EE1**) – (**EE4**) *and* (**EEi5**). *Then* $(\mathbf{V}, \mathbf{K})$ *is a complex.*

To relate epistemic entrenchment with irrevocable revision we introduce the following definition, where we write $\alpha < \beta$ when $\alpha \leq \beta$ and $\beta \not\leq \alpha$, and $\alpha =_{\leq} \beta$ when $\alpha \leq \beta$ and $\beta \leq \alpha$:

Definition 2.2 *Let* $\leq$ *be an entrenchment relation on a belief set* $\mathbf{K}$ *that satisfies* (**EE1**) – (**EE4**) *and* (**EEi5**). $*_{\leq}$ *is an* irrevocable entrenchment-based revision *if and only if* $(\mathbf{V}, \mathbf{K}) *_{\leq} \alpha = (\mathbf{V}_\alpha, \mathbf{K}_\alpha)$ *where*

$\beta \in \mathbf{V}_\alpha$ *if and only if* $\beta \in Cn(\mathbf{V} \cup \{\alpha\})$.

$\beta \in \mathbf{K}_\alpha$ *if and only if either* $\neg\alpha \in \mathbf{V}$ *or* $\alpha \to \neg\beta < \alpha \to \beta$. *[7, 8]*

$\beta \leq_\alpha \gamma$ *if and only if* $\alpha \to \beta \leq \alpha \to \gamma$. *[9]*

The last part of the definition allows $*_{\leq}$ to be iterable. $\leq_\alpha$ represents the new entrenchment order after the revision by α. Additional properties of this postulate can be found in [9].

Lemma 2.3 *Let* $\leq$ *be an entrenchment relation on a belief set* $\mathbf{K}$ *that satisfies* (**EE1**)–(**EE4**) *and* (**EEi5**). *Let* $*_{\leq}$ *be defined as in* **Definition 2.2**. *Then:*

(a) $(\mathbf{V}_\alpha, \mathbf{K}_\alpha)$ *is a complex.*
(b) *[9]* $\leq_\alpha$ *satisfies* (**EE1**) – (**EE4**)
(c) $\leq_\alpha$ *satisfies* (**EEi5**).

We introduce the following identity [6] to define irrevocable revision in terms of epistemic entrenchment:

$(C \leq_*)$ $\alpha \leq \beta$ if and only if $\alpha \in \mathbf{K}_{\neg(\alpha\vee\beta)}$ implies $\beta \in \mathbf{K}_{\neg(\alpha\vee\beta)}$.

The next theorems relate *irrevocable revision* with *irrevocable entrenchment-based revision*:

Theorem 2.4 *Let* $\leq$ *be an entrenchment relation on a belief set* $\mathbf{K}$ *that satisfies* (**EE1**) – (**EE4**) *and* (**EEi5**). *Let* $*_{\leq}$ *defined as in* **Definition 2.2**. *Then* $*_{\leq}$ *is an* irrevocable revision *defined as in* **Definition 1.2** *and* $(C \leq_*)$ *also holds.*

Theorem 2.5 *Let $*$ be an irrevocable revision on a complex* $(\mathbf{V}, \mathbf{K})$ *defined as in* **Definition 1.2**. *Furthermore let $\leq$ be the relation defined from $*$ by condition* $(C \leq_*)$. *Then $\leq$ satisfies* $(\mathbf{EE1}) - (\mathbf{EE4})$ *and* $(\mathbf{EEi5})$ *and $*$ is an* irrevocable entrenchment-based revision *defined as in* **Definition 2.2**.

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A Appendix: Proofs

The following properties will be helpful in the proofs

Property A.1 *[4] If $\vdash \alpha \leftrightarrow \alpha'$ and $\vdash \beta \leftrightarrow \beta'$, then: $\alpha \leq_{\mathbf{K}} \beta$ if and only if $\alpha' \leq_{\mathbf{K}} \beta'$.*

Property A.2 *[5] If $\leq$ satisfies* $(\mathbf{EE1})$, $(\mathbf{EE2})$ *and* $(\mathbf{EE3})$ *then, $\alpha \rightarrow \neg\beta < \alpha \rightarrow \beta$ if and only if $\neg\alpha < \alpha \rightarrow \beta$.*

Proof of Lemma 2.1.

We must prove that:

