An Improved LQR-based Controller for Switching Dc-dc Converters

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***Abstract*—In a previous paper, an account had been given to the various aspects for the control of PWM-type switching dc-dc converters, and an LQR-based digital combined observer controller had been proposed. This paper reports the progress on further research in this topic. It is found that the design of the state estimator (observer) according to the previous paper will lead to a steady-state error in the output voltage in comparison with the reference value and there are difficulties in designing state estimators for converters with uncertain parameters. This paper presents an improved design so that the drawbacks mentioned above can be overcome. By applying the theory of state-variable feedback, a general approach for controlling PWM-type switching dc-dc converters with or without "unstable" zeros, and capable of achieving good regulation and rejection to modest disturbances, will be presented. An application example based on a published Ćuk converter will also be given.**

I. Introduction

It is generally known that switching dc-dc converters are highly nonlinear plants with uncertain parameters, and subjected to inevitable and significant perturbations during operation. In a previous paper [1], some aspects of control for PWM-type switching dc-dc converters had been reviewed. The problem was then considered from the linear quadratic regulator (LQR) point of view. Under such a view, assumptions were added to the operating conditions as preliminaries so that the converter could be regarded as a linear plant with a single operating point subjected to small-signal perturbations. In this way, the problem was reduced to the design of fixed-parameter robust controllers capable of satisfying some specified control objectives under these perturbations. Linear quadratic regulator and observer techniques [2], [3] were then applied so that the closed-loop systems could achieve good transient responses and satisfactory robustness against perturbations to the system parameters. Note, however, that when the parameters of the plant are uncertain, the inevitable differences between the plant and the model assumed by the state estimator will lead to degradation of the overall system performance comparing with the full-state feedback case. With the inclusion of the observer, the attractive robustness properties of full-state feedback will no longer be guaranteed [4]. A study on the influence of the observer to the characteristics of the closed-loop system had thus been carried out [5]. It was found that the previously proposed configuration of observer will lead to a nonzero steady-state error in the output of the converter. In this paper, an improved and general approach for designing state-estimators based on concerns over transient response, zero steady-state error, and robustness will be presented. The result is a general methodology for designing digital combined observer controllers for switching dc-dc converters satisfying some prescribed control objectives. Due to its inherent advantages of flexibility, ease in fine tuning the controller parameters, and simplicity of implementation, the control law proposed is implemented digitally. Special attention is paid to the situation when the discrete-time plant model has open-loop zeros outside the unit circle of the z-plane. The design consideration will be discussed in Section II. In Section III, the details about the proposed LQR-based combined observer-controller will be presented. In Section IV, an application example based on a difficult plant of Ćuk converter will be given to illustrate the feasibility of the ideas in this paper and the performance of the designed controller.

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II. Design Consideration

The major difficulties in controlling switching dc-dc converters are due mainly to the difficulties in modeling the converters, their highly nonlinear nature in general, and possible migrations of open-loop zeros of the linearized model across the stability boundary in particular. With regard to these difficulties, a controller is to be designed such that the following three basic objectives can be achieved:

- *Objective 1*: stiff line and load regulations (dc responses).
- *Objective 2*: low output impedance and audio susceptibility (ac responses).
- *Objective 3*: robustness to uncertainties of plant parameters.

The achievement of these three objectives are usually reflected by the fulfillment of a defined specification for the controlled system. Simple specifications for an application example will be given in Section IV.

In accordance with *Objective 1*, zero steady-state error between the output voltage and the reference value is to

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be achieved even when a sustained deviation from the standard value in line voltage (V_i) or load current (I_o) is present. In *Objective 2*, low output impedance (v_o/i_o) and audio susceptibility (v_o/v_i) are equivalent to giving satisfactory transient responses when the perturbations to the system cause variations in load current (i_o) and line voltage (v_i), respectively. As will be further explained in Section II-A, the model of the switching dc-dc converter is only a small signal one. The acquisition of such a model often involves many approximations. Even when the model is obtained by using some (if they exist) very accurate modeling techniques, it is only an accurate representation of the converter at one particular operating point. When the switching dc-dc converter is subjected to internal or external perturbations (e.g., line and load variations), the operating point will be shifted from the nominal position and the assumed model will no longer be a good representation. This accounts for the uncertainties in the assumed plant parameters. The controller so designed must be robust enough to these uncertainties as stated in *Objective 3*. Based on the above discussion, considerations should be directed to a general strategy for designing an optimal and robust digital controller for PWM-type switching dc-dc converters. Special attention should be delivered to the robustness of the controller to the uncertainties of plant parameters, and particularly to the possible migrations of the open-loop zeros of the plant models across the stability boundary.

