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A SATELLITE SYSTEM SYNTHESIS MODEL FOR ORBITAL ARC ALLOTMENT OPTIMIZATION

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16. Abstract (Limit: 200 words)

We present a mixed integer programming formulation of a satellite system synthesis problem which we refer to as the arc allotment problem (AAP). Each satellite administration is to be allotted a weighted-length segment of the geostationary orbital arc within which its satellites may be positioned at any longitudes. The objective function maximizes the length of the unweighted arc segment allotted to every administration, subject to single-entry co-channel interference restrictions and constraints imposed by the visible arc for each administration. Useful relationships between special cases of AAP and another satellite synthesis problem are established. Solutions to two example problems are presented.

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INTRODUCTION

In this report, we consider the following satellite system synthesis problem: A weighted-length segment of the geostationary orbital (GSO) arc is to be allotted to each of a set of administrations for deploying satellites in the Fixed Satellite Service (FSS). To guarantee that inter-system interference does not exceed a specified acceptable level, minimum required satellite separations are enforced for each pair of arc segments. The arc segment allotted to each administration must be contained in its administration's visible arc. The objective is to maximize the length of the unweighted arc segment allotted to every administration. We refer to this problem as the arc allotment problem (AAP).

We present a mixed integer programming model for AAP and establish relationships between special cases of AAP and another satellite synthesis problem, the arc minimization problem (AMP), that may provide hints as to economical approaches for finding solutions to AAP. The objective in AMP, a point allotment synthesis problem, is to minimize the distance between the westernmost and easternmost allotted satellite locations; satellite separation and visible arc constraints are enforced. We also present solutions to two AAP example problems.

AAP is different from synthesis problems in which an orbital location is allotted to each of a set of satellites. The allotment of GSO arc segments to administrations has been suggested by Kiebler [9]. Kiebler employs empirical techniques to achieve a satisfactory allotment plan; no attempt is made at optimization. We make no frequency or polarization allotments as some synthesis models do [1-4,10-11,13,15]. Rather, we implicitly assume that all satellites deployed use a common, co-polarized channel in our approach to controlling inter-system interference.

This approach to satellite synthesis has certain advantages because the allotment of arcs provides more operational flexibility for satellite administrations. An administration can deploy any number of satellites that can be adequately accommodated in its arc segment. Satellite locations can be changed to meet changing communications needs. An administration can deploy additional satellites in its arc segment without creating excessive interference for any other administration. The problem of interference within an arc segment, like the allotment of frequencies and polarizations, becomes a domestic issue. Kiebler [9] discusses similar advantages of orbital arc allotments.

AAP has other advantages as well. The distribution of the GSO can account for differences in anticipated communications traffic, population, or service area size between administrations. Finally, the AAP model is smaller than point allotment models because AAP's size depends on the number of administrations, rather than the number of satellites.

Some mathematical programming models have been developed for solving synthesis problems in the Broadcasting Satellite Service (BSS) [3-4,10-11,13,15]; others were intended to be applied to FSS synthesis problems [7-8,12,14,17]. The solution techniques recommended for satellite synthesis models have been as diverse as the models themselves. Integer programming [1-2,7,10,12,14,17,19] and nonlinear programming [8,11,13,16] algorithms, as well as approximate methods [3-4,7,10,12,15,17], have been suggested. In some cases, more than one approach has been suggested for the same synthesis problem. For example, Ito at al. [8] have recommended a nonlinear programming model for AMP, while Reilly et al. [17] have suggested an integer programming model.

REQUIRED SATELLITE SEPARATIONS

The primary goal in satellite system synthesis models is to prevent excessive interference. Aggregate interference, the interference from all unwanted satellite signals, is the quantity of concern.

Our AAP model uses a required minimum satellite separation, measured in degrees of GSO arc, for each pair of administrations, to limit single-entry co-channel inter-system interferences. For our example problems, we have calculated separations with a procedure developed by Wang [19] for determining the minimum GSO separation between two satellites with elliptical-beam antennas that assures that single-entry co-channel carrier-to-interference (C/I) ratios at assumed ground stations (test points) served by the satellites are at least equal to some threshold. All feasible orbital locations are considered when calculating the required minimum separation values. The maximum of the separations calculated over the allowable range of orbital positions for each pair of administrations is used in AAP. Wang's separation concept has been applied to point allotment synthesis models

[7,12,14,17,19]. See Wang [19] or Levis et al. [12] for a more complete description of this separation concept.

