Comments and Corrections

Comments on "Comments on "A General Theory of Phase Noise in Electrical Oscillators""

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***Abstract*—A recently published correction to the expression of phase noise in general electrical oscillators, derived through a frequency-domain analysis, is incorrect. We show that the correct conclusion about phase noise is the original one.**

***Index Terms*—Impulse sensitivity function (ISF), oscillators, phase noise.**

Recently, a correction to the expression of phase noise in general electrical oscillators, derived through a frequency-domain analysis, has been published in [1]. However, the new expression in terms of the rms value of the impulse sensitivity function Γ is incorrect, while the original conclusion about phase noise [2] is correct.

As shown in [2], both the noise power around $n\omega_0 + \Delta\omega$ and $n\omega_0 - \Delta\omega$ generates two equal contributions at $\omega_0 \pm \Delta\omega$ with respect to the white noise. However, when n is equal to 0, only one noise contribution should be taken into account, so that [3, (19)] and [2, first equation] should be written as

$$\mathcal{L}\{\Delta\omega\} = 10\log\frac{\frac{\overline{i_{nw}^2}}{\Delta f}\cdot\left(\frac{c_0^2}{2}+\sum_{n=1}^{\infty}c_n^2\right)}{4q_{\max}^2\Delta\omega^2}. \tag{1}$$

This result has been obtained by Jannesari and Kamarei in [1, (7)] through a frequency-domain analysis.

The above result is based on the assumption that the Fourier series of Γ is expanded as

$$\Gamma(\phi) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cos(n\phi + \theta_n) \tag{2}$$

(see [3, (12)]), and according to Parseval's relation [4] we have

$$\frac{c_0^2}{2} + \sum_{n=1}^{\infty} c_n^2 = \frac{1}{\pi}\int_0^{2\pi} |\Gamma(\phi)|^2 d\phi = 2\Gamma_{\rm rms}^2. \tag{3}$$

Indeed, the expression for $\Gamma_{\rm rms}^2$ found in [3, (20)] is not correct. Nevertheless, the two errors in [3, (20)] and [2, first equation] cancel each other, resulting in the correct final, aesthetically pleasing, and most common expression for $\Lambda\ (\Delta\omega)$ given in [2, second equation] and reproduced here in (4):

$$\mathcal{L}\{\Delta\omega\} = 10\log\frac{\frac{\overline{i_{nw}^2}}{\Delta f}\cdot\Gamma_{\rm rms}^2}{2q_{\max}^2\Delta\omega^2} \tag{4}$$

as is immediate to check by substituting (3) in (1), while the expression in [1, (8)] is incorrect, since it uses the faulty expression of Parseval's relation given in [3, (20)].

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Author's Response

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The authors thank the commenters for their note. Our frequency domain analysis provides a new vision to express the phase noise formula precisely. The definition of $\Gamma_{\rm rms}$ in our formulas is based on its definition in the general theory of phase noise. As the $\Gamma_{\rm rms}$ is the root mean square (rms) value of the impulse sensitivity function (ISF), it is related to the Fourier expansion coefficients of ISF with Parseval's relation. The presented definition for $\Gamma_{\rm rms}$ in the general theory of phase noise is incorrect that is corrected in the present comment with precise Parseval relation. With this correction, the equation (8) of our comment has to be modified. With removing $(c_0/2)^2$ from (8), it will be corrected. This shows that when considering the phase noise based on $\Gamma_{\rm rms}$, the early proposed phase noise formula is by chance the same as the obtained result from our frequency domain analysis, but considering the phase noise formula not in the short form with $\Gamma_{\rm rms}$, the equation (7) of our comment is a precise formula.

References

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