A. Problem Definition

A switching dc-dc converter can be regarded as a multiple-input single-output (MISO) plant [6]. The output is the output voltage v_o of the switching converter. The control input is the signal represented by the PWM duty ratio d, where d is defined as the ratio of the power switch *ON* time to the switching period. The converter parameters are affected by the input line voltage v_i and output current i_o (output load R_L), which in conjunction with other minor variations such as EMI and stray effects, constitute the external disturbance inputs to the system. The steady-state values of d, v_o, v_i, and i_o, symbolized by D, V_o, V_i, and I_o, respectively, constitute the operating point of the switching converter. For these quantities constituting the operating point, the relationships between the instantaneous values, steady-state (dc) values, and the ac values are governed by the following four equations:

$$v_o = V_o + \tilde{v}_o \tag{1}$$

$$v_i = V_i + \tilde{v}_i \tag{2}$$

$$i_o = I_o + \tilde{i}_o \tag{3}$$

$$d = D + \tilde{d}. \tag{4}$$

where the "$\sim$" sign is used to represent small ac variations about the steady-state operating point. Switching dc-dc converters were known to possess the following nonlinearities [1]:

1) topology changes due to, perhaps, high temperature or components failure,
2) nonlinear characteristics of the electronic switches (fast dynamics), and
3) nonlinear plant parameter variations due to external disturbances (slow dynamics).

To cater to these nonlinearities, two approaches can be adopted.

1) Single Operating-point Approach (SOPA): Use an approximate linearized model to average out the effects of fast dynamics (nonlinearity 2)). This linearized model is usually accurate enough within the bandwidth of interest. However, owing to the nonlinearities 1) and 3), the following assumptions have to be made:

1) The regulated switching dc-dc converter has only one operating point,
2) the variations in line voltage and load current are infrequent and small enough to be tackled, and
3) other disturbances or the effects of topology changes are small and lie within the sensitivity tolerance of the controller (i.e., the controller is adequately robust).

2) Multiple Operating-point Approach (MOPA): Design a high quality adaptive controller that is capable of adapting to significant nonlinearities, and catering to multi-operating point situations.

It can be seen that *MOPA* is more general than *SOPA*, but the design and implementation of such a controller requires a more advanced control theory which is not as mature as *SOPA*. Also, *SOPA* is found to be sufficient in many cases practically, and we limit the scope of this paper by considering *SOPA* only. Under *SOPA*, a linearized ac small-signal model has to be considered. The closed-loop system can be represented by the block diagram as shown in Fig. 1. "Small-signal" is emphasized here, meaning that the highly nonlinear plant of switching dc-dc converter is modeled as a linear system working around its operating point. Perturbations to the system are assumed to be small signals such that the linearization can still be a valid approximation and will not affect the overall system stability. Perturbations to the system are modeled as the disturbance inputs $\tilde{\omega}_1$ and $\tilde{\omega}_2$ at the input and output sides of the plant, respectively. A controller is to be designed so as to drive the output $\tilde{v}_o$ to zero (i.e., $v_o \rightarrow V_o$ as $t \rightarrow \infty$), in the presence of perturbations (*Objective 1*), with good transient dynamics (*Objective 2*), and is robust enough to uncertainties in the plant parameters (*Objective 3*).

B. Design Procedures

The proposed design procedure can be summarized by the flow diagram as shown in Fig. 2. The first phase of circuit measurement shown is used to provide data for the second phase of system identification. Thus the switching dc-dc converter can be regarded as a black box, and only the open-loop control-to-output response data (i.e., data of $\tilde{d}$ and $\tilde{v}_o$), are necessary for modeling the plant. The

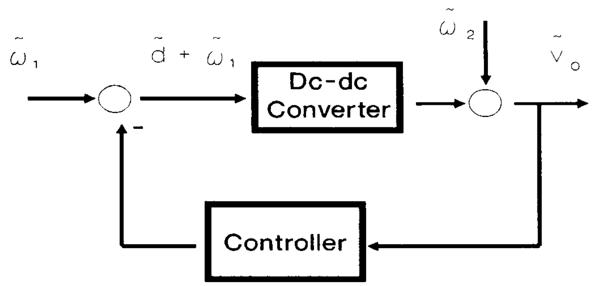

Fig. 1. Closed-loop dynamic model of a switching regulator.

