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An Approach to Simultaneous Control of Trajectory and Interaction Forces in Dual-Arm Configurations

Xiaoping Yun and Vijay R. Kumar

Abstract—**Multiple arm systems, multifingered grippers, and walking vehicles all have two common features. In each case, more than one actively coordinated articulation interacts with a passive object, thus forming one or more closed chains. For example, when two arms grasp an object simultaneously, the arms together with the object and the ground (base) form a closed chain. This induces kinematic and dynamic constraints and the resulting equations of motion are extremely nonlinear and coupled. Furthermore, the number of actuators exceeds the kinematic mobility of the chain in a typical case, which results in an underdetermined system of equations. An approach to control such constrained dynamic systems is described in this short paper. The basic philosophy is to utilize a minimal set of inputs to control the trajectory and the surplus inputs to control the constraint or interaction forces and moments in the closed chain. A dynamic control model is derived for the closed chain that is suitable for designing a controller, in which the trajectory as well as the interaction forces and moments are explicitly controlled. Nonlinear feedback techniques derived from differential geometry are then applied to linearize and decouple the nonlinear model. In this paper, these ideas are illustrated through a planar example in which two arms are used for cooperative manipulation. Results from a simulation are used to illustrate the efficacy of the method.**

I. Introduction

A. General

There are many tasks that require cooperative manipulation by two or more robot manipulators. In applications such as lifting a heavy object or assembling mating parts, the two manipulators must directly interact with each other. Any two manipulators, together with the grasped object(s), form a closed kinematic chain. In such a situation, the two manipulators are kinematically and dynamically constrained and the resulting dynamic equations of motion are extremely nonlinear and coupled. The control problem is further complicated since in a typical case, the number of actuators available far exceeds the mobility of the system (dimension of the task space). This scenario also occurs in a multifingered gripper, in which multiple fingers are used for fingertip grasping, and also in a walking vehicle where multiple legs are used to "manipulate" the vehicle body relative to the ground.

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X. Yun is with the Department of Computer and Information Science, University of Pennsylvania, Philadelphia, PA 19104.
V. R. Kumar is with the Department of Mechanical Engineering and Applied Mechanics, University of Pennsylvania, Philadelphia, PA 19104.

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B. Previous Work

In all the examples just discussed, the two key features are the closed chains that impose kinematic and dynamic constraints in the control equations and the redundancy in actuation. These characteristics have led to the development of a variety of control schemes for force control and hybrid control. With reference to multifingered grippers, the problem of static interdeterminacy and underdetermined nature of the equations of motion have been studied by Holzmann and McCarthy [3], Yoshikawa and Nagai [24], Hollerbach and Narasimhan [2], Kerr and Roth [5], and Li and Sastry [12]. The same problem in legged locomotion systems has been studied by Klein and Chung [7], and Kumar and Waldron [10]. However, in most of these works, the focus has been on optimizing contact conditions in a static, or at best quasistatic, mode of operation. Methods based on generalized inverses and linear programming were found to be effective.

The simplest approach to dual-arm control was based on a resolved motion rate control scheme in which manipulator dynamics was ignored and the inverses of the two Jacobian matrices were used to determine the joint velocities in response to specified end-effector trajectory [14]. Such an analysis is adequate for a static or at best quasistatic mode of operation since it does not account for dynamic coupling between the manipulators. The idea of hybrid position/force control was extended to the multi-arm case by Hayati [1]. Based on the equations of a motion constraint coordinate frame located at the manipulated object, a hybrid controller was designed for the coordination of multiple robots to ensure load sharing. More complete mathematical treatments can be found: a method for obtaining the dynamic equations of motion is described in [23]; the dynamic control problem has been analyzed in [21]; and a set of holonomic constraint equations relating positions, velocities, and accelerations have been derived and a method to compute joint torques with the aid of holonomic constraint equations has been developed by Zheng and Luh [15], [25]. Dynamic coordinated control is studied in [22] in which two control formulations are proposed: the closed kinematic chain formulation and the force feedback formulation. The first approach abandons the dichotomy of two arms, whereas the later emphasizes it. The former method may be more useful in tasks in which manipulators rigidly grasp the object, whereas the later could be preferred in tasks requiring loose coupling of manipulators. More recently, coordinated motion of two planar robots has been studied by Hemami and his co-workers [11]. In this work, the forces between the manipulators and the object are predicted from the model of the system and the current state, and linear state feedback is used for stabilization and control.

