Ordered Statistics Decoding of Linear Block Codes on the WSSUS Multipath Channel

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ABSTRATCT: The ordered statistics decoding algorithm proposed in [1] is applied to the wide-sense-stationary uncorrelated-scattering (WSSUS) Rayleigh fading channel [2], modeled as a frequency non-selective, slow-fading environment without inter-symbol interference (ISI). We derive the log-likelihood ratio of received symbols based on 2DPSK transmission. Simulations are conducted for the WSSUS channel at various interleaving degrees. At bit error rate (BER) 10^{-5}, 38 dB gains compared to uncoded 2DPSK are obtained for the decoding of the (128, 64, 22) extended BCH code with interleaving degree 50.

1. Introduction

In 1995, a computationally efficient soft-decision decoding algorithm based on statistics obtained from reordering the received symbols was introduced by Fossorier and Lin [1], referred to as ordered statistics decoding. It is shown in [5] that the application of ordered statistics decoding for Rayleigh fading channel with coherent detection provides a good trade-off between error performance and decoding complexity. However, the assumptions made in [5], i.e. perfect coherent detection and independent Rayleigh amplitudes (or equivalently, infinite degree of interleaving), are either difficult to achieve in practice or result in undesirable delay at the receiver. In this study, we apply ordered statistics decoding to mobile radio communication channels without previous assumptions. In particular, the error performance of ordered statistics decoding on the wide-sense-stationary uncorrelated-scattering (WSSUS) multipath channel [2] which exhibits uncorrelated dispersiveness in time delay and Doppler shifts is investigated.

2. System Configuration

The WSSUS channel characterized by multipath and fading exhibits bursty error features. To effectively combat the noisy channel, we propose the system which integrates ordered statistics decoding of linear block codes with a block interleaver [3, chap. 8, pp. 469]. The coded data is interleaved in such a way that the bursty channel is transformed to a channel having independent errors. In addition, a simple and effective non-coherent detection scheme known as 2DPSK [3, chap. 5, pp. 274-278] is adopted in our design. The block diagram of the proposed system is shown in Figure 1.

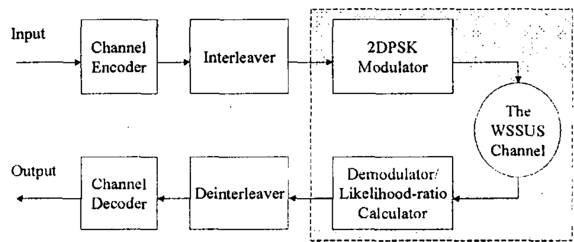

Figure 1. System block diagram.

3. The WSSUS Channel Model

The WSSUS channel model is determined by a two-dimensional scattering function in terms of the echo delay due to multipath effects and the Doppler frequency due to the mobile movement. The Monte Carlo based approximation [2] assumes that the time-varying channel is composed of M independent echoes. Each echo corresponds to a phase ϕ_n, a delay τ_n and a Doppler frequency f_{D_n}, where ϕ_n, τ_n and f_{D_n} are continuous, mutually independent random variables (RVs), $1 \le n \le M$. The pdfs of τ_n and f_{D_n} are shown to be proportional to the delay power spectrum and the Doppler power spectrum, respectively [2]. We use exponential distribution to model the delay power profile and the Clarke's spectrum [4, chap. 4, pp. 177-181] to model the Doppler power spectrum, whereas the phase ϕ_n is uniformly distributed from 0 to 2π. Then the instantaneous channel impulse response at time t to an impulse input at time $t - \tau$ is approximated as

$$h(\tau,t) = \lim_{M\to\infty} \frac{1}{\sqrt{M}} \sum_{n=1}^{M} e^{j(\phi_n + 2\pi f_{D_n} t)} \cdot \delta(\tau - \tau_n) \cdot \tag{1}$$

To derive the discrete-time channel representation, we denote the components of the overall baseband model as follows: $\{x_k\}$ is the data sequence, $g(t)$ and $g^*(-t)$ are the time-invariant impulse responses of the transmitter and receiver, $\tilde{n}(t)$ is a complex AWGN process, T is the symbol duration and $\{\tilde{z}_k\}$ is the sampled output. The time-varying overall transmitter-receiver plus channel impulse response is then given as

$$h_{total}(\tau,t)=\lim_{M\to\infty}\frac{1}{\sqrt{M}}\sum_{n=1}^{M}e^{j(\phi_n+2\pi f_{Dn}t)}\cdot g_{total}(\tau-\tau_n), \quad (2)$$

where $g_{total}(t)=g(t)\otimes g^*(-t)$.