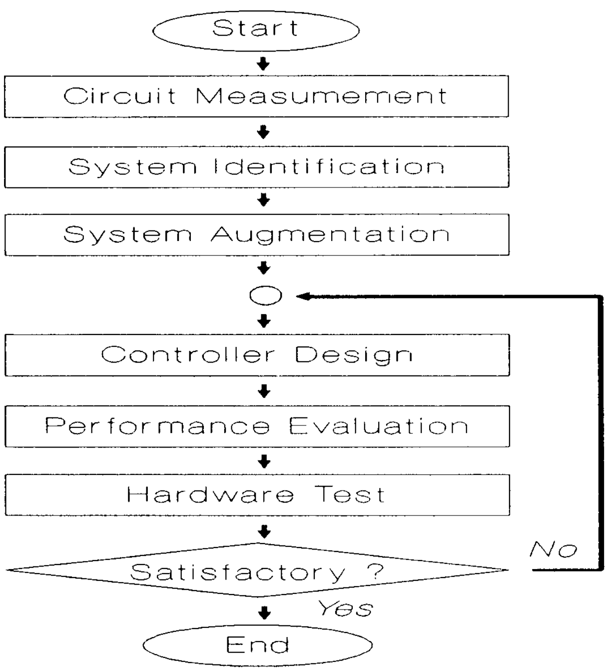

Fig. 2. A flow diagram showing the controller design procedures for switching dc-dc converters.

input data $\tilde{d}$ to this identification scheme ought to have small amplitude because a large input amplitude, due to the nonlinearities of the converter, will give different coefficients of the model which is no longer a small-signal linearized model. The linearized model is realized as a discrete-time open-loop control-to-output transfer function. The system model is then augmented to suit the combined observer-controller design. The details of these phases will be elaborated in Section III. Afterwards, its performance may be evaluated by first software simulation and then actual hardware implementation. Whenever necessary, further fine adjustments to the controller parameters are performed. The whole design procedure is a general procedure independent of the topologies of switching converters. With the sophisticated computing power available nowadays, it is possible to merge the first five phases shown in Fig. 2 into a stand-alone automatic CAD package [7].

III. LQR-Based Combined Observer-Controller Design

Based on the discussion in the previous sections, a robust digital LQR-based combined observer controller is proposed for PWM-type switching dc-dc converters. In this section, an account will be given to such a controller through discussions on three aspects:

1) the derivation of the optimal state-feedback control law,
2) the design of robust state estimator, and
3) the special case of plant models with possible migrations of open-loop zeros across the stability boundary.

C. Optimal State Feedback

With reference to Fig. 1, since the main interest is the regulation at the output node of the dc-dc converter, investigation on the system's response to the perturbation at the output node ($\tilde{\omega}_2$) will provide a more direct measure on the performance of the controller than that at the input node ($\tilde{\omega}_1$). So, as far as the controller design is concerned, $\tilde{\omega}_2$ (but not $\tilde{\omega}_1$) will be included for consideration. The converter (plant) can thus be represented by the following linearized state-space model:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\tilde{d} \tag{5}$$

$$\tilde{v}_o = \boldsymbol{C}\boldsymbol{x} + D\tilde{d} + \tilde{\omega}_2 \tag{6}$$

As far as a digital controller is concerned, the plant can be modelled by its discrete-time representation:

$$\boldsymbol{x}(k+1) = \boldsymbol{\Phi}\boldsymbol{x}(k) + \boldsymbol{\Gamma}\tilde{d}(k) \tag{7}$$

$$\tilde{v}_o(k) = \boldsymbol{C}\boldsymbol{x}(k) + D\tilde{d}(k) + \tilde{\omega}_2(k) \tag{8}$$

where $\tilde{d}$, $\tilde{v}_o$, and $\tilde{\omega}_2$ are scalar quantities. The scalar parameter D accounts for the possible link between the input and the output (e.g., the parasitic effects caused by the energy storage components [8]). As mentioned in Section II-A, the switching dc-dc converter is inevitably subjected to perturbations due to many reasons. In order to assure a full dc regulation ($\tilde{v}_o \to 0$ as $t \to \infty$) even when the unknown perturbation is a sustained and slowly varying one (i.e., $\tilde{\omega}_2(k+1) \approx \tilde{\omega}_2(k)$), the inclusion of an "integrator" in the feedback path is needed. To implement the "integral feedback," the system has to be augmented to a new system as follows:

$$\boldsymbol{x}_1(k+1) = \boldsymbol{\Phi}_1\boldsymbol{x}_1(k) + \boldsymbol{\Gamma}_1 u_1(k) \tag{9}$$

$$\tilde{v}_o(k) = \boldsymbol{C}_1\boldsymbol{x}_1(k) + \tilde{\omega}_2 \tag{10}$$

where

$$\boldsymbol{x}_1(k) = \begin{bmatrix} \boldsymbol{x}(k) \\ \tilde{d}(k) \end{bmatrix}$$

$$u_1(k) = \tilde{d}(k+1) - \tilde{d}(k)$$

$$\Phi_1 = \begin{bmatrix} \Phi & \Gamma \\ 0 & 1 \end{bmatrix}, \quad \Gamma_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_1 = [C \quad D].$$