A major shortcoming of all these methods is that they either do not address the force distribution (load balancing) issue directly or they involve *a priori* specification of the force distribution to combat the redundancy in the system. In the former case, the trajectory errors determine the force distribution that can result in large internal forces. This is also true when dual arm systems are treated as a master arm (leader) and a slave arm (follower). In the later case, the fraction of load on a particular actuator or arm is specified quite arbitrarily. On the other hand, Orin and Oh [18] describe an optimization method for computing the load distribution in robotic systems with closed chains. A linear program was used by them to solve the problem effectively, but the computational time was prohibitive. Other algorithms for optimal force distributions can be found on the literature on multifingered grippers and walking vehicle (see [9] for a list of references). These algorithms have been generalized to multiple robotic systems interacting with a common object [6], [8], [17]. Instead of determining optimal force distributions in order to specify the forces exerted by each of the manipulators on the object, it may be more practical and meaningful to actually control the internal forces in order to improve system performance. This is because high internal forces can crush the grasped object, whereas low internal forces can result in the object slipping [13], [16]. This is also reflected in a recent report by Pittelkau [19] in which a load-sharing force controller for two-armed manipulation that uses potential difference (PD) feedback of interaction forces was developed.

In the formulation of the control method, the equations governing the force distribution are algebraic, whereas the state equations are differential equations due to the rigid body assumption. The algebraic nature of the governing equations for forces leads to potentially unstable situations if the time delay caused by finite sampling rate is significant. Very little emphasis has been placed on *explicit control* of the force distribution.

C. Scope and Methodology

In this paper, we study the coordinated control of mechanisms with redundantly actuated closed chain. In particular, we consider manipulations by two-armed planar manipulators. We derive a dynamic control model of the system suitable for designing a controller in which the position of the grasped object and the constraint forces or the interaction forces between the manipulators are explicitly controlled. The interaction forces are similar to internal forces as defined by Mason and Salisbury [16]. (A more formal definition follows in Section II.) Nonlinear feedback techniques are then applied to linearize and decouple the nonlinear dynamic model. Standard techniques available for linear systems are employed to design the controller.

We consider, as an example, a planar case with two 2-R robots. The complete system can be kinematically modeled as a five-bar linkage. Results of a computer simulation on this model are presented in support of this coordination scheme.

II. Dynamic Model

We model the system as a closed kinematic chain. The kinematics of the two grippers and the grapsed object is modeled as a revolute pair. This is valid if the size of the grasped object is small in comparison to the link lengths and the interaction between the two arms can be reduced to a pure force. The links are assumed to be massless in comparison to the mass of the manipulated object. This is realistic in applications in which the two arms are used to lift a large mass. Thus the system is modeled by a five-bar linkage (as shown in Fig. 1) which has a mobility of two. That is, the task space is two-dimensional. Since the number of actuators is equal to four, we have a redundancy in the control problem.

The dynamic equations of motion can be easily obtained using Lagrange multipliers to account for the constraints induced by the closed chain:

$$H(\theta)\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix} + C(\theta,\dot{\theta}) + G(\theta) = \begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} + \Gamma_1^T\begin{bmatrix}\lambda_1\\ \lambda_2\end{bmatrix} \tag{1}$$

$$\begin{bmatrix}\tau_3\\ \tau_4\end{bmatrix} = \Gamma_2^T\begin{bmatrix}\lambda_1\\ \lambda_2\end{bmatrix} \tag{2}$$

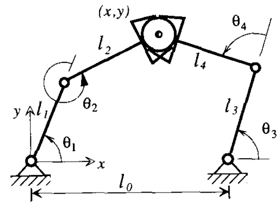


Fig. 1. Manipulation with two planar arms.

where H is the 2×2 inertia matrix, C is the 2×1 vector of Coriolis and centrifugal forces, G is the 2×1 vector of gravitational forces, τ_1, τ_2, τ_3, and τ_4 are the four joint torques, λ_1 and λ_2 are the two Lagrange multipliers, and Γ_1 and Γ_2 are the Jacobian matrices for the left and right arms, respectively. We consider the special case in which $l_0 = l_1 = l_2 = l_3 = l_4 = l$. If C_i, S_i, C_{ij}, and S_{ij} are used to denote $\cos\theta_i$, $\sin\theta_i$, $\cos(\theta_i + \theta_j)$ and $\sin(\theta_i + \theta_j)$, respectively, and the mass of the object is m, then

$$H = ml^2\begin{bmatrix}2(1 + C_2) & 1 + C_2\\ 1 + C_2 & 1\end{bmatrix}$$

$$G = mgl\begin{bmatrix}C_1 + C_{12}\\ C_{12}\end{bmatrix}$$

$$C = ml^2\begin{bmatrix}-2\dot{\theta}_1\dot{\theta}_2 S_2 - \dot{\theta}_2^2 S_2\\ \dot{\theta}_1^2 S_2\end{bmatrix}$$

$$\Gamma_1 = l\begin{bmatrix}-(S_1 + S_{12}) & -S_{12}\\ C_1 + C_{12} & C_{12}\end{bmatrix}$$

$$\Gamma_2 = l\begin{bmatrix}-(S_3 + S_{34}) & -S_{34}\\ C_3 + C_{34} & C_{34}\end{bmatrix}.$$

