MUI-Free Receiver for a Synchronous DS-CDMA System Based on Block Spreading in the Presence of Frequency-Selective Fading

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***Abstract*—We discuss a synchronous direct-sequence code-division multiple-access (DS-CDMA) system based on block spreading in the presence of frequency-selective fading. Note that block spreading, which is also known as chip interleaving, refers to a spreading of a data block sequence, which is obtained by dividing a data symbol sequence into consecutive blocks. For such a system, we develop a simple new receiver that completely removes the multiuser interference (MUI) without using any channel information (hence, the name MUI-free receiver). The MUI-free operation is obtained by the use of a shift-orthogonal set of code sequences on which this receiver is based. Within the framework of the MUI-free receiver, we further present a subspace deterministic blind single-user channel estimation algorithm. As a benchmark for the MUI-free receiver and the corresponding subspace deterministic blind single-user channel estimation algorithm, we consider the linear multiuser equalizer and the corresponding subspace deterministic blind multiuser channel estimation algorithm developed by Liu and Xu for a standard synchronous DS-CDMA system in the presence of frequency-selective fading. We show that the complexity of the MUI-free receiver using the corresponding subspace deterministic blind single-user channel estimation algorithm is much smaller than the complexity of the linear multiuser equalizer using the corresponding subspace deterministic blind multiuser channel estimation algorithm. We further show that the performance of the MUI-free receiver is comparable with the performance of the linear multiuser equalizer. This is for the case in which the channels are known as well as for the case in which the channels are estimated with the corresponding subspace deterministic blind channel estimation algorithm.**

***Index Terms*—Blind channel estimation, block spreading, code division multiple access, minimum mean square error equalization, zero forcing equalization.**

I. INTRODUCTION

IN HIGH-RATE direct-sequence code-division multiple-access (DS-CDMA) systems [1], [2], the multipath propagation causes the channels to be frequency selective (time dispersive). Therefore, the interchip interference (ICI) cannot be neglected and has to be suppressed by the receiver next to the multiuser interference (MUI). A popular receiver that combats ICI and MUI is the linear multiuser equalizer [3]–[5]. The design of the desired user's linear multiuser equalizer requires the knowledge of all the channels and code sequences. To estimate the desired user's channel based only on the knowledge of the desired user's code sequence, we can, for instance, use the popular subspace deterministic blind multiuser channel estimation algorithm [6]–[9], which is related to the standard work of [10] for TDMA systems with a single user per time channel. Note that in [9], it is shown that it is also possible to design the desired user's linear multiuser equalizer based only on the knowledge of the desired user's channel and code sequence (and not on the knowledge of all the channels and code sequences). However, even exploiting the ideas presented in this paper, designing the desired user's linear multiuser equalizer using the corresponding subspace deterministic blind multiuser channel estimation algorithm is computationally complex. Note that there also exist methods to directly estimate the desired user's linear multiuser equalizer in a blind fashion based only on the knowledge of the desired user's code sequence [11]–[14]. However, we will not focus on these methods here.

In this paper, we discuss a synchronous DS-CDMA system based on block spreading in the presence of frequency-selective fading. Note that block spreading, which is also known as chip interleaving [15], refers to a spreading of a data block sequence, which is obtained by dividing a data symbol sequence into consecutive blocks. For such a system, we develop a simple new receiver that completely removes the MUI without using any channel information (hence, the name MUI-free receiver). The MUI-free operation is obtained by the use of a shift-orthogonal set of code sequences on which this receiver is based. Within the framework of the MUI-free receiver, we further present a subspace deterministic blind single-user channel estimation algorithm. As a benchmark for the MUI-free receiver and the corresponding subspace deterministic blind single-user channel estimation algorithm, we consider the linear multiuser equalizer and the corresponding subspace deterministic blind multiuser channel estimation algorithm developed by Liu and Xu for a standard synchronous DS-CDMA system in the presence of frequency-selective fading [7].

In Section II, the data model of a synchronous DS-CDMA system based on block spreading in the presence of frequency-selective fading is presented. Section III then introduces the new MUI-free receiver. In Section IV, the corresponding subspace

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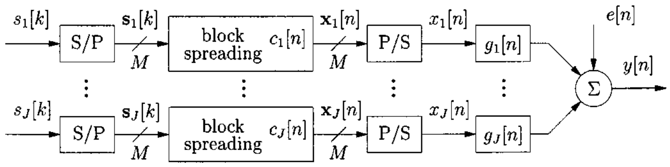


Fig. 1. DS-CDMA system based on block spreading.

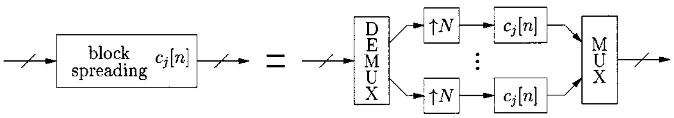


Fig. 2. Block spreading.

deterministic single-user blind channel estimation algorithm is discussed. Section V then reviews the linear multiuser equalizer and the corresponding subspace deterministic blind multiuser channel estimation algorithm developed by Liu and Xu for a standard synchronous DS-CDMA system in the presence of frequency-selective fading [7]. In Section VI, we then compare the complexity of the MUI-free receiver using the corresponding subspace deterministic blind single-user channel estimation algorithm with the complexity of the linear multiuser equalizer using the corresponding subspace deterministic blind multiuser channel estimation algorithm. Further, in Section VII, we compare the performance of the MUI-free receiver with the performance of the linear multiuser equalizer. This is for the case in which the channels are known as well as for the case in which the channels are estimated with the corresponding subspace deterministic blind channel estimation algorithm. We end with some conclusions in Section VIII.

II. Data Model

We first introduce some basic notation. We use lowercase boldface letters to denote vectors and uppercase boldface letters to denote matrices. In addition

- $(\cdot)^T$ transpose;
- $(\cdot)^*$ complex conjugate;
- $(\cdot)^H$ Hermitian transpose;
- $|\cdot|$ absolute value;
- $\|\cdot\|$ Frobenius norm.

Let us then describe a DS-CDMA system based on block spreading (see Figs. 1 and 2). The jth user ($j = 1, 2, \cdots, J$) first divides his data symbol sequence $s_j[k]$ into consecutive blocks of M data symbols, leading to the following data block sequence:

$$\mathbf{s}_j[k] = [s_j[kM] \quad s_j[kM+1] \quad \cdots \quad s_j[(k+1)M-1]]^T.$$

This data block sequence $\mathbf{s}_j[k]$ is then spread by a factor N with the length-N code sequence $c_j[n]$ ($c_j[n] \neq 0$ for $n = 0, 1, \cdots, N-1$, and $c_j[n] = 0$ for $n < 0$ and $n \geq N$), resulting into the chip block sequence $\mathbf{x}_j[n]$, which is given by

$$\mathbf{x}_j[n] = \mathbf{s}_j[k] c_j[n \bmod N], \quad \text{with } k = \lfloor n/N \rfloor. \tag{1}$$

The corresponding chip sequence $x_j[n]$, satisfying

$$\mathbf{x}_j[n] = [x_j[nM] \quad x_j[nM+1] \quad \cdots \quad x_j[(n+1)M-1]]^T$$

is then transmitted at the chip rate N/T, where T is the data symbol period. If we sample the receive antenna at the chip rate N/T, we obtain the following received sequence:

$$y[n] = \sum_{j=1}^{J} \sum_{n'=-\infty}^{+\infty} g_j[n'] x_j[n-n'] + e[n]$$

where $e[n]$ is the discrete-time additive noise at the receive antenna, and $g_j[n]$ is the discrete-time channel from the jth user to the receive antenna, including the transmit and receive filters. We model $g_j[n]$ as an FIR filter of order L_j with delay index δ_j ($g_j[n] \neq 0$ for $n = 0$ and $n = L_j$, $g_j[n] = 0$ for $n < 0$ and $n > L_j$). Note that the larger the order of $g_j[n]$, the more ICI for the jth user.

If we then divide the received sequence $y[n]$ into consecutive blocks of M received samples, leading to the following received block sequence:

$$\mathbf{y}[n] = [y[nM] \quad y[nM+1] \quad \cdots \quad y[(n+1)M-1]]^T$$

we can write

$$\mathbf{y}[n] = \sum_{j=1}^{J} \sum_{n'=-\infty}^{+\infty} \mathbf{G}_j[n'] \mathbf{x}_j[n-n'] + \mathbf{e}[n] \tag{2}$$

where $\mathbf{e}[n]$ is similarly defined as $\mathbf{y}[n]$, and $\mathbf{G}_j[n]$ is the discrete-time $M \times M$ matrix channel from the jth user to the receive antenna, given by the equation at the bottom of the page. Since $g_j[n]$ is an FIR filter of order L_j with delay index δ_j, $\mathbf{G}_j[n]$ is an $M \times M$ FIR matrix filter of order $\lceil(\delta_j + L_j)/M\rceil - \lfloor \delta_j/M \rfloor$ with delay index $\lfloor \delta_j/M \rfloor$ ($\mathbf{G}_j[n] \neq \mathbf{O}_M$ for $n = \lfloor \delta_j/M \rfloor$ and $n = \lceil(\delta_j + L_j)/M\rceil$, and $\mathbf{G}_j[n] = \mathbf{O}_M$ for $n < \lfloor \delta_j/M \rfloor$ and $n > \lceil(\delta_j + L_j)/M\rceil$). Note that the larger the order of $\mathbf{G}_j[n]$, the more inter chip block interference (ICBI) for the jth user.

In this paper, we focus on *synchronous communication*

$$\delta_j = 0, \quad \text{for } j = 1, 2, \cdots, J \tag{3}$$

$$\mathbf{G}_j[n] = \begin{bmatrix} g_j[Mn] & g_j[Mn-1] & \cdots & g_j[M(m-1)+1] \\ g_j[Mn+1] & g_j[Mn] & \cdots & g_j[M(-1)+2] \\ \vdots & \vdots & \ddots & \vdots \\ g_j[M(m+1)-1] & g_j[M(m+1)-2] & \cdots & g_j[Mn] \end{bmatrix}.$$

and *limited frequency-selective fading*

$$0 < L_j \ll N, \quad \text{for } j = 1, 2, \cdots, J. \tag{4}$$

We further assume that an overestimation L of the maximal channel order $L_{\max}$ is known ($L > L_{\max}$) and that L satifies $L \ll N$. If we then take the block size M equal to L, $\mathbf{G}_j[n]$ becomes an $L \times L$ FIR matrix filter of order 1 with delay index 0, where

$$\mathbf{G}_j[0] = \begin{bmatrix} g_j[0] & 0 & \cdots & 0 \\ g_j[1] & g_j[0] & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_j[L-1] & g_j[L-2] & \cdots & g_j[0] \end{bmatrix}$$

$$\mathbf{G}_j[1] = \begin{bmatrix} 0 & g_j[L-1] & \cdots & g_j[1] \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & g_j[L-1] \\ 0 & 0 & \cdots & 0 \end{bmatrix}.$$

Hence, (2) can then be written as

$$\mathbf{y}[n] = \sum_{j=1}^{J} \sum_{n'=0}^{1} \mathbf{G}_j[n']\mathbf{x}_j[n-n'] + \mathbf{e}[n]. \tag{5}$$

In Section VII, we will run some simulations for $J = 8$, $N = 17$, and $L = L_{\max} + 1 = 4$.

III. MUI-Free Receiver

Since $\mathbf{G}_j[n]$ is an $L \times L$ FIR matrix filter of order 1 with delay index 0, we can apply a block RAKE receiver consisting of a bank of two block correlators, where each block correlator for the jth user despreads the received block sequence $\mathbf{y}[k]$ matched to a different tap of the matrix channel $\mathbf{G}_j[n]$ using the code sequence $c_j[n]$, followed by a linear block combiner, which linearly combines the two block correlator outputs. Note that this block RAKE receiver is analogous to the RAKE receiver used in a standard DS-CDMA system [16].

The MUI-free receiver is a modified block RAKE receiver, consisting of a bank of two modified block correlators, where each modified block correlator for the jth user despreads the received block sequence $\mathbf{y}[k]$ matched to a different tap of the matrix channel $\mathbf{G}_j[n]$ using a code sequence that is slightly different from $c_j[n]$, followed by a linear block combiner, which linearly combines the two modified block correlator outputs (see Figs. 3 and 4). The use of a shift-orthogonal set of code sequences, on which this receiver is based, causes the two modified block correlators to completely remove the ICBI and MUI. Hence, the linear block combiner only has to suppress the remaining intersymbol interference (ISI).