(a) $\mathbf{V}$ is a belief set: Let $\delta \in Cn(\mathbf{V})$, we must prove that $\delta \in \mathbf{V}$. By compactness of the underlying logic there is a finite subset $\{\beta_1, ..., \beta_n\} \subseteq \mathbf{V}$, such that $\{\beta_1, ..., \beta_n\} \vdash \delta$. **Part 1.** We first show that $\beta_1 \wedge ... \wedge \beta_n \in \mathbf{V}$. For this purpose we are going to show that if $\beta_1 \in \mathbf{V}$ and $\beta_2 \in \mathbf{V}$ then $\beta_1 \wedge \beta_2 \in \mathbf{V}$. The rest follows by iteration of the same procedure. It follows from $(\mathbf{EE2})$ that $\beta_1 \leq \beta_1 \wedge \beta_2$ or $\beta_2 \leq \beta_1 \wedge \beta_2$; then by $(\mathbf{EE1})$ and $(\mathbf{EE5i})$, $\beta_1 \wedge \beta_2 \in \mathbf{V}$. **Part 2.** By repeated use of **Part 1**, we know that $\{\beta_1 \wedge ... \wedge \beta_n\} \in \mathbf{V}$. Since $\beta_1 \wedge ... \wedge \beta_n \vdash \delta$, by $(\mathbf{EE2})$ $\beta_1 \wedge ... \wedge \beta_n \leq \delta$, hence by $(\mathbf{EE1})$ and $(\mathbf{EEi5})$, $\delta \in \mathbf{V}$.

(b) $\mathbf{V} \subseteq \mathbf{K}$: Let $\beta \in \mathbf{V}$, then by $(\mathbf{EEi5})$ $\top \leq \beta$. If $\beta \leq \gamma$ for all γ it follows from $(\mathbf{EE1})$ that $\top \leq \gamma$ for all γ, and since $\top \in \mathbf{K}$, then by $(\mathbf{EE4})$, $\mathbf{K} = \mathbf{K}_\perp$. Hence $\beta \in \mathbf{K}$. If $\beta \leq \gamma$ for all γ is not satisfied, by $(\mathbf{EE4})$, $\beta \in \mathbf{K}$ ∎

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Proof of Lemma 2.3.

(a) $(\mathbf{V}_\alpha, \mathbf{K}_\alpha)$ **is a complex:** We must prove the following cases:

$\mathbf{K}_\alpha$ **is a belief set:** This proof is quite similar to the proof of **Lemma 2.1.a**. Let $\delta \in Cn(\mathbf{K}_\alpha)$. By compactness of the underlying logic there is a finite subset $\{\beta_1, ..., \beta_n\} \subseteq \mathbf{K}_\alpha$, such that $\{\beta_1, ..., \beta_n\} \vdash \delta$. If $\alpha \vdash \perp$, then it follows trivially from the definition of $\mathbf{K}_\alpha$ that $\beta_1 \wedge ... \wedge \beta_n \in \mathbf{K}_\alpha$ and $\delta \in \mathbf{K}_\alpha$. Let $\alpha \nvdash \perp$. Then: **Part 1.** We show first that $\beta_1 \wedge ... \wedge \beta_n \in \mathbf{K}_\alpha$. For this purpose we are going to show that if $\beta_1 \in \mathbf{K}_\alpha$ and $\beta_2 \in \mathbf{K}_\alpha$ then $\beta_1 \wedge \beta_2 \in \mathbf{K}_\alpha$. The rest follows by iteration of the same procedure. It follows from $\beta_1 \in \mathbf{K}_\alpha$ by the definition of $\mathbf{K}_\alpha$ that $(\alpha \rightarrow \neg\beta_1) < (\alpha \rightarrow \beta_1)$. Then by **Property A.2**, $\neg\alpha < (\alpha \rightarrow \beta_1)$. Then it follows from $\beta_2 \in \mathbf{K}_\alpha$ that $\neg\alpha < (\alpha \rightarrow \beta_2)$. By $(\mathbf{EE3})$, either $(\alpha \rightarrow \beta_1) \leq ((\alpha \rightarrow \beta_1) \wedge (\alpha \rightarrow \beta_2))$ or $(\alpha \rightarrow \beta_1) \leq ((\alpha \rightarrow \beta_1) \wedge (\alpha \rightarrow \beta_2))$. In the same way, by **Property A.1**, either $(\alpha \rightarrow \beta_1) \leq (\alpha \rightarrow (\beta_1 \wedge \beta_2))$ or

