



# A Kinematic Model of the Human Spine and Torso

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*Improving human motion models requires a more realistic model of the body, including a flexible torso and spine. Defining spine attributes and input parameters helps achieve inverse kinematic control of the spine.*

**R**esearchers have studied human figure models in computer graphics almost since the introduction of the medium. Through the last dozen years or so, the structure, flexibility, and fidelity of human models have increased dramatically—from the wireframe stick figure, through simple polyhedral models, to curved surfaces, and even finite-element models. Computer graphics modelers have tried to maximize detail and realism while maintaining a reasonable overall display cost.

The same issue pertains to control: Improving motion realism requires many degrees of freedom in the body linkage, and such redundancy strains effective and intuitively useful control methods. We can either simplify control by simplifying the model, thereby risking unrealistic movements, or complicate control with a complex model and hope the resulting motions appear more natural. The recent history of computer animation of human figures has focused on the quest to move the technology from the former



Figure 1. Contoured body bent to a target state vector: (flexion, axial, lateral) = (-52 degrees, 58 degrees, -7 degrees).



Figure 2. Contoured body bent to a target state vector: (flexion, axial, lateral) = (-1 degree, -60 degrees, -6 degrees).

situation towards the latter while simultaneously forcing the control complexity into algorithms rather than skilled manual manipulation.

This point of view motivates our efforts in human figure modeling and animation, as well as those of several other groups. In particular, notable algorithms for greater animation power have addressed kinematics, dynamics, inverse kinematics, available torque, global optimization, locomotion, deformation, and gestural and directional control.<sup>1</sup> Throughout this range of studies, however, the human models themselves tended to be rather simplified versions of real human flexibility. In the early 1980s we warned that increased realism in the models would demand ever more accurate and complicated motion control. Now that the control regimes are improving, it is time to return to the human models and ask if we must reevaluate their structures to take advantage of algorithmic improvements. When we considered this question, we determined that a more accurate model of the human spine and torso would be essential to further realism in human motion.

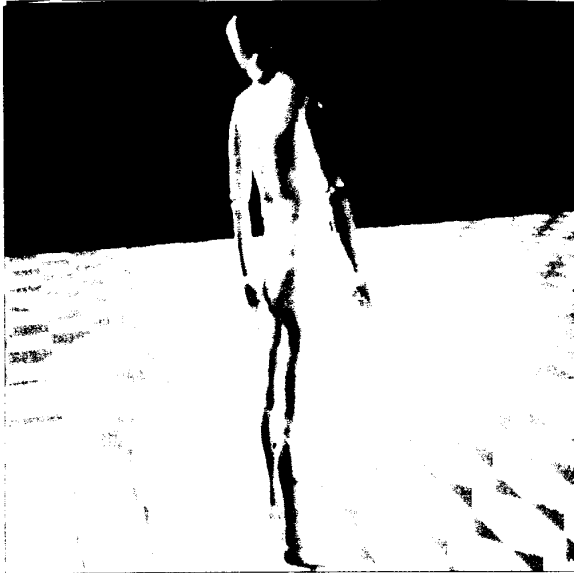
Although many models have appeared to have a flexible torso, they have been computer constructions of the surface shape manipulated by skilled animators.<sup>2</sup> We needed a torso suitable for animation that also satisfied our research project requirements for anthropometric scalability.<sup>3</sup> Thus, we could not accept a single model of fixed proportions when we would potentially need to model vast differences among human body types.