A. Bank of Modified Block Correlators

As already mentioned, in contrast with a block correlator for the jth user, a modified block correlator for the jth user despreads the received block sequence $\mathbf{y}[k]$ using a code sequence that is slightly different from $c_j[n]$. More specifically, the first modified block correlator for the jth user despreads the received

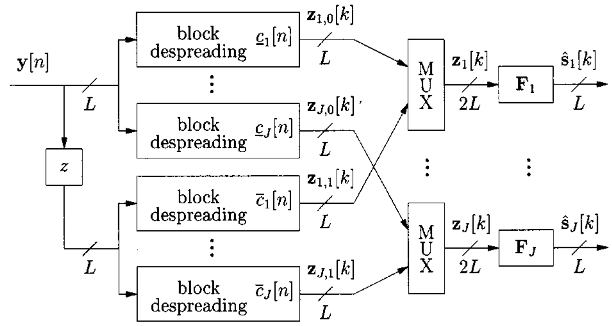


Fig. 3. MUI-free receiver.

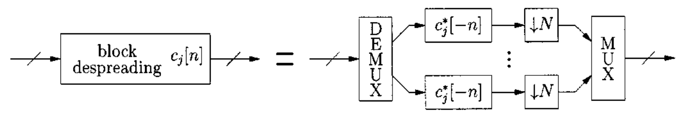


Fig. 4. Block despreading.

block sequence $\mathbf{y}[k]$ matched to $\mathbf{G}_j[0]$ using the code sequence $\underline{c}_j[n]$, where

$$\underline{c}_j[n] = \begin{cases} c_j[n], & \text{for } n \neq 0 \\ 0, & \text{for } n = 0 \end{cases}$$

whereas the second modified block correlator for the jth user despreads the received block sequence $\mathbf{y}[k]$ matched to $\mathbf{G}_j[1]$ using the code sequence $\overline{c}_j[n]$, where

$$\overline{c}_j[n] = \begin{cases} c_j[n], & \text{for } n \neq N-1 \\ 0, & \text{for } n = N-1. \end{cases}$$

Then, defining the $(N-1) \times 1$ code vectors $\underline{\mathbf{c}}_j$ and $\overline{\mathbf{c}}_j$ as

$$\underline{\mathbf{c}}_j = [c_j[1] \quad c_j[2] \quad \cdots \quad c_j[N-1]]^T$$
$$\overline{\mathbf{c}}_j = [c_j[0] \quad c_j[1] \quad \cdots \quad c_j[N-2]]^T$$

the output of the first modified block correlator can be written as

$$\mathbf{z}_{j,0}[k] = \sum_{n=0}^{N-1} \underline{c}_j^*[n]\mathbf{y}[kN+n] = \sum_{n=1}^{N-1} c_j^*[n]\mathbf{y}[kN+n]$$
$$= \underline{\mathbf{c}}_j^H \underline{\mathbf{c}}_j \mathbf{G}_j[0]\mathbf{s}_j[k] + \mathbf{q}_{j,0}[k] + \mathbf{p}_{j,0}[k] + \mathbf{n}_{j,0}[k] \tag{6}$$

whereas the output of the second modified block correlator can be written as

$$\mathbf{z}_{j,1}[k] = \sum_{n=0}^{N-1} \overline{c}_j^*[n]\mathbf{y}[kN+n+1]$$
$$= \sum_{n=0}^{N-2} c_j^*[n]\mathbf{y}[kN+n+1]$$
$$= \overline{\mathbf{c}}_j^H \overline{\mathbf{c}}_j \mathbf{G}_j[1]\mathbf{s}_j[k] + \mathbf{q}_{j,1}[k] + \mathbf{p}_{j,1}[k] + \mathbf{n}_{j,1}[k]. \tag{7}$$

In these formulas, $\mathbf{q}_{j,0}[k]$ and $\mathbf{q}_{j,1}[k]$ represent the residual ICBI:

$$\mathbf{q}_{j,0}[k] = \underline{\mathbf{c}}_j^H \overline{\mathbf{c}}_j \mathbf{G}_j[1]\mathbf{s}_j[k]$$
$$\mathbf{q}_{j,1}[k] = \overline{\mathbf{c}}_j^H \underline{\mathbf{c}}_j \mathbf{G}_j[0]\mathbf{s}_j[k]$$

$\mathbf{p}_{j,0}[k]$ and $\mathbf{p}_{j,1}[k]$ represent the residual MUI:

$$\mathbf{p}_{j,0}[k] = \sum_{\substack{j'=1\\ j'\neq j}}^{J} \underline{\mathbf{c}}_j^H \underline{\mathbf{c}}_{j'} \mathbf{G}_{j'}[0]\mathbf{s}_{j'}[k] + \underline{\mathbf{c}}_j^H \overline{\mathbf{c}}_{j'} \mathbf{G}_{j'}[1]\mathbf{s}_{j'}[k]/$$
$$\mathbf{p}_{j,1}[k] = \sum_{\substack{j'=1\\ j'\neq j}}^{J} \overline{\mathbf{c}}_j^H \underline{\mathbf{c}}_{j'} \mathbf{G}_{j'}[0]\mathbf{s}_{j'}[k] + \overline{\mathbf{c}}_j^H \overline{\mathbf{c}}_{j'} \mathbf{G}_{j'}[1]\mathbf{s}_{j'}[k]/$$

and $\mathbf{n}_{j,0}[k]$ and $\mathbf{n}_{j,1}[k]$ represent the residual additive noise:

$$\begin{aligned}\mathbf{n}_{j,0}[k] &= \sum_{n=0}^{N-1} \underline{c}_j^*[n]\mathbf{e}[kN+n]\\ &= \sum_{n=1}^{N-1} c_j^*[n]\mathbf{e}[kN+n]\\ \mathbf{n}_{j,1}[k] &= \sum_{n=0}^{N-1} \overline{c}_j^*[n]\mathbf{e}[kN+n+1]\\ &= \sum_{n=0}^{N-2} c_j^*[n]\mathbf{e}[kN+n+1].\end{aligned}$$

Note that the residual ICBI and MUI formulas only contain terms in the time index k.

Assume now that a shift orthogonal set of code sequences is used.

Definition 1: A set of J length-N code sequences $\{c_j[n]\}_{j=1}^J$ is shift-orthogonal if and only if

$$\begin{cases} \underline{\mathbf{c}}_j^H \underline{\mathbf{c}}_{j'} = \overline{\mathbf{c}}_j^H \overline{\mathbf{c}}_{j'} = \nu\delta[j-j'] \\ \underline{\mathbf{c}}_j^H \overline{\mathbf{c}}_{j'} = \overline{\mathbf{c}}_j^H \underline{\mathbf{c}}_{j'} = 0 \end{cases}, \text{ for } j, j' = 1, 2, \cdots, J \quad (8)$$

where $\nu = (N-1)/N$, and $\delta[\cdot]$ is the discrete-time impulse function.

Remark 1: Note that (8) can only be satisfied if $J \leq \lfloor (N-1)/2 \rfloor$ (since $2J$ vectors of size $(N-1)\times 1$ can only be mutually orthogonal if $2J \leq N-1$).

Then it is clear that the two modified block correlators completely remove the ICBI and MUI, and (6) and (7) can be written as

$$\begin{aligned}\mathbf{z}_{j,0}[k] &= \nu\mathbf{G}_j[0]\mathbf{s}_j[k] + \mathbf{n}_{j,0}[k]\\ \mathbf{z}_{j,1}[k] &= \nu\mathbf{G}_j[1]\mathbf{s}_j[k] + \mathbf{n}_{j,1}[k].\end{aligned}$$

Note that if $e[n]$ is white with variance σ_e^2, we get

$$\begin{aligned}\mathbf{R}_{\mathbf{n}_{j,0}} &= \mathrm{E}\{\mathbf{n}_{j,0}[k]\mathbf{n}_{j,0}^H[k]\} = \nu\sigma_e^2\mathbf{I}_L\\ \mathbf{R}_{\mathbf{n}_{j,1}} &= \mathrm{E}\{\mathbf{n}_{j,1}[k]\mathbf{n}_{j,1}^H[k]\} = \nu\sigma_e^2\mathbf{I}_L\\ \mathbf{R}_{\mathbf{n}_{j,0}\mathbf{n}_{j,1}} &= \mathrm{E}\{\mathbf{n}_{j,0}[k]\mathbf{n}_{j,1}^H[k]\} = \mathbf{R}_{\mathbf{n}_{j,1}\mathbf{n}_{j,0}}\\ &= \mathrm{E}\{\mathbf{n}_{j,1}[k]\mathbf{n}_{j,0}^H[k]\} = \mathbf{O}_L\end{aligned}$$

where $\mathrm{E}\{\cdot\}$ represents the expectation, and $\mathbf{O}_n$ and $\mathbf{I}_n$ denote the $n\times n$ zero matrix and the $n\times n$ identity matrix, respectively. Stacking the two modified block correlator outputs

$$\mathbf{z}_j[k] = [\mathbf{z}_{j,0}^T[k] \quad \mathbf{z}_{j,1}^T[k]]^T$$

we obtain

$$\mathbf{z}_j[k] = \nu\mathbf{G}_j\mathbf{s}_j[k] + \mathbf{n}_j[k] \quad (9)$$

where $\mathbf{n}_j[k]$ is similarly defined as $\mathbf{z}_j[k]$, and $\mathbf{G}_j$ is the $2L\times L$ channel matrix for the jth user, which is given by

$$\mathbf{G}_j = [\mathbf{G}_j^T[0] \quad \mathbf{G}_j^T[1]]^T.$$

Note that if $e[n]$ is white with variance σ_e^2, we get

$$\mathbf{R}_{\mathbf{n}_j} = \mathrm{E}\{\mathbf{n}_j[k]\mathbf{n}_j^H[k]\} = \nu\sigma_e^2\mathbf{I}_{2L}. \quad (10)$$

B. Linear Block Combiner

After the two modified block correlators have completely removed the ICBI and the MUI, we linearly combine the two modified block correlator outputs $\mathbf{z}_{j,0}[k]$ and $\mathbf{z}_{j,1}[k]$ to suppress the remaining ISI:

$$\begin{aligned}\hat{\mathbf{s}}_j[k] &= \mathbf{F}_{j,0}\mathbf{z}_{j,0}[k] + \mathbf{F}_{j,1}\mathbf{z}_{j,1}[k]\\ &= \nu(\mathbf{F}_{j,0}\mathbf{G}_j[0] + \mathbf{F}_{j,1}\mathbf{G}_j[1])\mathbf{s}_j[k]\\ &\quad + \mathbf{F}_{j,0}\mathbf{n}_{j,0}[k] + \mathbf{F}_{j,1}\mathbf{n}_{j,1}[k]\end{aligned}$$

where $\mathbf{F}_{j,0}$ and $\mathbf{F}_{j,1}$ are the $L\times L$ linear block combiner weights. Using the notation of (9), we can also write

$$\hat{\mathbf{s}}_j[k] = \mathbf{F}_j\mathbf{z}_j[k] = \nu\mathbf{F}_j\mathbf{G}_j\mathbf{s}_j[k] + \mathbf{F}_j\mathbf{n}_j[k]$$

where $\mathbf{F}_j$ is the $L\times 2L$ linear block combiner for the jth user, which is given by

$$\mathbf{F}_j = [\mathbf{F}_{j,0} \quad \mathbf{F}_{j,1}].$$

We now focus on the calculation of the signal-to-interference-plus-noise ratio (SINR) and bit error rate (BER) for the jth user at the output of the linear block combiner $\mathbf{F}_j$. First of all, observe that

$$\mathbf{R}_{\mathbf{z}_j} = \mathrm{E}\{\mathbf{z}_j[k]\mathbf{z}_j^H[k]\} = \nu^2\mathbf{G}_j\mathbf{R}_{\mathbf{s}_j}\mathbf{G}_j^H + \mathbf{R}_{\mathbf{n}_j} \quad (11)$$

where $\mathbf{R}_{\mathbf{n}_j}$ is similarly defined as $\mathbf{R}_{\mathbf{z}_j}$, and

$$\mathbf{R}_{\mathbf{s}_j} = \mathrm{E}\{\mathbf{s}_j[k]\mathbf{s}_j^H[k]\}.$$

If we then define the $L\times L$ matrix $\mathbf{A}_j$ as

$$\mathbf{A}_j = \nu\mathbf{F}_j\mathbf{G}_j$$

and the $L\times L$ matrix $\overline{\mathbf{A}}_j$ as the matrix that is obtained by zeroing the diagonal elements of $\mathbf{A}_j$, the SINR[1] and BER for the jth user at the output of the linear block combiner $\mathbf{F}_j$ can be expressed as

$$\mathrm{SINR}_j = \frac{\dfrac{1}{L}\displaystyle\sum_{l=1}^{L} |\mathbf{A}_j(l,l)|^2 \mathbf{R}_{\mathbf{s}_j}(l,l)}{\dfrac{1}{L}\displaystyle\sum_{l=1}^{L} \left([\overline{\mathbf{A}}_j\mathbf{R}_{\mathbf{s}_j}\overline{\mathbf{A}}_j^H](l,l) + [\mathbf{F}_j\mathbf{R}_{\mathbf{n}_j}\mathbf{F}_j^H](l,l)\right)} \quad (12)$$