Here it is assumed that the effects of all disturbance inputs will cause the output to deviate from the desired operating point, and the controller output is to bring the perturbed output of the converter back to its reference condition. Based on this system augmentation, an LQR approach was proposed [1] to derive an optimal state-feedback control law and is summarized in the Appendix.

B. Robust State Estimator

As discussed in Section I, for the sake of generality and the fact that not all states of the plant are easily measurable, a state estimator (observer) is used to estimate the states for feedback. The asymptotic state estimator can be represented by the following expression:

$$\hat{x}_1(k+1) = \Phi_1 \hat{x}_1(k) + \Gamma_1 u_1(k) + L(\tilde{v}_o(k) - C_1 \hat{x}_1(k)) \tag{11}$$

where $\hat{x}_1$ denotes the observed state variables of the augmented system. With the presence of the state estimator, a complete control scheme is represented by the block diagram as shown in Fig. 3. From (11) it can be seen that the design of the state estimator involves the determination of the vector L in accordance with the augmented system represented by Φ_1, Γ_1, and C_1. The values of the components of vector L play an important role in the overall performance of the combined observer controller. A good design of L should satisfy the following two conditions:

- *Condition 1*: satisfactory transient dynamics of the controlled system when it is subjected to perturbations, and
- *Condition 2*: robustness of the overall system to the uncertainties in the plant parameters.

In fact, these two conditions are related to the extent of achieving *Objective 2* and *Objective 3* discussed in Section II. As far as the design of the state estimator is concerned, two issues have to be considered in order to satisfy the above two conditions. The first one is the choice of the assumed plant model, which will be further discussed in the next subsection. The second one is the derivation of the vector L which will be presented as follows.

Based on the arguments in [5], the vector L can be derived using an approach similar to the derivation of the feedback control law of the linear quadratic regulator (LQR) problem. Consider the completely controllable and observable discrete-time shift-invariant system given by

$$x(k+1) = \Phi_1^T x(k) + C_1^T u(k). \tag{12}$$

The optimal feedback control law $u(k) = -L^T x(k)$ for this system with a quadratic performance index

$$J = \sum_{k=0}^{\infty} (u(k)^2 R + x(k)^T Q x(k)) \tag{13}$$

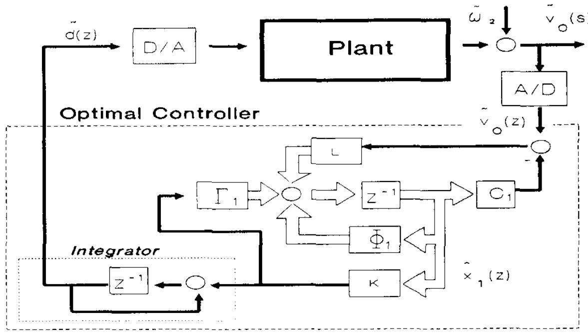

Fig. 3. Final control scheme for a dc-dc converter.

is known to give all roots of

$$\det(zI - \Phi_1^T + C_1^T L^T) = \det(zI - \Phi_1 + LC_1) = 0 \tag{14}$$

inside the unit circle of the z-plane [10]. Also, the choice of the ratio $\|Q\|/|R|$ is associated with a tradeoff between the degree of satisfaction of *Condition 1* and that of *Condition 2*. By properly choosing the positive scalar R, and the symmetric and nonnegative definite matrix Q, the vector L can be determined by using the theory of linear optimal control. The choices of Q and R will be illustrated in the application example in Section IV.

C. Possible Migrations of Zeros Across the Stability Boundary

Attention has to be paid to the situation when perturbations to the system will cause the plant model's zeros to drift from inside to outside the unit-circle of the z-plane and vice versa. In this case, as far as the controller design is concerned, two factors have to be considered:

1) the model for designing the feedback gain vector K may be invalid, and
2) the model for designing the observer gain vector L may be invalid.