The minimum pairwise satellite separations calculated by Wang's procedure are based on single-entry interference, or the interference caused by one unwanted satellite at a time at each test point. In practice, the aggregate interference requirement can be satisfied by imposing a more stringent requirement, typically an additional 5 dB, on the single-entry C/I ratios. For example, if we require aggregate C/I ratios of at least 25 dB, appropriate satellite separations can be calculated assuming a single-entry co-channel protection ratio of about 30 dB. Such a procedure was adopted for the 1977 World Administrative Radio Conference [5] and has proven to be valid in some point allotment test problems [11,16].

ARC ALLOTMENT MODEL FORMULATION

For our AAP model, we define the following parameters and decision variables.

Parameters:

n = number of satellite administrations.

E_i, W_i = easternmost or westernmost feasible location for satellites serving administration i, in degrees west longitude. (Note that $E_i < W_i$.)
$i=1,2,...,n$

F_i = weighting factor for the length of administration i's allotted arc segment.
$i=1,2,...,n$

Δ_{ij} = minimum required separation between satellites serving administrations i and j, in degrees longitude.

$i=1,2,\ldots,n-1;\ j=i+1,i+2,\ldots,n$

Decision variables:

e_i = eastern endpoint of the GSO arc segment for administration i (in degrees west longitude).

$i=1,2,\ldots,n$

w_i = western endpoint of the GSO arc segment for administration i (in degrees west longitude).

$i=1,2,\ldots,n$

a = length of the unweighted arc segment allotted to every administration (in degrees of GSO arc).

$$x_{ij} = \begin{cases} 1 & \text{if } w_i > w_j \\ 0 & \text{if } w_i < w_j \end{cases}$$

$i=1,2,\ldots,n-1;\ j=i+1,i+2,\ldots,n$ such that $\Delta_{ij}>0$.

When $\Delta_{ij}>0$ for some administrations i and j, the nearest endpoints of the arc segments allotted to these administrations must be separated by at least Δ_{ij} degrees. If $\Delta_{ij}=0$ for some administrations i and j, then their satellites may be collocated without causing excessive interference. In this case, the arc segments allotted to administrations i and j can intersect, or even coincide. Zero-valued satellite separations would allow at most a common endpoint for the administrations' arcs. Hence, we do not enforce required satellite separations between arc segments allotted to any administrations i and j for which $\Delta_{ij}=0$. The binary variables x_{ij} are therefore defined only for those pairs of administrations i and j for which $\Delta_{ij}>0$.

AAP can be formulated as a mixed integer program as follows:

$$\text{Maximize} \quad z = a \qquad\qquad (1)$$

$$\text{Subject to} \quad w_i - e_i - F_i a = 0 \qquad i=1,2,\ldots,n \qquad (2)$$

$$e_i - w_j + (E_i - W_j - \Delta_{ij})x_{ij} > E_i - W_j \qquad \begin{array}{l} i=1,2,\ldots,n-1 \\ j=i+1,i+2,\ldots,n \\ \text{such that } \Delta_{ij}>0 \end{array} \qquad (3)$$

$$e_j - w_i - (E_j - W_i - \Delta_{ij})x_{ij} > \Delta_{ij} \qquad \begin{array}{l} i=1,2,\ldots,n-1 \\ j=i+1,i+2,\ldots,n \\ \text{such that } \Delta_{ij}>0 \end{array} \qquad (4)$$

$$e_i > E_i \qquad i=1,2,\ldots,n \qquad (5)$$

$$w_i < W_i \qquad i=1,2,\ldots,n \qquad (6)$$

$$a > 0 \qquad\qquad (7)$$

$$x_{ij} \,\varepsilon \{0,1\} \qquad \begin{array}{l} i=1,2,\ldots,n-1 \\ j=i+1,i+2,\ldots,n \\ \text{such that } \Delta_{ij}>0 \end{array} \qquad (8)$$

The objective function (1) maximizes the length of the unweighted arc segment allotted to every administration. Each administration is allotted a weighted-length arc segment based on the length of the unweighted arc segment, a, by constraint set (2). These constraints also ensure that the western endpoint of each arc is located west of its eastern endpoint. Constraint sets (3) and (4) guarantee that potentially interfering satellites are sufficiently separated. Every location in each allotted arc segment is guaranteed to be feasible for the associated administration by constraint sets (5) and (6). The remaining constraints, (7) and (8), enforce nonnegativity and integrality restrictions on decision variables. For a problem with n administrations, this model has 2n+1 continuous variables, and at most, n^2+2n structural constraints and n(n-1)/2 binary variables. The precise numbers of binary variables and structural constraints depend on the number of non-zero Δ_{ij}'s.