[1]The only interference we have here is ISI.

$$\mathrm{BER}_j \approx \frac{1}{L}\sum_{l=1}^{L} \mathrm{Q}\left\{\frac{|\mathbf{A}_j(l,l)|^2 \mathbf{R}_{\mathbf{s}_j}(l,l)}{[\overline{\mathbf{A}}_j \mathbf{R}_{\mathbf{s}_j} \overline{\mathbf{A}}_j^H](l,l) + [\mathbf{F}_j \mathbf{R}_{\mathbf{n}_j} \mathbf{F}_j^H](l,l)}\right\} \tag{13}$$

where $\mathrm{Q}\{\cdot\}$ is the well-known BER for signaling over an additive white Gaussian noise (AWGN) channel as a function of the SNR per data symbol for the chosen modulation scheme [17]. Note that the approximation in (13) becomes an equality if the interference plus noise at the output of the linear block combiner $\mathbf{F}_j$ is Gaussian. We now discuss two linear block combiners in more detail: the zero-forcing (ZF) linear block combiner and the minimum mean-square error (MMSE) linear block combiner. The existence of the ZF linear block combiner is based on the fact that the $2L \times L$ channel matrix $\mathbf{G}_j$ has full column rank L.

1) ZF Linear Block Combiner: We say that a linear block combiner $\mathbf{F}_j$ is ZF if

$$\mathbf{A}_j = \nu \mathbf{F}_j \mathbf{G}_j = \mathbf{I}_L.$$

The ZF linear block combiner (we take the one that minimizes the MSE $\mathrm{E}\{\|\hat{\mathbf{s}}_j[k] - \mathbf{s}_j[k]\|^2\}$) can be expressed as

$$\mathbf{F}_{j,ZF} = \nu^{-1}(\mathbf{G}_j^H \mathbf{R}_{\mathbf{n}_j}^{-1} \mathbf{G}_j)^{-1} \mathbf{G}_j^H \mathbf{R}_{\mathbf{n}_j}^{-1}. \tag{14}$$

If the data symbol sequence $s_j[k]$ is white with variance 1 ($\mathbf{R}_{\mathbf{s}_j} = \mathbf{I}_L$) and the additive noise $e[n]$ is white with variance σ_e^2 ($\mathbf{R}_{\mathbf{n}_j} = \nu\sigma_e^2 \mathbf{I}_{2L}$), (12)–(14) become

$$\begin{aligned} \mathbf{F}_{j,ZF} &= \nu^{-1}(\mathbf{G}_j^H \mathbf{G}_j)^{-1} \mathbf{G}_j^H \\ \mathrm{SINR}_{j,ZF} &= \frac{1}{\nu^{-1}\sigma_e^2 \dfrac{1}{L}\displaystyle\sum_{l=1}^{L} [(\mathbf{G}_j^H \mathbf{G}_j)^{-1}](l,l)} \\ \mathrm{BER}_{j,ZF} &\approx \frac{1}{L}\sum_{l=1}^{L} \mathrm{Q}\left\{\frac{1}{\nu^{-1}\sigma_e^2 [(\mathbf{G}_j^H \mathbf{G}_j)^{-1}](l,l)}\right\}. \end{aligned} \tag{15}$$

2) MMSE Linear Block Combiner: We say that a linear block combiner $\mathbf{F}_j$ is MMSE if the MSE

$$\mathrm{E}\{\|\hat{\mathbf{s}}_j[k] - \mathbf{s}_j[k]\|^2\}$$

is minimized. The MMSE linear block combiner is given by

$$\mathbf{F}_{j,\mathrm{MMSE}} = (\nu^2 \mathbf{G}_j^H \mathbf{R}_{\mathbf{n}_j}^{-1} \mathbf{G}_j + \mathbf{R}_{\mathbf{s}_j}^{-1})^{-1} \nu \mathbf{G}_j^H \mathbf{R}_{\mathbf{n}_j}^{-1}. \tag{16}$$

If the data symbol sequence $s_j[k]$ is white with variance 1 ($\mathbf{R}_{\mathbf{s}_j} = \mathbf{I}_L$) and the additive noise $e[n]$ is white with variance σ_e^2 ($\mathbf{R}_{\mathbf{n}_j} = \nu\sigma_e^2 \mathbf{I}_{2L}$), (12), (13), and (16) become

$$\begin{aligned} &\mathbf{F}_{j,\mathrm{MMSE}} \\ &\quad = (\nu \mathbf{G}_j^H \mathbf{G}_j + \sigma_e^2 \mathbf{I}_L)^{-1} \mathbf{G}_j^H \\ &\mathrm{SINR}_{j,\mathrm{MMSE}} \\ &\quad = \frac{\dfrac{1}{L}\displaystyle\sum_{l=1}^{L} [(\mathbf{I}_L + \nu^{-1}\sigma_e^2 (\mathbf{G}_j^H \mathbf{G}_j)^{-1})^{-1}](l,l)}{\dfrac{1}{L}\displaystyle\sum_{l=1}^{L} 1 - [(\mathbf{I}_L + \nu^{-1}\sigma_e^2 (\mathbf{G}_j^H \mathbf{G}_j)^{-1})^{-1}](l,l)} \\ &\mathrm{BER}_{j,\mathrm{MMSE}} \\ &\quad \approx \frac{1}{L}\sum_{m=1}^{L} \mathrm{Q}\left\{\frac{[(\mathbf{I}_L + \nu^{-1}\sigma_e^2 (\mathbf{G}_j^H \mathbf{G}_j)^{-1})^{-1}](l,l)}{1 - [(\mathbf{I}_L + \nu^{-1}\sigma_e^2 (\mathbf{G}_j^H \mathbf{G}_j)^{-1})^{-1}](l,l)}\right\}. \end{aligned} \tag{17}$$

C. Shift-Orthogonal Code Design

For $N = d + 1$ and $J = \lfloor (N-1)/2 \rfloor = d/2$ (d power of 2), we show how to design a shift-orthogonal set of J length-N code sequences $\{c_j[n]\}_{j=1}^J$ (see Definition 1). Defining the $n \times n$ matrix $\mathbf{J}_n$ as

$$\mathbf{J}_n = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \tag{18}$$

we first introduce the following definition.

Definition 2: A $d \times d/2$ (d power of 2) matrix $\mathbf{C}_d$ is admissible if and only if

- $\mathbf{C}_d^H \mathbf{C}_d = d\mathbf{I}_{d/2}$;
- $\mathbf{C}_d^H \mathbf{J}_d \mathbf{C}_d = \mathbf{O}_{d/2}$.

We can then state the following theorem.

Theorem 1: If a $d \times d/2$ (d power of 2) matrix $\mathbf{C}_d$ is admissible, then for $N = d + 1$ and $J = \lfloor (N-1)/2 \rfloor = d/2$, the following set of J length-N code sequences $\{c_j[n]\}_{j=1}^J$ is shift-orthogonal:

$$c_j[n] = \begin{cases} (1/\sqrt{N})\mathbf{C}_d(N-1, j), & \text{for } n = 0 \\ (1/\sqrt{N})\mathbf{C}_d(n, j), & \text{for } n = 1, 2, \cdots, N-1 \\ 0, & \text{for } n < 0 \text{ and } n \geq N \end{cases}$$
$$\text{for } j = 1, 2, \cdots, J. \tag{19}$$

Proof: The proof follows directly from Definitions 1 and 2 as well as from the fact that $\underline{\mathbf{c}}_j = (1/\sqrt{N})\mathbf{C}_d(:, j)$, and $\overline{\mathbf{c}}_j = (1/\sqrt{N})\mathbf{J}_d \mathbf{C}_d(:, j)$ ∎

Hence, it only remains to show how to design an admissible $d \times d/2$ (d power of 2) matrix $\mathbf{C}_d$. Defining the $n \times n$ matrix $\mathbf{D}_n$ as

$$\mathbf{D}_n = \begin{bmatrix} -1 & 0 & 0 & 0 & \cdots \\ 0 & 1 & 0 & 0 & \cdots \\ 0 & 0 & -1 & 0 & \cdots \\ 0 & 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

we therefore introduce the following definition.

Definition 3: A $d \times d/2$ (d power of 2) matrix $\mathbf{C}_d = [\mathbf{C}_{d,1}\ \mathbf{C}_{d,2}]$, where $\mathbf{C}_{d,1}$ and $\mathbf{C}_{d,2}$ have size $d \times d/4$, is reducible if and only if $\mathbf{C}_{d,2} = \mathbf{D}_d \mathbf{C}_{d,1}$.

This gives rise to the following theorem.

Theorem 2: If a $d \times d/2$ (d power of 2) matrix $\mathbf{C}_d = [\mathbf{C}_{d,1}\ \mathbf{C}_{d,2}]$, where $\mathbf{C}_{d,1}$ and $\mathbf{C}_{d,2}$ have size $d \times d/4$, is admissible and reducible, then the $2d \times d$ matrix

$$\begin{aligned} \mathbf{C}_{2d} &= [\mathbf{C}_{2d,1} \quad \mathbf{C}_{2d,2}] \\ &= \begin{bmatrix} \mathbf{C}_{d,1} & \mathbf{J}_d \mathbf{C}_{d,1} & \mathbf{C}_{d,2} & -\mathbf{J}_d \mathbf{C}_{d,2} \\ \mathbf{C}_{d,2} & -\mathbf{J}_d \mathbf{C}_{d,2} & \mathbf{C}_{d,1} & \mathbf{J}_d \mathbf{C}_{d,1} \end{bmatrix} \end{aligned}$$

where $\mathbf{C}_{2d,1}$ and $\mathbf{C}_{2d,2}$ have size $2d \times d/2$, is also admissible and reducible.

Proof: See Appendix A. ■

Based on this theorem, we can design an admissible and reducible $d \times d/2$ (d power of 2) matrix $\mathbf{C}_d$ in a recursive way, starting, for instance, with

$$\mathbf{C}_4 = \begin{bmatrix} +1 & -1 \\ +1 & +1 \\ -1 & +1 \\ +1 & +1 \end{bmatrix} \quad \text{or}$$

$$\mathbf{C}_4 = \frac{1}{\sqrt{2}} \begin{bmatrix} +1-i & -1+i \\ +1+i & +1+i \\ -1+i & +1-i \\ +1+i & +1+i \end{bmatrix}$$

resulting in a set of BPSK or QPSK code sequences, respectively. Observe from Theorem 1 that the J length-N code sequences defined in (19) are actually based on J length-$(N-1)$ code sequences, extended with a cyclic prefix of 1 code symbol ($c_j[0] = c_j[N-1]$). The insertion of a cyclic prefix is a well-known procedure in discrete multitone (DMT) systems [18].

IV. Blind Channel Estimation

Within the framework of the MUI-free receiver, we present a subspace deterministic blind single-user channel estimation algorithm. This algorithm is performed on the model (9). We first describe how the algorithm works and then give a performance analysis. We finally present a related channel gain estimation method. Note that in the context of a TDMA system with a single user per time channel and applying repetition coding, a similar approach is presented in [19]. However, no performance analysis or related channel gain estimation method is presented there.