The control scheme has to be determined through comparing different combinations of the above two factors under the standard operating conditions and the worst-case conditions. A qualitative assessment can be summarized by Table I. In this table, IC and OC refer to the algorithms assuming that the system zeros are inside and outside the unit circle of the z-plane respectively (so OC in the second column corresponds to *Algorithm A-2* stated in the Appendix). The control laws derived under these combinations were then examined in situations with migrations of open-loop zeros in order to achieve a general qualitative assessment of the controllers' performance. They are shown in the last column of Table I which can be explained as follows:

1) When the locations of zeros are actually outside the unit circle, both algorithms for determining K and L refer to invalid models in choice 1, and the closed-

TABLE I
COMPARISON OF DIFFERENT COMBINATIONS OF CONTROLLER AND OBSERVER ALGORITHMS FOR PLANT MODELS WITH POSSIBLE MIGRATIONS OF ZEROS ACROSS THE STABILITY BOUNDARY

Choice	Algorithm for Designing $\boldsymbol{K}$	Algorithm for Designing $\boldsymbol{L}$	Performance
1	IC	IC	Very poor
2	IC	OC	Satisfactory
3	OC	IC	Poor
4	OC	OC	Satisfactory

IC = Algorithm assuming that the system zeros are inside the unit-circle of the z-plane.
OC = Algorithm assuming that the system zeros are outside the unit-circle of the z-plane.

loop system performance becomes very poor; whereas in the other choices, at least one model is valid, resulting in a better performance.

2) Experimental results indicate that among choices 2, 3, and 4, choice 3 gives a poorer response and stability margin. This shows that the invalidity of the model for observer gain design plays a major role for performance degradation.
3) Both choices 2 and 4 give satisfactory results. In this case, the final choice may depend on which side (inside or outside the unit circle) the system zeros lie more frequently during the operation.

Simple quantitative analyses (measures of the relative stability margins) of the performances concerning the different combinations in Table I, with respect to an application example, will be given in Section IV.

IV. APPLICATION EXAMPLE

The example plant is a published Ćuk converter [11] as shown in Fig. 4 under the standard operating point, viz. $V_i = 25$ V, $D = 0.55$, $V_0 = -30$ V, and $R_L = 30\ \Omega$. This is a 4th order system with the interesting property that at the desired operating point, its z-domain linearized model has two complex zeros inside the unit circle, but when the operating condition alters, e.g., due to load changes, the zeros may shift to outside the unit circle. Such a shifting of open-loop zeros across the stability boundary and its relatively high order make it a difficult plant to be controlled by other common methods. An LQR-based digital combined observer controller is to be designed achieving the following closed-loop system specifications:

1) Range of input voltage variation: 21 V–29 V,
2) range of output load variation: 25 Ω–34 Ω,
3) line and load regulations: less than 1%,
4) percentage overshoot due to load variations: less than 5% of reference voltage (−30 V), and
5) steady-state output ripple: within ±0.5% tolerance to reference voltage.

In this paper, discrete-time ac small-signal models (z-transfer functions) are obtained automatically by using

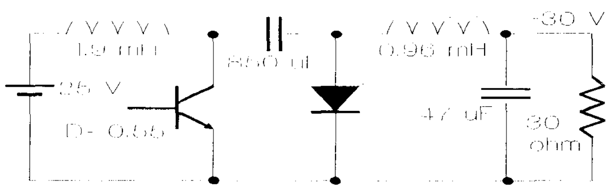

Fig. 4. A −30 V/1 A Ćuk converter circuit.

the CAD techniques mentioned in Section II-B. The results are the numerical control-to-output transfer functions ($\tilde{v}_o/\tilde{d}$) for the plant at different operating points with the relations between parametrical figures and the circuit components' values being unnecessary to be known. The sampling frequency was chosen to be 10 kHz, under the criterion that the sampling time is at least 10 times smaller than the largest time constant of the system. The control-to-output transfer function at standard operating point can then be transformed to the following discrete-time state-space representation.

$$\boldsymbol{x}(k+1) = \begin{bmatrix} 3.6662 & -5.2431 & 3.4865 & -0.9099 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \boldsymbol{x}(k) + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tilde{d}(k) \tag{15}$$

$$\tilde{v}_o(k) = [10.9239\ \ -18.1095\ \ 3.5938\ \ 3.6405]\boldsymbol{x}(k) \tag{16}$$