This AAP model will yield "balanced" solutions, solutions in which the lengths of all weighted-length arc segments are based on the length of a common unweighted arc segment, a. A balanced solution to AAP provides every administration with equal operational flexibility per weighting unit.

SOLUTIONS TO TEST PROBLEMS

In this section, we present solutions to two AAP examples. The two example problems include six South American administrations: Argentina (ARG), Bolivia (BOL), Chile (CHL), Paraguay (PRG), Peru (PRU), and Uruguay (URG). Each administration is assumed to have the same easternmost feasible satellite location, 80°W. The westernmost feasible satellite location for each administration is assumed to be 110°W. In the first problem, each administration's weighting factor is unity. The weighting factors in the second problem are recent population figures [18], expressed in millions.

The minimum satellite separation values used are shown in Table 1. These separation values are based on a single-entry co-channel protection ratio of 30 dB and are the same satellite separations used in synthesis example problems by Levis et al. [12], Reilly et al. [17], and Wang [19]. The solutions to the two problems are displayed in Tables 2 and 3.

The optimal lengths of the unweighted arc segments in the two problems are 3.827° and 0.294°, respectively. Note that the east-to-west ordering of the administrations' arc segments is the same in both problems. More of the GSO arc is allotted in Problem 1, 22.962°, than in Problem 2, 21.771°. We will refer to these example problems and their solutions in the next section.

A linear programming-based branch-and-bound [6] code was used to solve these example problems. The solution times for the two problems were 14.86 and 7.09 CPU seconds, respectively, on an IBM 3081-D computer at The Ohio State University.

Table 1. Minimum Satellite Separations

(in degrees of GSO arc)

	BOL	CHL	PRG	PRU	URG
ARG	4.17	4.19	4.32	1.41	4.14
BOL		4.57	4.04	4.26	0.94
CHL			2.00	3.94	1.59
PRG				1.10	2.46
PRU					0.37

Table 2. Solution to Problem 1

Adminis-tration	Arc Weighting Factor	Eastern Limit (°W)	Western Limit (°W)	Arc Length (degrees)
BOL	1.0	80.000	83.827	3.827
URG	1.0	84.766	88.593	3.827
CHL	1.0	90.183	94.010	3.827
PRG	1.0	96.010	99.837	3.827
PRU	1.0	100.936	104.763	3.827
ARG	1.0	106.173	110.000	3.827

Table 3. Solution to Problem 2

Adminis-tration	Arc Weighting Factor	Eastern Limit (°W)	Western Limit (°W)	Arc Length (degrees)
BOL	6.1	80.000	81.790	1.790
URG	2.9	82.730	83.581	0.851
CHL	12.1	86.351	89.910	3.550
PRG	3.3	91.910	92.878	0.968
PRU	19.7	93.978	99.758	5.780
ARG	30.1	101.168	110.000	8.832

RELATIONSHIPS TO ARC MINIMIZATION PROBLEM (AMP)

Recall that AMP is a point allotment satellite synthesis problem whose objective is to minimize the distance between the westernmost and easternmost allotted satellite locations, subject to satellite separation and visible arc constraints. We will establish important and useful relationships between special cases of AAP and AMP in this section. See Reilly et al. [17] and Ito et al. [8] for possible formulations of AMP.

We consider AAP and AMP problems in which $E_i=E$ and $W_i=W$ for $i=1,2,...,n$, $\Delta_{ij} > 0$ for $i=1,2,...,n-1$ and $j=i+1,i+2,...,n$, and each administration has one satellite. Let $x^*(i)$ denote the longitude of the i-th satellite in the sequence of satellites in an optimal solution to

AMP, and let $x^*_{(n)}-x^*_{(1)}$ be the distance between the westernmost and easternmost allotted satellites locations. (Throughout this section, subscripts with parentheses will refer to places in the optimal AMP ordering.)