Snakes exhibit a similar type of flexible figure,<sup>4</sup> but the an-

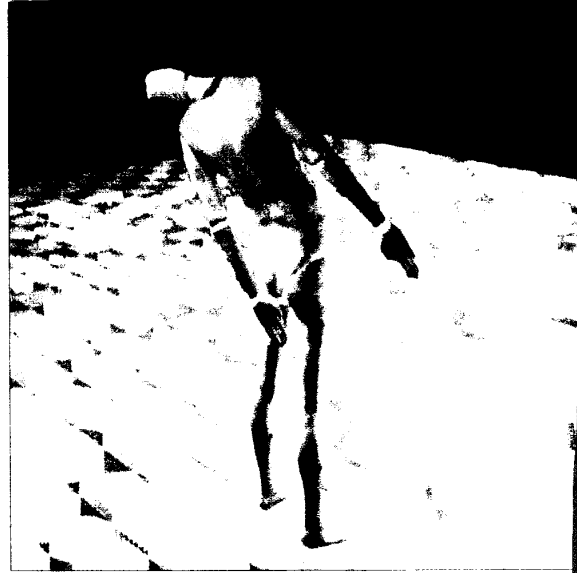


Figure 3. Contoured body bent to a target state vector: (flexion, axial, lateral) = (3 degrees, -42 degrees, 23 degrees).

thropometry issues do not arise. Miller based his animation approach on dynamics. Because humans do not need to locomote with their torsos, we deemed a kinematics model



**Figure 4. Contoured body bent to a target state vector: (flexion, axial, lateral) = (-10 degrees, 108 degrees, -45 degrees).**



**Figure 5. Contoured body bent to a target state vector: (flexion, axial, lateral) = (18 degrees, 108 degrees, -68 degrees).**



**Figure 6. Contoured body bent to a target state vector: (flexion, axial, lateral) = (-33 degrees, 39 degrees, 6 degrees).**

adequate. On the other hand, Zeltzer and Stredney's "George" skeleton model had a detailed vertebral column, but it was not articulated nor did it bend during kinematic animation.<sup>5</sup> Willmert simulated limited neck vertebral motion in the sagittal plane.<sup>6</sup>

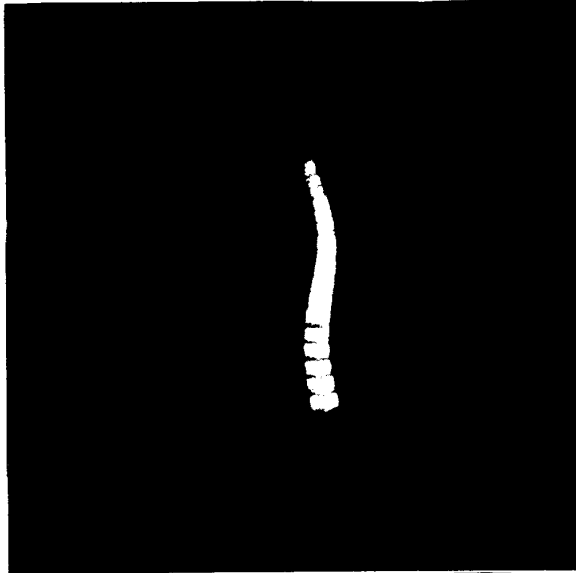
The default polyhedral figures used in "Jack" (the software system developed at the University of Pennsylvania for human

figure modeling and manipulation<sup>7</sup>) lacked much detail in the human torsos. The graphically displayed bodies appeared to be impersonating robots with stiff backs, bending only from the waist. A five-segment torso with more possibilities of articulation was created a year ago. But the back was modeled without any curves, contrary to actual human anatomy. If the spine were realistically modeled, then the torso, a vessel connected and totally dependent on the spine, could be viewed and manipulated interactively. So one of us (Monheit) undertook the development of a far more satisfactory and highly flexible vertebral model of the spine and its associated torso shape.

The conceptual model of the spinal column is derived from medical data and heuristics related to human kinesiology. The spine is a collection of vertebrae connected by ligaments, small muscles, vertebral joints (called processes), and intervertebral disks.<sup>8</sup> Nature has designed the spine for<sup>9,10</sup>

1. Support of the body's weight
2. Stability of the torso
3. Flexibility of motion
4. Protection of the spinal cord

The spine moves as a series of vertebrae connected by dependent joints,<sup>11</sup> meaning that it is impossible to isolate movement of one vertebral joint from the surrounding vertebrae. Muscle groups of the head, neck, abdomen, and back initiate movement of the spine, and the interconnecting ligaments allow the movement of neighboring vertebrae.<sup>8,12</sup>



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Figures 7-12. Wireframe polygonal body with visible spine moves through a rolling sequence. Zero interpolation flag is set to "no."