λ_1 and λ_2 are related to the constraint forces in the system. Referring to Fig. 2, if F_1 and F_2 are forces exerted on the object by the left and the right arm, respectively,

$$\lambda_1 = F_{2x} \qquad \lambda_2 = F_{2y}. \tag{3}$$

Equations (1)–(3) can be written compactly in the form:

$$\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix} = A\tau + B \tag{4}$$

where

$$A = \left[H^{-1}\ H^{-1}\Gamma_1^T(\Gamma_2^T)^{-1}\right] \qquad (\text{a } 2 \times 4 \text{ matrix})$$

$$B = -H^{-1}(C + G)$$

$$\tau = [\tau_1\tau_2\tau_3\tau_4]^T.$$

The interaction force F_I between the two manipulators [9] is defined as shown in Fig. 2:

$$F_I = F_1 - F_2. \tag{5}$$

Clearly, the larger interaction force components, the more the object is squeezed.

Using (3) we can rewrite this as

$$F_I = \begin{bmatrix}F_{1x} - F_{2x}\\ F_{1y} - F_{2y}\end{bmatrix} = D\tau \tag{6}$$

where D, a 2×4 matrix, is defined by

$$D = \left[(\Gamma_1^T)^{-1}\ \ -(\Gamma_2^T)^{-1}\right].$$

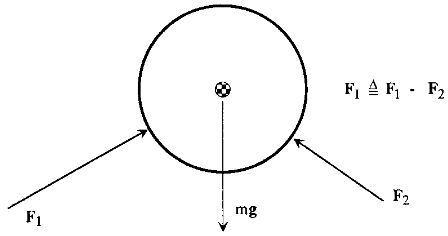


Fig. 2. Forces acting on the object.

From Fig. 2, the equations of motion for the object may be written as

$$F_1 + F_2 - mg = m\begin{bmatrix} \ddot{p}_x \\ \ddot{p}_y \end{bmatrix} \tag{7}$$

where p_x and p_y are the coordinates of the object. In this equation, there are four unknowns (F_{1x}, F_{2x}, F_{1y}, F_{2y}), but there are only two equations. At this stage, a computed torque scheme is not feasible, as for a given acceleration, we cannot determine F_1 and F_2 uniquely. Alternatively, from a different view point, we cannot uniquely determine the joint torques (inputs) from (4). The redundancy in actuation (mobility = 2, but the number of actuators = 4) can be seen in (4) and (7).

Note that if a desired interaction force is specified, the redundancy is automatically resolved. As discussed earlier, trying to maintain a desired interaction force is meaningful, since low interaction forces may result in instability of the grasp [16], whereas high interaction forces may damage the object.

In Section III, we propose a coordination scheme that exploits the redundancy in the control problem effectively, so that it is possible to control the interaction forces *as well as* the trajectory.

III. Nonlinear Feedback Control

In Section II we derived the motion equations of the closed kinematic chain formed by the two manipulators. The objective here is to design a control system to control the trajectory as well as the interaction forces in the system. The equations of motion are nonlinear and coupled and the need to control the interaction forces further complicates the system of equations. In the past, linearization of the model about an operating point has been used to reduce the design of the controller to determining a linear state feedback [11], leading to unacceptable performance at points far away from the operating point. We propose a method that is more exact and therefore more robust.

An effective way of attacking control problems in robotics is to simplify—or more precisely—linearize, the motion equations by using nonlinear feedback. The computed torque method designed for position control of robot manipulators achieves linearization by simply canceling the nonlinearity in the motion equations. In this case, however, both position and force are to be controlled and it is not easy to cancel the nonlinearity in the motion equations. Therefore, we apply systematic *nonlinear* feedback techniques to this problem—the objective is to find a nonlinear feedback to linearize the motion equations of the closed chain.

In the problem formulation, we introduce the following state variables (the rationale for the choice is explained later):

$$\begin{aligned} x_1 &= \theta_1 & x_2 &= \theta_2 \\ x_3 &= \dot{\theta}_1 & x_4 &= \dot{\theta}_2 \\ x_5 &= \tau_1 & x_6 &= \tau_2 \\ x_7 &= \tau_3 & x_8 &= \tau_4. \end{aligned}$$

We also use the following block notation

$$\begin{aligned} x^1 &= \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ x^2 &= \begin{bmatrix} x^3 \\ x^4 \end{bmatrix} \\ x^3 &= \begin{bmatrix} x_5 & x_6 & x_7 & x_8 \end{bmatrix}^T \\ x &= \left[(x^1)^T \;\; (x^2)^T \;\; (x^3)^T \right]^T. \end{aligned}$$

Using the notation above, the motion equations (4) of the two manipulators can be written as

$$\dot{x} = \begin{bmatrix} \dot{x}^1 \\ \dot{x}^2 \\ \dot{x}^3 \end{bmatrix} = \begin{bmatrix} x^2 \\ A(x^1)x^3 + B(x^1, x^2) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ I_4 \end{bmatrix} u. \tag{8}$$