Given an optimal solution to AMP, we can construct a feasible solution to the AAP that has the same problem parameters. Each satellite can be allotted a weighted-length arc segment based on the length of an unweighted arc segment with minimum length

$$a_L = (W - E - (x^*_{(n)} - x^*_{(1)}))/ \sum_{i=1}^{n} F_i$$

degrees, assuming the satellite (administration) ordering prescribed by the solution to AMP is preserved.

It may be possible to allot arc segments based on a longer unweighted arc segment because the separation between some pairs of adjacent satellites in the AMP ordering may exceed the pair's minimum required orbital separation. The arc segment allotted to an administration whose arc is positioned between those of two strong interferers may separate the arcs of the interferers sufficiently, making the interferers' separation constraint nonbinding in AAP. When the administrations are ordered as prescribed by AMP, the maximum possible length of the unweighted arc segment on which allotments are based is therefore

$$a_U = a_L + \sum_{j=1}^{n-1} \max(x^*_{(j+1)} - x^*_{(j)} - \Delta_{(j)(j+1)}, 0)/ \sum_{i=1}^{n} F_i$$

degrees. A sufficient condition for determining whether arcs whose lengths are based on an unweighted arc segment of length a_U can be allotted is given below.

Result 1: If $a_L > \left(\Delta(j)(j+k) - \sum_{m=j}^{j+k-1} \Delta(m)(m+1)\right) / \sum_{m=j+1}^{j+k-1} F(m)$

for all $j=1,2,\ldots,n-2$ and $k=j+2,j+3,\ldots,n$ such that $x^*(j+k) - x^*(j) = \Delta(j)(j+k)$, then arc segments based on an unweighted arc segment of length a_U can be allotted to every administration.

Proof: (By contradiction) Suppose the longest arc segment that can be allotted to each satellite has length r, where $a_L<r<a_U$. Then, $r \sum_{i=1}^{n} F_i < a_U \sum_{i=1}^{n} F_i = (W - E) - \sum_{j=1}^{n-1} \Delta(j)(j+1)$. Therefore, the available GSO arc is not fully consumed by the allotted arc segments and the required separations between adjacent satellite arcs. Hence, there exists a j and k such that

$$r < \left(\Delta(j)(j+k) - \sum_{m=j}^{j+k-1} \Delta(m)(m+1)\right) / \sum_{m=j+1}^{j+k-1} F(m)$$

Contradiction. QED.

The following corollary applies to cases where the arc weighting factors are the same for all administrations.

Corollary: If $a_L > \max_{j=1,2,\ldots,n-1} \{x^*(j+1) - x^*(j) - \Delta(j)(j+1)\}$

and $F_i=F$ for $i=1,2,\ldots,n$, then unweighted arc segments of length a_U can be allotted to every administration.

Proof: The proof is similar to the proof for Result 1.

Recall the first AAP example problem presented earlier. Suppose that instead of solving the AAP we solve the AMP with the same problem parameters. The optimal solution to this AMP prescribes the following satellite locations: 80.00° for BOL, 80.94° for URG, 84.57° for CHL, 86.57° for PRG, 88.51° for PRU, and 90.89° for ARG. (The AMP solution was found in 5.43 CPU seconds with the same branch-and-bound code used to solve the AAP examples.) Hence, $a_L = (110 - 80 - (90.89 - 80))/ 6 = 3.185°$. However, there is superfluous separation between some pairs of adjacent satellites: 2.04° between URG and CHL, 0.84° between PRG and PRU, and 0.97° between PRU and ARG. Since, $3.185 > \max\{2.04, 0.84, 0.97\}$, arcs of length $a_U = 3.185 + (2.04 + 0.84 + 0.97)/ 6 = 3.827°$ can be allotted to each satellite (corollary to Result 1). This solution is known to be optimal to AAP (see Table 2).