The following sections describe both the model and the representation of the model within Jack.

## Spinal motion

The model of spinal motion is based on

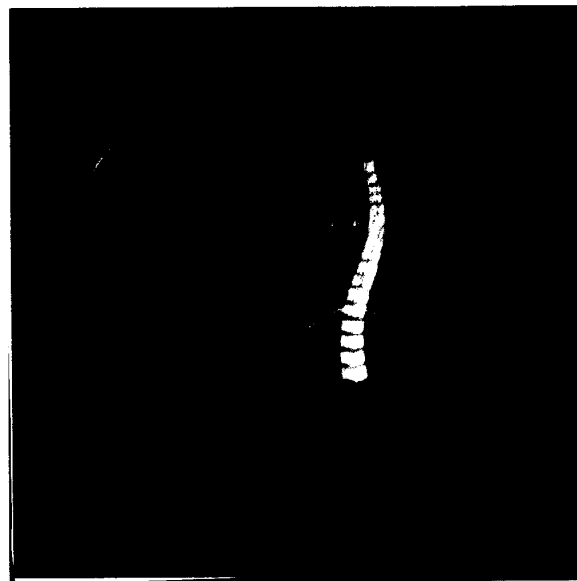
- Anatomy of the physical vertebrae and disks
- Range of movement of each vertebra
- Effect of the surrounding ligaments and muscles

### Anatomy of vertebrae and disks

The spinal column consists of 33 vertebrae organized into five regions<sup>8</sup>:

- Cervical
- Thoracic
- Lumbar
- Sacral
- Coccyx

The vertebrae are labeled by medical convention: C1 - C7, T1 - T12, L1 - L5, and S1 - S5. The regions listed above are in vertical descending order. Which regions should we consider part of the torso? The cervical spine lies within the neck. Also, the sacrum and coccyx contain vertebrae fixed through fusion.<sup>9</sup> So, the mobile part of the torso includes the 12 thoracic and 5



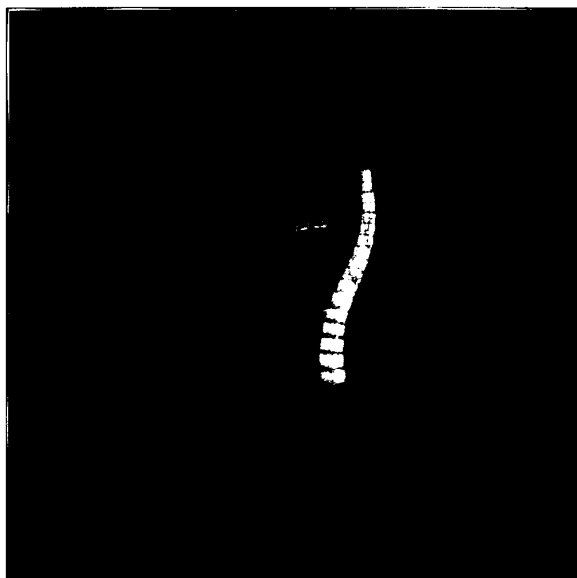
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lumbar vertebrae. Therefore, the torso model includes 17 vertebrae and 18 joints of movement.

Each vertebra is uniquely sized and shaped, but all vertebrae contain a columnar body and an arch. The body is relatively large and cylindrical, supporting most of the weight of the entire spine. The vertebral bodies increase gradually in size from the cervical to the lumbar region.<sup>9</sup>

The arch supports seven processes: four articular, two transverse, and one spinous.<sup>9</sup> The processes are bony protrusions on

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12

the vertebra that aid and limit the vertebral motion. The transverse and spinous processes serve as levers for both muscles and ligaments.<sup>8</sup> The articular processes provide a joint facet for the joint between successive vertebral arches. These processes, due to their geometry, cause the vertebrae to rotate with three degrees of freedom. Ligaments and small muscles span successive vertebral processes, giving the spinal column its stability. Because of this strong interconnectivity, we model spinal movement as interdependent movement of neighboring joints.