For a frequency non-selective, slow fading environment without inter-symbol interference (ISI), the channel output at instant $t = kT$ given x_k is transmitted is

$$\tilde{z}_k=\tilde{h}_k x_k+\tilde{n}_k, \quad (3.1)$$

with

$$\tilde{h}_k=h_{total}(0,kT)=\lim_{M\to\infty}\frac{1}{\sqrt{M}}\sum_{n=1}^{M}e^{j(\phi_n+2\pi f_{Dn}kT)}\cdot g_{total}(-\tau_n), \quad (3.2)$$

where $x_k=(-1)^{d_k}=(-1)^{b_k\oplus d_{k-1}}\in\{+1,-1\}$ is an interleaved code bit, $b_k, d_k\in GF(2)$. And $\tilde{h}_k$ is a complex, zero-mean Gaussian process with Rayleigh distributed amplitude A and uniformly distributed phase θ. Also, $\tilde{n}_k=n_{k,r}+jn_{k,i}$ is a sample of the noise process $\tilde{n}(t)\otimes g^*(-t)$, modeled as a zero-mean, complex Gaussian RV with independent, identical components of variance $N_0/2$. The discrete-time representation is shown in Figure 2.

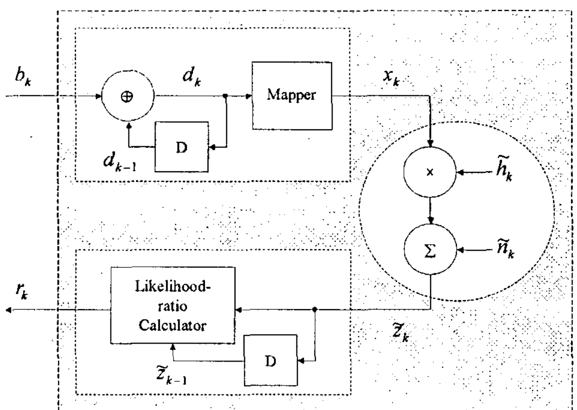

Figure 2. Discrete-time representation of the 2DPSK modulator, the WSSUS channel and the receiver.

4. Log-Likelihood Ratio

The value of received symbol is proportional to the log-likelihood ratio for BPSK transmission on AWGN channels [1]. Hence, the hard-decision reliability of the received symbol is its absolute value. As a result, ordered statistics decoder operates on a block of received sequence directly for such systems. In our case, the likelihood ratio has to be re-derived to cope with different channel characteristics and transmission strategy. Because successive tap gains, $\tilde{h}_{k-1}$, $\tilde{h}_k$, $\tilde{h}_{k+1}$, etc., are highly correlated due to slow fading, we assume that the channel tap gain $\tilde{h}_k=A\cdot e^{j\theta}$ at time $t = kT$ equals to the previous tap gain $\tilde{h}_{k-1}$. That is, $\tilde{z}_{k-1}=A\cdot e^{j\theta}\cdot x_{k-1}+\tilde{n}_{k-1}$, and $\tilde{z}_k=A\cdot e^{j\theta}\cdot x_k+\tilde{n}_k$. It is derived that

$$\begin{cases}\tilde{n}_k-\tilde{n}_{k-1}=\tilde{z}_k-\tilde{z}_{k-1} & \text{for } x_k=x_{k-1}\\ \tilde{n}_k+\tilde{n}_{k-1}=\tilde{z}_k+\tilde{z}_{k-1} & \text{for } x_k\neq x_{k-1}\end{cases}. \quad (4)$$