A. Algorithm

For a burst length K (K multiple of L), we first define the $2L \times K/L$ matrix $\mathbf{Z}_j$ as

$$\mathbf{Z}_j = [\mathbf{z}_j[0] \quad \mathbf{z}_j[1] \quad \cdots \quad \mathbf{z}_j[K/L-1]].$$

Using (9), we then obtain

$$\mathbf{Z}_j = \nu \mathbf{G}_j \mathbf{S}_j + \mathbf{N}_j \tag{20}$$

where $\mathbf{N}_j$ is similarly defined as $\mathbf{Z}_j$, and $\mathbf{S}_j$ is the $L \times K/L$ matrix given by

$$\mathbf{S}_j = [\mathbf{s}_j[0] \quad \mathbf{s}_j[1] \quad \cdots \quad \mathbf{s}_j[K/L-1]].$$

The algorithm is based on the fact that the $2L \times L$ channel matrix $\mathbf{G}_j$ is tall and has full column rank L and on the assumption that the $L \times K/L$ matrix $\mathbf{S}_j$ has full row rank L (hence, we need $L \leq K/L$).

First, let us assume that there is no additive noise present in (20). Because $\mathbf{G}_j$ has full column rank L and $\mathbf{S}_j$ has full row rank L, the $2L \times K/L$ matrix $\mathbf{Z}_j$ has rank L. Defining the $2L \times L$ matrix $\mathbf{U}_j^{\perp}$ as the collection of the L left singular vectors of $\mathbf{Z}_j$ corresponding to the L zero singular values (because $\mathbf{G}_j$ is tall, $\mathbf{U}_j^{\perp}$ is not empty), the columns of $\mathbf{U}_j^{\perp}$ then form an orthonormal basis of the left null space of $\mathbf{G}_j$:

$$\mathbf{U}_j^{\perp H} \mathbf{G}_j = \mathbf{O}_L. \tag{21}$$

Defining the $L \times L^2$ matrix $\mathcal{U}_j^{\perp}$ as

$$\mathcal{U}_j^{\perp} = [\mathbf{U}_{j,1}^{\perp} \quad \mathbf{U}_{j,2}^{\perp} \quad \cdots \quad \mathbf{U}_{j,L}^{\perp}]$$

where $\mathbf{U}_{j,l}^{\perp}$ ($l = 1, 2, \cdots, L$) is the $L \times L$ matrix given by

$$\mathbf{U}_{j,l}^{\perp} = \mathbf{U}_j^{\perp}(l : l+L-1, :)$$

(21) can also be written as

$$\mathcal{U}_j^{\perp H} \mathbf{g}_j = \mathbf{0} \tag{22}$$

where $\mathbf{g}_j$ is the $L \times 1$ channel vector for the jth user, which is given by

$$\mathbf{g}_j = [g_j[0] \quad g_j[1] \quad \cdots \quad g_j[L-1]]^T.$$

We have the following identifiability result [19].

Theorem 3: The channel $g_j[n]$ can be uniquely (up to a complex scaling factor) determined from (22).

Proof: See [19]. ■

Second, let us assume there is additive noise present in (20). Defining the $2L \times L$ matrix $\hat{\mathbf{U}}_j^{\perp}$ as the collection of the L left singular vectors of $\mathbf{Z}_j$ corresponding to the L smallest singular values and defining the $L \times L$ matrix $\hat{\mathbf{U}}_{j,l}^{\perp}$ and the $L \times L^2$ matrix $\hat{\mathcal{U}}_j^{\perp}$ in a similar fashion as $\mathbf{U}_{j,l}^{\perp}$ and $\mathcal{U}_j^{\perp}$, we then consider the following minimization problem:

$$\min_{\mathbf{g}_j} \{ \| \hat{\mathcal{U}}_j^{\perp H} \mathbf{g}_j \|^2 \}. \tag{23}$$

To avoid the all-zero solution, some nontriviality constraint is imposed on $\mathbf{g}_j$. If we impose a unit norm constraint on $\mathbf{g}_j$, then the left singular vector of $\hat{\mathcal{U}}_j^{\perp}$ corresponding to the smallest singular value represents a possible solution. Since this vector can be interpreted as an estimate of $\mathbf{g}_j^o$, which is the left singular vector of $\mathcal{U}_j^{\perp}$ corresponding to the smallest singular value, we will denote this solution as $\hat{\mathbf{g}}_j^o$. As an estimate for the channel vector $\mathbf{g}_j$, we then consider

$$\hat{\mathbf{g}}_j = \hat{\gamma}_j \hat{\mathbf{g}}_j^o \tag{24}$$

where $\hat{\gamma}_j$ is an estimate of γ_j, which is given by $\mathbf{g}_j = \gamma_j \mathbf{g}_j^o$. This γ_j can be estimated from some short known headers that are transmitted, or we can blindly estimate $|\gamma_j|$, which is equal to the channel gain $\|\mathbf{g}_j\|$ (see Section IV-C) and use an appropriate differential modulation scheme to get rid of the phase ambiguity [17]. However, for simplicity, we will estimate γ_j as

$$\hat{\gamma}_j = \mathbf{g}_j \hat{\mathbf{g}}_j^{o\dagger}.$$

This leads to an estimate $\hat{\mathbf{g}}_j$ that is optimal in the LS sense (since $\mathbf{g}_j$ is not known, this is, of course, not feasible in practice).

B. Performance Analysis

The performance analysis is given in the following theorem.

Theorem 4: Assume that the channel vector estimate $\hat{\mathbf{g}}_j$ is obtained as in (24) with $\hat{\gamma}_j = \gamma_j$. Further, assume that the data symbol sequence $s_j[k]$ is white with variance 1 and that the additive noise $e[n]$ is white with variance σ_e^2. Then, only considering the first-order approximation of $\Delta\mathbf{g}_j = \hat{\mathbf{g}}_j - \mathbf{g}_j$ in $\mathbf{N}_j$, the bias and the normalized MSE (NMSE) of $\hat{\mathbf{g}}_j$ can be expressed as

$$\begin{aligned}\text{bias}_j &= \mathbf{0}\\ \text{NMSE}_j &\approx \frac{M\sigma_e^2}{B\nu\|\mathbf{g}_j\|^2}\|\mathcal{U}_j^{\perp\dagger H}\|^2\end{aligned}$$

where $(\cdot)^\dagger$ represents the Moore-Penrose pseudo-inverse.

Proof: See Appendix B. ■

C. Channel Gain Estimation

In this section, we show how to estimate $|\gamma_j|$, which is equal to the channel gain $\|\mathbf{g}_j\|$, from $\mathbf{Z}_j$. Assume that the data symbol sequence $s_j[k]$ is white with variance 1 ($\mathbf{R}_{\mathbf{s}_j} = \mathbf{I}_L$) and the additive noise $e[n]$ is white with variance σ_e^2 ($\mathbf{R}_{\mathbf{n}_j} = \nu\sigma_e^2\mathbf{I}_{2L}$). Then, we can write (11) as

$$\begin{aligned}\mathbf{R}_{\mathbf{z}_j} &= \mathrm{E}\{\mathbf{z}_j[k]\mathbf{z}_j^H[k]\} = \nu^2\mathbf{G}_j\mathbf{G}_j^H + \nu\sigma_e^2\mathbf{I}_{2L}\\ &= \nu^2|\gamma_j|^2\mathbf{G}_j^o\mathbf{G}_j^{oH} + \nu\sigma_e^2\mathbf{I}_{2L}\end{aligned}$$

where $\mathbf{G}_j^o$ is similarly defined as $\mathbf{G}_j$, using $\mathbf{g}_j^o$ instead of $\mathbf{g}_j$. Hence, we can write

$$|\gamma_j|^2 = \nu^{-2}\frac{1}{L}\,\mathrm{tr}\{\mathbf{G}_j^{o\dagger}(\mathbf{R}_{\mathbf{z}_j} - \nu\sigma_e^2\mathbf{I}_{2L})\mathbf{G}_j^{o\dagger H}\}$$

where $\mathrm{tr}\{\cdot\}$ represents the trace. This leads to the following estimate of $|\gamma_j|^2$:

$$|\hat{\gamma}_j|^2 = \nu^{-2}\frac{1}{L}\,\mathrm{tr}\{\hat{\mathbf{G}}_j^{o\dagger}(\hat{\mathbf{R}}_{\mathbf{z}_j} - \nu\hat{\sigma}_e^2\mathbf{I}_{2L})\hat{\mathbf{G}}_j^{o\dagger H}\} \tag{25}$$

where $\hat{\mathbf{G}}_j^o$ is similarly defined as $\mathbf{G}_j^o$; using $\hat{\mathbf{g}}_j^o$ instead of $\mathbf{g}_j^o$, $\hat{\mathbf{R}}_{\mathbf{z}_j}$ is defined as

$$\hat{\mathbf{R}}_{\mathbf{z}_j} = \frac{L}{K}\mathbf{Z}_j\mathbf{Z}_j^H$$

and $\hat{\sigma}_e^2$ is, for instance, obtained as ν^{-1} times the average of the L smallest eigenvalues of $\hat{\mathbf{R}}_{\mathbf{z}_j}$. Note that this estimate of the noise variance can also be used in the MMSE linear block combiner when the noise variance σ_e^2 is not known.

V. Benchmark

As a benchmark for the MUI-free receiver and the corresponding subspace deterministic blind single-user channel estimation algorithm, we consider the linear multiuser equalizer and the corresponding subspace deterministic blind multiuser channel estimation algorithm developed by Liu and Xu for a standard synchronous DS-CDMA system in the presence of frequency-selective fading [7]. In this section, we briefly review the key ideas behind these techniques.

A standard DS-CDMA system can be described as in Section II, taking $M = 1$. Like before, we focus on synchronous communication [see (3)] and limited frequency-selective fading [see (4)]. Like before, we further assume that an overestimation L of the maximal channel order $L_{\max}$ is known ($L > L_{\max}$) and that L satisfies $L \ll N$. As discussed in [7], if we then introduce

$$\bar{\mathbf{y}}[k] = [y[kN+L-1] \quad y[kN+L] \quad \cdots \quad y[(k+1)N-1]]^T$$

we can write

$$\bar{\mathbf{y}}[k] = \sum_{j=1}^{J} \bar{\mathbf{h}}_j s_j[k] + \bar{\mathbf{e}}[k]$$

where $\bar{\mathbf{e}}[k]$ is similarly defined as $\bar{\mathbf{y}}[k]$, and $\bar{\mathbf{h}}_j$ is the $(N-L+1)\times 1$ *composite* channel vector, which is given by

$$\bar{\mathbf{h}}_j = \mathbf{C}_j\mathbf{g}_j$$

with

$$\mathbf{C}_j = \begin{bmatrix} c_j[L-1] & \cdots & c_j[0]\\ c_j[L] & \cdots & c_j[1]\\ \vdots & & \vdots\\ c_j[N-1] & \cdots & c_j[N-L]\end{bmatrix}.$$

Note that $\bar{\mathbf{y}}[k]$ can also be written as

$$\bar{\mathbf{y}}[k] = \mathbf{H}\mathbf{s}[k] + \bar{\mathbf{e}}[k] \tag{26}$$

where $\mathbf{H}$ is the $(N-L+1)\times J$ *composite* channel matrix, which is given by

$$\mathbf{H} = [\bar{\mathbf{h}}_1 \quad \bar{\mathbf{h}}_2 \quad \cdots \quad \bar{\mathbf{h}}_J]$$

and $\mathbf{s}[k]$ is defined as

$$\mathbf{s}[k] = [s_1[k] \quad s_2[k] \quad \cdots \quad s_J[k]]^T.$$

Since $\bar{\mathbf{y}}[k]$ is free from ISI by construction, we only have to suppress the remaining MUI. This can be done as follows:

$$\hat{s}_j[k] = \mathbf{f}_j\bar{\mathbf{y}}[k] = \mathbf{f}_j\mathbf{H}\mathbf{s}[k] + \mathbf{f}_j\bar{\mathbf{e}}[k]$$

where $\mathbf{f}_j$ is the $(N-L+1)\times 1$ linear multiuser equalizer for the jth user. See [3]–[5] for details on the ZF and MMSE linear multiuser equalizer. The existence of the ZF linear multiuser equalizer is based on the assumption that the $(N-L+1)\times J$ composite channel matrix $\mathbf{H}$ has full column rank J (hence, we need $J \le N-L+1$).