Intervertebral disks separate each vertebra. A disk has three parts<sup>11</sup>:

- **nucleus pulposus**: the sphere in the center, consisting of 85 percent water
- **annulus fibrosus**: the fibers running as concentric cylinders around the nucleus
- **cartilaginous plates**: a thin wall separating the disk from the vertebral body

The disk changes shape as the neighboring vertebrae bend. But, since the nucleus is 85 percent water, very little compression occurs. The disk can bulge out spherically as force is applied to the columnar body above or below. Therefore, overall the disk does not function as a spring, but as a deformable cylindrical separation between vertebrae. This supports the theory that the vertebrae do not slide, but rotate around an axis.<sup>11</sup>

### Vertebral range of movement

Vertebral movement is limited by the relative size of the disks, the attached ligaments, and the shape and slant of the processes and facet joints. Statistics for joint limits between each successive vertebra have been recorded and compiled.<sup>11</sup> Also, the spine has a natural shape at rest position. We use the initial joint position of each vertebra as input to the model.

The range of movement differs for each region of the spine. For instance, the optimum movement of the lumbar region is flexion or extension. The thoracic area easily moves laterally, while in the sagittal plane flexion and extension are limited. The cervical area is very flexible for both axial twisting and lateral

Table 1. Rotation of the spine.		
Flexion/extension	Forward and backward bending	Rotation around the x axis
Axial rotation	Twisting	Rotation around the y axis
Lateral bending	Side bending	Rotation around the z axis

bending. The joint limits for each region affect how much that joint can participate in any given movement. The posture of the torso results from the specialization of the spinal regions.<sup>13</sup>

### Effect of ligaments and muscles

The vertebrae are interconnected by a complex web of surrounding ligaments and muscles. If the force initiated by a muscle group is applied at one joint, that joint moves, as do the neighboring joints to a lesser degree. Some joints farther away might not be affected by the initiator joint's movement.

It is possible, by simultaneous contractions of extensor and flexor muscles around the spinal column, to deactivate joints not initiating the movement.<sup>13</sup> Depending on the force of these resisting muscles, the joints on or near the joint closest to the resistor will move less than they would if the resisting force had not been applied. The final position of the spine is a function of the initiator force, the resisting muscle, and the resistance.

### Input parameters

We modeled the spine as a black box with an initial state, input parameters, and an output state. To initiate movement of the spine, we introduced several input parameters:

- **joint range “from” and “to”:** Within the total number of joints in the spine, any nonempty contiguous subset of vertebral joints can be specified by two joint indices. These joints indicate which part of the spine is active in movement. For example, the user specifies movement in the range between T5 and T10. All other joints are frozen during the movement.
- **initiator joint:** The joint where movement begins, usually the joint with greatest motion.
- **resistor joint:** The joint that resists the movement. This can be equated to a muscle that contracts and tries to keep part of the spine immobile.
- **resistance:** The amount of resistance provided by the resistor joint.
- **spine target position:** A 3D vector describing the target position after rotation around the x, y, and z axes. The target position is the sum of all joint position vectors in the spine after movement succeeds.
- **zero interpolation:** A “yes” indicates that movement is interpolated through the joint rest position. A “no” indicates that only the joint limits are used to interpolate movement.

### Vertebral joint movement

The joint between each vertebra has three degrees of rotation. The spine will move toward the target position by rotating around the three possible axes,<sup>11</sup> as shown in Table 1.

The position of the flexion rotational axis for each vertebral joint has been measured from cadavers. It is not equidistant to the two adjacent vertebrae, but lies closer to the bottom vertebra.<sup>11</sup> The origin of the axis of movement determines how the vertebrae move. When the torso is modeled on the spine, the axis also directly determines how the torso changes shape.

