Partially Adaptive Beamforming for Correlated Interference Rejection

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***Abstract*— Conventional linearly constrained adaptive beamformers often suffer from severe signal cancellation in the presence of interferers correlated with the signal. In this paper, we propose a partially adaptive beamforming technique for correlated interference rejection in broadband signal environments. The beamformer output mean squared error is decomposed into an interference mean squared error term and an additional signal cancellation term that is due to the presence of correlated interference. Both mean squared errors depend on the adaptation space. The partially adaptive beamforming technique proposed here chooses an adaptation space which results in little signal cancellation while maintaining satisfactory interference cancellation. It is shown that, for a given interference scenario, a partially adaptive beamformer can be designed such that maximum interference cancellation is achieved without any signal cancellation. In practice, an approximate design procedure is provided to accommodate a set of likely interference scenarios. Analysis of the feasibility of this approach is presented. The effectiveness of the technique is demonstrated through examples.**

I. Introduction

IT IS well-known that signal cancellation results when conventional linearly constrained adaptive beamformers are employed in environments containing interferers that are correlated with the signal [1]. In addition to cancelling uncorrelated interferers, the beamformer uses the correlated interferers to cancel part of the desired signal in order to achieve its goal of minimum output power. This often leaves the beamformer output virtually useless. Correlated interferers can occur due to multipath propagation or "smart jamming." Successful adaptive beamforming in correlated interference environments becomes, therefore, a very important issue.

Various methods for preventing signal cancellation have been proposed [2]–[4]. The predominant approach is use of averaging to destroy the correlation between signal and interference prior to beamforming. There are two primary means to achieve this: spatial averaging and frequency-domain averaging. The spatial averaging technique [2], [3] uses a bank of subarrays. The subarray data covariance matrices are averaged to reduce the correlation between the signal and interferers. Obviously, this technique only works for uniform array structures. If the correlated interferers originate from directions close to that of the desired signal, then large numbers of subarrays or long inter-subarray displacements are necessary in order to reduce the correlation to an acceptable level. Since the effective array aperture is only that of the subarray, the beamformer's interference cancellation capability is often severely compromised. In the case of broadband signals, correlation reduction may also be carried out via averaging in the frequency domain, the so-called coherent signal subspace (CSS) technique [4]. However, the CSS transformation preprocessor that implements frequency domain averaging is dependent on preliminary estimates of signal directions. Moreover, frequency domain averaging is only applicable to situations where the time delays between the desired signal and correlated interferers are greater than half of the reciprocal of the signal bandwidth. Both averaging techniques represent a two-stage procedure: a correlation reduction preprocessing step followed by conventional beamforming.

More recently, a split-polarity transformation (SPT) technique has been proposed [5]. The SPT processor reverses the phases of the interferers in the data using *a priori* information about the interference environment. Decorrelation is achieved by averaging the covariance matrices of the original and processed data. Since the SPT processor must maintain the original phase of the desired signal while reversing the phase of the interferers, the method is effective only if the desired signal and interferers are not closely located.

Here we propose a one-stage partially adaptive beamforming approach. The beamformer output mean squared error is defined and used to evaluate the performance of a beamformer in the presence of correlated interferers. Minimum mean squared error is shown to be equivalent to minimum output power in the absence of correlated interferers. In correlated interference environments, the mean squared error consists of the usual interference component and an additional signal cancellation component. Both terms are dependent on the choice of the adaptation space. Hence, we propose choosing the adaptive degrees of freedom subject to a constraint on the maximum signal cancellation over a set of likely interference scenarios. A constructive procedure for obtaining an adaptation space that satisfies the constraint is proposed. Note that this approach does not require a uniform array structure. Time and frequency domain analyses reveal the general conditions under which partially adaptive beamformers can successfully cancel interferers while preserving the signal. The analyses show that the partially adaptive beamforming approach is only effective with broadband signals. Simulations are provided to demonstrate the utility of the partially adaptive beamforming approach. Good performance is obtained with relatively small

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spatial separations and short time delays between the signal and correlated interferers.

The paper is organized as follows. Section II introduces the class of linearly constrained minimum variance beamformers and the concept of partially adaptive beamforming. The mean squared error criterion for evaluating adaptive beamformer performance in correlated interference environments is established in Section III. In Section IV, the partially adaptive beamforming solution is developed. An analysis of this approach is provided in Section V. Examples illustrating the effectiveness of partially adaptive beamforming approach are furnished in Section VI, and a summary is given in Section VII. Throughout the paper, lower and upper case boldface symbols represent vectors and matrices respectively. Superscript H denotes complex conjugate transpose.

II. Adaptive Beamforming

A. Linearly Constrained Minimum Variance Beamforming

Let the n-dimensional vector $\boldsymbol{x}$ represent the data received at the sensor outputs (and delayed versions of the sensor outputs if FIR filters are employed). The beamformer output y is an inner product of the beamformer weight vector $\boldsymbol{w}$ and the data vector $\boldsymbol{x}$

$$y = \boldsymbol{w}^H \boldsymbol{x}. \tag{1}$$

The linearly constrained minimum variance (LCMV) criterion [6] for choosing $\boldsymbol{w}$ is

$$\min_{\boldsymbol{w}} \boldsymbol{w}^H \boldsymbol{R}_x \boldsymbol{w} \quad \text{subject to} \quad \boldsymbol{C}^H \boldsymbol{w} = \boldsymbol{g} \tag{2}$$

where $\boldsymbol{R}_x = E\{\boldsymbol{x}\boldsymbol{x}^H\}, \boldsymbol{C}$ is the $n \times m$ constraint matrix, and $\boldsymbol{g}$ is the m dimensional response vector. Minimization of output power reduces the contributions of interference and noise to the beamformer output while the m linear constraints are employed to prevent distortion of the desired signal. A generalized sidelobe canceller (GSC) representation [7] for the LCMV weight vector is

$$\boldsymbol{w} = \boldsymbol{w}_o - \boldsymbol{C}_n \boldsymbol{w}_a. \tag{3}$$

The GSC structure is illustrated in Fig. 1. Here, $\boldsymbol{w}_o$ is a non-adaptive weight vector that satisfies the constraints $\boldsymbol{C}^H \boldsymbol{w}_o = \boldsymbol{g}$. The $n \times q\,(q = n - m)$ full rank matrix $\boldsymbol{C}_n$ satisfies $\boldsymbol{C}^H \boldsymbol{C}_n = \boldsymbol{O}$. Therefore, the q dimensional adaptive weight $\boldsymbol{w}_a$ is unconstrained and the minimization problem (2) becomes

$$\min_{\boldsymbol{w}_a} (\boldsymbol{w}_o - \boldsymbol{C}_n \boldsymbol{w}_a)^H \boldsymbol{R}_x (\boldsymbol{w}_o - \boldsymbol{C}_n \boldsymbol{w}_a). \tag{4}$$

The solution to (4) is obtained as

$$\boldsymbol{w}_a = (\boldsymbol{C}_n^H \boldsymbol{R}_x \boldsymbol{C}_n)^{-1} \boldsymbol{C}_n^H \boldsymbol{R}_x \boldsymbol{w}_o. \tag{5}$$

$\boldsymbol{C}_n$ is termed the signal blocking matrix.[1] We refer to $range(\boldsymbol{C}_n)$ as the adaptation space because $\boldsymbol{w}$ can only adapt the components that lie in this space.

[1] The portion of $\boldsymbol{x}(k)$ due to the signal generally lies in the space spanned by the columns of $\boldsymbol{C}$.

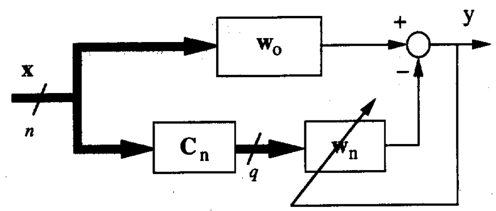

Fig. 1. The GSC beamformer implementation.

B. Partially Adaptive Beamforming

In a partially adaptive beamformer, the original adaptation space $range(\boldsymbol{C}_n)$ is mapped into a subspace through a fixed $q \times p\,(p < q)$ full-rank transformation matrix $\boldsymbol{T}$ to yield a lower dimensional adaptation space $range(\boldsymbol{T}_n)$ where

$$\boldsymbol{T}_n = \boldsymbol{C}_n \boldsymbol{T}. \tag{6}$$

A pictorial illustration of the GSC structure for a partially adaptive beamformer is obtained from Fig. 1 by replacing $\boldsymbol{C}_n$ by $\boldsymbol{T}_n$. The corresponding optimal adaptive weight vector is

$$\boldsymbol{w}_a = (\boldsymbol{T}_n^H \boldsymbol{R}_x \boldsymbol{T}_n)^{-1} \boldsymbol{T}_n^H \boldsymbol{R}_x \boldsymbol{w}_o. \tag{7}$$

In uncorrelated interference environments, partially adaptive beamformers are employed to reduce the computational complexity and improve the convergence rate of adaptive algorithms involving large number of sensor arrays [6], [8], [9].