Given the optimal AMP ordering, we find that $a_L = (110 - 80 - (90.89 - 80))/ 74.2 = 0.2575°$ and $a_U = 0.2575 + (2.04 + 0.84 + 0.97)/ 74.2 = 0.3094°$ for the second AAP example problem. The optimal solution to this problem (see Table 3) indicates that the weighted-length arcs based on an unweighted arc segment of length a_U can not be allotted. This is not surprising since $0.2575 < (4.57 - (0.94 + 1.59))/ 2.9$ (Result 1 for j=1 and k=2). The weighted-length allotted arc segments are not long enough to allow recovery of all of the superfluous separation between adjacent pairs of arc segments. This explains why more of the GSO arc was allotted in Problem 1 than in Problem 2.

In both examples, the AAP ordering of administrations is the same as the AMP ordering. However, it is not known whether the optimal orderings in the AAP and AMP problems are always identical. Intuitively, we expect the two orderings to be similar. Long arc segments can be allotted when the satellites are ordered in such a way that they can be allotted points on a short arc segment without causing excessive interference. The following result gives sufficient, but not necessary, conditions for determining whether the optimal AAP and AMP satellite orderings are identical.

Result 2: The optimal AAP ordering is the same as the optimal AMP ordering if

(a) $x^*(n) - x^*(1) = W - E$, or

(b) $x^*(j+1) - x^*(j) = \Delta(j)(j+1)$ for $j=1,2,\ldots,n-1$

Proof: In case (a), the AMP solution fills the entire available orbital arc. Therefore, $a^* = 0$, and the AMP ordering must be optimal for AAP.

In case (b), all pairs of adjacent satellites are separated by their minimum required orbital separations. Adjacent arc segments in AAP must be separated by their administrations' minimum required satellite separations, and the largest possible portion of the available orbital arc remains for allotment to the administrations. Therefore, the AMP ordering is optimal for AAP. QED.

The solution to the AMP example does not satisfy either of the sufficient conditions given in Result 2, yet the optimal AAP and AMP orderings are identical.

The relationships between the AAP and AMP problems which we have established may be useful in finding good solutions to AAP. Suppose we have used the integer programming formulation of Reilly et al. [17] or the nonlinear programming formulation of Ito et al. [8] to find a solution, not necessarily an optimal solution, to AMP. The AMP solution could be converted to an AAP solution. The quality of the AAP solution would depend upon the quality of the AMP solution. The AMP solution transformation could be beneficial if good solutions to AMP can be found faster than good solutions to AAP.

CONCLUSIONS

We have presented a mixed integer programming model for the arc allotment problem (AAP), the problem of allotting the longest possible weighted-length segment of the geostationary orbital (GSO) arc to each of a set of satellite administrations. Because AAP provides an orbital arc segment for each satellite, rather than a point on the GSO arc, a solution to this model provides satellite administrations with operational flexibility. Provided that its allotted arc segment is of sufficient length, an administration can deploy any number of satellites, can add new satellites, and can reposition satellites within its allotted arc segment as its communications needs change.

AAP has additional advantages. The distribution of the GSO can account for differences in anticipated communications traffic, population, or service area size between administrations. The problems of interference and frequency and polarization allotments are reduced almost completely to domestic concerns. AAP's size is determined by the number of administrations, not the number of satellites; hence, AAP is smaller than many integer programming synthesis models for point allotments [7,12,14,17,19].

The AAP model we have presented could be used to allot arc segments to groups of administrations, rather than to individual administrations. (The allotment of arcs to groups of administrations has been suggested by Kiebler [9] in his Orbital Arc Segmentation and Technical Standards (OASTS) approach to satellite synthesis.) The parameters and variables would have to be redefined, but the model itself would be essentially unaffected.

We have established relationships between special cases of AAP and the arc minimization problem (AMP). These relationships may be useful in converting an AMP solution to an AAP solution. Given an optimal solution to AMP, lower and upper bounds on the optimal length of the unweighted arc segment allotted to every administration in AAP can be calculated. Sufficient conditions for determining whether the length of the optimal unweighted arc segment is equal to its upper bound and whether the optimal ordering for AMP is the optimal ordering for AAP are given.

REFERENCES

[1] Baybars, I., "Optimal Assignment of Broadcasting Frequencies", EUROPEAN JOURNAL OF OPERATIONAL RESEARCH, Vol. 9, No. 3, pp. 257-263, March 1982.

[2] Cameron, S., "The Solution of the Graph-Coloring Problem as a Covering Problem", IEEE TRANSACTIONS ON ELECTROMAGNETIC COMPATIBILITY, Vol. EMC-19, No. 3, August 1977.