The corresponding subspace deterministic blind multiuser channel estimation algorithm is explained in [7]. For a burst length K, we first define the $(N-L+1)\times K$ matrix $\mathbf{Y}$ as

$$\mathbf{Y} = [\bar{\mathbf{y}}[0] \quad \bar{\mathbf{y}}[1] \quad \cdots \quad \bar{\mathbf{y}}[K-1]].$$

Using (26), we then obtain

$$\mathbf{Y} = \mathbf{H}\mathbf{S} + \mathbf{E} \tag{27}$$

where $\mathbf{E}$ is similarly defined as $\mathbf{Y}$, and $\mathbf{S}$ is the $J\times K$ matrix given by

$$\mathbf{S} = [\mathbf{s}[0] \quad \mathbf{s}[1] \quad \cdots \quad \mathbf{s}[K-1]].$$

The algorithm is based on the assumption that the $(N-L+1)\times J$ composite channel matrix $\mathbf{H}$ is tall and has full column rank J (hence, we need $J < N-L+1$) and on the assumption that the $J\times K$ matrix $\mathbf{S}$ has full row rank J (hence, we need $J \le K$). First, let us assume there is no additive noise present in (27). The counterpart of (22) then is

$$\mathbf{U}^{\perp H}\mathbf{C}_j\mathbf{g}_j = \mathbf{0}$$

where $\mathbf{U}^{\perp}$ is the $(N-L+1)\times(N-L+1-J)$ matrix defined as the collection of the $N-L+1-J$ left singular vectors of $\mathbf{Y}$ corresponding to the $N-L+1-J$ zero singular values. See [7] for the identifiability result. Second, let us assume that there is additive noise present in (27). The counterpart of (23) then is

$$\min_{\mathbf{g}_j}\{\|\hat{\mathbf{U}}^{\perp H}\mathbf{C}_j\mathbf{g}_j\|^2\}$$

where $\hat{\mathbf{U}}^{\perp}$ is the $(N-L+1)\times(N-L+1-J)$ matrix defined as the collection of the $N-L+1-J$ left singular vectors of $\mathbf{Y}$ corresponding to the $N-L+1-J$ smallest singular values. See [7] for a performance analysis and a related channel gain estimation method.

VI. Complexity Analysis

In this section, we compare the complexity of the MUI-free receiver (see Section III) using the corresponding subspace deterministic blind single-user channel estimation algorithm (see Section IV) with the complexity of the linear multi-user equalizer (see [7] and Section V) using the corresponding subspace deterministic blind multi-user channel estimation algorithm (see [7] and Section V). We assume that the burst length K is very large.

Let us first focus on the MUI-free receiver (see Section III). The design of the jth user's MUI-free receiver is determined by the design of the jth user's linear block combiner. The design of the jth user's linear block combiner only requires the knowledge of the jth user's channel. To estimate the jth user's channel, we use the subspace deterministic blind single-user channel estimation algorithm (see Section IV), which only requires the jth user's code sequence, namely, through the fact that this algorithm makes use of the data at the output of the bank of two modified block correlators. For this, we first have to compute the subspace decomposition of the $2L\times K/L$ matrix $\mathbf{Z}_j$, which has a complexity of O$\{LK\}$. Then, we have to compute the left singular vector of the $L\times L^2$ matrix $\hat{\mathcal{U}}_j^{\perp}$ corresponding to the smallest singular value, which has a complexity of O$\{L^4\}$. Hence, the complexity to estimate the jth user's channel is O$\{LK\}$ (since K is very large). Next, we have to calculate the pseudo-inverse of the $2L\times L$ matrix $\mathbf{G}_j$ (we focus on a ZF linear block combiner), which results in a complexity of O$\{L^3\}$. To conclude, the design of the jth user's linear block combiner has a complexity of O$\{LK\}$ (since K is very large). Hence, the design of the jth user's MUI-free receiver also has a complexity of O$\{LK\}$. Note that the design of a complete bank of J MUI-free receivers (e.g., at a base station) results in a complexity of O$\{JLK\}$.

Let us next focus on the linear multiuser equalizer (see [7] and Section V). The design of the jth user's linear multiuser equalizer requires the knowledge of all the channels and code sequences. To estimate the jth user's channel, we use the subspace deterministic blind multiuser channel estimation algorithm (see [7] and Section V), which only requires the jth user's code sequence. For this, we first have to compute a subspace decomposition of the $(N-L+1)\times K$ matrix $\mathbf{Y}$, which has a complexity of O$\{N^2K\}$. Note that this subspace decomposition is user-independent and, therefore, still results into a complexity of O$\{N^2K\}$ if we want to estimate all the channels. Then, we have to compute the left singular vector of the $L\times(N-L+1-J)$ matrix $\mathbf{C}_j^H\hat{\mathbf{U}}^{\perp}$ corresponding to the smallest singular value, which has a complexity of O$\{L^2(N-L+1-J)\}$. Note that this subspace decomposition is user-dependent and, therefore, results in a complexity of O$\{JL^2(N-L+1-J)\}$ if we want to estimate all the channels. Hence, the complexity to estimate all the channels is O$\{N^2K\}$ (since K is very large). Next, we have to calculate the pseudo-inverse of the $(N-L+1)\times J$ matrix $\mathbf{H}$ (we focus on a ZF linear multi-user equalizer), which results in a complexity of O$\{NJ^2\}$. To conclude, the design of the jth user's linear multiuser equalizer has a complexity of O$\{N^2K\}$ (since K is very large). Note that the design of a complete bank of J multiuser equalizers (e.g., at a base station) still results in a complexity of O$\{N^2K\}$. Even using the ideas presented in [9], where it is shown that it is possible to design the desired user's linear multiuser equalizer based only on the knowledge of the desired user's channel and code sequence (and not on the knowledge of all the channels and code sequences), it is not possible to decrease this complexity of $\mathcal{O}\{N^2K\}$.

Comparing the obtained complexities, we observe that the complexity to design the MUI-free receiver ($\mathcal{O}\{LK\}$) is *much smaller* than the complexity to design the linear multiuser equalizer (O$\{N^2K\}$). This is because $L \ll N$. Moreover, the complexity to design a complete bank of J MUI-free receivers (O$\{JLK\}$) is also much smaller than the complexity to design a complete bank of J linear multiuser equalizers (O$\{N^2K\}$), although the latter is of the same order as the complexity to design only one linear multiuser equalizer. This is because J is of the same order as N and $L \ll N$.

Finally, note that the estimation of the desired user's data symbol sequence based on the calculated MUI-free receiver or linear multiuser equalizer results in a complexity per data symbol period of O$\{N\}$ (this complexity is more or less the same for both receivers).

VII. Simulation Results

In this section, we perform some simulations on a DS-CDMA system based on block spreading and a standard DS-CDMA system. We assume that the data symbol sequences $\{s_j[k]\}_{j=1}^J$ are QPSK modulated mutually uncorrelated and white with variance 1. We further assume that the additive noise $e[n]$ is white Gaussian with variance σ_e^2. We define the received energy per data symbol period for the jth user as

$$E_j = \|\mathbf{g}_j\|^2$$

TABLE I
NORMALIZED CHANNELS

n	0	1	2	3
$g_1[n]$	$+0.1513 - 0.1291i$	$-0.1327 + 0.6884i$	$-0.3550 - 0.1626i$	$+0.1228 + 0.5490i$
$g_2[n]$	$-0.2588 + 0.4124i$	$+0.3581 - 0.0455i$	$-0.4241 + 0.5462i$	$-0.3760 - 0.1147i$
$g_3[n]$	$+0.4066 - 0.5422i$	$+0.5321 + 0.2291i$	$+0.4098 + 0.1556i$	$+0.1024 - 0.0487i$
$g_4[n]$	$-0.2163 + 0.2912i$	$-0.5332 - 0.4866i$	$-0.0811 - 0.4955i$	$+0.0781 + 0.2985i$
$g_5[n]$	$-0.3083 + 0.0279i$	$+0.8654 + 0.0949i$	$-0.0394 - 0.0426i$	$+0.0033 - 0.3779i$
$g_6[n]$	$-0.0638 - 0.5494i$	$+0.2137 + 0.3858i$	$+0.2448 + 0.5350i$	$+0.1602 + 0.3574i$
$g_7[n]$	$+0.1898 - 0.1522i$	$+0.1045 - 0.1316i$	$+0.3813 - 0.3089i$	$-0.5015 - 0.6482i$
$g_8[n]$	$-0.0609 + 0.7390i$	$-0.5202 + 0.0305i$	$-0.1328 - 0.3271i$	$-0.1992 - 0.1195i$

and we assume that all users that are interfering with this jth user have the same received energy per data symbol period

$$E_{j'} = \|\mathbf{g}_{j'}\|^2 = E, \quad \text{for } j' \neq j.$$

We then define the signal-to-noise ratio (SNR) and the near-far ratio (NFR) for the jth user at the input of the receiver as $\mathrm{SNR}_j = E_j/\sigma_e^2$ and $\mathrm{NFR}_j = E/E_j$.

We here consider an eight-user DS-CDMA system based on block spreading and a standard DS-CDMA system ($J = 8$) with $N = 17$. We design a shift-orthogonal set of J length-N code sequences $\{c_j[n]\}_{j=1}^{J}$, as explained in Section III-C, and use this set of code sequences for the DS-CDMA system based on block spreading as well as for the standard DS-CDMA system. We start from (see Section III-C for details)

$$\mathbf{C}_4 = \begin{bmatrix} +1 & -1 \\ +1 & +1 \\ -1 & +1 \\ +1 & +1 \end{bmatrix}$$

resulting into a set of BPSK code sequences. For every user j, we generate a random channel $g_j[n]$ of order $L_j = 3$ with delay index $\delta_j = 0$ (note that (3) and (4) are then satisfied). We take $L = L_{\max} + 1 = 4$. The normalized channels are listed in Table I. For all simulations, we will conduct 5000 trials using bursts of $K = 200$ data symbols.

Simulation 1: First, we compare the MUI-free receiver with ZF and MMSE linear block combining (see Section III) with the ZF and MMSE linear multiuser equalizer (see [7] and Section V). For the moment, we assume that all the channels $\{g_j[n]\}_{j=1}^{J}$ and the noise variance σ_e^2 (necessary for both MMSE receivers) are known. Fig. 5 shows the average theoretical and simulated BER per user as a function of the SNR for an NFR of 0 dB. Fig. 6 shows the same results as a function of the NFR for an SNR of 10 dB. First of all, we observe that the simulation results are well predicted by the theoretical results. Second, we know from the previous sections that the MUI-free receiver should always be NFR-independent,[2] irrespective of the linear block combiner. This is clearly illustrated in Fig. 6. From this figure, it is also clear that the ZF linear multiuser equalizer is NFR-independent, whereas the MMSE linear multiuser equalizer is not. For a very high NFR, the performance of the MMSE linear multiuser equalizer is equal to the performance of the ZF linear multiuser equalizer. For a very low NFR, the performance of the MMSE linear multiuser equalizer approaches the performance of the coherent RAKE receiver [21], which is, for such a very low NFR, better than the performance of the ZF linear multiuser equalizer. Note that the NFR region where the transition takes place is positioned around the inverse of the SNR. Further, we observe that the performance of the MUI-free receiver with ZF linear block combining is comparable with the performance of the ZF linear multiuser equalizer. We know that the performance of the MUI-free receiver with MMSE linear block combining is better than the performance of the MUI-free receiver with ZF linear block combining and that the difference between those performances decreases with the SNR. Therefore, the performance of the MUI-free receiver with MMSE linear block combining is also better than the performance of the MMSE linear multiuser equalizer at very high NFR, and the difference between those performances also decreases with the SNR. The somewhat surprising result that the MUI-free receiver can perform better than the linear multiuser equalizer is due to the fact that the linear multiuser equalizer we consider here only

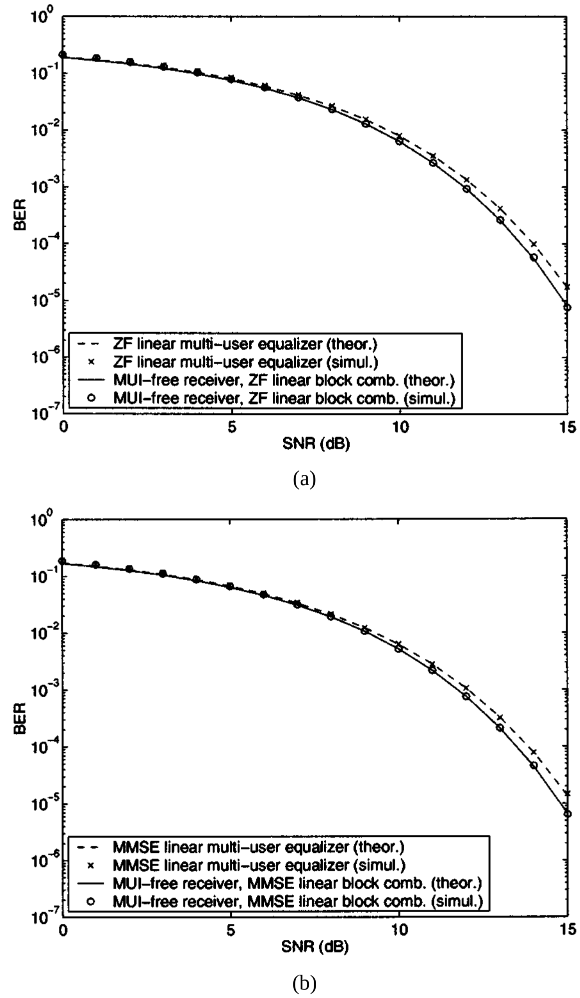


Fig. 5. Average theoretical and simulated BER per user as a function of the SNR for an NFR of 0 dB (known channels and noise variance).