More compactly,

$$\dot{x} = f(x) + gu \tag{9}$$

where $f(x)$ and g can be easily identified, matrices A and B are defined in Section II, and $u(=\dot{x}^3)$ is the reference input to the system. To control the position of the object and the forces applied by manipulators, output equations must contain quantities representing the position and the forces. In our case we choose θ_1 and θ_2 in order to control position, and $F_{1x} - F_{2x}$, and $F_{1y} - F_{2y}$, the two interaction force components. That is, the output equations are

$$y = \begin{bmatrix} h_1(x^1) \\ h_2(x^1) \\ h_3(x) \\ h_4(x) \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ D_1(x^1)x^3 \\ D_2(x^1)x^3 \end{bmatrix} = \begin{bmatrix} h^1(x^1) \\ h^2(x) \end{bmatrix} \tag{10}$$

where

$$h^1 = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \qquad h^2 = \begin{bmatrix} h_3 \\ h_4 \end{bmatrix}$$

and D_1 and D_2 are the two rows of the matrix D defined in the previous section.

By writing the motion equations in the form of (9), we have introduced an integrator in each input channel. The reason for doing so is to eliminate the direct input terms in output equations as shown below. Nonlinear feedback techniques are developed for systems whose outputs depend only on states. Since we had included forces in the output equations, the expressions for the output contained terms which were (direct) functions of the torques. We enlarged the state space by introducing integrators in input channels so that our output equation (10) depended only on the state. This now allows us to use standard algorithms to derive the required nonlinear feedback for system linearization.

We now have an affine nonlinear system described by the state equation, (9), and output equation, (10). Nonlinear feedback techniques may be applied to linearize the system if it is known that the system *is*, in fact, linearizable. There are necessary and sufficient conditions regarding linearizability to affine nonlinear systems [4], [20] that can be checked for the present system. However, these conditions are very tedious. Instead, we directly derive a nonlinear feedback and verify that the nonlinear feedback *does* indeed linearize the system.

The derivation of the nonlinear feedback requires the computation of the so-called decoupling matrix. We first define the notation of Lie derivatives. If $f(x)$ is a vector field and $h(x)$ is a function, the

Lie derivative of h along f is defined as [4]

$$L_f h = \frac{\partial h}{\partial x} f.$$

The second-order Lie derivative of h along f is

$$L_f^2 h = \frac{\partial L_f h}{\partial x} f.$$

The higher order Lie derivatives are defined in the same way. The relative degree of the system is defined as [4]

$$\rho_i = \min\left\{s \mid L_g L_f^{s-1} h_i \neq 0\right\}, \qquad i = 1, 2, 3, 4. \tag{11}$$

The decoupling matrix of the system is then defined by [4]

$$\Phi(x) = \begin{bmatrix} L_g L_f^{\rho_1-1} h_1 \\ \vdots \\ l_g L_f^{\rho_4-1} h_4 \end{bmatrix}. \tag{12}$$

We carry out the computation of the decoupling matrix for the two manipulators as follows.

$$L_g h^1 = \frac{\partial h^1}{\partial x} g = 0$$

$$L_f h^1 = \frac{\partial h^1}{\partial x} f = x^2$$

$$L_g L_f h^1 = \frac{\partial L_f h^1}{\partial x} g = 0$$

$$L_f^2 h^1 = \frac{\partial L_f h^1}{\partial x} f = A(x^1)x^3 + B(x^1, x^2)$$

$$L_g L_f^2 h^1 = \frac{\partial L_f^2 h^1}{\partial x} g = A(x^1)$$

$$L_g h^2 = \frac{\partial h^2}{\partial x} g = D(x^1).$$

Therefore, the decoupling matrix for the present system is

$$\Phi(x) = \begin{bmatrix} L_g L_f^2 h^1 \\ L_g h^2 \end{bmatrix} = \begin{bmatrix} A(x^1) \\ D(x^1) \end{bmatrix}. \tag{13}$$

Having obtained the decoupling matrix, the required nonlinear feedback is [4] (see Fig. 3 for a block diagram)

$$u = \alpha(x) + \beta(x)v \tag{14}$$

with $\alpha(x)$ and $\beta(x)$ being defined by

$$\Phi(x)\alpha(x) = -\begin{bmatrix} L_f^3 h^1 \\ L_f h^2 \end{bmatrix} \tag{15}$$

$$\Phi(x)\beta(x) = I. \tag{16}$$

As long as the decoupling matrix $\Phi(x)$ is nonsingular, $\alpha(x)$ and $\beta(x)$ are well defined. To simultaneously achieve output decoupling, we need to transform the state space by using a diffeomorphic transformation [4], which, in our case, is defined as

$$\begin{aligned} z &= T(x) \\ &= \begin{bmatrix} h_1 & L_f h_1 & L_f^2 h_1 & h_2 & L_f h_2 & L_f^2 h_2 & h_3 & h_4 \end{bmatrix} \end{aligned} \tag{17}$$

where z is the new state variables for the state space in which the system will be linear and output decoupled. Applying the nonlinear feedback (14) and employing the nonlinear transformation (17), the motion equations of the two manipulators are converted into the following linear and decoupled systems.