III. Beamformer Performance Criterion

In this section, the beamformer mean squared error is defined. This mean squared error is then used to evaluate the performance of an adaptive beamformer with adaptation space $range(\boldsymbol{T}_n)$ in the presence of correlated interferers.

A. Beamformer Mean Squared Error

Let $\boldsymbol{s}$ and $\boldsymbol{n}$ be the portions of $\boldsymbol{x}$ due to the desired signal and interference respectively, i.e.

$$\boldsymbol{x} = \boldsymbol{s} + \boldsymbol{n} \tag{8}$$

then

$$\boldsymbol{R}_x = \boldsymbol{R}_s + \boldsymbol{R}_{sn} + \boldsymbol{R}_{ns} + \boldsymbol{R}_n \tag{9}$$

where

$$\begin{aligned} \boldsymbol{R}_s &= E\{\boldsymbol{s}\boldsymbol{s}^H\} \\ \boldsymbol{R}_n &= E\{\boldsymbol{n}\boldsymbol{n}^H\} \\ \boldsymbol{R}_{sn} &= \boldsymbol{R}_{ns}^H = E\{\boldsymbol{s}\boldsymbol{n}^H\}. \end{aligned}$$

The terms $\boldsymbol{R}_{sn}$ and $\boldsymbol{R}_{ns}$ represent the correlation between the desired signal and the interference.

The desired beamformer output is

$$s = \boldsymbol{w}_o^H \boldsymbol{s} = \boldsymbol{w}^H \boldsymbol{s}. \tag{10}$$

Hence, the desired output signal power is

$$P_s = \boldsymbol{w}_o^H \boldsymbol{R}_s \boldsymbol{w}_o. \tag{11}$$

The beamformer mean squared error MSE is defined as the mean squared distance between the desired signal and the beamformer output, i.e.

$$MSE = E\{|s - \boldsymbol{w}^H \boldsymbol{x}|^2\}. \tag{12}$$

An effective beamformer should exhibit small MSE. Substituting (10) and (8) into (12) results in

$$MSE = E\{|\boldsymbol{w}^H \boldsymbol{n}|^2\} = \boldsymbol{w}^H \boldsymbol{R}_n \boldsymbol{w}. \tag{13}$$

B. MSE Decomposition for Optimal Weight Vector

In order to see the impact of correlated interference on beamformer performance, the MSE associated with the optimal beamformer weight vector (7) is decomposed into two terms: the term due to the interference and an additional term due to the presence of correlation between the signal and interference.

Since the constraint matrix $\boldsymbol{C}$ is designed to provide specified response to the desired signal, $\boldsymbol{s} \in range(\boldsymbol{C}) = range^{\perp}(\boldsymbol{C}_n) \subset range^{\perp}(\boldsymbol{T}_n)$, and consequently

$$\boldsymbol{T}_n^H \boldsymbol{R}_{sn} = \boldsymbol{O}.$$

Subsititution of (9) into (7) yields

$$\boldsymbol{w}_a = (\boldsymbol{T}_n^H \boldsymbol{R}_n \boldsymbol{T}_n)^{-1} \boldsymbol{T}_n^H (\boldsymbol{R}_n + \boldsymbol{R}_{ns}) \boldsymbol{w}_o. \tag{14}$$

Note that the weight vector in (14) is decomposed into two components as

$$\boldsymbol{w}_a = \boldsymbol{w}_n + \boldsymbol{w}_s \tag{15}$$

where

$$\boldsymbol{w}_n = (\boldsymbol{T}_n^H \boldsymbol{R}_n \boldsymbol{T}_n)^{-1} \boldsymbol{T}_n^H \boldsymbol{R}_n \boldsymbol{w}_o \tag{16}$$

$$\boldsymbol{w}_s = (\boldsymbol{T}_n^H \boldsymbol{R}_n \boldsymbol{T}_n)^{-1} \boldsymbol{T}_n^H \boldsymbol{R}_{ns} \boldsymbol{w}_o. \tag{17}$$

For a given adaptation space, $\boldsymbol{w}_n$ is the optimal adaptive weight vector obtained in the absence of correlated interference. It is considered the "best" adaptive weight vector available in the sense that it provides the maximum interference cancellation without causing any signal cancellation. The term $\boldsymbol{w}_s$ is due to the presence of correlated interference. It is responsible for the signal cancellation.

As illustrated in Fig. 2, the signal and interference components at the output of the nonadaptive beamformer $\boldsymbol{w}_o$ are $s = \boldsymbol{w}_o^H \boldsymbol{s}$ and $n_o = \boldsymbol{w}_o^H \boldsymbol{n}$, respectively. The output of the adaptive branch consists of $n_n = (\boldsymbol{T}_n \boldsymbol{w}_n)^H \boldsymbol{n}$ and $n_s = (\boldsymbol{T}_n \boldsymbol{w}_s)^H \boldsymbol{n}$. Define

$$n_i = n_o - n_n = (\boldsymbol{w}_o - \boldsymbol{T}_n \boldsymbol{w}_n)^H \boldsymbol{n}. \tag{18}$$

In the absence of correlated interference, n_i is the interference portion of the beamformer output. The term n_s is the additional component due to the presence of correlated interference; it is primarily used to cancel the desired signal. One can easily verify that

$$E\{n_i^* n_s\} = 0. \tag{19}$$

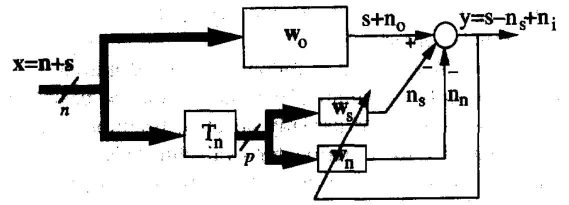

Fig. 2. Decomposition of signals in GSC structure.

Therefore, the MSE associated with the optimal adaptive weight (14) is

$$MSE = E\{|n_i - n_s|^2\} = E_i + E_s \tag{20}$$

$$E_i = E\{|n_i|^2\} \tag{21}$$

$$E_s = E\{|n_s|^2\}. \tag{22}$$

E_i and E_s are respectively termed the interference MSE and the signal cancellation MSE.[2] Let

$$P_{no} = E\{|n_o|^2\} = \boldsymbol{w}_o^H \boldsymbol{R}_n \boldsymbol{w}_o \tag{23}$$

$$P_{nn}(\boldsymbol{T}_n) = E\{|n_n|^2\} = \boldsymbol{w}_o^H \boldsymbol{R}_n \boldsymbol{T}_n (\boldsymbol{T}_n^H \boldsymbol{R}_n \boldsymbol{T}_n)^{-1} \cdot \boldsymbol{T}_n^H \boldsymbol{R}_n \boldsymbol{w}_o \tag{24}$$

$$P_{sc}(\boldsymbol{T}_n) = \boldsymbol{w}_o^H \boldsymbol{R}_{sn} \boldsymbol{T}_n (\boldsymbol{T}_n^H \boldsymbol{R}_n \boldsymbol{T}_n)^{-1} \boldsymbol{T}_n^H \boldsymbol{R}_{ns} \boldsymbol{w}_o. \tag{25}$$

It can be shown easily that

$$E_i = P_{no} - P_{nn}(\boldsymbol{T}_n) \tag{26}$$

$$E_s = P_{sc}(\boldsymbol{T}_n). \tag{27}$$

Note that P_{no} is the interference power at the nonadaptive branch output. $P_{nn}(\boldsymbol{T}_n)$ is the interference power at the adaptive branch output in the absence of correlated interference; it represents the beamformer's interference cancellation capability. $P_{sc}(\boldsymbol{T}_n)$ represents the beamformer's signal cancellation capability in the presence of correlated interference; it is equal to the signal cancellation MSE.