Elongation and compression are absent from the model. The hydrophilic intervertebral disk, when submitted to prolonged compression, induces a slight decrease in height due to fluid leakage. Conversely, after a long period of rest or zero gravity, the spine elongates by maximum filling of the nucleus pulposus (at the center of the disk).<sup>11</sup> Dehydration during a day's activity can result in a loss of height of 2 centimeters in an adult. In any short duration of movement the disk is essentially incompressible, and therefore elongation is imperceptible.<sup>14</sup>

Shearing or sliding (translational movements) of the vertebrae would lead to variation in the intervertebral separation. The mechanics of the intervertebral disk would not allow this.<sup>11</sup> Therefore, we assume that for normal activities the three degrees of rotational movement are the only ones possible for each vertebral joint.

### Spine database

Any human figure can have a wide variety of torso shapes. Also, each person has a different degree of flexibility and range of movement. To model the position and shape changes of an individual's spine, we designed a database for creating a unique set of features for the spine and torso. Medical data provides the database elements for an average person.<sup>11</sup>

Database elements include

- Size of vertebra—x, y, z dimension
- Intervertebral disk size—separation between vertebrae
- Joint limits—three rotations, two limits per rotation
- Joint rest position—initial joint position of the spine

### Applying the motion model

Consider a stationary spine and its attributes. Each vertebra has a current position defined by the joint position between each vertebra for each of the three degrees of rotation. The spinal database also includes definitions of joint rest positions and six joint limits for every joint. If each attribute is summed up for all joints, then 3D vectors are defined for current position, joint rest position, and two joint limits for the global spine. The target position—the 3D vector sum of final joint positions—is supplied as an input parameter. Movement towards the target position is either bending or unbending, meaning either towards the joint limits or towards the spine's rest position.

tion. Motion is defined as an interpolation between the current position and either the spine's position of maximum limit or the spine's rest position.

Three rotations are calculated independently and then merged into one. For example, a 3D orientation vector (such as bend forward 45 degrees, twist 20 degrees left, and side bend 15 degrees right) can be accomplished in one function with three loop iterations. We assumed for the model that the maximum vertebral joint limit in one dimension will not affect the joint limits of another dimension.

The model includes the spine's rest position because it is the position of highest comfort and stability. If the spine is unbending in one dimension of movement, it will move towards that position of highest comfort in that rotational dimension. What determines how much each vertebra bends as the spine moves? First, consider one-dimensional rotation, then apply the principles to the 3D model. Within one dimension are several factors:

- Current position
- Target position
- Direction of movement: unbending or bending
- Position of the vertebra
- Initiator joint
- Resistor joint
- Amount of resistance
- Whether this joint is frozen or participating
- Whether the position is calculated past the joint limit. If so, set the position to the joint limit.

### Participation of the spine

A participation vector is derived from the spine's current position, target position, and maximum position. This global participation represents a 3D vector of the ratio of spine movement to the maximum range of movement. Participation is used to calculate the joint weights.

The following formulas are defined in each of three degrees of freedom:

let

*Target* = spine target position

*Current* = spine current position

*Max* = spine sum of joint limits

*Rest* = spine sum of joint rest positions

*P* = participation

then if spine is bending

$$P = \frac{Target - Current}{Max - Current}$$

else if spine is unbending

$$P = \frac{Target - Current}{Rest - Current}$$

### Calculation of joint weights

The joint positions of the entire spine must sum up to the target position. To determine how much the joint participates, a set of weights is calculated for each joint. The participation weight is a function of the joint number, the initiator joint, and the global participation derived above. Also, a resistance weight is based on the resistor joint, degree of resistance, and global participation. To calculate the weight for a given joint,

let

*i* = joint number

*j* = joint position

*limit* = the joint limit

*rest* = the joint's rest position

*w* = weight

*p* = participation weight

*r* = resistance weight

then if spine is bending

$$w_i = p_i \cdot r_i \cdot (limit_i - j_i)$$

else if spine is unbending

$$w_i = p_i \cdot r_i \cdot (rest_i - j_i)$$

The range of weights is from 0 to 1. A weight of 1 specifies that the movement will go 100 percent of the differential between the current position and either the joint limit (for bending) or the joint rest position (for unbending). A weight of 0 means that the joint will move 0 percent of the differential (none at all). The weights are a function of the input parameters and global participation.