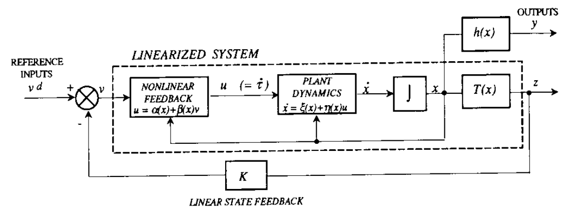


Fig. 3. Block diagram of nonlinear feedback control of the two-arm closed chain.

$$z^1 = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \tag{18}$$

$$\dot{z}^1 = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v_1 \tag{19}$$

$$y_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} z^1 \tag{20}$$

$$z^2 = \begin{bmatrix} z_4 \\ z_5 \\ z_6 \end{bmatrix} \tag{21}$$

$$\dot{z}^2 = \begin{bmatrix} \dot{z}_4 \\ \dot{z}_5 \\ \dot{z}_6 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_4 \\ z_5 \\ z_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v_2 \tag{22}$$

$$y_2 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} z^2 \tag{23}$$

$$\dot{z}_7 = [0]z_7 + [1]v_3 \tag{24}$$

$$y_3 = z_7 \tag{25}$$

$$\dot{z}_8 = [0]z_8 + [1]v_4 \tag{26}$$

$$y_4 = z_8. \tag{27}$$

To verify that we do obtain the linear system above after applying the nonlinear feedback, let us compute the derivative of the state z with respect to time.

$$\begin{aligned} \dot{z}^1 &= \frac{d}{dt}\begin{bmatrix} h_1 \\ L_f h_1 \\ L_f^2 h_1 \end{bmatrix} = \frac{\partial}{\partial x}\begin{bmatrix} h_1 \\ L_f h_1 \\ L_f^2 h_1 \end{bmatrix} \frac{dx}{dt} \\ &= \frac{\partial}{\partial x}\begin{bmatrix} h_1 \\ L_f h_1 \\ L_f^2 h_1 \end{bmatrix} (f(x) + gu) \\ &= \begin{bmatrix} \dfrac{\partial h_1}{\partial x} f + \dfrac{\partial h_1}{\partial x} gu \\ \dfrac{\partial L_f h_1}{\partial x} f + \dfrac{\partial L_f h_1}{\partial x} gu \\ \dfrac{\partial L_f^2 h_1}{\partial x} f + \dfrac{\partial L_f^2 h_1}{\partial x} gu \end{bmatrix} \\ &= \begin{bmatrix} L_f h_1 \\ L^2 h_1 \\ L_f^3 h_1 \end{bmatrix} + \begin{bmatrix} L_g h_1 \\ L_g L_f h_1 \\ L_g L_f^2 h_1 \end{bmatrix} u \end{aligned}$$

$$= \begin{bmatrix} z_2 \\ z_3 \\ L_f^3 h_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ L_g L_f^2 h_1 \end{bmatrix} u$$

that is,

$$\dot{z}_1 = z_2$$
$$\dot{z}_2 = z_3$$
$$\dot{z}_3 = L_f^3 h_1 + L_g L_f^2 h_1 u.$$

Likewise, we have

$$\dot{z}_4 = z_5$$
$$\dot{z}_5 = z_6$$
$$\dot{z}_6 = L_f^3 h_2 + L_g L_f^2 h_2 u.$$

We also have

$$\dot{z}_7 = \frac{d}{dt} z_7 = \frac{dh_3}{dt} = \frac{\partial h_3}{\partial x}\frac{dx}{dt} = \frac{\partial h_3}{\partial x}\left(f(x) + gu\right)$$
$$= L_f h_3 + L_g h_3 u$$
$$\dot{z}_8 = L_f h_4 + L_g h_4 u.$$