In uncorrelated interference environments $\boldsymbol{R}_{sn} = \boldsymbol{O}$, hence $P_{sc}(\boldsymbol{T}_n) = 0$, and the beamformer output power is

$$P_o = \boldsymbol{w}_o^H \boldsymbol{R}_s \boldsymbol{w}_o + \boldsymbol{w}^H \boldsymbol{R}_n \boldsymbol{w} = P_s + MSE. \tag{28}$$

The minimum output power is $P_o^{min} = P_s + E_i$. Therefore, minimum $MSE = E_i$ is achieved via output power minimization. However, in correlated interference environments

$$P_o = P_s + MSE + \boldsymbol{w}_o^H \boldsymbol{R}_{sn} \boldsymbol{w} + \boldsymbol{w}^H \boldsymbol{R}_{ns} \boldsymbol{w}_o. \tag{29}$$

The minimum output power is now

$$P_o^{\min} = P_s + P_{no} - P_{nn}(\boldsymbol{T}_n) - P_{sc}(\boldsymbol{T}_n) + \boldsymbol{w}_o^H \boldsymbol{R}_{sn} \boldsymbol{w}_o + \boldsymbol{w}_o^H \boldsymbol{R}_{ns} \boldsymbol{w}_o - P_{ns}(\boldsymbol{T}_n) - P_{ns}^*(\boldsymbol{T}_n) \tag{30}$$

where

$$P_{ns}(\boldsymbol{T}_n) = \boldsymbol{w}_o^H \boldsymbol{R}_n \boldsymbol{T}_n (\boldsymbol{T}_n^H \boldsymbol{R}_n \boldsymbol{T}_n)^{-1} \boldsymbol{T}_n^H \boldsymbol{R}_{ns} \boldsymbol{w}_o. \tag{31}$$

[2] E_s actually represents both the signal cancellation and changes in interference cancellation that occur as a result of correlated interference between the signal and interference. However, the signal cancellation is usually the dominant and most significant effect. Hence, we refer to E_s as the signal cancellation MSE.

Due to the presence of the cross terms, both signal and interference are suppressed when minimum output power is achieved. Hence, output power minimization does not yield small MSE in the presence of correlated interference.

IV. Partially Adaptive Beamforming Solution

The dependence of signal and interference cancellation on the adaptation space is demonstrated in the previous section. Here we propose restricting the degrees of freedom in the beamformer to limit signal cancellation capability.

A. Motivation

It is straightforward to show that

$$P_{nn}(\boldsymbol{T}_n) \le P_{nn}(\boldsymbol{C}_n) \tag{32}$$

$$P_{sc}(\boldsymbol{T}_n) \le P_{sc}(\boldsymbol{C}_n) \tag{33}$$

if $range(\boldsymbol{T}_n) \subset range(\boldsymbol{C}_n)$. That is, both signal cancellation capability and interference cancellation capability degrade when the dimension of the adaptation space is reduced. The goal is to choose the adaptation space so that the signal cancellation capability is greatly reduced while any loss in interference cancellation capability is minimized. If this goal is achieved, then the output power minimization criterion will yield a small MSE.

It is easy to show that

$$P_{nn}(\boldsymbol{T}_n) = \boldsymbol{z}_n^H \boldsymbol{P}(\boldsymbol{T}_n)\boldsymbol{z}_n \tag{34}$$

$$P_{sc}(\boldsymbol{T}_n) = \boldsymbol{z}_s^H \boldsymbol{P}(\boldsymbol{T}_n)\boldsymbol{z}_s \tag{35}$$

where

$$\boldsymbol{P}(\boldsymbol{T}_n) = \boldsymbol{R}_n^{H/2}\boldsymbol{T}_n(\boldsymbol{T}_n^H\boldsymbol{R}_n\boldsymbol{T}_n)^{-1}\boldsymbol{T}_n^H\boldsymbol{R}_n^{1/2} \text{ is the projection matrix onto } range(\boldsymbol{R}_n^{H/2}\boldsymbol{T}_n) \tag{36}$$

$$\boldsymbol{z}_n = \boldsymbol{P}(\boldsymbol{C}_n)\boldsymbol{R}_n^{H/2}\boldsymbol{w}_o \tag{37}$$

$$\boldsymbol{z}_s = \boldsymbol{P}(\boldsymbol{C}_n)\boldsymbol{R}_n^{-(1/2)}\boldsymbol{R}_{ns}\boldsymbol{w}_o. \tag{38}$$

Note that $\boldsymbol{P}(\boldsymbol{T}_n)\boldsymbol{P}(\boldsymbol{C}_n) = \boldsymbol{P}(\boldsymbol{T}_n)$ since $range(\boldsymbol{P}(\boldsymbol{C}_n)) \subset range(\boldsymbol{P}(\boldsymbol{T}_n))$. The goal of partially adaptive beamformer design is to choose $\boldsymbol{T}_n$ such that $P_{sc}(\boldsymbol{T}_n) \approx 0$ and $P_{nn}(\boldsymbol{T}_n) \approx P_{nn}(\boldsymbol{C}_n)$.

There is no signal cancellation if $\boldsymbol{P}(\boldsymbol{T}_n)\boldsymbol{z}_s = \boldsymbol{o}$. This is achieved if

$$\boldsymbol{T}_n^H\boldsymbol{R}_{ns}\boldsymbol{w}_o = \boldsymbol{o}. \tag{39}$$

Recall $\boldsymbol{T}_n = \boldsymbol{C}_n\boldsymbol{T}$, so (39) implies $\boldsymbol{T}^H\boldsymbol{C}_n^H\boldsymbol{R}_{ns}\boldsymbol{w}_o = \boldsymbol{o}$. Such a $\boldsymbol{T}$ always exists provided $p \le q-1$. Thus, only one adaptive degree of freedom must be removed from the fully adaptive beamformer to prevent signal cancellation.

Removal of this degree of freedom will generally decrease the beamformer's interference cancellation. However, while this degree of freedom must be excluded from the adaptive portion of the GSC to prevent signal cancellation, it can be included in the nonadaptive portion to enhance interference cancellation, as suggested in [10]. Let $\boldsymbol{U}_n$ be the unused adaptation space, i.e.

$$range(\boldsymbol{T}_n) \oplus range(\boldsymbol{U}_n) = range(\boldsymbol{C}_n) \tag{40}$$

where $\oplus$ denotes the direct sum of subspaces. If the partially adaptive beamformer has nonadaptive weight vector

$$\boldsymbol{w}_q = \boldsymbol{w}_o - \boldsymbol{U}_n(\boldsymbol{U}_n^H\boldsymbol{R}_m\boldsymbol{U}_n)^{-1}\boldsymbol{U}_n\boldsymbol{R}_m\boldsymbol{w}_o \tag{41}$$

where

$$\boldsymbol{R}_m = \boldsymbol{R}_n^{1/2}(\boldsymbol{I} - \boldsymbol{P}(\boldsymbol{T}_n))\boldsymbol{R}_n^{H/2}, \tag{42}$$

then it can be shown that $P_{nn}(\boldsymbol{w}_q, \boldsymbol{T}_n) = P_{nn}(\boldsymbol{w}_o, \boldsymbol{C}_n)$, while $P_{sc}(\boldsymbol{w}_q, \boldsymbol{T}_n) = P_{sc}(\boldsymbol{w}_o, \boldsymbol{T}_n)$. Hence, when the nonadaptive weight vector is chosen according to (41), fully adaptive interference cancellation is obtained with partially adaptive signal cancellation.

B. Partially Adaptive Beamformer Design

The above discussion assumes the interference scenario is known; in practice it is generally unknown. Here we use a parameterized interference model to accomodate uncertainty in the interference scenario. Let the vector $\boldsymbol{\Theta}$ parameterize the interference environment. For example, $\boldsymbol{\Theta}$ may represent the number of interferers, their locations, spectral characteristics, etc. $\boldsymbol{R}_n$ and $\boldsymbol{R}_{ns}$ are assumed to be completely determined by $\boldsymbol{\Theta}$, and their explicit dependence on $\boldsymbol{\Theta}$ is indicated by the notation $\boldsymbol{R}_n(\boldsymbol{\Theta})$ and $\boldsymbol{R}_{ns}(\boldsymbol{\Theta})$. The set of interference scenarios over which $\boldsymbol{T}_n$ is designed is represented by a discrete set $\mathcal{Q} = \{\boldsymbol{\Theta}_k, k = 1, 2, \cdots, K\}$. Here K is chosen so that the $\boldsymbol{\Theta}_k$ adequately sample the range of interference scenarios of interest.