$(\alpha \to \beta_2) \leq (\alpha \to (\beta_1 \wedge \beta_2))$. In the first case, we use (**EE1**) and $\neg\alpha < (\alpha \to \beta_1)$ to obtain $\neg\alpha < (\alpha \to (\beta_1 \wedge \beta_2))$ and in the second we use $\neg\alpha < (\alpha \to \beta_2)$ to obtain the same result. It follows that $\beta_1 \wedge \beta_2 \in \mathbf{K}_\alpha$. **Part 2.** By repeated use of **Part 1.**, we know that $\{\beta_1 \wedge ... \wedge \beta_n\} \in \mathbf{K}_\alpha$. Let $\vdash \beta \leftrightarrow \beta_1 \wedge ... \wedge \beta_n$. We also have $\vdash \beta \to \delta$, then by the definition of $\mathbf{K}_\alpha$ $(\alpha \to \neg\beta) < (\alpha \to \beta)$. Since $\vdash (\alpha \to \beta) \to (\alpha \to \delta)$ and $\vdash (\alpha \to \neg\delta) \to (\alpha \to \neg\beta)$, (**EE2**) yields $(\alpha \to \beta) \leq (\alpha \to \delta)$ and $(\alpha \to \neg\delta) \leq (\alpha \to \neg\beta)$. We can apply (**EE1**) to $(\alpha \to \neg\delta) \leq (\alpha \to \neg\beta)$, $(\alpha \to \neg\beta) < (\alpha \to \beta)$ and $(\alpha \to \beta) \leq (\alpha \to \delta)$ to obtain $(\alpha \to \neg\delta) < (\alpha \to \delta)$. Hence by the definition of $\mathbf{K}_\alpha$, $\delta \in \mathbf{K}_\alpha$.
$\mathbf{V}_\alpha \subseteq \mathbf{K}_\alpha$: Let $\beta \in \mathbf{V}_\alpha$. by the definition of $\mathbf{V}_\alpha$, $\alpha \to \beta \in \mathbf{V}$. If $\alpha \to \neg\beta \in \mathbf{V}$, then $\neg\alpha \in \mathbf{V}$, hence $\mathbf{V}_\alpha \subseteq \mathbf{K}_\alpha = \mathbf{K}_\perp$. Let $\alpha \to \neg\beta \notin \mathbf{V}$, then $\alpha \to \neg\beta < \alpha \to \beta$, hence $\beta \in \mathbf{K}_\alpha$ from which it follows that $\mathbf{V}_\alpha \subseteq \mathbf{K}_\alpha$.

(b) (**EE1**) – (**EE4**) See [9].

(c) (**EEi5**) If $\beta \in Cn(\mathbf{V} \cup \{\alpha\})$ then $\alpha \to \beta \in \mathbf{V}$, then by (**EEi5**), for all γ $\alpha \to \alpha \leq \alpha \to \beta$, hence $\gamma \leq_\alpha \beta$ for all γ. If $\gamma \leq_\alpha \beta$ for all γ, then in particular $\alpha \leq_\alpha \beta$, $\alpha \to \alpha \leq \alpha \to \beta$. It follows by (**EE1**) and (**EE2**) that, then $\alpha \to \beta \in \mathbf{V}$, hence $\beta \in Cn(\mathbf{V} \cup \{\alpha\})$ ∎

Proof of Theorem 2.4

($*$1) See **Lemma 2.3.a**.

($*$2) If $\neg\alpha \in \mathbf{V}$, then it follows trivially from the definition of $\mathbf{K}_\alpha$ that $\alpha \in \mathbf{K}_\alpha$. Let $\neg\alpha \notin \mathbf{V}$. By (**EEi5**) $\neg\alpha < (\neg\alpha \vee \alpha)$ or equivalently by **Property A.1** $(\alpha \to \neg\alpha) < (\alpha \to \alpha)$. Hence by the definition of $\mathbf{K}_\alpha$, $\alpha \in \mathbf{K}_\alpha$.

($*$b) It follows trivially from the definition of $\mathbf{V}_\alpha$.

($*$3) Let $\neg\alpha \notin \mathbf{K}$. We must prove that $\mathbf{K}_\alpha = Cn(\mathbf{K} \cup \{\alpha\})$. We will prove this identity by double inclusion. For the first direction let $\beta \in \mathbf{K}_\alpha$. We want to show that $\beta \in Cn(\mathbf{K} \cup \{\alpha\})$, which can be done by showing that $\alpha \to \beta \in \mathbf{K}$. by the definition of $\mathbf{K}_\alpha$, since $\beta \in \mathbf{K}_\alpha$, $(\alpha \to \neg\beta) < (\alpha \to \beta)$; hence by (**EE4**), $(\alpha \to \beta) \in \mathbf{K}$. For the other direction, let $\beta \in Cn(\mathbf{K} \cup \{\alpha\})$. Then $\alpha \to \beta \in \mathbf{K}$. Due to $\neg\alpha \notin \mathbf{K}$, $Cn(\mathbf{K} \cup \{\alpha\}) \neq \mathbf{K}_\perp$, then $\neg\beta \notin Cn(\mathbf{K} \cup \{\alpha\})$, then $\alpha \to \neg\beta \notin \mathbf{K}$; and it follows by (**EE4**) that $(\alpha \to \neg\beta) < (\alpha \to \beta)$. Hence, by the definition of $\mathbf{K}_\alpha$, $\beta \in \mathbf{K}_\alpha$.

($*$4) Let $\mathbf{V} \nvdash \neg\alpha$ and assume that $\mathbf{K} = \mathbf{K}_\perp$. Then by the definition of $\mathbf{K}_\alpha$, $(\alpha \to \neg\perp) < (\alpha \to \perp)$. By **Property A.1** $\top < \perp$. Contradiction by (**EE2**). Hence $\mathbf{K} \neq \mathbf{K}_\perp$.

($*$5) Let $\vdash \alpha \leftrightarrow \beta$. If $\neg\alpha \in \mathbf{V}$ then $\beta \in \mathbf{V}$, hence by the definition of $\mathbf{K}_\alpha$ and $\mathbf{K}_\beta$, $\mathbf{K}_\alpha = \mathbf{K}_\beta = \mathbf{K}_\perp$. By **Property A.1** it follows for all γ that $(\alpha \to \neg\gamma) < (\alpha \to \gamma)$ if and only if $(\beta \to \neg\gamma) < (\beta \to \gamma)$; hence $\mathbf{K}_\alpha = \mathbf{K}_\beta$.