We now write the equations for $\dot{z}_3$, $\dot{z}_6$, $\dot{z}_7$, and $\dot{z}_8$ together

$$\begin{bmatrix} \dot{z}_3 \\ \dot{z}_6 \\ \dot{z}_7 \\ \dot{z}_8 \end{bmatrix} = \begin{bmatrix} L_f^3 h_1 \\ L_f^3 h_2 \\ L_f h_3 \\ L_f h_4 \end{bmatrix} + \begin{bmatrix} L_g L_f^2 h_1 \\ L_g L_f^2 h_2 \\ L_g h_3 \\ L_g h_4 \end{bmatrix} u$$
$$= \begin{bmatrix} L_f^3 h^1 \\ L_f h^2 \end{bmatrix} + \begin{bmatrix} L_g L_f^2 h^1 \\ L_g h^2 \end{bmatrix} u$$
$$= \begin{bmatrix} L_f^3 h^1 \\ L_f h^2 \end{bmatrix} + \Phi(x)\left(\alpha(x) + \beta(x) v\right)$$
$$= \begin{bmatrix} L_f^3 h^1 \\ L_f h^2 \end{bmatrix} + \Phi(x)\left(-\Phi^{-1}(x)\begin{bmatrix} L_f^3 h^1 \\ L_f h^2 \end{bmatrix} + \Phi^{-1}(x) v\right) = v$$

which shows that the system of the two manipulators is linearized and decoupled, and the linearized system has the structure as stated above. Now the controller design for the two manipulators reduces to the familiar design problem for linear systems as shown in Fig. 3. Note that the force control loop is first order. The integrator in the loop acts as a causal filter, which enhances the stability of the system and decreases the adverse effect of time delay (large sampling interval) on the performance of the system.

IV. Simulation

A. General

We consider as an example the cooperative manipulation of a 10-kg mass with two manipulators as shown in Fig. 1. The length of each of the links is 1 m, whereas their mass is assumed to be zero. We specify interaction force components to be 1 N in the x-direction and 1.5 N in the y direction, respectively. We demonstrate the performance of the control scheme with two examples.

In the first example, we consider a case in which the desired trajectory is a straight line from (0.1, 0.1) to (0.9, 0.1) with an initial position of (0.05, 0.05) as shown in Fig. 4. (All length units are in meters.) As a second example, the manipulator is commanded to move the object in a circle with the center at (0.5, 1.55) with a radius 0.25. The initial position is (0.8, 1.55), which does not lie on the circle as shown in Fig. 10.

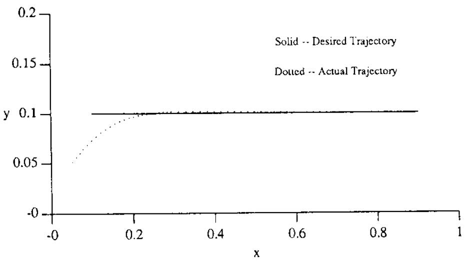


Fig. 4. Straight line trajectory.

In the next subsection, we briefly describe the rationale behind the design of the linear feedback, K (see Fig. 3). We then discuss the results of the simulation.

B. Pole Placement

As we showed earlier, application of nonlinear feedback transformed the nonlinear dynamics of the two-arm chain into a linear and decoupled system. For proper system performance, we apply a constant linear feedback to the linearized system in order to place poles in the desired locations.

The subsystems controlling the interaction forces are of first order. We simply use a proportional feedback for these two subsystems. The proportional feedback gains are taken as 300 in these two loops. The subsystems controlling position are of third order. The basic approach we adopt in designing the feedback, is to make the subsystems appear as first order systems. To this end, the three poles are placed at -10, $86.67 + j50$, $86.67 - j50$.

C. Results and Discussion

Figs. 4–10 illustrate the performance of the proposed scheme for two cases. In Fig. 4, a simple straight line trajectory is considered, whereas Fig. 10 depicts the performance for a circular trajectory. Notice that, in both cases, the control algorithm converges to the desired trajectory in spite of the fact that the actual initial starting location is not on the desired trajectory.

In the first example, the time interval for the straight line path is specified to be 1 s. As seen from Figs. 4–9, the control algorithm brings the object to the desired trajectory within one third of a second, after which the desired and actual paths are identical. The steady-state error is virtually zero. Figs. 7 and 8 show the torque requirements for the four actuators. In the simulation, limits were placed on the actuator torques to simulate a real-world system with actuators of finite capacity. This limit was chosen to be 150 Nm. Since the system starts from rest, the torque requirements are initially high. The saturation for the torque at joint 1 (at 150 Nm) can be seen from Fig. 7. Also, from Fig. 9 it can be seen that the interaction force components are maintained at the desired values, except for the first tenth of a second.

In the second example, the circular trajectory in Figs. 10–15 is traced in 2 s. The system converges to the desired trajectory within half a second after which a steady state is reached. Once more the desired interaction forces are maintained.

The system robustness and its sensitivity to modeling imperfections were tested by letting the actual mass differ from the nominal mass. Simulations were conducted by setting the actual mass 10% to 200% different from the nominal mass. In all cases, trajectory tracking errors are insensitive to the load perturbation. Results of

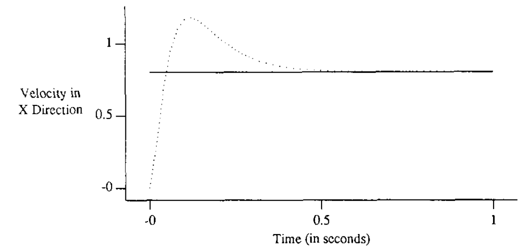


Fig. 5. Velocity in x direction for the straight line trajectory (solid line—desired trajectory, dotted line—actual trajectory).