An average mean squared error minimization partially adaptive beamformer design procedure was proposed in [11]. However, this procedure results in a very complicated optimization problem. An alternate solution is pursued here by limiting the beamformer's signal cancellation for each scenario within the set $\mathcal{Q}$. That is, we choose $\boldsymbol{T}_n$ to satisfy

$$P_{sc}(\boldsymbol{T}_n, \boldsymbol{\Theta}) \le \delta_o, \forall \boldsymbol{\Theta} \in \mathcal{Q} \tag{43}$$

for some positive δ_o. Note that δ_o is the upper bound on signal cancellation for any interference scenario in $\mathcal{Q}$. It may be chosen as a small fraction of the expected signal output power. Satisfaction of (43) generally requires removal of multiple degrees of freedom from the fully adaptive beamformer adaptation space.

The procedure proposed here for choosing a $\boldsymbol{T}_n$ that satisfies (43) is similar in spirit to the point design procedure described in [12] for partially adaptive beamformer design in noncorrelated interference environments. The strategy is to remove from the original basis of $range(\boldsymbol{C}_n)$ those basis vectors that lead to severe signal cancellation. At an interference scenario $\boldsymbol{\Theta}$ of interest, $P_{sc}(\boldsymbol{T}_n, \boldsymbol{\Theta})$ is computed using the current adaptation space $range(\boldsymbol{T}_n)$. If $P_{sc}(\boldsymbol{T}_n, \boldsymbol{\Theta})$ exceeds the specified tolerance, the adaptation space is reduced by one dimension, i.e., a column in $range(\boldsymbol{T}_n)$ is extracted. This process is carried out in a sequential order through all possible scenarios. Since $P_{sc}(\boldsymbol{T}_n, \boldsymbol{\Theta})$ only decreases as the adaptation space dimension is reduced, removal of additional columns never enhances the beamformer's signal cancellation capability.

In order to achieve condition (39) at a given interference scenario $\boldsymbol{\Theta}$, we propose removing $\boldsymbol{t}_s(\boldsymbol{\Theta}) = \boldsymbol{T}_n\boldsymbol{T}_n^H\boldsymbol{R}_{ns}(\boldsymbol{\Theta})\boldsymbol{w}_o$ from $\boldsymbol{T}_n$. Many methods can be used to remove a given basis vector from a subspace spanned by a set of basis vectors. We adopt the following QR decomposition technique. Let

$$[\boldsymbol{t}_s(\boldsymbol{\Theta}) \quad \boldsymbol{T}_n] = \boldsymbol{QR} = [\boldsymbol{q} \quad \boldsymbol{T}_n' \quad \boldsymbol{V}_n]\boldsymbol{R} \tag{44}$$

where $\boldsymbol{Q}$ is a unitary matrix and $\boldsymbol{R}$ is an upper triangular matrix. Since $\boldsymbol{R}$ is upper triangular and $\boldsymbol{t}_s(\boldsymbol{\Theta}) \neq \boldsymbol{o}$, the first column of $\boldsymbol{Q}, \boldsymbol{q}$, represents the same 1–D space as $\boldsymbol{t}_s$. $\boldsymbol{T}_n'$ contains the columns of $\boldsymbol{Q}$ corresponding to nonzero rows of $\boldsymbol{R}$ (except the first column $\boldsymbol{q}$) while $\boldsymbol{V}_n$ contains the columns of $\boldsymbol{Q}$ corresponding to zero rows of $\boldsymbol{R}$. Thus, $range(\boldsymbol{T}_n')$ is the new adaptation space. Note that the removal of $\boldsymbol{t}_s(\boldsymbol{\Theta})$ not only completely eliminates signal cancellation at scenario $\boldsymbol{\Theta}$ but also generally reduces the signal cancellation capability at neighboring scenarios. Hence, usually only a few components of the original adaptation space need to be removed in order to satisfy the signal cancellation constraint over the interference scenario set $\mathcal{Q}$.

The multilevel design procedure described in [12] may also be employed to approximately minimize the number of components removed. Define a sequence of L performance levels $\delta_1 > \delta_2 > \cdots > \delta_L = \delta_o$. We begin with δ_1 and proceed to δ_L. The performance levels are often chosen as a set of decreasing functions of the expected signal output power. As an example, we may choose $\delta_1 = 0.5\boldsymbol{w}_o^H\boldsymbol{R}_s\boldsymbol{w}_o, \delta_2 = 0.3\boldsymbol{w}_o^H\boldsymbol{R}_s\boldsymbol{w}_o$, and $\delta_3 = \delta_o = 0.1\boldsymbol{w}_o^H\boldsymbol{R}_s\boldsymbol{w}_o$. This constrains the worst case signal cancellation to be less that 10% of the signal output power. Values of L ranging from two to four generally yield satisfactory results. At each level we only remove those components that cause signal cancellation levels higher than the current performance level. This multilevel design procedure essentially removes components from the original adaptation space based on their relative signal cancellation contributions. Components which result in the most severe signal cancellation are removed first. The following is a pseudo-code description of this procedure:

$\boldsymbol{T}_n = \boldsymbol{C}_n$
for $= l = 1$ to L
  for $k = 1$ to K
    if $= P_{sc}(\boldsymbol{T}_n, \boldsymbol{\Theta}_k) > \delta_l$
      perform QR decomposition $[\boldsymbol{T}_s(\boldsymbol{\Theta}) \quad \boldsymbol{T}_n]$
      $= [\boldsymbol{q} \quad \boldsymbol{T}_n' \quad \boldsymbol{V}_n]\boldsymbol{R}$ as in (44)
      set $\boldsymbol{T}_n = \boldsymbol{T}_n'$.
    end-of-if
  end-loop-on-k
end-loop-on-l.

Next, the nonadaptive weight vector $\boldsymbol{w}_o$ is modified to improve the beamformer's interference cancellation capability over the set of likely interference scenarios. That is, we choose a new nonadaptive weight vector $\boldsymbol{w}_q$ that solves

$$\min_{\boldsymbol{w}_q} \sum_{\boldsymbol{\Theta}\in\mathcal{Q}} \boldsymbol{w}_q^H[\boldsymbol{R}_n(\boldsymbol{\Theta}) - \boldsymbol{R}_n(\boldsymbol{\Theta})\boldsymbol{T}_n(\boldsymbol{T}_n^H\boldsymbol{R}_n(\boldsymbol{\Theta})\boldsymbol{T}_n)^{-1} \cdot \boldsymbol{T}_n^H\boldsymbol{R}_n(\boldsymbol{\Theta})]\boldsymbol{w}_q \tag{45}$$

where $\boldsymbol{w}_q$ is constrained to be of the form $\boldsymbol{w}_q = \boldsymbol{w}_o - \boldsymbol{U}_n\boldsymbol{w}_m$. Here $\boldsymbol{U}_n$ is defined in (40) and $\boldsymbol{w}_m$ represents the $p - q$ available degrees of freedom in $\boldsymbol{w}_q$. Since $\boldsymbol{w}_q$ is nonadaptive, it is independent of $\boldsymbol{\Theta}$ and (45) is equivalent to the quadratic minimization problem

$$\min_{\boldsymbol{w}_m} (\boldsymbol{w}_o - \boldsymbol{U}_n\boldsymbol{w}_m)^H\boldsymbol{R}^{av}(\boldsymbol{w}_o - \boldsymbol{U}_n\boldsymbol{w}_m) \tag{46}$$

where

$$\boldsymbol{R}^{av} = \sum_{\boldsymbol{\Theta}\in\mathcal{Q}} [\boldsymbol{R}_n(\boldsymbol{\Theta}) - \boldsymbol{R}_n(\boldsymbol{\Theta})\boldsymbol{T}_n(\boldsymbol{T}_n^H\boldsymbol{R}_n(\boldsymbol{\Theta})\boldsymbol{T}_n)^{-1} \cdot \boldsymbol{T}_n^H\boldsymbol{R}_n(\boldsymbol{\Theta}). \tag{47}$$

Solving (47) for $\boldsymbol{w}_m$ gives the nonadaptive weight vector

$$\boldsymbol{w}_q = \boldsymbol{w}_o - \boldsymbol{U}_n(\boldsymbol{U}_n^H\boldsymbol{R}^{av}\boldsymbol{U}_n)^{-1}\boldsymbol{U}_n^H\boldsymbol{R}^{av}\boldsymbol{w}_o. \tag{48}$$

V. Performance Analysis

As noted in the previous section, reducing the number of adaptive degrees of freedom leads to loss in both signal and interference cancellation capabilities. In this section we assess the relative effects of partially adaptive beamforming on signal and interference cancellation from both time and frequency domain perspectives. This analysis does not consider the effects of modifying the beamformer's nonadaptive weight vector.