With regard to the control strategies discussed in this paper, the following state-space model at $R_L = 34\ \Omega$ is also recorded:

$$\boldsymbol{x}(k+1) = \begin{bmatrix} 3.6336 & -5.1196 & 3.3375 & -0.8517 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \boldsymbol{x}(k) + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tilde{d}(k) \tag{17}$$

$$\tilde{v}_o(k) = [13.0378\ \ -27.8917\ \ 16.4579\ \ -1.5617]\boldsymbol{x}(k) \tag{18}$$

For this example Ćuk converter circuit, it is found that when the load changes from 30 Ω to 32 Ω or higher, two open-loop zeros will shift outside the unit circle in the z-plane. The specifications listed above are so mild that the assumptions behind the successful application of *SOPA* are valid. Due to the inherent change of operating point in general and significant shifts of open-loop zeros in particular, a compromise has to be adopted on which the plant model should be chosen for the design of $\boldsymbol{K}$ and $\boldsymbol{L}$. Since the operating point of this example Ćuk converter may change from the standard operating point ($R_L = 30\ \Omega$) to the worst case with open-loop zeros outside the unit circle ($R_L = 34\ \Omega$), the four combinations of plant models assumed in the design algorithms for $\boldsymbol{K}$ and $\boldsymbol{L}$ as listed in Table I have to be considered. In order to determine the best combination of the assumed plant models, the stability of the system under the worst-case conditions, viz. the actual plant zeros are outside the unit circle ($R_L = 34\ \Omega$), and the relative stability margins of the system under the standard operating conditions were examined and measured. The results are summarized in Table II. From Table II, it can be seen that when the zeros of the assumed plant model for designing $\boldsymbol{L}$ are inside the unit circle ($R_L = 30\ \Omega$, choices 1 and 3), the system fails to remain stable under the worst-case conditions. When the zeros of the assumed plant models for designing $\boldsymbol{L}$ are outside the unit circle and those for designing $\boldsymbol{K}$ are on either side (choices 2 and 4), the system remains to be stable in the worst-case conditions and the relative stability margins are similar in both cases. Since it is reasonable to assume that the converter is more often in the standard operating conditions, choice 2 of Table II is adopted in choosing the assumed plant models for the design of the LQR-based combined observer controller. Thus $\boldsymbol{K}$ was designed based on the model with $R_L = 30\ \Omega$; and for $\boldsymbol{L}$, the model with $R_L = 34\ \Omega$ was used.

1) Design of the State-feedback Vector $\boldsymbol{K}$*:* Based on Algorithm A-1 in the Appendix, three dominant poles of the closed-loop system were chosen in such a way that two complex poles were close to the two complex zeros in the z-plane, and one dominant real pole was around 1000 Hz in the frequency domain. The performance index to be optimized in the design of the LQR-based controller is given by (22) and (23), in which $\boldsymbol{Q}$ is defined by (23). R and σ should be chosen such that they are small compared with the elements of $\boldsymbol{Q}$. However, if R and σ were too small, it would lead to a large $\boldsymbol{K}$ without significant improvement to the transient responses [2], [9]. Experimentally, satisfactory results were found when choosing R to be 0.01 and σ to be 0.1 in this example. The vector $\boldsymbol{K}$ is then given by:

$$\boldsymbol{K} = [-0.7438,\ 2.2930,\ -2.3604,\ 0.8106,\ -1.8291]. \tag{19}$$

2) Design of the State-estimator Gain Vector $\boldsymbol{L}$*:* When the state estimator was designed, the model assumed was that with a load of 34 Ω. With the proper system augmentation as discussed in Section III-A, the method discussed in Section III-B for designing the state estimator was applied. The quadratic performance index of (13) was specified, in which $\boldsymbol{Q}$ was chosen to be the identity matrix $\boldsymbol{I}$ (of order 5) and a number of values of R was tried so as to compare the results in terms of the transient responses and the stability margins. After the necessary tradeoff between these two factors, a satisfactory result was obtained for $R = 100\ 000$, which gave the following vector $\boldsymbol{L}$:

$$\boldsymbol{L} = [4.8925,\ 4.6085,\ 4.3454,\ 4.0997,\ 0.0026]^T \tag{20}$$

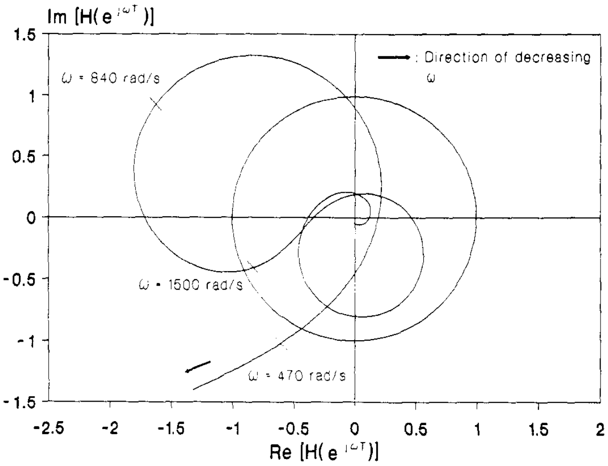

Fig. 5. A Nyquist plot of the loop transfer function $\boldsymbol{H}(e^{j\omega T})$ for the controlled Ćuk converter under nominal operating conditions.