($*$df) If $\vdash \alpha$, then $\vdash (\alpha \vee \beta) \leftrightarrow \beta$ and the rest follows from the previous proof of **($*$5)**. Equivalently if $\vdash \beta$. Let $\nvdash \alpha$ and $\nvdash \beta$. We have three subcases[8]:
(a) $\neg\alpha < \neg\beta$. It follows from $\neg\alpha < \neg\beta$ and (**EE3**) that $\neg\alpha =_\leq (\neg\alpha \wedge \neg\beta)$. Then $\neg\alpha \notin \mathbf{V}$. We will prove that $\mathbf{K}_{(\alpha\vee\beta)} = \mathbf{K}_\alpha$. For one direction let $\delta \in \mathbf{K}_\alpha$. It follows from the definition of $\mathbf{K}_\alpha$ that $(\alpha \to \neg\delta) < (\alpha \to \delta)$. Then by **Property A.2**, $\neg\alpha < (\alpha \to \delta)$. Since (**EE2**) yields $\neg\beta < (\beta \to \delta)$, we use (**EE1**) to obtain

[8] We write $\alpha =_\leq \beta$ if and only if $\alpha \leq \beta$ and $\beta \leq \alpha$.

both $(\neg\alpha \wedge \neg\beta) < (\alpha \rightarrow \delta)$ and $(\neg\alpha \wedge \neg\beta) < (\beta \rightarrow \delta)$. (**EE2**) and (**EE3**) yield $(\neg\alpha \wedge \neg\beta) < ((\alpha \vee \beta) \rightarrow \delta)$. Hence $((\alpha \vee \beta) \rightarrow \neg\delta) < ((\alpha \vee \beta) \rightarrow \delta)$ from which it follows that $\delta \in \mathbf{K}_{(\alpha\vee\beta)}$.

For the other direction, let $\delta \in \mathbf{K}_{(\alpha\vee\beta)}$. It follows by $\neg\alpha =_{\leq} (\neg\alpha \wedge \neg\beta)$ that $\not\vdash (\neg\alpha \wedge \neg\beta)$; then by the definition of $\mathbf{K}_\alpha$, $(\neg\alpha \wedge \neg\beta) < ((\alpha \vee \beta) \rightarrow \delta)$. By (**EE2**) $((\alpha \vee \beta) \rightarrow \delta) \leq (\alpha \rightarrow \delta)$. (**EE1**) yields $\neg\alpha < (\alpha \rightarrow \delta)$, hence $\alpha \rightarrow \neg\delta < \alpha \rightarrow \delta$ from which it follows that $\delta \in \mathbf{K}_\alpha$.

(b) $\neg\beta < \neg\alpha$: Similar to case **(a)**; $\mathbf{K}_{(\alpha\vee\beta)} = \mathbf{K}_\beta$.

(c) $\neg\alpha =_{\leq} \neg\beta$. Then $\neg\alpha =_{\leq} \neg\beta =_{\leq} (\neg\alpha \wedge \neg\beta)$ that implies that $\neg\alpha \in \mathbf{V}$ if and only if $\neg\beta \in \mathbf{V}$ if and only if $(\neg\alpha \wedge \neg\beta) \in \mathbf{V}$, hence if $\neg\alpha \in \mathbf{V}$, then $\mathbf{K}_{(\alpha\vee\beta)} = \mathbf{K}_\alpha = \mathbf{K}_\beta = \mathbf{K}_\perp$. Let $\neg\alpha \notin \mathbf{V}$. Then: $\delta \in \mathbf{K}_\alpha \cap \mathbf{K}_\beta$ if and only if (by the definition of $\mathbf{K}_\alpha$ and $\mathbf{K}_\beta$), $\neg\alpha < (\alpha \rightarrow \delta)$ and $\neg\beta < (\alpha \rightarrow \delta)$ if and only if (by (**EE1**)) $(\neg\alpha \wedge \neg\beta) < (\alpha \rightarrow \delta)$ and $(\neg\alpha \wedge \neg\beta) < (\alpha \rightarrow \delta)$ if and only if (by (**EE2**) and by (**EE3**)) $(\neg\alpha \wedge \neg\beta) < ((\alpha \vee \beta) \rightarrow \delta)$ if and only if (by the definition of $\mathbf{K}_{(\alpha\vee\beta)}$) $\delta \in \mathbf{K}_{(\alpha\vee\beta)}$.