A. Time Domain Analysis

If $\boldsymbol{T}_n$ is chosen to satisfy (39), then

$$range(\boldsymbol{P}(\boldsymbol{C}_n)) = range(\boldsymbol{P}(\boldsymbol{T}_n)) \oplus range(\boldsymbol{z}_s). \tag{49}$$

Thus

$$\boldsymbol{z}_n = \boldsymbol{P}(\boldsymbol{C}_n)\boldsymbol{z}_n = \boldsymbol{P}(\boldsymbol{T}_n)\boldsymbol{z}_n + \frac{\boldsymbol{z}_s^H\boldsymbol{z}_n}{\boldsymbol{z}_s^H\boldsymbol{z}_s}\boldsymbol{z}_s \tag{50}$$

and

$$\begin{aligned} P_{nn}(\boldsymbol{C}_n) &= \boldsymbol{z}_n^H\boldsymbol{z}_n = \boldsymbol{z}_n^H\boldsymbol{P}(\boldsymbol{T}_n)\boldsymbol{z}_n + \frac{|\boldsymbol{z}_s^H\boldsymbol{z}_n|^2}{\boldsymbol{z}_s^H\boldsymbol{z}_s} \\ &= P_{nn}(\boldsymbol{T}_n) + P_{nn}(\boldsymbol{C}_n)\cos^2\phi_z \end{aligned} \tag{51}$$

where ϕ_z is the angle between $\boldsymbol{z}_n$ and $\boldsymbol{z}_s$. We then obtain

$$P_{nn}(\boldsymbol{T}_n) = P_{nn}(\boldsymbol{C}_n)(1 - \cos^2\phi_z) \tag{52}$$

$$P_{sc}(\boldsymbol{T}_n) = 0. \tag{53}$$

The partially adaptive beamformer experiences very little loss in interference cancellation if

$$\cos^2\phi_z \approx 0. \tag{54}$$

The success of partially adaptive beamforming depends on the relative orientation of the vectors $\boldsymbol{z}_n$ and $\boldsymbol{z}_s$. If $\boldsymbol{z}_n$ and $\boldsymbol{z}_s$ are orthogonal, then choosing $\boldsymbol{T}_n$ to satisfy (39) yields $\boldsymbol{z}_s \in range^{\perp}(\boldsymbol{P}(\boldsymbol{T}_n))$ and $\boldsymbol{z}_n \in range(\boldsymbol{P}(\boldsymbol{T}_n))$. Thus, $\boldsymbol{P}(\boldsymbol{T}_n)\boldsymbol{z}_s = \boldsymbol{o}$ and $\boldsymbol{P}(\boldsymbol{T}_n)\boldsymbol{z}_n = \boldsymbol{z}_n$. In this case, fully adaptive interference cancellation ($P_{nn}(\boldsymbol{T}_n) = P_{nn}(\boldsymbol{C}_n)$) is obtained with zero signal cancellation ($P_{sc}(\boldsymbol{T}_n) = 0$). On the other hand, if $\boldsymbol{z}_n$ and $\boldsymbol{z}_s$ are collinear, then any choice for $\boldsymbol{T}_n$

will reduce P_{nn} and P_{sc} equally and the partially adaptive approach does not yield any performance improvement in the sense that $P_{nn}(\boldsymbol{T}_n)/P_{sc}(\boldsymbol{T}_n) = P_{nn}(\boldsymbol{C}_n)/P_{sc}(\boldsymbol{C}_n)$. In practice, $\boldsymbol{z}_n$ and $\boldsymbol{z}_s$ will generally be neither orthogonal nor collinear; the partially adaptive beamformer's signal cancellation capability is completely disabled while its interference cancellation capability is reduced by the factor $1 - \cos^2 \phi_z$. Note that

$$\cos^2 \phi_z = \frac{|\boldsymbol{z}_n^H \boldsymbol{z}_s|^2}{\boldsymbol{z}_n^H \boldsymbol{z}_n \boldsymbol{z}_s^H \boldsymbol{z}_s} = \frac{|P_{ns}(\boldsymbol{C}_n)|^2}{P_{nn}(\boldsymbol{C}_n) P_{sc}(\boldsymbol{C}_n)} \tag{55}$$

where P_{ns} is defined in (31).

Recall that n_n and n_s are the components of the adaptive branch in the GSC used to cancel the interferece n_o and signal s, respectively, at the nonadaptive beamformer output. Let $r(n_n, n_s)$ be the cross correlation coefficient between n_n and n_s for a fully adaptive beamformer ($\boldsymbol{T}_n = \boldsymbol{C}_n$), i.e.

$$r(n_n, n_s) = \frac{E\{n_n^* n_s\}}{\sqrt{E\{n_n^* n_n\} E\{n_s^* n_s\}}}. \tag{56}$$

Straightforward calculation shows

$$\cos^2 \phi_z = |r(n_n, n_s)|^2. \tag{57}$$

Hence, good interference cancellation can be obtained with zero signal cancellation if the components n_s and n_n from the adaptive branch of the fully adaptive beamformer are weakly correlated. This result is intuitively satisfying. If n_n and n_s are uncorrelated, then $\boldsymbol{T}_n$ can be chosen so that only n_n passes through the adaptive branch. This results in fully adaptive interference cancellation and zero signal cancellation. In contrast, if n_n and n_s are correlated, then the portion of n_n that is correlated with n_s will not be present at the adaptive branch output when $\boldsymbol{T}_n$ is chosen to prevent signal cancellation and less of the interference at the nonadaptive beamformer output is cancelled.

If there is strong correlation between an interferer and the desired signal such that complete signal cancellation occurs($s \approx n_s$) and the beamformer attains nearly complete interference cancellation ($n_o \approx n_n$), then we obtain

$$\cos^2 \phi_z \approx \frac{|\boldsymbol{w}_o^H \boldsymbol{R}_{ns} \boldsymbol{w}_o|^2}{P_{no} P_s} \tag{58}$$

by substituting n_o and s for n_n and n_s in (56) and (57). Equation (58) indicates that the relative magnitude of $\boldsymbol{w}_o^H \boldsymbol{R}_{ns} \boldsymbol{w}_o = E\{n_o s^*\}$ is the determining factor for the potential effectiveness of the partially adaptive beamforming approach. If the correlation between n_o and s is small relative to the total power in n_o and s, then an effective partially adaptive beamformer can be designed.

B. Frequency Domain Analysis

An alternate perspective of the effectiveness of the partially adaptive beamforming approach is obtained by analyzing the capability of the adaptive branch frequency response to match the nonadaptive branch frequency response.

Assume all signals of interest lie in the frequency band in $[\omega_1, \omega_2]$ and for ease of exposition that the environment consists of the desired signal and a single correlated interferer arriving from directions θ_s and θ_c, respectively, in white noise of power σ^2. (This analysis is extended in the Appendix to include additional uncorrelated interferers.) Let $S^2(\omega)$ and $C^2(\omega)$ be the power spectral densities of the desired signal and correlated interferer with $S(\omega)$ and $C(\omega)$ their positive square roots and $\rho(\omega)$ be the cross correlation coefficient between signal and interferer ($|\rho(\omega)| \leq 1$). Under these assumptions

$$\boldsymbol{R}_n = \int_{\omega_1}^{\omega_2} C^2(\omega) \boldsymbol{d}(\omega, \theta_c) \boldsymbol{d}^H(\omega, \theta_c)\, d\omega + \sigma^2 \boldsymbol{I} \tag{59}$$

$$\boldsymbol{R}_{ns} = \int_{\omega_1}^{\omega_2} \rho(\omega) C(\omega) S(\omega)\, \boldsymbol{d}(\omega, \theta_c)\, \boldsymbol{d}^H(\omega, \theta_s)\, d\omega \tag{60}$$

where $\boldsymbol{d}(\omega, \theta)$ denotes the array response vector [6] in direction θ and at frequency ω. P_{nn} and P_{sc} are now rewritten as

$$P_{nn} = \int_{\omega_1}^{\omega_2} q_n(\omega) \boldsymbol{v}^H(\omega)\, d\omega \left[\int_{\omega_1}^{\omega_2} \boldsymbol{v}(\omega) \boldsymbol{v}^H(\omega)\, d\omega + \sigma^2 \boldsymbol{I} \right]^{-1} \cdot \int_{\omega_1}^{\omega_2} \boldsymbol{v}(\omega) q_n^*(\omega)\, d\omega \tag{61}$$

$$P_{sc} = \int_{\omega_1}^{\omega_2} q_s(\omega) \boldsymbol{v}^H(\omega)\, d\omega \left[\int_{\omega_1}^{\omega_2} \boldsymbol{v}(\omega) \boldsymbol{v}^H(\omega)\, d\omega + \sigma^2 \boldsymbol{I} \right]^{-1} \cdot \int_{\omega_1}^{\omega_2} \boldsymbol{v}(\omega) q_s^*(\omega)\, d\omega \tag{62}$$

where

$$q_n(\omega) = C(\omega) \boldsymbol{w}_o^H \boldsymbol{d}(\omega, \theta_c), \tag{63}$$

$$q_s(\omega) = \rho^*(\omega) S(\omega) \boldsymbol{w}_o^H \boldsymbol{d}(\omega, \theta_s), \tag{64}$$

$$\boldsymbol{v}(\omega) = C(\omega) \boldsymbol{T}_n^H \boldsymbol{d}(\omega, \theta_c). \tag{65}$$