### Resistance

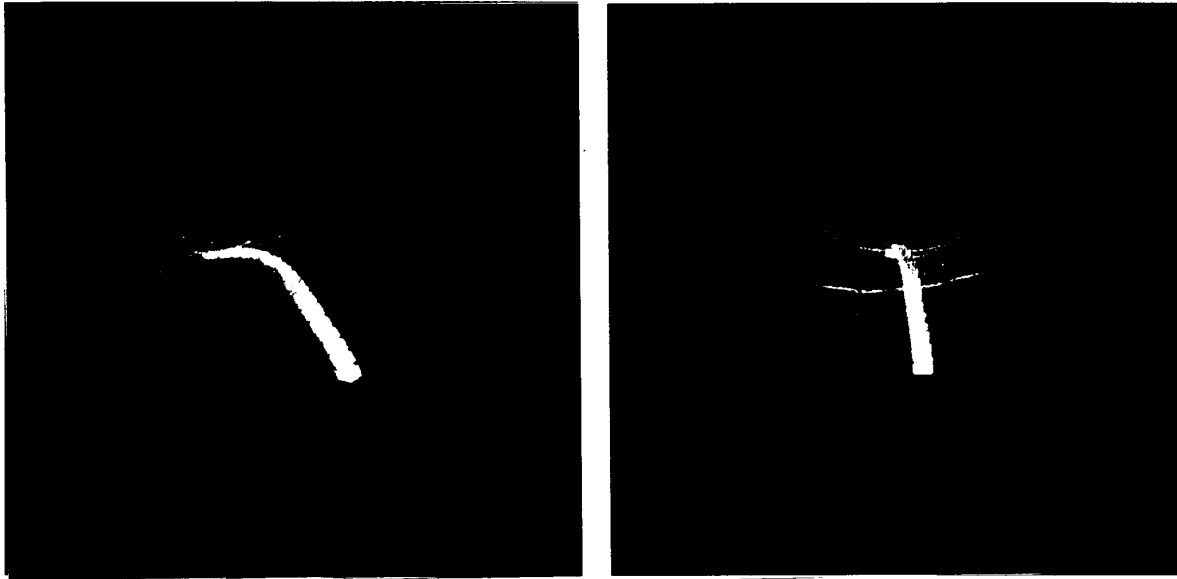
To understand resistance, divide the spine into two regions split at the resistor joint. The region of higher activity contains the initiator. Label these regions active and resistive. The effect of resistance is that joints in the resistive region will resist participating in the movement specified by the parameter degree of resistance. Also, joints between the initiator and resistor will have less activity, depending on the degree of resistance.

Resistance does not freeze any of the joints. Even at 100 percent resistance, the active region will move until all joints reach their joint limits. Then, if there is no other way to satisfy the target position, the resistive region begins to participate.

### Calculation of joint positions

If the desired movement is from the current position to one of two maximally bent positions, then the weights calculated should be 1.0 for each joint participating. The algorithm interpolates correctly to either maximally bent position. It also inter-

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Figures 13-14. Wireframe polygonal body performs circular motion.

polates correctly to the position of highest comfort. To calculate the position of each joint after movement succeeds.

let  
 $j$  = joint position  
 $j^*$  = new joint position  
 $Target$  = spine target position  
 $Current$  = spine current position  
 $M = Target - Current$  = incremental movement of the spine

then

$$j_i^* = j_i + \frac{M w_i}{\sum w_i}$$

Prove  $\sum j_i^* = Target$ :

$$\sum j_i^* = \sum \left( j_i + \frac{M w_i}{\sum w_i} \right)$$

$$= \sum j_i + \sum \frac{M w_i}{\sum w_i}$$

$$= Current + M \frac{\sum w_i}{\sum w_i}$$

$$= Current + M$$

$$= Target$$

### Bendspine function

Given the inputs

- initiator joint,
- rotational movement vector,
- resistor joint, and
- resistance

and a database of joint positions and limits, the Bendspine function outputs the new position of each vertebra after movement.

### Movespine function

This function drives spinal movement. It captures the joint limits and positions and, after prompting for input parameters, calls Bendspine. Upon returning the new joint positions, Jack sets the joint angles and redraws the window.