Let $\tilde{z}_{k-1}$ and $\tilde{z}_k$ denote the received random symbols at time index k-1 and k, and define event $E=\{\tilde{z}_k<\tilde{Z}_k<\tilde{z}_k+d\tilde{z},\tilde{z}_{k-1}<\tilde{Z}_{k-1}<\tilde{z}_{k-1}+d\tilde{z}\}$, where $d\tilde{z}$ is a very small deviation in the complex plane. Thus, the log-likelihood ratio of the differentially encoded bit is given by

$$L(\tilde{z}_k,\tilde{z}_{k-1})=\ln\frac{P(E\mid X_k=X_{k-1})}{P(E\mid X_k\neq X_{k-1})}, \quad (5.1)$$

$$=\ln\frac{p_{\tilde{N}_k-\tilde{N}_{k-1}}(\tilde{z}_k-\tilde{z}_{k-1})}{p_{\tilde{N}_k+\tilde{N}_{k-1}}(\tilde{z}_k+\tilde{z}_{k-1})}, \quad (5.2)$$

$$\propto \operatorname{Re}(\tilde{z}_k^*\tilde{z}_{k-1}), \quad (5.3)$$

where X_{k-1} and X_k are the transmitted random symbols and $p_{\tilde{N}_k\pm\tilde{N}_{k-1}}(\cdot)$ denotes the pdfs of the noise RVs $\tilde{N}_k\pm\tilde{N}_{k-1}$. Consequently, the associated reliability measure is defined as $|r_k|$, where

$$r_k=\operatorname{Re}(\tilde{z}_k^*\tilde{z}_{k-1}). \quad (6)$$

With the new reliability measure due to 2DPSK modulation, the ordered statistics decoding is employed.

5. Ordered Statistics Decoding

Ordered statistics decoding reduces search space for maximum likelihood decoding (MLD) performance by taking advantage of the reliability information from the received symbols. To illustrate the idea, consider the binary transmission system described in Section 3 where a binary (N, K, d_H) linear block code C is used for error control over the discrete-time channel. For each block of K input bits, a codeword $\bar{c}=(c_1, c_2,\ldots, c_N)$ in C is generated at the encoder output, where c_i is an element of $GF(2)$, $1\leq i\leq N$. The $\{b_k\}$ sequence is formed by interleaving successive codewords. With 2DPSK modulation, the sequence is differentially encoded, mapped into the bipolar sequence $\{x_k\}$ and sent through the channel. At the output of the likelihood-ratio calculator, the corresponding real number sequence $\{r_k\}$ is received. By deinterleaving $\{r_k\}$, the block of received symbols $\bar{r}$, associated with the transmitted codeword $\bar{c}$, is obtained, where $\bar{r}=(r_1, r_2,\ldots, r_N)$. The components of $\bar{r}$ are independent for sufficient degree of interleaving. The hard-decision of each symbol r_i is based on the sign of r_i, whereas the associated reliability is determined by $|r_i|$.

For each $\bar{r}$, the ordered statistics decoder performs two permutations (λ_1, λ_2), followed by two decoding steps, i.e., order-0 and order-l decoding, where l is an integer, $l > 0$. λ_1 reorders the components of each $\bar{r}$ based on their reliabilities, while λ_2 reorders them again to find the K

most reliable independent (MRI) positions [1]. Thereafter, Order-0 decoding constructs a codeword corresponding to the hard decision of the MRI positions. This codeword is expected to have as few information bits in error as possible. Furthermore, order-l decoding improves the result obtained from order-0 decoding progressively until the asymptotically optimum error performance is achieved. For codes and channel signal-to-noise ratios (SNRs) of practical interests, the optimum codeword candidate $\bar{c}^*$,

$$\bar{c}^* = \arg \max_{\bar{c}} \sum_{i=1}^{N} (-1)^{c_i} r_i \,, \tag{7}$$

will most likely be found at a small value of l [4].

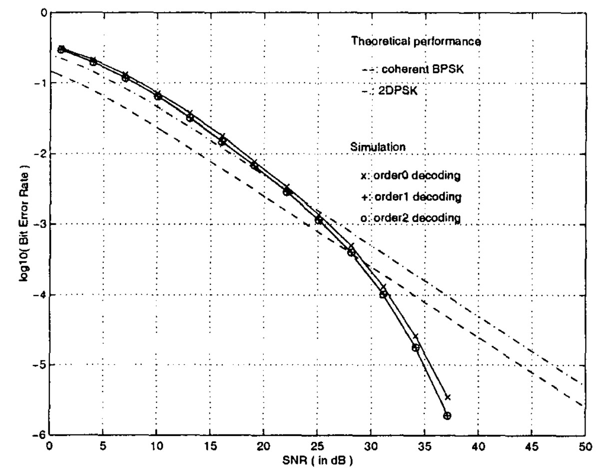

Figure 3. Performance of the (24, 12, 8) extended Golay code with interleaving degree 1.