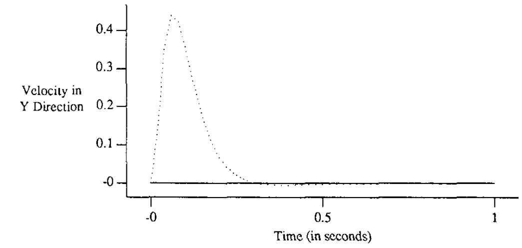


Fig. 6. Velocity in y direction for the straight line trajectory (solid line—desired trajectory, dotted line—actual trajectory).

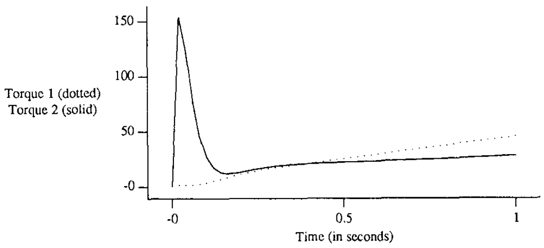


Fig. 7. Joint torques of the left arm for the straight line trajectory.

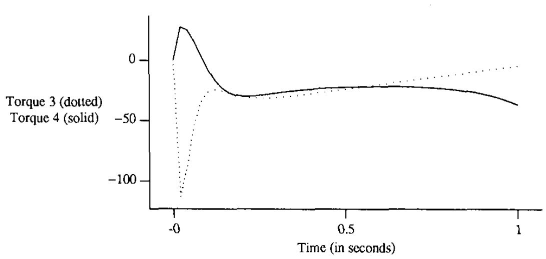


Fig. 8. Joint torques of the right arm for the straight line trajectory.

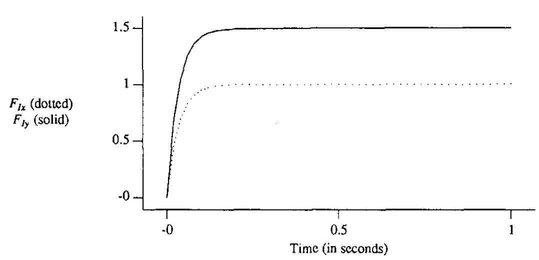


Fig. 9. Interaction forces for the straight line trajectory. The desired values of F_{Ix} and F_{Iy} are 1.0 and 1.5, respectively.

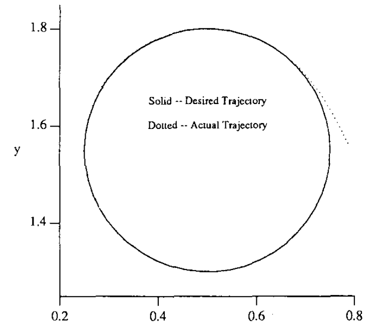


Fig. 10. Actual and desired circular trajectory.

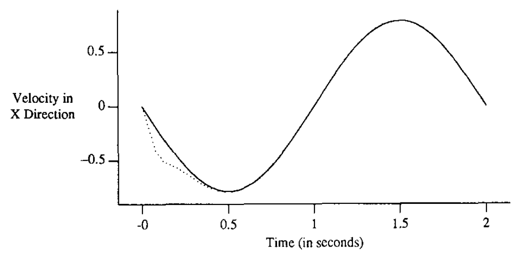


Fig. 11. Velocity in x direction for the circular trajectory (solid line—desired trajectory, dotted line—actual trajectory).

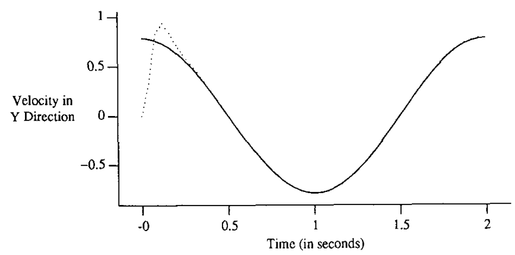


Fig. 12. Velocity in y direction for the circular trajectory (solid line—desired trajectory, dotted line—actual trajectory).

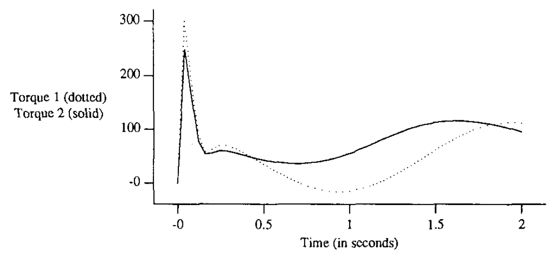
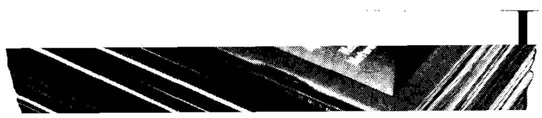


Fig. 13. Joint torques of the left arm for the circular trajectory.

such a simulation with the actual mass being 100% different from the nominal mass are seen in Figs. 16 and 17. It is evident from the figure that the difference between the commanded trajectory and the response is insignificant. However, the interaction force during the first tenth of a second becomes large as the load perturbation increases.