($*$c) Let $((\mathbf{V}, \mathbf{K}) * \alpha) * \beta = (\mathbf{V}', \mathbf{K}')$. We will use double inclusion to prove this: For the first direction, let $\gamma \in \mathbf{K}'$. We have two cases:

(a) $\neg\beta \in \mathbf{V}_\alpha$. Then $\neg\beta \in Cn(\mathbf{V} \cup \{\alpha\})$, from which it follows that $\neg\alpha \vee \neg\beta \in \mathbf{V}$, then by the definition of $\mathbf{K}_{(\alpha\wedge\beta)}$, $\mathbf{K}_{(\alpha\wedge\beta)} = \mathbf{K}_\perp$, hence $\mathbf{K}' \subseteq \mathbf{K}_{(\alpha\wedge\beta)}$.

(b) $\neg\beta \notin \mathbf{V}_\alpha$. It follows by the definition of $\mathbf{K}'$ that $(\beta \rightarrow \neg\gamma) <_\alpha (\beta \rightarrow \gamma)$. Then by the definition of $\leq_\alpha$, $(\alpha \rightarrow (\beta \rightarrow \neg\gamma)) < (\alpha \rightarrow (\beta \rightarrow \gamma))$. By **Property A.1** this is equivalent to $((\alpha \wedge \beta) \rightarrow \neg\gamma) <_\alpha ((\alpha \wedge \beta) \rightarrow \gamma)$; then by the definition of $\mathbf{K}_{(\alpha\wedge\beta)}$, $\gamma \in \mathbf{K}_{(\alpha\wedge\beta)}$; hence $\mathbf{K}' \subseteq \mathbf{K}_{(\alpha\wedge\beta)}$.

For the second direction let $\gamma \in \mathbf{K}_{(\alpha\wedge\beta)}$. We have two cases:

(c) $\neg(\alpha \wedge \beta) \in \mathbf{V}$. Then it follows that $\neg\beta \in \mathbf{V}_\alpha = Cn(\mathbf{V} \cup \{\alpha\})$, then by the definition of $\mathbf{K}'$, $\mathbf{K}' = \mathbf{K}_\perp$, hence $\mathbf{K}_{(\alpha\wedge\beta)} \subseteq \mathbf{K}'$.

(d) $\neg(\alpha \wedge \beta) \notin \mathbf{V}$. It follows by the definition of $\mathbf{K}_{(\alpha\wedge\beta)}$ that $((\alpha \wedge \beta) \rightarrow \neg\gamma) < ((\alpha \wedge \beta) \rightarrow \gamma)$, then by **Property A.1**, $(\alpha \rightarrow (\beta \rightarrow \neg\gamma)) < (\alpha \rightarrow (\beta \rightarrow \gamma))$, then by the definition of $<_\alpha$, $\beta \rightarrow \neg\gamma <_\alpha \beta \rightarrow \gamma$. Hence $\gamma \in \mathbf{K}'$ and $\mathbf{K}_{(\alpha\wedge\beta)} \subseteq \mathbf{K}'$.

($C \leq_*$) Let $\alpha \leq \beta$ and $\alpha \in \mathbf{K}_{\neg(\alpha\wedge\beta)}$. There are two subcases according to the definition of $\mathbf{K}_{\neg(\alpha\wedge\beta)}$: **(a)** $\neg(\alpha \wedge \beta) \in \mathbf{V}$: Hence $\beta \in \mathbf{K}_{\neg(\alpha\wedge\beta)} = \mathbf{K}_\perp$. (b) $\neg(\alpha \wedge \beta) \notin \mathbf{V}$: Then $(\neg(\alpha \wedge \beta) \rightarrow \neg\alpha) < (\neg(\alpha \wedge \beta) \rightarrow \alpha)$, then by **Property A.1**, $(\beta \vee \neg\alpha) < \alpha$. By (**EE2**), $\beta \leq (\beta \vee \neg\alpha)$, it then follows by (**EE1**) that $\beta < \alpha$. We obtain a contradiction, hence the second case is not possible. The other direction can be proved by showing that **(a)** if $\beta < \alpha$, then $\alpha \in \mathbf{K}_{\neg(\alpha\wedge\beta)}$ and **(b)** if $\beta <_{\mathbf{K}} \alpha$, then $\beta \notin \mathbf{K}{*}\neg(\alpha \wedge \beta)$.

(a) We can to this by showing $\neg(\alpha \wedge \beta) \rightarrow \neg\alpha < \neg(\alpha \wedge \beta) \rightarrow \alpha$, or equivalently, $\beta \vee \neg\alpha < \alpha$. Suppose for *reductio* that this is not the case. Then $\alpha \leq \beta \vee \neg\alpha$. Since $\alpha \leq \alpha$, (**EE3**) yields $\alpha \leq \alpha \wedge (\beta \vee \neg\alpha)$, hence $\alpha \leq \alpha \wedge \beta$, so that by (**EE1**) $\alpha \leq \beta$, contrary to the conditions.