[2] Note that NFR-independence is not exactly the same as near-far resistance, which is defined in [20].

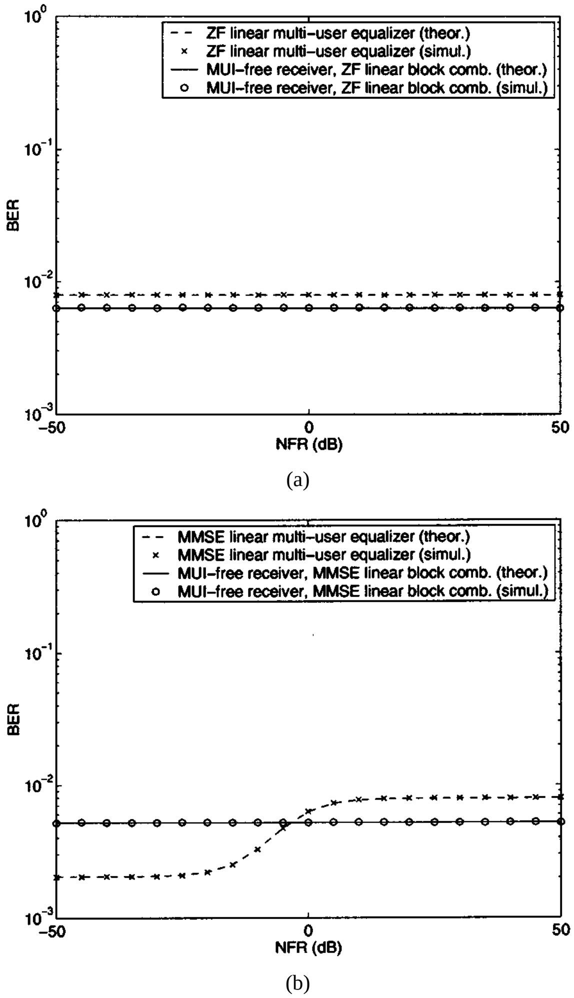


Fig. 6. Average theoretical and simulated BER per user as a function of the NFR for an SNR of 10 dB (known channels and noise variance).

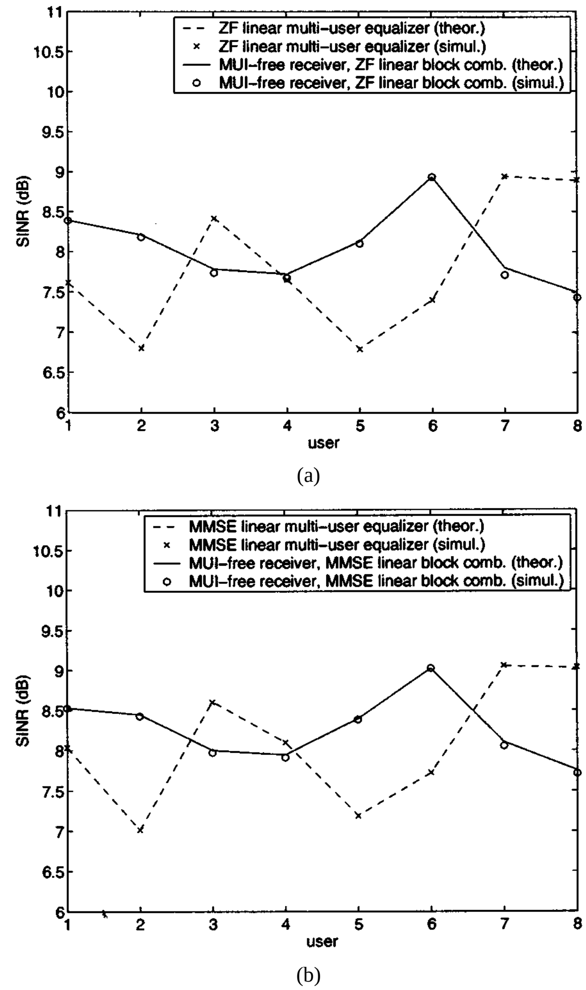


Fig. 7. Theoretical and simulated SINR as a function of the desired user for an SNR of 10 dB and an NFR of 0 dB (known channels and noise variance).

has length $N-L+1=14$. Hence, it removes $L-1=3$ out of $N=17$ received samples, whereas in the MUI-free receiver, every modified block correlator only removes 1 out of $N=17$ received samples (since every modified block correlator only removes 1 out of $N=17$ received blocks). The performance of a much longer (hence, much more expensive) linear multiuser equalizer will always be somewhat better than the performance of the MUI-free receiver. Finally, Fig. 7 shows the theoretical and simulated SINR as a function of the desired user for an SNR of 10 dB and an NFR of 0 dB. Again, we observe that the simulation results are well predicted by the theoretical results.

Simulation 2: Next, we compare the subspace deterministic blind single-user channel estimation algorithm (see Section III) with the subspace deterministic blind multiuser channel estimation algorithm (see [7] and Section V). Fig. 8 shows the average theoretical and simulated NMSE per user of the channel estimates as a function of the SNR for an NFR of 0 dB. Fig. 9 shows the same results as a function of the NFR for an SNR of 20 dB. First of all, we see that the theoretical performances of the single-user and multiuser algorithm are comparable. From Fig. 8, we observe that for an SNR above 10 dB, the simulation results are well predicted by the theoretical results (below 10

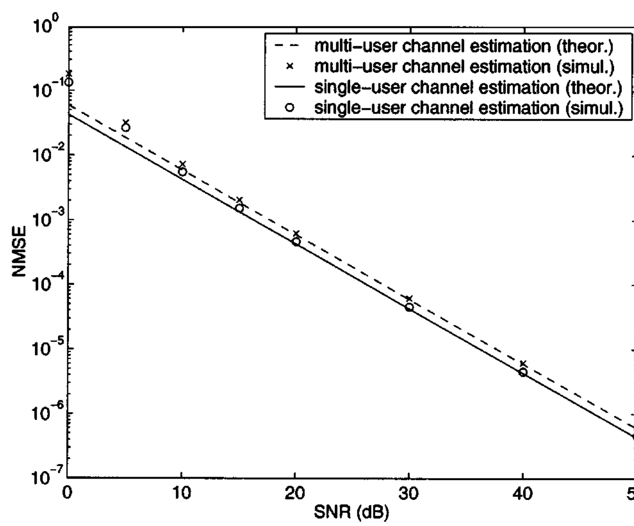


Fig. 8. Average theoretical and simulated NMSE per user of the channel estimates as a function of the SNR for an NFR of 0 dB.

dB, the additive noise influence is too large to predict the simulation results using only first-order subspace perturbation analysis). From Fig. 9, we notice that for the single-user algorithm, the above statement is true for all values of the NFR, whereas

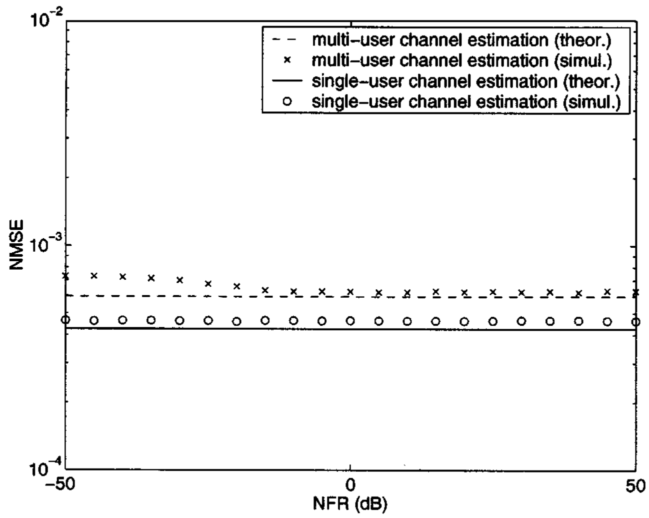


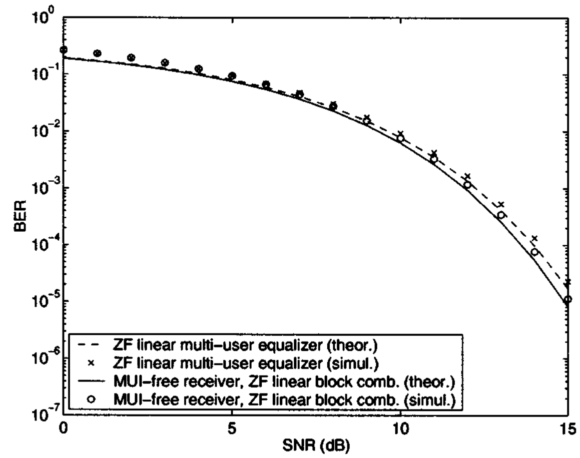
Fig. 9. Average theoretical and simulated NMSE per user of the channel estimates as a function of the NFR for an SNR of 20 dB.

for the multiuser algorithm, it is only true for an NFR above the inverse of the SNR. When the NFR drops below the inverse of the SNR, the $(N-L+1)\times(N-L+1-J)$ matrix $\hat{\mathbf{U}}^{\perp}$ that is used to estimate the channel vector $\mathbf{g}_j$ (see [7] and Section V) is severely influenced by additive noise, and the first-order perturbation analysis is not accurate enough to predict the simulation results. We know from the previous sections that the single-user algorithm should always be NFR independent. This is clearly illustrated in Fig. 9 by the theoretical as well as the simulation results. The multiuser algorithm, on the other hand, only appears to be theoretically NFR independent. The simulation results show that this is not the case in practice.

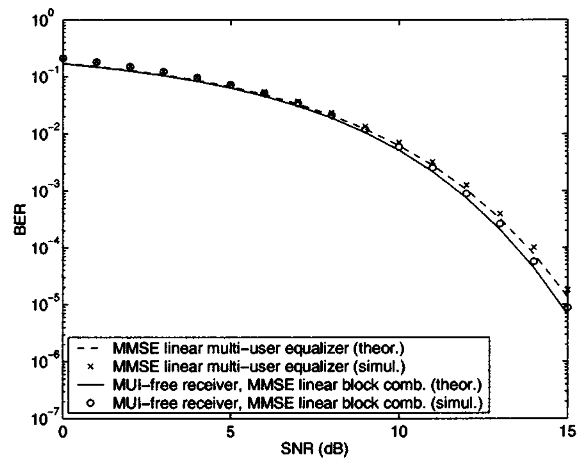
Simulation 3: We now repeat Simulation 1, but for each receiver, we will use the corresponding subspace deterministic blind channel estimation algorithm. Moreover, for each of both MMSE receivers, we will use the corresponding estimate of the noise variance σ_e^2. Fig. 10 shows the average theoretical (known channels and noise variance) and simulated (estimated channels and noise variance) BER per user as a function of the SNR for an NFR of 0 dB. Fig. 11 shows the same results as a function of the NFR for an SNR of 10 dB. We observe that estimating the channels and the noise variance (necessary in both MMSE receivers) decreases the performances of all receivers. Note that the design of the desired user's ZF or MMSE linear multiuser equalizer requires the knowledge of all the channels (we do not use the ideas presented in [9]). When the NFR drops below the inverse of the SNR, the channel estimates of the interfering users become less accurate. For the ZF linear multiuser equalizer, this property increases the difference in BER with Simulation 1. For the MMSE linear multiuser equalizer, this property does not change the difference in BER with Simulation 1 since the MMSE linear mutliuser equalizer is, in that region of the NFR, more steered by the additive noise.