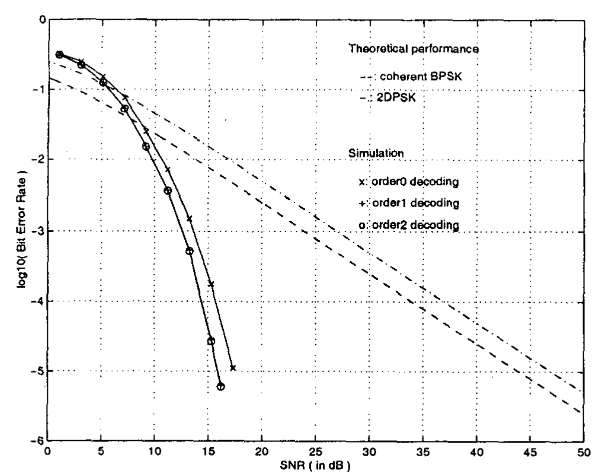

Figure 4. Performance of the (24, 12, 8) extended Golay code with interleaving degree 40.

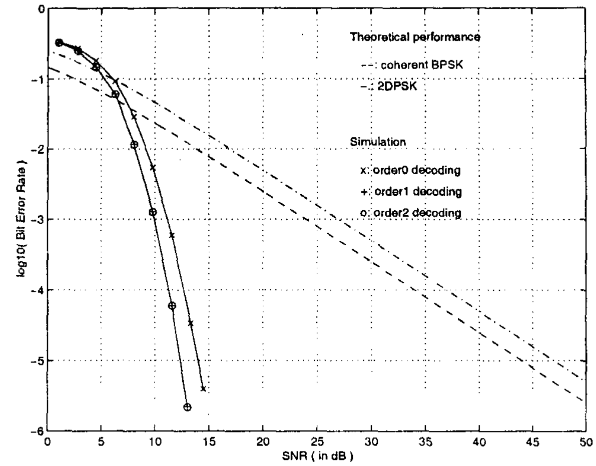

Figure 5. Performance of the (24, 12, 8) extended Golay code with interleaving degree 200.

6. Simulation Results

Simulation of the mobile radio channel in this study is based on a number of parameters. The number of echoes N is 50 with a mean delay spread of 10^{-6} second. The maximum Doppler frequency due to vehicle movement is 80 Hz (assuming mobile speed 60 mph and carrier frequency 900 MHz). And the rolloff factor of the raised cosine pulse shaping filter is chosen as 0.5. The target data rate is set at 32 kbit/s. The (24, 12, 8) extended Golay code and the (128, 64, 22) extended BCH code are used for error control with various interleaving degrees v.

Figures 3 to 8 depict the error performance of the (24, 12, 8) extended Golay code and the (128, 64, 22) extended BCH code with various v's. For each code and a particular v, simulation results for various orders of decoding are plotted in terms of BER vs. SNR, defined as $E_b < A^2 > / N_0$, where E_b is the energy per information bit, $< A^2 >$ is the time average of the Rayleigh amplitude. For both codes considered, increasing error performance is observed with increasing v. For the Golay code, Figures 3 to 5 demonstrate that order-1 decoding already achieves the practically optimum performance, no significant improvement can be obtained with higher order of decoding. For the BCH code, Figures 6 to 8 show that order-3 decoding is practically optimum.

At BER 10^{-5}, order-1 decoding of the Golay code with v equal to 1, 40, 200 has 12, 31 and 34.5 dB coding gain, respectively, compared to uncoded 2DPSK. Note that $v = 1$ is equivalent to the case where no interleaver is utilized. On the other hand, order-3 decoding of the BCH code with v equal to 1, 10, 50 achieves 18, 25.5 and 28.5 dB coding gain over uncoded 2DPSK at BER 10^{-4}.