(b) Suppose to the contrary that $\beta < \alpha$ and $\beta \in \mathbf{K}_{\neg(\alpha\wedge\beta)}$. There are two cases: **(b1)** $\alpha \wedge \beta \in \mathbf{V}$. Then $\beta \in \mathbf{V}$, hence by (**EEi5**) $\alpha \leq \beta$, contrary to the conditions. **(b2)** $\neg(\alpha \wedge \beta) \rightarrow \neg\beta < \neg(\alpha \wedge \beta) \rightarrow \beta$, or equivalently by **Property A.1**, to $\alpha \vee \neg\beta < \beta$, which is impossible since by (**EE2**), $\alpha \leq \alpha \vee \neg\beta$. ∎

Proof of Theorem 2.5

(**EE1**) Let $\alpha \leq \beta$, $\beta \leq \gamma$ and $\alpha \in \mathbf{K}_{\neg(\alpha\wedge\gamma)}$. We must prove that $\gamma \in \mathbf{K}_{\neg(\alpha\wedge\gamma)}$.

(a) $\alpha \in \mathbf{K}_{\neg(\alpha\wedge\beta)}$: Then by $(C \le_*)$, $\beta \in \mathbf{K}_{\neg(\alpha\wedge\beta)}$. By **(∗1)** $\alpha\wedge\beta \in \mathbf{K}_{\neg(\alpha\wedge\beta)}$. It follows by **(∗1)** that $\mathbf{K} \neq \mathbf{K}_\perp$ and by **(∗4)** $\alpha\wedge\beta \in \mathbf{V}$, then $\beta \in V$. **(∗1)** and **(∗b)** yield $\beta \in \mathbf{K}_{\neg(\beta\wedge\gamma)}$. Then by $(C \le_*)$, $\gamma \in \mathbf{K}_{\neg(\beta\wedge\gamma)}$ and by the same reasoning we arrive at $\gamma \in \mathbf{V}$. Hence by **(∗1)** and **(∗b)**, $\gamma \in \mathbf{K}_{\neg(\alpha\wedge\gamma)}$.

(b) $\alpha \notin \mathbf{K}_{\neg(\alpha\wedge\beta)}$. Let $\gamma \notin \mathbf{K}_{\neg(\alpha\wedge\gamma)}$. Then by **(∗1)** and **(∗b)**, $\gamma \notin \mathbf{V}$, and it follows that $\beta\wedge\gamma \notin \mathbf{V}$, then by **(∗2)** and **(∗4)**, $\beta\wedge\gamma \notin \mathbf{K}_{\neg(\beta\wedge\gamma)}$. Since $\beta \le \gamma$ and $(C \le_*)$, $\beta \notin \mathbf{K}_{\neg(\beta\wedge\gamma)}$. We will arrive at an absurd by proving **(b1)** $\alpha \in \mathbf{K}_{\neg(\alpha\wedge\beta\wedge\gamma)}$ and **(b2)** $\alpha \notin \mathbf{K}_{\neg(\alpha\wedge\beta\wedge\gamma)}$:

(b1) Since $\vdash \neg(\alpha\wedge\beta\wedge\gamma) \leftrightarrow (\neg(\alpha\wedge\gamma) \vee (\alpha\wedge\neg\beta))$, it follows by **(∗df)** and **(∗5)** that $\mathbf{K}_{\neg(\alpha\wedge\gamma)} \cap \mathbf{K}_{(\alpha\wedge\neg\beta)} \subseteq \mathbf{K}_{\neg(\alpha\wedge\beta\wedge\gamma)}$. By hypothesis $\alpha \in \mathbf{K}_{\neg(\alpha\wedge\gamma)}$ and by **(∗1)** and **(∗2)** $\alpha \in \mathbf{K}_{(\alpha\wedge\neg\beta)}$; hence $\alpha \in \mathbf{K}_{\neg(\alpha\wedge\beta\wedge\gamma)}$.

(b2) Due to the hypothesis condition $\alpha \notin \mathbf{K}_{\neg(\alpha\wedge\beta)}$ it is enough to prove that $\mathbf{K}_{\neg(\alpha\wedge\beta\wedge\gamma)} \subseteq \mathbf{K}_{\neg(\alpha\wedge\beta)}$. Due to **(∗df)**, **(∗5)** and $\vdash \neg(\alpha\wedge\beta\wedge\gamma) \leftrightarrow (\neg(\alpha\wedge\beta)\vee\neg\gamma)$ it enough to prove that $\alpha\wedge\beta \notin \mathbf{K}_{\neg(\alpha\wedge\beta\wedge\gamma)}$. Since $\alpha \notin \mathbf{K}_{\neg(\alpha\wedge\beta)}$, then by **(∗1)** and **(∗b)** $\alpha \notin \mathbf{V}$ and consequently $(\alpha\wedge\beta\wedge\gamma) \notin \mathbf{V}$; then by **(∗4)** $(\alpha\wedge\beta\wedge\gamma) \notin \mathbf{K}_{\neg(\alpha\wedge\beta\wedge\gamma)}$. Then by **(∗1)** either $(\alpha\wedge\beta) \notin \mathbf{K}_{\neg(\alpha\wedge\beta\wedge\gamma)}$ or $\gamma \notin \mathbf{K}_{\neg(\alpha\wedge\beta\wedge\gamma)}$. In the first case we already have what we need. In the second case, it follows from **(∗1)** that $(\beta\wedge\gamma) \notin \mathbf{K}_{\neg(\alpha\wedge\beta\wedge\gamma)}$; then by **(∗df)** and **(∗5)** $\mathbf{K}_{\neg(\alpha\wedge\beta\wedge\gamma)} \subseteq \mathbf{K}_{\neg(\beta\wedge\gamma)}$ and since $\beta \notin \mathbf{K}_{\neg(\beta\wedge\gamma)}$, $\beta \notin \mathbf{K}_{\neg(\alpha\wedge\beta\wedge\gamma)}$, hence by **(∗1)** $(\alpha\wedge\beta) \notin \mathbf{K}_{\neg(\alpha\wedge\beta\wedge\gamma)}$ that concludes the proof.