VIII. CONCLUSIONS

In this paper, we have discussed a synchronous DS-CDMA system based on block spreading in the presence of frequency-selective fading. For such a system, we have developed a simple new receiver that completely removes the MUI without using any channel information (hence, the name MUI-free receiver). The MUI-free operation is obtained by the use of a shift-orthogonal set of code sequences on which this receiver is based. Within the framework of the MUI-free receiver, we have further presented a subspace deterministic blind single-user channel estimation algorithm. As a benchmark for the MUI-free receiver and the corresponding subspace deterministic blind single-user channel estimation algorithm, we have considered the linear multiuser equalizer and the corresponding subspace deterministic blind multiuser channel estimation algorithm developed by Liu and Xu for a standard synchronous DS-CDMA system in the presence of frequency-selective fading [7]. We have shown that the complexity of the MUI-free receiver using the corresponding subspace deterministic blind single-user channel estimation algorithm is much smaller than the complexity of the linear multiuser equalizer using the corresponding subspace deterministic blind multiuser channel estimation algorithm. We have further shown that the performance of the MUI-free receiver is comparable with the performance of the linear multiuser equalizer. This is for the case in which the channels are



(a)



(b)

Fig. 10. Average theoretical (known channels and noise variance) and simulated (estimated channels and noise variance) BER per user as a function of the SNR for an NFR of 0 dB.

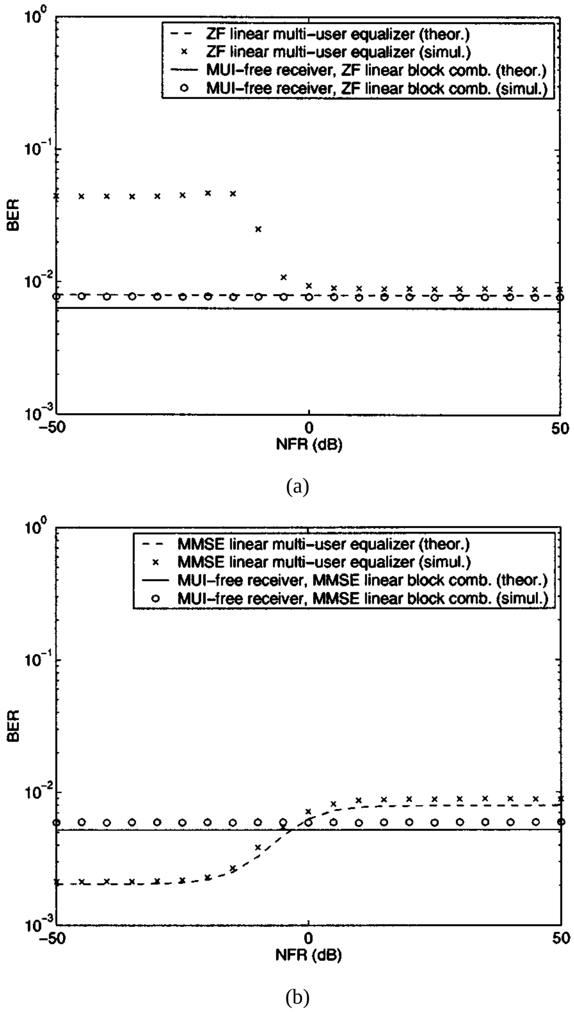


Fig. 11. Average theoretical (known channels and noise variance) and simulated (estimated channels and noise variance) BER per user as a function of the NFR for an SNR of 10 dB.

known as well as for the case in which the channels are estimated with the corresponding subspace deterministic blind channel estimation algorithm. An interesting topic for further research is the extension of the MUI-free receiver to multiple receive antennas, aiming at an increase of the user capacity of the system. Finally, note that for a discrete multitone CDMA (DMT-CDMA) system based on block spreading in the presence of frequency-selective fading, a simple extension of the MUI-free receiver is presented in [22].

APPENDIX A
PROOF OF THEOREM 2

Assume that the $d \times d/2$ (d power of 2) matrix $\mathbf{C}_d = [\mathbf{C}_{d,1}\ \mathbf{C}_{d,2}]$, where $\mathbf{C}_{d,1}$ and $\mathbf{C}_{d,2}$ have size $d \times d/4$, is admissible and reducible. Using $\mathbf{C}_{d,2} = \mathbf{D}_d\mathbf{C}_{d,1}$, it is easy to check that $\mathbf{C}_{2d,2} = \mathbf{D}_{2d}\mathbf{C}_{2d,1}$. Next, using $\mathbf{C}_d^H\mathbf{C}_d = d\mathbf{I}_{d/2}$ and $\mathbf{C}_d^H\mathbf{J}_d\mathbf{C}_d = \mathbf{O}_{d/2}$, it can be shown by simple calculation that $\mathbf{C}_{2d}^H\mathbf{C}_{2d} = 2d\mathbf{I}_d$. Finally, using $\mathbf{C}_d^H\mathbf{C}_d = d\mathbf{I}_{d/2}$, $\mathbf{C}_d^H\mathbf{J}_d\mathbf{C}_d = \mathbf{O}_{d/2}$, and $\mathbf{C}_{d,2} = \mathbf{D}_d\mathbf{C}_{d,1}$, we now prove that $\mathbf{C}_{2d}^H\mathbf{J}_{2d}\mathbf{C}_{2d} = \mathbf{O}_d$. Defining the $n \times n$ matrix $\mathbf{T}_n$ as

$$\mathbf{T}_n = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

we first observe that

$$\mathbf{C}_{2d}^H\mathbf{J}_{2d}\mathbf{C}_{2d} = \begin{bmatrix} \mathbf{C}_{d,1}^H & \mathbf{C}_{d,2}^H \\ \mathbf{C}_{d,1}^H\mathbf{J}_d^H & -\mathbf{C}_{d,2}^H\mathbf{J}_d^H \\ \mathbf{C}_{d,2}^H & \mathbf{C}_{d,1}^H \\ -\mathbf{C}_{d,2}^H\mathbf{J}_d^H & \mathbf{C}_{d,1}^H\mathbf{J}_d^H \end{bmatrix} \cdot \begin{bmatrix} \mathbf{J}_d - \mathbf{T}_d & \mathbf{T}_d \\ \mathbf{T}_d & \mathbf{J}_d - \mathbf{T}_d \end{bmatrix} \cdot \begin{bmatrix} \mathbf{C}_{d,1} & \mathbf{J}_d\mathbf{C}_{d,1} & \mathbf{C}_{d,2} & -\mathbf{J}_d\mathbf{C}_{d,2} \\ \mathbf{C}_{d,2} & -\mathbf{J}_d\mathbf{C}_{d,2} & \mathbf{C}_{d,1} & \mathbf{J}_d\mathbf{C}_{d,1} \end{bmatrix}.$$

Writing $\mathbf{C}_{2d}^H\mathbf{J}_{2d}\mathbf{C}_{2d}$ as

$$\mathbf{C}_{2d}^H\mathbf{J}_{2d}\mathbf{C}_{2d} = \begin{bmatrix} \mathbf{B}_{1,1} & \mathbf{B}_{1,2} & \mathbf{B}_{1,3} & \mathbf{B}_{1,4} \\ \mathbf{B}_{2,1} & \mathbf{B}_{2,2} & \mathbf{B}_{2,3} & \mathbf{B}_{2,4} \\ \mathbf{B}_{3,1} & \mathbf{B}_{3,2} & \mathbf{B}_{3,3} & \mathbf{B}_{3,4} \\ \mathbf{B}_{4,1} & \mathbf{B}_{4,2} & \mathbf{B}_{4,3} & \mathbf{B}_{4,4} \end{bmatrix}$$

where $\mathbf{B}_{m,n}$ $(m, n = 1, 2, 3, 4)$ is the $d/4 \times d/4$ submatrix of $\mathbf{C}_{2d}^H\mathbf{J}_{2d}\mathbf{C}_{2d}$ at position (m, n), it is then clear that the $d/4 \times d/4$ submatrices $\mathbf{B}_{3,1}$, $\mathbf{B}_{3,2}$, $\mathbf{B}_{3,3}$, $\mathbf{B}_{3,4}$, $\mathbf{B}_{4,1}$, $\mathbf{B}_{4,2}$, $\mathbf{B}_{4,3}$, and $\mathbf{B}_{4,4}$ are, respectively, equal to the $d/4 \times d/4$ submatrices $\mathbf{B}_{1,3}$, $\mathbf{B}_{1,4}$, $\mathbf{B}_{1,1}$, $\mathbf{B}_{1,2}$, $\mathbf{B}_{2,3}$, $\mathbf{B}_{2,4}$, $\mathbf{B}_{2,1}$ and $\mathbf{B}_{2,2}$. Hence, only the latter have to be calculated:

Submatrix $\mathbf{B}_{1,1}$:

$$\begin{aligned}\mathbf{B}_{1,1} =& \mathbf{C}_{d,1}^H(\mathbf{J}_d - \mathbf{T}_d)\mathbf{C}_{d,1} + \mathbf{C}_{d,2}^H\mathbf{T}_d\mathbf{C}_{d,1} \\ &+ \mathbf{C}_{d,1}^H\mathbf{T}_d\mathbf{C}_{d,2} + \mathbf{C}_{d,2}^H(\mathbf{J}_d - \mathbf{T}_d)\mathbf{C}_{d,2} \\ =& -\mathbf{C}_{d,1}^H\mathbf{T}_d\mathbf{C}_{d,1} + \mathbf{C}_{d,1}^H\mathbf{D}_d\mathbf{T}_d\mathbf{C}_{d,1} \\ &+ \mathbf{C}_{d,1}^H\mathbf{T}_d\mathbf{D}_d\mathbf{C}_{d,1} - \mathbf{C}_{d,1}^H\mathbf{D}_d\mathbf{T}_d\mathbf{D}_d\mathbf{C}_{d,1} \\ =& -\mathbf{C}_{d,1}^H\mathbf{T}_d\mathbf{C}_{d,1} - \mathbf{C}_{d,1}^H\mathbf{T}_d\mathbf{C}_{d,1} \\ &+ \mathbf{C}_{d,1}^H\mathbf{T}_d\mathbf{C}_{d,1} + \mathbf{C}_{d,1}^H\mathbf{T}_d\mathbf{C}_{d,1} = \mathbf{O}_{d/4}.\end{aligned}$$

Submatrix $\mathbf{B}_{1,2}$:

$$\begin{aligned}\mathbf{B}_{1,2} =& \mathbf{C}_{d,1}^H(\mathbf{J}_d - \mathbf{T}_d)\mathbf{J}_d\mathbf{C}_{d,1} + \mathbf{C}_{d,2}^H\mathbf{T}_d\mathbf{J}_d\mathbf{C}_{d,1} \\ &- \mathbf{C}_{d,1}^H\mathbf{T}_d\mathbf{J}_d\mathbf{C}_{d,2} - \mathbf{C}_{d,2}^H(\mathbf{J}_d - \mathbf{T}_d)\mathbf{J}_d\mathbf{C}_{d,2} \\ =& \mathbf{C}_{d,1}^H\mathbf{J}_d^2\mathbf{C}_{d,1} - \mathbf{C}_{d,1}^H\mathbf{T}_d\mathbf{J}_d\mathbf{C}_{d,1} + \mathbf{C}_{d,1}^H\mathbf{D}_d\mathbf{T}_d\mathbf{J}_d\mathbf{C}_{d,1} \\ &- \mathbf{C}_{d,1}^H\mathbf{T}_d\mathbf{J}_d\mathbf{D}_d\mathbf{C}_{d,1} - \mathbf{C}_{d,1}^H\mathbf{D}_d\mathbf{J}_d^2\mathbf{D}_d\mathbf{C}_{d,1} \\ &+ \mathbf{C}_{d,1}^H\mathbf{D}_d\mathbf{T}_d\mathbf{J}_d\mathbf{D}_d\mathbf{C}_{d,1} \\ =& \mathbf{C}_{d,1}^H\mathbf{J}_d^2\mathbf{C}_{d,1} - \mathbf{C}_{d,1}^H\mathbf{T}_d\mathbf{J}_d\mathbf{C}_{d,1} - \mathbf{C}_{d,1}^H\mathbf{T}_d\mathbf{J}_d\mathbf{C}_{d,1} \\ &+ \mathbf{C}_{d,1}^H\mathbf{T}_d\mathbf{J}_d\mathbf{C}_{d,1} - \mathbf{C}_{d,1}^H\mathbf{J}_d^2\mathbf{C}_{d,1} \\ &+ \mathbf{C}_{d,1}^H\mathbf{T}_d\mathbf{J}_d\mathbf{C}_{d,1} = \mathbf{O}_{d/4}.\end{aligned}$$

Submatrix $\mathbf{B}_{1,3}$: similar calculation as for $\mathbf{B}_{1,1}$.
Submatrix $\mathbf{B}_{1,4}$: similar calculation as for $\mathbf{B}_{1,2}$.