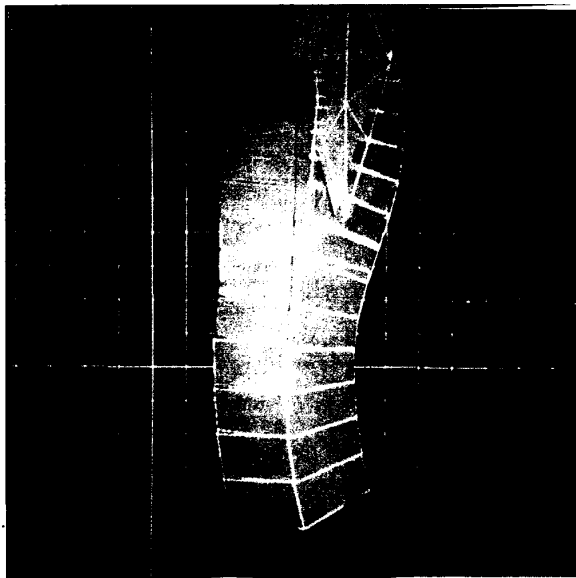
### Design for a new torso

First, we created a torso using 17 segments corresponding with 18 vertebral joints. Each segment is (arbitrarily) a hexagonal slice, with sites (points of attachment) located at the posterior side of the torso. These sites correspond to the spinal vertebral joints. This torso serves as an approximation of human anatomy to display movement efficiently.

Next, we converted more detailed human models based on biostereometrically scanned real (and hence anatomically correct) subjects to bodies with 17 contoured slices replacing the original one-segment torso. (Original subject data was supplied by Kathleen Robinette of the Armstrong Aerospace Medical Research Laboratory at Wright-Patterson Air Force Base.)



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**Figure 15.** Breathing torso indicates movement over a 5.0-second breath cycle. Green is 0.0 seconds, red is 1.6 seconds, yellow is 4.0 seconds.

These slices were designed to overlap, preventing gaps from showing up as the torso bends. Each slice was then tiled and tested in the Movespine and Bendspine functions. The final result is a realistic torso that moves according to the kinematic properties of the spine.

## Examples

Our examples show a range of postures and motion sequences involving a bendable torso and spine. The acid test of the model is real-time and video animation. We are continuing our efforts toward realistic human performance.

Figures 1 through 6 show human models based on the 95th-percentile-height male of a sample of biostereometrically scanned real subjects. The torsos are displayed in the Jack software system and bent using the interactive functions Movespine and Bendspine. The target state (x, y, z) indicates the sum of joint positions in three degrees of freedom for the 18-joint spine.

In Figures 7 through 12, after the zero interpolation flag is set to "no," the spine is rolled forward and backward around its local x axis. Note the figure-eight path drawn by the site at the top of the head. Three other vertebral joints are traced through the space while rolling the spine.

In Figures 13 and 14, while Jack traces the movement of three vertebral joints, the spine is circled by varying all three degrees of freedom in coordination. The knees and ankles bend as the spine moves backwards.

In Figure 15, as the body breathes, an extension of the upper torso occurs with flexion of the lower torso. The pattern reverses on exhalation. On a 5.0-second breath cycle, the green outline indicates positions at 0.0 seconds, red at 1.6 seconds, and yellow at 4.0 seconds.

## Future work

We plan to use the Movespine function to generate a vocabulary of torso gestures, postures, and choreography. Several new torsos will be designed and incorporated into Jack. Body linkages will include a flexible neck with seven cervical vertebrae. The model and software can discriminate each cervical joint without coding changes. In fact, we can input any number of vertebral joints to the system. Motion involving interactive reach<sup>15</sup> will be redefined to include the spine model. Joint limits of one rotational degree of freedom should be affected by position within other degrees of freedom. In addition, we should enhance the model to indicate the interdependence of lateral inclination and axial rotation due to the obliquity of the posterior articular facets.<sup>11</sup> Overall, we must attempt not to oversimplify the model of movement. Experiments so far, however, have justified the effort put into the model and appear to have added considerably to realistic human figure animation. □

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