(**EE2**) Let $\vdash \alpha \to \beta$, and $\alpha \in \mathbf{K}_{\neg(\alpha\wedge\beta)}$. Then by **(∗1)** $\beta \in \mathbf{K}_{\neg(\alpha\wedge\beta)}$; hence by $(C \le_*)$ $\alpha \le \beta$.

(**EE3**) We have three subcases:

(a) $\alpha \notin \mathbf{K}_{\neg(\alpha\wedge\beta)}$. Then by **(∗5)** $\alpha \notin \mathbf{K}_{\neg(\alpha\wedge(\alpha\wedge\beta))}$, hence by $(C \le_*)$ $\alpha \le (\alpha\wedge\beta)$.

(b) $\beta \notin \mathbf{K}_{\neg(\alpha\wedge\beta)}$. In the same way as **(a)**, $\beta \le (\alpha\wedge\beta)$.

(c) $\alpha \in \mathbf{K}_{\neg(\alpha\wedge\beta)}$ and $\beta \in \mathbf{K}_{\neg(\alpha\wedge\beta)}$. Then by **(∗1)**, $(\alpha\wedge\beta) \in \mathbf{K}_{\neg(\alpha\wedge\beta)}$. Hence by $(C \le_*)$, $\alpha \le (\alpha\wedge\beta)$ and $\beta \le (\alpha\wedge\beta)$.

(**EE4**) From left to right, let $\alpha \notin \mathbf{K}$. Then for all β by **(∗3)** $\mathbf{K}_{\neg(\alpha\wedge\beta)} = Cn(\mathbf{K} \cup \{\neg(\alpha\wedge\beta)\})$. Suppose that $\alpha \in \mathbf{K}_{\neg(\alpha\wedge\beta)}$. Then $(\neg(\alpha\wedge\beta) \to \alpha) \in \mathbf{K}$, and since $\mathbf{K}$ is logically closed, $\alpha \in \mathbf{K}$. Contradiction, then for all β $\alpha \notin \mathbf{K}_{\neg(\alpha\wedge\beta)}$; hence by $(C \le_*)$ for all β, $\alpha \le \beta$. For the other direction let $\alpha \le \beta$ for all β; then in particular $\alpha \le \neg\alpha$. Then by $(C \le_*)$ if $\alpha \in \mathbf{K}_{\neg(\alpha\wedge\neg\alpha)}$ then $\neg\alpha \in \mathbf{K}_{\neg(\alpha\wedge\neg\alpha)}$. By **(∗3)**, since $\mathbf{K}$ is consistent, $\mathbf{K}_{\neg(\alpha\wedge\neg\alpha)} = \mathbf{K}$. Then if $\alpha \in \mathbf{K}$, then $\neg\alpha \in \mathbf{K}$. Hence $\alpha \notin \mathbf{K}$.

(**EEi5**) For one direction, let $\beta \le \alpha$ for all β. Then, in particular, $\top \le \alpha$. Then by $(C \le_*)$ "if $\top \in \mathbf{K}_{\neg(\alpha\wedge\top)}$ then $\alpha \in \mathbf{K}_{\neg(\alpha\wedge\top)}$". Then by **(∗1)** $\alpha \in \mathbf{K}_{\neg(\alpha\wedge\top)}$ that is equivalent by **(∗5)** to $\alpha \in \mathbf{K}_{\neg\alpha}$. Hence by **(∗2)** and **(∗4)** $\alpha \in \mathbf{V}$.

For the other direction, let $\alpha \in \mathbf{V}$, then by **(∗b)**, $\alpha \in \mathbf{V}_{\alpha\wedge\beta}$ for all β. Then by **(∗1)**, $\alpha \in \mathbf{K}_{\alpha\wedge\beta}$ for all β, hence by $(C \le_*)$, $\beta \le \alpha$ for all β.

$*$ **is an irrevocable entrenchment-based revision** We must prove that $\mathbf{V}_\alpha$, $\mathbf{K}_\alpha$ and $\le_\alpha$ are as in **Definition 2.2**.

$\mathbf{V}_\alpha$**:** It follows directly from postulate **(∗b)**.