$\boldsymbol{v}(\omega)$ and $q_n(\omega)$ represent the frequency content of the correlated interferer at the output of the blocking matrix $\boldsymbol{T}_n$ and nonadaptive beamformer $\boldsymbol{w}_o$, respectively. $q_s(\omega)$ is the frequency content of the portion of the desired signal at the nonadaptive beamformer output that is correlated with the interferer.

The expressions for P_{nn} and P_{sc} are simplified by representing $\boldsymbol{v}(\omega)$ in terms of an orthonormal set of basis functions. Let $\boldsymbol{R}_c = \int_{\omega_1}^{\omega_2} \boldsymbol{v}(\omega) \boldsymbol{v}^H(\omega)\, d\omega$ have the eigendecomposition

$$\boldsymbol{R}_c = \boldsymbol{V} \boldsymbol{\Sigma}^2 \boldsymbol{V}^H \tag{66}$$

where $\boldsymbol{\Sigma}^2$ is a diagonal matrix of eigenvalues and the columns of $\boldsymbol{V}$ are the corresponding eigenvectors. We assume the eigenvalues are ordered

$$\sigma_1^2 \geq \sigma_2^2 \geq \cdots \geq \sigma_p^2 \geq 0. \tag{67}$$

Define the transformed basis for the frequency content of the blocking matrix output

$$\boldsymbol{u}(\omega) = \boldsymbol{\Sigma}^{-1} \boldsymbol{V}^H \boldsymbol{v}(\omega). \tag{68}$$

The case where some of the eigenvalues are zero is easily accommodated by replacing the inverse in (68) with a psuedoinverse. Note that a zero eigenvalue indicates that $\boldsymbol{v}(\omega)$

has no component in the space spanned by the corresponding eigenvector. The orthonormality of $\boldsymbol{u}(\omega)$ is easily verified

$$\int_{\omega_1}^{\omega_2} \boldsymbol{u}(\omega)\boldsymbol{u}^H(\omega)\,d\omega = \boldsymbol{\Sigma}^{-1}\boldsymbol{V}^H\int_{\omega_1}^{\omega_2}\boldsymbol{v}(\omega)\boldsymbol{v}^H(\omega)\,d\omega\boldsymbol{V}\boldsymbol{\Sigma} = \boldsymbol{\Sigma}^{-1}\boldsymbol{V}^H\boldsymbol{R}_c\boldsymbol{V}\boldsymbol{\Sigma} = \boldsymbol{I}. \tag{69}$$

Substituting $\boldsymbol{V}\boldsymbol{\Sigma}\boldsymbol{u}(\omega)$ for $\boldsymbol{v}(\omega)$ in (61) and (62) yields

$$P_{nn} = \boldsymbol{h}_n^H\boldsymbol{\Sigma}\boldsymbol{V}^H[\boldsymbol{V}(\boldsymbol{\Sigma}^2+\sigma^2\boldsymbol{I})\boldsymbol{V}^H]^{-1}\boldsymbol{V}\boldsymbol{\Sigma}\boldsymbol{h}_n \tag{70}$$
$$P_{sc} = \boldsymbol{h}_s^H\boldsymbol{\Sigma}\boldsymbol{V}^H[\boldsymbol{V}(\boldsymbol{\Sigma}^2+\sigma^2\boldsymbol{I})\boldsymbol{V}^H]^{-1}\boldsymbol{V}\boldsymbol{\Sigma}\boldsymbol{h}_s \tag{71}$$

where

$$\boldsymbol{h}_n = \int_{\omega_1}^{\omega_2}\boldsymbol{u}(\omega)q_n^*(\omega)\,d\omega \tag{72}$$
$$\boldsymbol{h}_s = \int_{\omega_1}^{\omega_2}\boldsymbol{u}(\omega)q_s^*(\omega)\,d\omega. \tag{73}$$

Simplifying (70) and (71), we obtain

$$P_{nn} = \sum_{i=1}^{p}\frac{\sigma_i^2}{\sigma_i^2+\sigma^2}|h_{ni}|^2 \tag{74}$$
$$P_{sc} = \sum_{i=1}^{p}\frac{\sigma_i^2}{\sigma_i^2+\sigma^2}|h_{si}|^2. \tag{75}$$

If we assume that $\boldsymbol{R}_c$ is approximately rank r, that is, the eigenvalues satisfy

$$\sigma_1^2 \ge \sigma_2^2 \ge \cdots \ge \sigma_r^2 \gg \sigma^2 \gg \sigma_{r+1}^2 \ge \cdots \ge \sigma_p^2 \tag{76}$$

then (74) and (75) are

$$P_{nn} \approx (\boldsymbol{h}_n^r)^H\boldsymbol{h}_n^r \tag{77}$$
$$P_{sc} \approx (\boldsymbol{h}_s^r)^H\boldsymbol{h}_s^r \tag{78}$$

where $\boldsymbol{h}_n^r$ and $\boldsymbol{h}_s^r$ contain the first r elements of $\boldsymbol{h}_n$ and $\boldsymbol{h}_s$, respectively.

The vectors $\boldsymbol{h}_n$ and $\boldsymbol{h}_s$ are the coordinates of $q_n(\omega)$ and $q_s(\omega)$ in the space spanned by the components $\{u_i(\omega)\}$ of the vector function $\boldsymbol{u}(\omega)$. Hence, the interference cancellation level is determined by the projection of the nonadaptive branch interference frequency content onto the space spanned by the adaptive branch interference frequency content. Similarly, the signal cancellation level is determined by the projection of the portion of the signal frequency content in the nonadaptive branch that is correlated with the interferer onto the space spanned by the adaptive branch interference frequency content. The goal is to choose $\boldsymbol{T}_n$ and $\boldsymbol{w}_o$ so that there is a good match between the nonadaptive and adaptive branch interference frequency content and a poor match between the adaptive branch interference frequency content and the correlated portion of the signal in the nonadaptive branch. Replacing $\boldsymbol{C}_n$ with $\boldsymbol{T}_n$ shrinks the space spanned by the basis $\boldsymbol{u}(\omega)$. Modification of $\boldsymbol{w}_o$ changes the projections of $q_n(\omega)$ and $q_s(\omega)$ onto this space.

Similar to (77) and (78)

$$P_{ns} \approx (\boldsymbol{h}_n^r)^H\boldsymbol{h}_s^r. \tag{79}$$

Hence, assuming $\boldsymbol{T}_n = \boldsymbol{C}_n$ and $\boldsymbol{R}_c$ is rank r, (55) is rewritten as

$$\cos^2\phi_z \approx \frac{|(\boldsymbol{h}_n^r)^H\boldsymbol{h}_s^r|^2}{(\boldsymbol{h}_n^r)^H\boldsymbol{h}_n^r(\boldsymbol{h}_s^r)^H\boldsymbol{h}_s^r}. \tag{80}$$

Here we see that $\cos^2\phi_z$ corresponds to the cosine squared of the angle between $\boldsymbol{h}_n^r$ and $\boldsymbol{h}_s^r$.

If $q_n(\omega) = \alpha q_s(\omega)$ for some constant α, then $\boldsymbol{h}_n = \alpha^*\boldsymbol{h}_s$. In this case $\cos^2\phi_z \approx 1$ and partially adaptive beamforming is ineffective. In particular, if the signal and interferer are narrowband, i.e., $\omega_1 = \omega_2$, we have $q_n(\omega_1) = \alpha q_s(\omega_1)$. Indeed, since $\boldsymbol{R}_c = \boldsymbol{v}(\omega_1)\boldsymbol{v}^H(\omega_1)$ is rank one in this case, it is easy to verify that $u(\omega) = 1$. Consequently, P_{nn} and P_{sc} are independent of the adaptation space represented by $\boldsymbol{T}_n$ and we conclude that partially adaptive beamforming cannot succeed with narrowband signals.