For the decoding of the Golay code with v = 200, each received r_k is separated from previous symbol r_{k-1} by 200/32000 sec = 6.25 ms, which is half of the coherence time ($\approx$ 1/80 sec = 12.5 ms) of the channel. For the decoding of the BCH code with v = 50, the separation between two successive received symbols within a block is 50/32000 sec = 1.56 ms. Hence for both cases, only a small fraction of successive received symbols within a block are affected together by the channel. Simulation results show that no significant performance improvement can be obtained in both cases by increasing v.

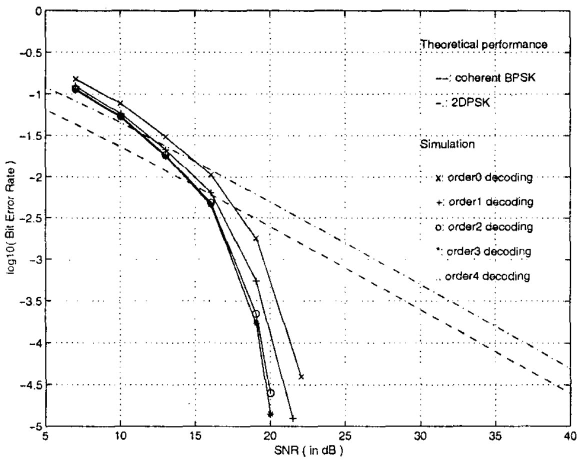

Figure 6. Performance of the (128, 64, 22) extended BCH code with interleaving degree 1.

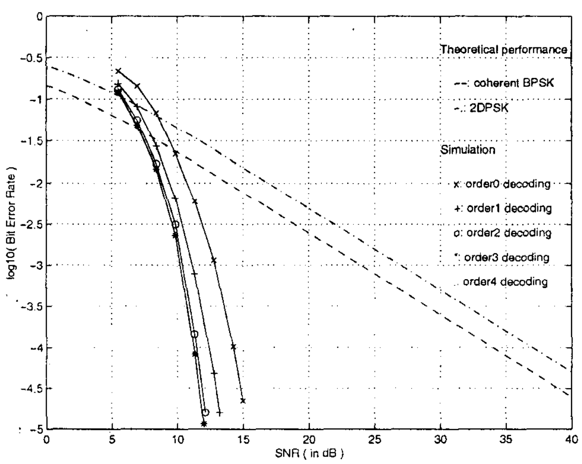

Figure 7. Performance of the (128, 64, 22) extended BCH code with interleaving degree 10.

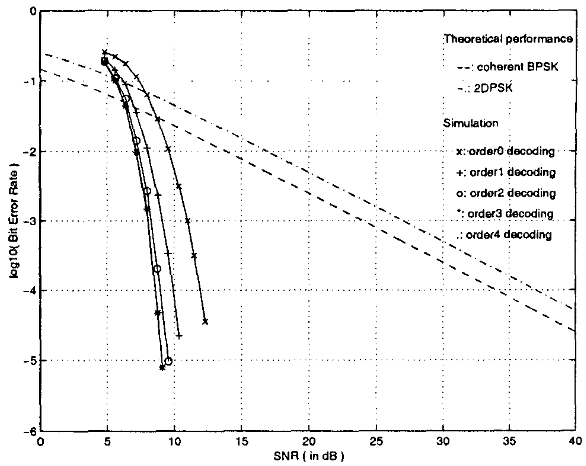

Figure 8. Performance of the (128, 64, 22) extended BCH code with interleaving degree 50.

7. Conclusion

In this study, ordered statistics decoding is applied to the WSSUS multipath channel to protect data transmitted at 32 kbit/s. Simulations are conducted over the channel with 2DPSK transmission for ordered statistics decoding of the (24, 12, 8) extended Golay code and the (128, 64, 22) extended BCH code. The log-likelihood ratio due to 2DPSK and the WSSUS channel is derived. Results demonstrate that the purpose of error protection over the WSSUS channel can be effectively achieved by combining an interleaver with ordered statistics decoding of channel codes. This is important for delay constrained applications for which ARQ is not feasible due to intolerable round trip delay.

References

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