TABLE II
COMPARISON OF DIFFERENT COMBINATIONS OF CONTROLLER AND OBSERVER ALGORITHMS FOR THE EXAMPLE ĆUK CONVERTER UNDER THE WORST-CASE CONDITIONS ($R_L = 34\ \Omega$) AND STANDARD OPERATING CONDITIONS ($R_L = 30\ \Omega$)

Choice	Algorithm for Designing K	Algorithm for Designing L	Stability under Worst-Case Conditions	Standard Operating Conditions GM	PM
1	IC	IC	Unstable	8 dB	27°
2	IC	OC	Stable	9 dB	20°
3	OC	IC	Unstable	9 dB	41°
4	OC	OC	Stable	6 dB	23°

GM = Gain Margin
PM = Phase Margin
IC = Algorithm assuming that the system zeros are inside the unit-circle of the z-plane ($R_L = 30\ \Omega$)
OC = Algorithm assuming that the system zeros are outside the unit-circle of the z-plane ($R_L = 34\ \Omega$)

To investigate the robustness of the controller to the uncertainties of plant parameters, a Nyquist plot for the loop transfer function $\boldsymbol{H}(e^{j\omega T})$ ($T = 0.1$ ms for a controller sampling frequency of 10 kHz) of the proposed combined observer controller and the standard plant model ($V_i = 25$ V, $R_L = 30\ \Omega$) was obtained and shown in Fig. 5. From Fig. 5, the gain margin is about 9 dB and the phase margin is about 20°. These results are better than those with controllers based on state-estimator design using direct eigenvalues assignment.

Plots of the transient responses of the output voltages, when the controlled Ćuk converter is subjected to a load change from 30 Ω to 34 Ω, and an input voltage change from 25 V to 29 V, are shown in Figs. 6(a) and (b), respectively. It can be seen that the change in load results in a relatively small perturbation to the system and the corresponding transient response is better, when compared with the change in input voltage from which a large variation in the output voltage is found. The response for the change in input voltage can be viewed as an illustration of the robustness of the controller when the plant is subjected to relatively large perturbations (16% change in input voltage), which also means greater uncertainties in the plant parameters. Also, it can be seen from Fig. 6(a) that robustness can be retained even though there are possible migrations of system zeros across the stability boundary due to load variations. In all cases, the steady-state value of the output voltage is successfully clamped to the reference value of -30 V with $\pm 0.5\%$ tolerance for ripples, and all the specifications listed at the head of this section have been met. Noting the difficulties in controlling a fourth order system of Ćuk converter with possible migrations of plant model's zeros, the performance of the controller is found satisfactory. For a comparison, the performance of a standard PID controller for controlling the same plant was investigated. It was found that under some controller parameters, the PID controller might perform better than the proposed controller in terms of transient responses and relative stability margins. However, the derivations of these parameter values of the PID controller involve a tedious multi-dimensional search process under a trial-and-error basis. For a PID controller derived based on some standard means, e.g., the method by Ziegler and Nichols [12], the performance is unacceptably poor.