In the case of strong correlated interference $|\rho(\omega)|^2 \approx 1$. Again assuming $\boldsymbol{T}_n = \boldsymbol{C}_n$ and nearly complete signal and interference cancellation, we have

$$P_{no} = \int_{\omega_1}^{\omega_2} C^2(\omega)|\boldsymbol{w}_o^H\,\boldsymbol{d}(\omega,\theta_c)|^2\,d\omega = \int_{\omega_1}^{\omega_2}|q_n(\omega)|^2\,d\omega \tag{81}$$

$$P_s = \int_{\omega_1}^{\omega_2} S^2(\omega)|\boldsymbol{w}_o^H\,\boldsymbol{d}(\omega,\theta_s)|^2\,d\omega \approx \int_{\omega_1}^{\omega_2}|\rho(\omega)|^2S^2(\omega)|\boldsymbol{w}_o^H\,\boldsymbol{d}(\omega,\theta_s)|^2\,d\omega = \int_{\omega_1}^{\omega_2}|q_s(\omega)|^2\,d\omega \tag{82}$$

$$\boldsymbol{w}_o^H\boldsymbol{R}_{ns}\boldsymbol{w}_o = \int_{\omega_1}^{\omega_2}\rho(\omega)C(\omega)S(\omega)\boldsymbol{w}_o^H\,\boldsymbol{d}(\omega,\theta_c)\boldsymbol{d}^H(\omega,\theta_s)\cdot\boldsymbol{w}_o\,d\omega = \int_{\omega_1}^{\omega_2}q_n(\omega)q_s^*(\omega)\,d\omega \tag{83}$$

and (58) is expressed in the alternate form

$$\cos^2\phi_z \approx \frac{\left|\int_{\omega_1}^{\omega_2}q_n(\omega)q_s^*(\omega)\,d\omega\right|^2}{\int_{\omega_1}^{\omega_2}|q_n(\omega)|^2\,d\omega\int_{\omega_1}^{\omega_2}|q_s(\omega)|^2\,d\omega}. \tag{84}$$

Here $\cos^2\phi_z$ corresponds to the cosine squared of the angle between the functions $q_n(\omega)$ and $q_s(\omega)$. Equation (84) also follows from (80). If nearly complete signal and interference cancellation is attained, then both $q_n(\omega)$ and $q_s(\omega)$ lie almost entirely in the space spanned by the elements of $\boldsymbol{u}(\omega)$. $\boldsymbol{u}(\omega)$ is an orthonormal basis so the inner product involving $\boldsymbol{h}_n^r$ and $\boldsymbol{h}_s^r$ are equivalent to the inner product involving $q_n(\omega)$ and $q_s(\omega)$.

VI. Examples

The performance of two partially adaptive beamformers obtained through the multilevel point design (PD) procedures described in Section IV is evaluated in this section and compared to that of a fully adaptive beamformer and the CSS beamformer.

The array employed has nine sensors in an equally spaced linear geometry with nine tap FIR filters in each sensor channel, resulting in a total of 81 weights. The uniform structure is chosen for convenience, and is not required by the partially adaptive beamforming technique. The tap spacing is chosen in accordance with the Nyquist sampling rate so that frequency is normalized on the interval $[0, \pi]$, i.e., the sampling time is normalized to 1. The distance between adjacent sensors is one-half the wavelength of the highest signal frequency. All signals are assumed to lie in the frequency band $[9/11\pi, \pi]$. The desired signal arrives from the direction perpendicular to the array, i.e., at a direction sine of 0. Five of the available 81 degrees of freedom are used to provide a unit gain and linear phase response in the desired signal direction. The interference spectra are assumed white on $[9/11\pi, \pi]$. The desired output signal power (11) is normalized to 1 in this example.

A. Fully Adaptive Beamforming

Here we illustrate the signal cancellation phenomenon when fully adaptive beamforming (FAB) is employed in a correlated interference scenario. Assume there are two uncorrelated interferers with direction sines 0.45 (26.74°) and -0.55 ($-33.37°$), each of power level 10 relative to signal power spectral density. A correlated interferer which is a delayed (0.1 sec) and scaled (0.8) version of the desired signal is also present at direction sine -0.22 ($-12.71°$). The background white noise level is 0.01.

If fully adaptive beamforming is employed, the beamformer output power is $P_o = 0.0012$ which indicates severe signal cancellation. In this case, $P_{no} = 0.2357, P_{nn} = 0.2353$, and $P_{sc} = 0.9989$. Therefore, fully adaptive beamforming not only cancels the uncorrelated interferers but cancels the desired signal as well. The corresponding $MSE = 0.9993$. This signal cancellation is also evident in the adapted beampattern. As shown in Fig. 3, unit gain is obtained at direction sine 0 as the result of linear constraints. Two nulls are formed at direction sines -0.55 and 0.45, corresponding to the two uncorrelated interferers. The beamformer does not put a null at direction sine -0.22, the correlated interferer's direction. Instead, the correlated interferer is amplified by 1/0.8 and phase shifted appropriately to cancel the desired signal.

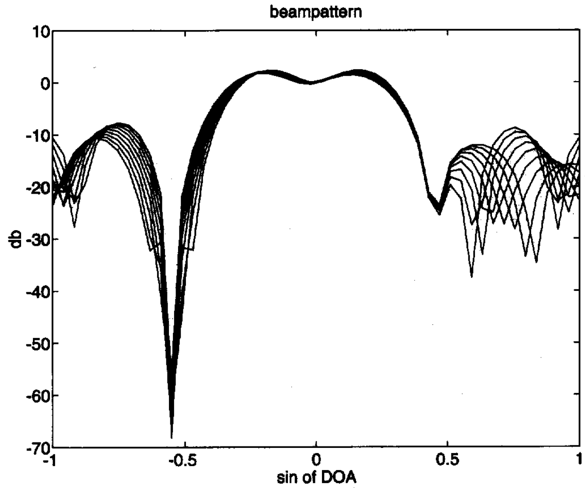

Fig. 3. Beampattern of fully adaptive beamformer. Each line represents the response at a different frequency in the band $9/11\pi, \pi]$.

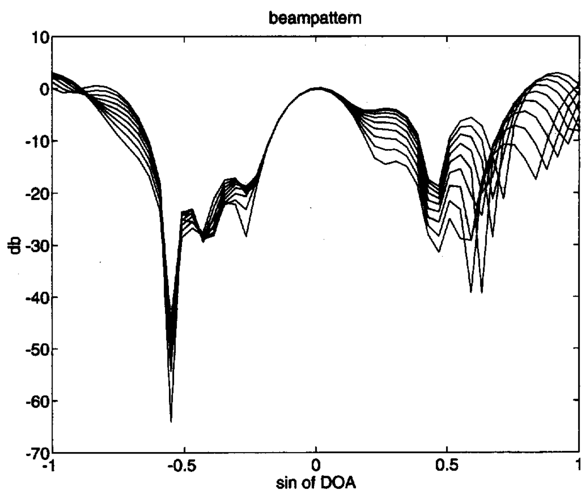

Fig. 4. Beampattern of partially adaptive beamformer. Each line represents the response at a different frequency in the band $[9/11\pi, \pi]$.

B. Partially Adaptive Beamforming

A partially adaptive beamformer is now designed as described in Section IV. For the design process, two uncorrelated interferers are assumed to be present with direction sines in the intervals [0.2 1] ([11.54°, 90°]) and [−1, −0.5] [−90°, −30°], respectively. The correlated interferer's direction sine is assumed to be in the interval [−0.4, −0.2] [−23.58°, −11.54°]. All other interference information is assumed known. Using a 2-level ($\delta_1 = 0.3, \delta_2 = 0.1$) point design procedure, a partially adaptive beamformer of dimension 71 is obtained. Evaluating the partially adaptive beamformer for the same interference scenario used for the fully adaptive beamformer, we have $P_{no} = 0.2357, P_{nn} = 0.2315$, and $P_{sc} = 0.0001$, resulting in $MSE = 0.0103$. The beamformer output power is $P_o = 0.9047$. Therefore, severe signal cancellation is avoided at the expense of a slight degradation in interference cancellation capability. The adapted beampattern shown in Fig. 4 indicates that an approximate null is placed at the direction sine of each interferer.