Submatrix $\mathbf{B}_{2,1}$:

$$\begin{aligned}\mathbf{B}_{2,1} =& \mathbf{C}_{d,1}^H\mathbf{J}_d^H(\mathbf{J}_d-\mathbf{T}_d)\mathbf{C}_{d,1} - \mathbf{C}_{d,2}^H\mathbf{J}_d^H\mathbf{T}_d\mathbf{C}_{d,1}\\ &+ \mathbf{C}_{d,1}^H\mathbf{J}_d^H\mathbf{T}_d\mathbf{C}_{d,2} - \mathbf{C}_{d,2}^H\mathbf{J}_d^H(\mathbf{J}_d-\mathbf{T}_d)\mathbf{C}_{d,2}\\ =& d\mathbf{I}_{d/4} - \mathbf{C}_{d,1}^H\mathbf{J}_d^H\mathbf{T}_d\mathbf{C}_{d,1} - \mathbf{C}_{d,1}^H\mathbf{D}_d\mathbf{J}_d^H\mathbf{T}_d\mathbf{C}_{d,1}\\ &+ \mathbf{C}_{d,1}^H\mathbf{J}_d^H\mathbf{T}_d\mathbf{D}_d\mathbf{C}_{d,1} - d\mathbf{I}_{d/4}\\ &+ \mathbf{C}_{d,1}^H\mathbf{D}_d\mathbf{J}_d^H\mathbf{T}_d\mathbf{D}_d\mathbf{C}_{d,1}\\ =& -\mathbf{C}_{d,1}^H\mathbf{J}_d^H\mathbf{T}_d\mathbf{C}_{d,1} - \mathbf{C}_{d,1}^H\mathbf{J}_d^H\mathbf{T}_d\mathbf{C}_{d,1}\\ &+ \mathbf{C}_{d,1}^H\mathbf{J}_d^H\mathbf{T}_d\mathbf{C}_{d,1} + \mathbf{C}_{d,1}^H\mathbf{J}_d^H\mathbf{T}_d\mathbf{C}_{d,1} = \mathbf{O}_{d/4}.\end{aligned}$$

Submatrix $\mathbf{B}_{2,2}$:

$$\begin{aligned}\mathbf{B}_{2,2} =& \mathbf{C}_{d,1}^H\mathbf{J}_d^H(\mathbf{J}_d-\mathbf{T}_d)\mathbf{J}_d\mathbf{C}_{d,1} - \mathbf{C}_{d,2}^H\mathbf{J}_d^H\mathbf{T}_d\mathbf{J}_d\mathbf{C}_{d,1}\\ &- \mathbf{C}_{d,1}^H\mathbf{J}_d^H\mathbf{T}_d\mathbf{J}_d\mathbf{C}_{d,2} + \mathbf{C}_{d,2}^H\mathbf{J}_d^H(\mathbf{J}_d-\mathbf{T}_d)\mathbf{J}_d\mathbf{C}_{d,2}\\ =& -\mathbf{C}_{d,1}^H\mathbf{J}_d^H\mathbf{T}_d\mathbf{J}_d\mathbf{C}_{d,1} - \mathbf{C}_{d,1}^H\mathbf{D}_d\mathbf{J}_d^H\mathbf{T}_d\mathbf{J}_d\mathbf{C}_{d,1}\\ &- \mathbf{C}_{d,1}^H\mathbf{J}_d^H\mathbf{T}_d\mathbf{J}_d\mathbf{D}_d\mathbf{C}_{d,1} - \mathbf{C}_{d,1}^H\mathbf{D}_d\mathbf{J}_d^H\mathbf{T}_d\mathbf{J}_d\mathbf{D}_d\mathbf{C}_{d,1}\\ =& -\mathbf{C}_{d,1}^H\mathbf{J}_d^H\mathbf{T}_d\mathbf{J}_d\mathbf{C}_{d,1} - \mathbf{C}_{d,1}^H\mathbf{J}_d^H\mathbf{T}_d\mathbf{J}_d\mathbf{C}_{d,1}\\ &+ \mathbf{C}_{d,1}^H\mathbf{J}_d^H\mathbf{T}_d\mathbf{J}_d\mathbf{C}_{d,1} + \mathbf{C}_{d,1}^H\mathbf{J}_d^H\mathbf{T}_d\mathbf{J}_d\mathbf{C}_{d,1} = \mathbf{O}_{d/4}.\end{aligned}$$

Submatrix $\mathbf{B}_{2,3}$: similar calculation as for $\mathbf{B}_{2,1}$.
Submatrix $\mathbf{B}_{2,4}$: similar calculation as for $\mathbf{B}_{2,2}$.
From these calculations, it is clear that $\mathbf{C}_{2d}^H\mathbf{J}_{2d}\mathbf{C}_{2d} = \mathbf{O}_d$, which concludes the proof.

Appendix B
Proof of Theorem 4

Using a result from subspace perturbation analysis [23], the first-order approximation of $\Delta\mathbf{g}_j = \hat{\mathbf{g}}_j - \mathbf{g}_j$ in $\mathbf{N}_j$ can be written as $\Delta^{(1)}\mathbf{U}_j^\perp = -\mathbf{Z}_j^{\dagger H}\mathbf{N}_j^H\mathbf{U}_j^\perp$. The first-order approximation of $\Delta\mathbf{U}_{j,l}^\perp = \hat{\mathbf{U}}_{j,l}^\perp - \mathbf{U}_{j,l}^\perp$ $(l = 1,2,\cdots,L)$ in $\mathbf{N}_j$ is therefore given by $\Delta^{(1)}\mathbf{U}_{j,l}^\perp = -\mathbf{Z}_{j,l}^{\dagger H}\mathbf{N}_j^H\mathbf{U}_j^\perp$, where $\mathbf{Z}_{j,l}^\dagger = \mathbf{Z}_j^\dagger(:,l : l+L-1)$. Thus, the first-order approximation of $\Delta\mathcal{U}_j^\perp = \hat{\mathcal{U}}_j^\perp - \mathcal{U}_j^\perp$ in $\mathbf{N}_j$ is

$$\begin{aligned}\Delta^{(1)}\mathcal{U}_j^\perp =& [\Delta^{(1)}\mathbf{U}_{j,1}^\perp \quad \cdots \quad \Delta^{(1)}\mathbf{U}_{j,L}^\perp]\\ =& -[\mathbf{Z}_{j,1}^{\dagger H}\mathbf{N}_j^H\mathbf{U}_j^\perp \quad \cdots \quad \mathbf{Z}_{j,L}^{\dagger H}\mathbf{N}_j^H\mathbf{U}_j^\perp].\end{aligned}$$

If $\hat{\mathbf{g}}_j$ is obtained as in (24) with $\hat{\gamma}_j = \gamma_j$, then we can use the same result from subspace perturbation analysis as before to determine the first-order approximation of $\Delta\mathbf{g}_j = \hat{\mathbf{g}}_j - \mathbf{g}_j$ in $\mathbf{N}_j$:

$$\begin{aligned}\Delta^{(1)}\mathbf{g}_j =& -\mathcal{U}_j^{\perp\dagger H}\Delta^{(1)}\mathcal{U}_j^{\perp H}\mathbf{g}_j = \mathcal{U}_j^{\perp\dagger H}\begin{bmatrix}\mathbf{U}_j^{\perp H}\mathbf{N}_j\mathbf{Z}_{j,1}^\dagger\\ \vdots\\ \mathbf{U}_j^{nH}\mathbf{N}_j\mathbf{Z}_{j,L}^\dagger\end{bmatrix}\mathbf{g}_j\\ =& [\mathcal{U}_{j,1}^{\perp\dagger H}\mathbf{U}_j^{\perp H} \quad \cdots \quad \mathcal{U}_{j,L}^{\perp\dagger H}\mathbf{U}_j^{\perp H}]\\ &\cdot\begin{bmatrix}\mathbf{N}_j & & \\ & \ddots & \\ & & \mathbf{N}_j\end{bmatrix}\begin{bmatrix}\mathbf{Z}_{j,1}^\dagger\mathbf{g}_j\\ \vdots\\ \mathbf{Z}_{j,L}^\dagger\mathbf{g}_j\end{bmatrix}\end{aligned}$$

where $\mathcal{U}_{j,l}^{\perp\dagger}$ $(l = 1,2,\cdots,L)$ is the $L \times L$ matrix given by

$$\mathcal{U}_{j,l}^{\perp\dagger} = \mathcal{U}_j^{\perp\dagger}((l-1)L+1 : lL, :).$$

Only considering the first-order approximation of $\Delta\mathbf{g}_j$ in $\mathbf{N}_j$, the bias and the MSE of $\hat{\mathbf{g}}_j$ can be expressed as $\text{bias}_j = \mathrm{E}\{\Delta^{(1)}\mathbf{g}_j\}$ and $\text{MSE}_j = \mathrm{E}\{\Delta^{(1)}\mathbf{g}_j^H\Delta^{(1)}\mathbf{g}_j\}$. Assume that the data symbol sequence $s_j[k]$ is white with variance 1 and the additive noise $e[n]$ is white with variance σ_e^2. The bias of $\hat{\mathbf{g}}_j$ then becomes

$$\text{bias}_j = \mathrm{E}\{\Delta^{(1)}\mathbf{g}_j\} = \mathbf{0}.$$

The MSE of $\hat{\mathbf{g}}_j$ then becomes

$$\begin{aligned}\text{MSE}_j =& \mathrm{E}\{\Delta^{(1)}\mathbf{g}_j^H\Delta^{(1)}\mathbf{g}_j\}\\ =& \nu\sigma_e^2\sum_{m=1}^{M}\Bigg(\|\mathcal{U}_{j,m}^{n\dagger H}\|^2\,\mathrm{E}\{\|\mathbf{Z}_{j,l}^\dagger\mathbf{g}_j\|^2\}\\ &+ \sum_{\substack{l'=1\\ l'\neq l}}^{L}\mathrm{tr}\{\mathcal{U}_{j,l}^{\perp\dagger H}\mathcal{U}_{j,l'}^{\perp\dagger}\}\,\mathrm{E}\{\mathbf{g}_j^H\mathbf{Z}_{j,l}^{\dagger H}\mathbf{Z}_{j,l'}^\dagger\mathbf{g}_j\}\Bigg).\end{aligned}$$

Following a similar reasoning as in [23], we can further prove that

$$\begin{cases}\mathrm{E}\{\|\mathbf{Z}_{j,l}^\dagger\mathbf{g}_j\|^2\} \approx \dfrac{L}{K\nu^2}, & \text{for } l = 1,2,\cdots,L\\ \mathrm{E}\{\mathbf{g}_j^H\mathbf{Z}_{j,l}^{\dagger H}\mathbf{Z}_{j,l'}^\dagger\mathbf{g}_j\} \approx 0, & \text{for } l,l' = 1,2,\cdots,L\\ & \text{with } l \neq l'.\end{cases}$$

Therefore, the NMSE of $\hat{\mathbf{g}}_j$ can be written as

$$\text{NMSE}_j \approx \frac{L\sigma_e^2}{K\nu\|\mathbf{g}_j\|^2}\sum_{l=1}^{L}\|\mathcal{U}_{j,l}^{\perp\dagger H}\|^2 = \frac{L\sigma_e^2}{K\nu\|\mathbf{g}_j\|^2}\|\mathcal{U}_j^{\perp\dagger H}\|^2.$$

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