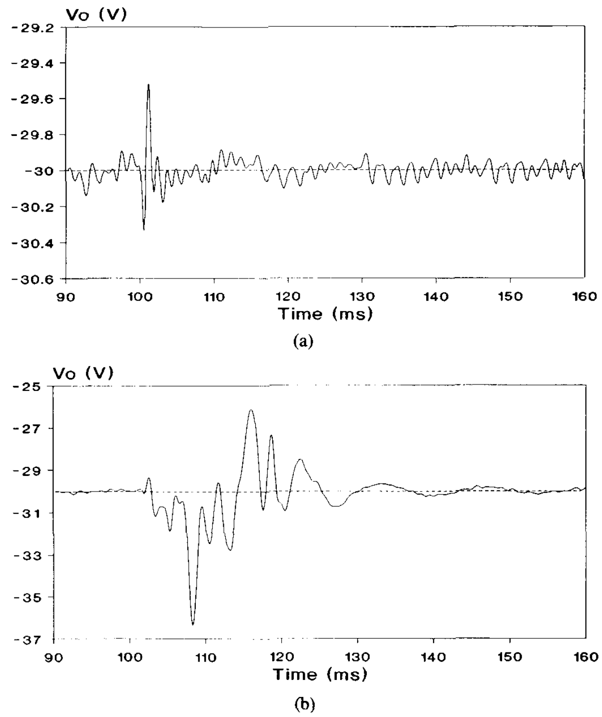

Fig. 6. Transient response of the controlled Ćuk converter for (a) a load change from 30 Ω to 34 Ω, and (b) an input voltage change from 25 V to 29 V, at 100 ms.

V. Conclusion

A general approach for controlling PWM-type switching dc-dc converters digitally using state-feedback techniques and linear optimal control theory has been reported. Upon investigating the methodology of designing the state estimator, a method derived from the general LQR problem is proposed which is found to offer better transient responses and robustness to uncertainties in plant parameters when compared with the typical eigenvalues assignment method. With the difficulties of control properly addressed, special attention has been directed to plant models with possible migrations of the open-loop zeros across the stability boundary during the operation. Results of applying these techniques to a published Ćuk converter have been reported as illustrations to different points of interest.

Appendix

To guarantee the regulator to have good closed-loop behavior and to be relatively insensitive to the uncertainties of plant parameters (which implies satisfactions of *Objective 1* to *Objective 3* of Section II), the controller feedback gain vector $\boldsymbol{K}$ has to be determined optimally through minimizing an appropriate performance index. By linear optimal control theory, the closed-loop poles can be assigned such that the dominant poles are close to the desired locations and the remaining poles are nondominant near to the origin of the z-plane. Such an optimal controller provides a guaranteed reduction in sensitivity to plant parameter variations when compared with an equivalent open-loop system [2], [3]. In this problem, a suitable quadratic performance index for the digital controller can be determined using the design steps of the following algorithm (adapted from [2], [3], [9]):

Algorithm A-1:

Step 1: Form a polynomial $m(z)$ such that it has the desired dominant poles p_i as its roots.

$$m(z) = \prod_{i=1}^{n'} (z - p_i) \tag{19}$$

where n' is the number of dominant poles.

Step 2: Solve for the vector $\boldsymbol{d}$ and form the matrix $\mathbf{Q}$ according to:

$$\boldsymbol{d}^T(z\boldsymbol{I} - \boldsymbol{\Phi})\boldsymbol{\Gamma} = m(z)/\det(z\boldsymbol{I} - \boldsymbol{\Phi}) \tag{20}$$

$$\mathbf{Q} = \boldsymbol{d}\boldsymbol{d}^T \tag{21}$$

Step 3: Check that $[\boldsymbol{\Phi}, \boldsymbol{d}]$ is completely observable.

Step 4: Define the performance index:

$$\underset{k=0}{\overset{\infty}{J}} = \sum \left(u_1(k)^2 \sigma + \boldsymbol{x}_1(k)^T \mathbf{Q}_1 \boldsymbol{x}_1(k) \right) \tag{22}$$

$$\text{where } \mathbf{Q}_1 = \begin{bmatrix} \mathbf{Q} & 0 \\ 0 & \mathbf{R} \end{bmatrix} \tag{23}$$

$\mathbf{Q}$ is symmetric, nonnegative definite and of order n (order of the open-loop unaugmented system). $\mathbf{R}$ and σ are positive scalars. $\mathbf{R}$ is chosen to be sufficiently small so that it gives a more significant weight to the state error weighing matrix **Q**. σ is chosen to be between the value of **R** and the largest eigenvalue of **Q**.

Step 5: Determine the optimal feedback gain vector **K**.

For a system with all zeros inside the unit circle in the z-plane, the quadratic performance index can be determined with dominant poles close to the zeros of the system. However, when the zeros are outside the unit circle in the z-plane (this is quite common for plants of dc-dc converters), the dominant poles cannot be chosen to be close to the "unstable" zeros. Our approach to cater to this situation is as follows:

Algorithm A-2:

For any "unstable" zero z_i in the z-plane, choose the corresponding dominant pole to be located near the position $z = 1/z_i$. If necessary, modify the location of the dominant pole to sufficiently far from the stability boundary.

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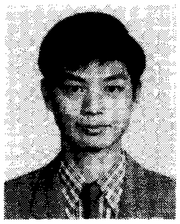

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