The partially adaptive beamformer performance is further evaluated using 1500 interference scenarios uniformly distributed on the intervals assumed for design. The maximum MSE is 0.0382 and the average is 0.0070. Hence, the partially adaptive beamformer performs very well for all assumed interference scenarios.

C. Comparison with Frequency Domain Averaging Method

The partially adaptive beamforming (PAB) technique proposed here is effective for broadband signals. An alternate technique for correlation reduction in broadband signal environments is the frequency domain averaging or CSS technique

TABLE I
MSE PERFORMANCE AS A FUNCTION OF CORRELATED INTERFERER TIME DELAY FOR PARTIALLY ADAPTIVE (PAB), CSS, AND FULLY ADAPTIVE (FAB) BEAMFORMERS

	PAB MSE		CSS MSE		FAB MSE	
delay	max	ave	max	ave	max	ave
0.0000	0.0154	0.0080	1.0590	1.0167	1.0000	1.0000
0.1111	0.0160	0.0082	1.01445	1.0027	1.0000	1.0000
0.2222	0.0169	0.0085	0.9932	0.9605	1.0000	0.9999
0.3333	0.0174	0.0086	0.9477	0.8897	0.9995	0.9994
0.4444	0.0174	0.0088	0.8935	0.7916	0.9980	0.9974
0.5556	0.0171	0.0091	0.8226	0.6719	0.9937	0.9922
0.6667	0.0164	0.0093	0.7328	0.5388	0.9843	0.9810
0.7778	0.0157	0.0093	0.6284	0.4096	0.9667	0.9607
0.8889	0.0159	0.0094	0.5146	0.2891	0.9374	0.9278
10.0000	0.0171	0.0100	0.4046	0.1996	0.8933	0.8797

[4]. Here the performance of adaptive beamformers designed using these two techniques are compared.

As is shown in the previous example, an effective partially adaptive beamformer can be designed for a wide range of interference scenarios. However, the CSS method requires preliminary estimates of the interferer and signal directions. These estimates result in much smaller direction-of-arrival (DOA) intervals than those used for the previous example. Therefore, a new set of interference scenarios based on assumed DOA estimates are used here to design the partially adaptive beamformer. Three unity power interferers are embedded in -30 dB white noise: two uncorrelated interferers (one with incident angle in the interval $[-52°, -48°]$ and the other in the interval $[8°, 12°]$) and a correlated interferer whose incident angle is in the interval $[28°, 32°]$. The latter is a delayed version of the desired signal. The possible time delay of the correlated interferer is assumed to range from 0 to 10 time units.

The CSS method uses $-52°$, $-48°$, $-1°$, $1°$, $8°$, $12°$, $28°$, and $32°$ as the DOA estimates. In order to construct a square (9×9) full rank focusing transformation matrix, a fictitious signal from direction $80°$ is also included. While a time domain approximation of the CSS transformation is proposed in [13], we use the exact frequency domain transformation to optimize the CSS method performance. Note that the exact transformation cannot be physically implemented.

A 62-dimensional partially adaptive beamformer is designed for a set of likely interference scenarios that consists of all combinations of three interferers from the same intervals used by the CSS processor ($[-52°, -48°]$, $[8°, 12°]$, $[28°, 32°]$) with the correlated interferer's time delay in the range $[0, 10]$.

Partially adaptive, CSS, and fully adaptive beamformer performance is evaluated at 1250 distinct interference scenarios obtained by varying uniformly within the set of likely interference scenarios. The result is summarized in Table I as a function of the time delay. The average MSE's for each time delay value are obtained by averaging over 125 scenarios.

The partially adaptive beamforming technique significantly outperforms the frequency domain averaging technique over this range of time delays between the desired signal and correlated interferer. The frequency domain averaging technique avoids severe signal cancellation only when the time delays are relatively long. This is because that the frequency domain averaging technique requires the time delay to be much longer than the reciprocal of the signal bandwidth ($= 11/2\pi = 1.7507$ in this example) even when the exact interference directions are available [4]. The partially adaptive beamformer is effective for situations where the time delay between the desired signal and interferers is very short, even where the time delay is zero. Furthermore, it accommodates a wider variation in the interferer's DOA's.

VII. SUMMARY

In this paper, we propose a partially adaptive beamforming technique to prevent signal cancellation in the presence of correlated interferers. By decomposing the beamformer's output mean squared error into an interference mean squared error term and a signal cancellation term, a partially adaptive beamformer design criterion is proposed. This criterion constrains the beamformer signal cancellation level for a set of likely interference scenarios. A procedure is given for designing a partially adaptive beamformer that satisfies the criterion. Analyses in both time domain and frequency domain reveal that the effectiveness of the approach is dependent on the cosine squared of the angle between interference and signal vectors. This method is only effective for broadband signal environments. Simulations demonstrate that the partially adaptive beamforming approach is effective even when the time delay between the desired signal and correlated interferer is very short.

VIII. APPENDIX

Here we extend the frequency domain analysis to include additional uncorrelated interferers. For simplicity, we only assume one additional uncorrelated interferer from direction θ_n with power spectral density $N^2(\omega)$. The case of multiple noncorrelated interferers follows directly. The interference covariance matrix now becomes

$$\boldsymbol{R}_n = \int_{\omega_1}^{\omega_2} C^2(\omega)\, \boldsymbol{d}(\omega, \theta_c)\, \boldsymbol{d}^H(\omega, \theta_c) + N^2(\omega) \boldsymbol{d}(\omega, \theta_n) \cdot \boldsymbol{d}^H(\omega, \theta_n)\, d\omega + \sigma^2 \boldsymbol{I}. \tag{85}$$

Let the eigendecomposition of

$$\int_{\omega_1}^{\omega_2} C^2(\omega) \boldsymbol{T}_n^H\, \boldsymbol{d}(\omega, \theta_c)\, \boldsymbol{d}^H(\omega, \theta_c) \boldsymbol{T}_n + N^2(\omega) \boldsymbol{T}_n^H \cdot \boldsymbol{d}(\omega, \theta_n)\, \boldsymbol{d}^H(\omega, \theta_n) \boldsymbol{T}_n\, d\omega$$

be $\boldsymbol{V}\boldsymbol{\Sigma}^2\boldsymbol{V}^H$ and define

$$\boldsymbol{q}_n(\omega) = [C(\omega) \boldsymbol{w}_o^H\, \boldsymbol{d}(\omega, \theta_c); N(\omega) \boldsymbol{w}_o^H\, \boldsymbol{d}(\omega, \theta_n)] \tag{86}$$

$$\boldsymbol{q}_s(\omega) = [\rho^*(\omega) \boldsymbol{w}_o^H\, \boldsymbol{d}(\omega, \theta_s); 0] \tag{87}$$

$$\boldsymbol{U}(\omega) = [\boldsymbol{u}_c(\omega); \boldsymbol{u}_n(\omega)] = \boldsymbol{\Sigma}^{-1} \boldsymbol{V}^H [C(\omega) \boldsymbol{T}_n^H\, \boldsymbol{d}(\omega, \theta_c); N(\omega) \boldsymbol{T}_n^H\, \boldsymbol{d}(\omega, \theta_n)]. \tag{88}$$

It is easy to verify that

$$\int_{\omega_1}^{\omega_2} \boldsymbol{U}(\omega) \boldsymbol{U}^H(\omega) = \boldsymbol{I}_r. \tag{89}$$

Therefore, as before, we have

$$P_{nn} = \boldsymbol{h}_n^H diag\left(\frac{\sigma_i^2}{\sigma_i^2+\sigma^2}\right)\boldsymbol{h}_n \tag{90}$$

$$P_{sc} = \boldsymbol{h}_s^H diag\left(\frac{\sigma_i^2}{\sigma_i^2+\sigma^2}\right)\boldsymbol{h}_s \tag{91}$$

where

$$\boldsymbol{h}_n = \int_{\omega_1}^{\omega_2} \boldsymbol{U}(\omega)\boldsymbol{q}_n^H(\omega)\,d\omega \tag{92}$$

$$\boldsymbol{h}_s = \int_{\omega_1}^{\omega_2} \boldsymbol{U}(\omega)\boldsymbol{q}_s^H(\omega)\,d\omega \tag{93}$$

and $diag(\sigma_i^2/[\sigma_i^2+\sigma^2])$ denotes a diagonal matrix whose ith diagonal element is $\sigma_i^2/(\sigma_i^2+\sigma^2)$.

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