$\mathbf{K}_\alpha$**:** For the left to right direction, let $\beta \in \mathbf{K}_\alpha$ and $\neg\alpha \notin \mathbf{V}$, then by **(∗1)** $(\alpha\to\beta) \in \mathbf{K}_\alpha$ and by **(∗4)** and **(∗2)** $(\alpha\to\neg\beta) \notin \mathbf{K}_\alpha$. Then by **(∗5)** $(\alpha\to\beta) \in \mathbf{K}_{((\alpha\to\beta)\wedge(\alpha\to\neg\beta))}$ and $(\alpha\to\neg\beta) \notin \mathbf{K}_{((\alpha\to\beta)\wedge(\alpha\to\neg\beta))}$. Hence by $(C \le_*)$, $(\alpha\to\neg\beta) \le (\alpha\to\beta)$ and $(\alpha\to\beta) \not\le (\alpha\to\neg\beta)$; i.e., $(\alpha\to\neg\beta) < (\alpha\to\beta)$. For

the other direction if $\neg\alpha \in \mathbf{V}$. It follows by $(*\mathbf{b})$ that $\neg\alpha \in \mathbf{V}_\alpha$, and by $(*\mathbf{1})$ that $\neg\alpha \in \mathbf{K}_\alpha$, hence by $(*\mathbf{2})$, $\beta \in \mathbf{K}_\alpha = \mathbf{K}_\perp$. Let $(\alpha \to \neg\beta) < (\alpha \to \beta)$. Then by $(C \le_*)$ and $(*\mathbf{5})$ $(\alpha \to \beta) \in \mathbf{K}_\alpha$; hence by $(*\mathbf{1})$ and $(*\mathbf{2})$ $\beta \in \mathbf{K}_\alpha$.
$\le_\alpha$**:** For one direction, let $\beta \le_\alpha \gamma$. It follows by $(C \le_*)$ that "if $\beta \in \mathbf{K}_{\alpha\ \neg(\beta\wedge\gamma)}$, then $\gamma \in \mathbf{K}_{\alpha\ \neg(\beta\wedge\gamma)}$". Then by $(*\mathbf{c})$ "if $\beta \in \mathbf{K}_{\alpha\wedge\neg(\beta\wedge\gamma)}$, then $\gamma \in \mathbf{K}_{\alpha\wedge\neg(\beta\wedge\gamma)}$". Due to $(*\mathbf{1})$ and $(*\mathbf{2})$, it follows that "$\beta \in \mathbf{K}_{\alpha\wedge\neg(\beta\wedge\gamma)}$ if and only if $\alpha \to \beta \in \mathbf{K}_{\alpha\wedge\neg(\beta\wedge\gamma)}$" and "$\gamma \in \mathbf{K}_{\alpha\wedge\neg(\beta\wedge\gamma)}$ if and only if $\alpha \to \gamma \in \mathbf{K}_{\alpha\wedge\neg(\beta\wedge\gamma)}$". Then "if $\alpha \to \beta \in \mathbf{K}_{\alpha\wedge\neg(\beta\wedge\gamma)}$ then $\alpha \to \gamma \in \mathbf{K}_{\alpha\wedge\neg(\beta\wedge\gamma)}$". Since $\vdash (\alpha\wedge\neg(\beta\wedge\gamma)) \leftrightarrow (\neg((\alpha \to \beta) \wedge (\alpha \to \gamma)))$, hence by $(*\mathbf{5})$ and $(C \le_*)$, we conclude that $\alpha \to \beta \le \alpha \to \gamma$.
For the other direction, if $\gamma \in Cn(\mathbf{V} \cup \{\alpha\})$. Then by $(*\mathbf{b})$, $\gamma \in \mathbf{V}_\alpha$. By $(*\mathbf{b})$ and $(*\mathbf{1})$, $\gamma \in \mathbf{K}_{\alpha\ \neg(\beta\wedge\gamma)}$ for all β, hence by $(C \le_*)$, $\beta \le_\alpha \gamma$. If $\alpha \to \beta \le \alpha \to \gamma$ then by $(C \le_*)$ and $(*\mathbf{5})$, "if $\alpha \to \beta \in \mathbf{K}_{\alpha\wedge\neg(\beta\wedge\gamma)}$ then $\alpha \to \gamma \in \mathbf{K}_{\alpha\wedge\neg(\beta\wedge\gamma)}$". It follows by $(*\mathbf{c})$ that "if $\alpha \to \beta \in \mathbf{K}_{\alpha\ \neg(\beta\wedge\gamma)}$ then $\alpha \to \gamma \in \mathbf{K}_{\alpha\ \neg(\beta\wedge\gamma)}$". Then by $(*\mathbf{1})$ and $(*\mathbf{2})$, "if $\beta \in \mathbf{K}_{\alpha\ \neg(\beta\wedge\gamma)}$ then $\gamma \in \mathbf{K}_{\alpha\ \neg(\beta\wedge\gamma)}$"; hence by $(C \le_*)$, $\beta \le_\alpha \gamma$. ∎

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