The first-order response for the interaction forces is to be expected, since the subsystems controlling the interaction forces are of



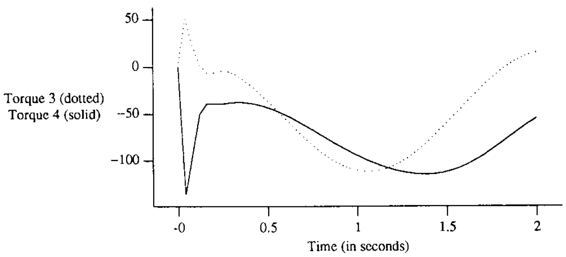


Fig. 14. Joint torques of the right arm for the circular trajectory.

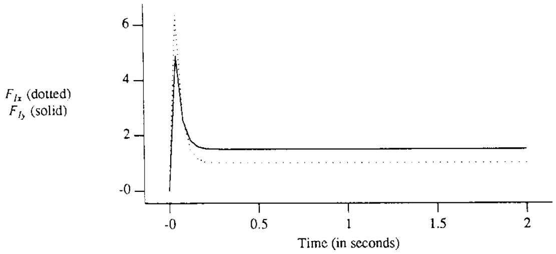


Fig. 15. Interaction forces for the circular trajectory. The desired values of F_{Ix} and F_{Iy} are 1.0 and 1.5 respectively.

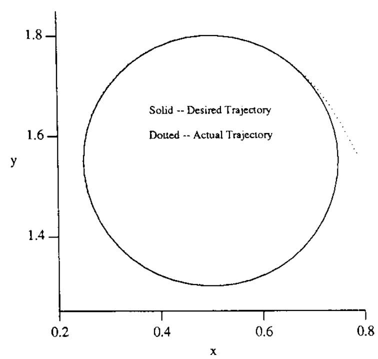


Fig. 16. Actual and desired circular trajectory with 100% error in mass of the load.

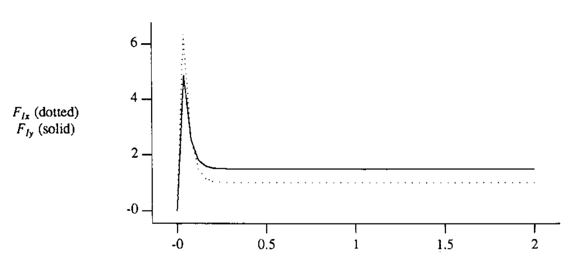


Fig. 17. Interaction forces for the circular trajectory with 100% percent error in mass of the load. Desired values of F_{Ix} and F_{Iy} are 1.0 and 1.5, respectively.

first order. Although it is somewhat less obvious from the figure, the trajectory exhibits the response of a third-order system.

V. Concluding Remarks

A coordination scheme for cooperative manipulation with two arms was presented in this paper. The kinematic and dynamic constraints together with the redundancy in actuation make the control of such a device a formidable problem. The approach presented in this paper embodies three key ideas. First, the surplus control inputs are used to control the interaction or constraint forces. In general, in a closed chain with n actuators and mobility m, only m actuators (inputs) are required to control the trajectory and the other $n-m$ actuators can be utilized to control the force distribution. Second, nonlinear feedback techniques were used to deal with the complex, nonlinear coupled model. This is in contrast to simplified linearized models with constant linear feedback that have been used in the past for problems in robotics. Finally, the explicit control of interaction forces results in a first-order system of equations, thus alleviating instability problems.

A simulation of a simplified model of a planar model has been presented to illustrate some of the advantages of this scheme. The assumption of massless links is not overly restrictive if we consider high strength to weight ratio arms, which are fast becoming a reality. Furthermore, it is possible to perform the same analysis with links with finite mass—the equations only become more cumbersome. Since this does not serve to improve our insight into the problem, we have presented a relatively simple case. Similarly, the modeling of the gripper-object interaction by a revolute joint is only for the sake of simplicity. The possibility of these equations becoming more complicated naturally brings up the point of computational loads in a single-processor environment. Currently, this problem is under investigation.

Preliminary numerical experiments performed by varying the actual mass demonstrated that the scheme is fairly insensitive to modeling imperfections. However, more work needs to be done in order to verify this. Also, the fact that the system converges to the desired trajectory from a point away from the trajectory is encouraging, once more indicative of robustness.

This work is a preliminary study on robotic system with closed chains and redundancy. The general ideas presented here could be applied to multifingered grippers, walking vehicles, or any other system with parallelism in actuation.

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