[3] Chouinard, G. and M. Vachon, "A Synthesis of a Plan by Computer Using an Assignment Method and Inductive Selection", ITU/CITEL/ DOC-Canada Preparatory Seminar for 1983 RARC, Ottawa, Canada, May 1981.

[4] Christensen, J., "BSS CAPS, A System Description", ITU/CITEL/ DOC-Canada Preparatory Seminar for 1983 RARC, Ottawa, Canada, May 1981.

[5] Final Acts of the World Administrative Radio Conference for the Planning of the Broadcasting-Satellite Service in Frequency Bands 11.7 - 12.2 GHz (in Regions 2 and 3) and 11.7 - 12.5 GHz (in Region 1), Annex 9, "Criteria for Sharing Between Services", Geneva, Switzerland, p. 104, 1977.

[6] Garfinkel, R. and G. Nemhauser, INTEGER PROGRAMMING, John Wiley and Sons, New York, 1972.

[7] Gonsalvez, D., "On Orbital Allotments for Geostationary Satellites", Ph.D. Dissertation, The Ohio State University, December 1986.

[8] Ito, Y., T. Mizuno, and T. Muratani, "Effective Utilization of Geostationary Orbit Through Optimization", IEEE TRANSACTIONS ON COMMUNICATIONS, Vol. COM-27, No. 10, pp. 1551-1558, October 1979.

[9] Kiebler, J., "NASA Studies on the U.S. Approach for Fixed Satellite Management at the 1985/88 Space WARC", Document AH 178-229, Ad Hoc Committee 178 of the Interdepartmental Radio Advisory Committee.

[10] Levis, C., C. Martin, C. Wang, and D. Gonsalvez, "Engineering Calculations for Communications Satellite Systems Planning", The Ohio State University, ElectroScience Laboratory, First Summary Report 713533-2 for Grant NAG 3-159, March 1983.

[11] Levis, C., C. Martin, D. Gonsalvez, and C. Wang, "Engineering Calculations for Communications Satellite Systems Planning", The Ohio State University, ElectroScience Laboratory, Second Interim Report 713533-3 for Grant NAG 3-159, June 1983.

[12] Levis, C., C. Wang, Y. Yamamura, C. Reilly, and D. Gonsalvez, "The Role of Service Areas in the Optimization of FSS Orbital and Frequency Assignments", A COLLECTION OF TECHNICAL PAPERS, AIAA 11th Communications Satellite Systems Conference, pp. 190-198, 1986.

[13] Martin, C., D. Gonsalvez, C. Levis, and C. Wang, "Engineering Calculations for Communications Satellite Systems Planning", The Ohio State University, ElectroScience Laboratory, Interim Report 713533-4 for Grant NAG 3-159, December 1985.

[14] Mount-Campbell, C., C. Reilly, and D. Gonsalvez, "A Mixed Integer Linear Programming Formulation of the FSS Synthesis Problem Using Minimum Required Pair-Wise Separations", The Ohio State University, Department of Industrial and Systems Engineering, Working Paper 1986-006.

[15] Nedzela, M. and J. Sidney, "A Synthesis Algorithm for Broadcasting Satellite Service", ITU/CITEL/DOC-Canada Preparatory Seminar for 1983 RARC, Ottawa, Canada, May 1981.

[16] Reilly, C., C. Mount-Campbell, D. Gonsalvez, C. Martin, C. Levis, and C. Wang, "Broadcasting Satellite Synthesis Using Gradient and Cyclic Coordinate Search Procedures", A COLLEC-TION OF TECHNICAL PAPERS, AIAA 11th Communications Satellite Systems Conference, pp 237-245, 1986.

[17] Reilly, C., C. Mount-Campbell, D. Gonsalvez, and C. Levis, "Alternative Mathematical Programming Formulations for FSS Synthesis", The Ohio State University, Department of Industrial and Systems Engineering, Working Paper 1986-005.

[18] THE EUROPA YEAR BOOK 1986, Volumes I and II, Europa Publications Limited, London, 1986.

[19] Wang, C., "Optimization of Orbital Assignment and Specification of Service Areas in Satellite Communications, Ph.D. Dissertation, The Ohio State University, June 1986.