A Stable Adaptive Fuzzy Sliding-Mode Control for Affine Nonlinear Systems with Application to Four-Bar Linkage Systems

Chih-Lyang Hwang and Chia-Ying Kuo

***Abstract*—In this paper, a stable adaptive fuzzy sliding-mode control for affine highly nonlinear systems is developed. First, the external part of a transformed system via a feedback linearizing control evolves a linear dynamic system with uncertainties. A reference model with the desired amplitude and phase properties is given to obtain an error model. Because the uncertainties are assumed to be large, a fuzzy model is employed to model these uncertainties. A learning law with e-modification for the weight of a fuzzy model is considered to ensure the boundedness of learning weight without the requirement of persistent excitation condition. Then, an equivalent control using the known part of system dynamics and the learning fuzzy model is designed to achieve the desired control behavior. Furthermore, the uncertainties caused by the approximation of fuzzy model and the error of learning weight are tackled by a switching control. Under some mild conditions, the stability of the internal part of the transformed system is guaranteed. Finally, the stability of the overall system is verified by the Lyapunov theory so that the ultimately bounded tracking is accomplished. Simulations and experiments of velocity control of four-bar-linkage system are also presented to verify the usefulness of the proposed control.**

***Index Terms*—Adaptive fuzzy control, four-bar-linkage system, Lyapunov stability, sliding-mode control.**

I. Introduction

FUZZY (or adaptive) control has widened its applicability to many engineering fields, it is increasing the need of theoretic analysis, e.g., stability, robustness, and performance. It is generally applicable to the systems that are mathematically poorly modeled. However, the major disadvantages of fuzzy (or adaptive) control are the lack of systematic design, without the insurance of stability of closed-loop system in the presence of uncertainties, and a poor performance due to the probable drift of learning weight [1]–[9].

Although many papers discuss the stable adaptive fuzzy controls [3], [5], [6], [9], they have made many assumptions. For example, Wang [6] used a Lyapunov-based learning law to improve a probable local minimum of error measure. Despite its advantages, the control method in [6] has three substantial drawbacks: i) The controller must adapt itself to every change of reference signal. ii) The method is limited to the system where the Lie-derivative of system output is constant. iii) To ensure the convergence of weight, fuzzy basis function must be persistently excited. Furthermore, Su and Stepanenko [3] have presented a modified version of Sanner and Slotine [10]. It has assumed that the Lie-derivative of system output is not only a constant, but also is known in advance. They use a modulation function to combine a robust control scheme outside of a compact set and an adaptive scheme inside the compact set. Hence, the global stability of overall system is guaranteed. However, the above method has the following disadvantages: i) The scheme is too complex to realize. ii) The possibility of discontinuous control occurs. iii) The compact set for the proposed control is unclear. The paper discussed by Spooner and Passsino [5] have investigated stable indirect and direct adaptive fuzzy controller using *a priori* knowledge about the r times derivative of system output. That is, the r times derivative of system output contains known part and unknown part of system dynamics. However, its uncertain term is too specially and the first $r-1$ derivatives of system output must be available. In 1999, Fischle and Schroder [9] present the solutions to the above problems. For instance, the controller does not necessarily adapt itself to every change of reference signal, the method is not limited to the system where the Lie-derivative of system output is constant. However, they must satisfy the following conditions: i) The relative degree r must be equal to the order of system n. ii) The first $n-1$ derivatives of system output must be available for the learning algorithm. iii) For the convergence of weight, the fuzzy basic functions must be persistently excited.

It is well known that sliding-mode control provides a robust means for controlling a nonlinear dynamic system with uncertainties [11]–[15]. It often results in a chattering control input due to its discontinuous switching control used to deal with the uncertainties. The larger uncertainties take place, the larger switching control happens. In the current paper, the nonlinear functions include known part (i.e., nominal system) and unknown part (i.e., uncertainties). The known part is achieved by deriving from the physical law, e.g., Lagrange's dynamic principle. Then, a coordinate transformation satisfying some conditions is employed to achieve a transformed system, including an external part with the order that equals the system relative degree and an internal part [16], [17]. The external part of transformed system via a feedback linearizing control becomes a linear dynamic system with uncertainties. Then, a prescribed reference model is designed to obtain an error model. A fuzzy model is applied to model these large uncertainties. A learning law with e-modification for the weight of the fuzzy model is used to ensure the boundedness of learning weight without the

Manuscript received October 14, 1999; revised September 27, 2000. This work was supported by Tatung University by Grant B88-1100-05 and by the National Science Council of R.O.C. by Grant NSC-88-2212-E-036-0007.

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Publisher Item Identifier S 1063-6706(01)02817-X.

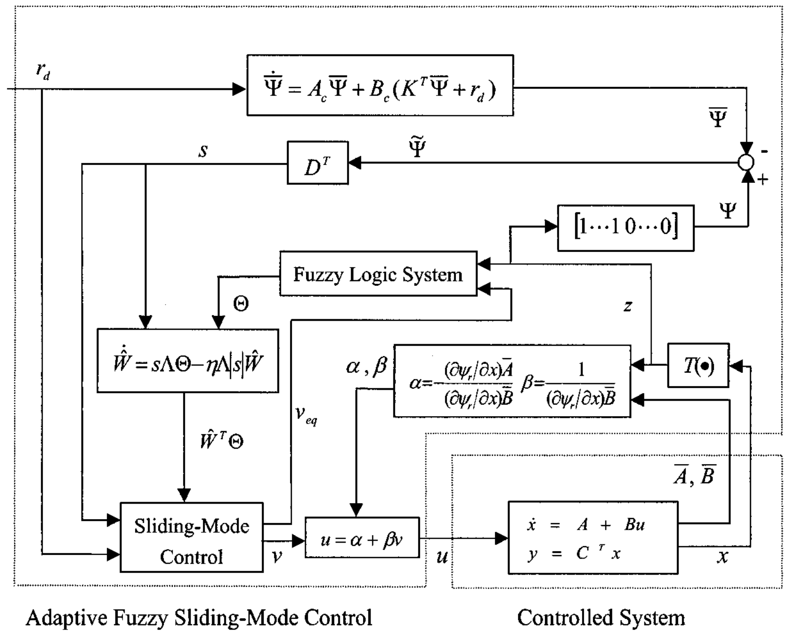


Fig. 1. Control block diagram.

requirement of persistent excitation condition [18]–[20]. Then, an equivalent control using the known part of system dynamics and the learning fuzzy-model is applied to achieve the desired control behavior. Because the fuzzy model is not applied to the whole nonlinear functions, the resolution of the fuzzy model increases or a good description of system uncertainties is accomplished. Furthermore, the uncertainties caused by the approximation of the fuzzy model and the error of learning weight are tackled by the switching control. The system performance is much improved as compared with traditional fuzzy (or adaptive) control because the uncertainties are reduced by the previous equivalent control. The proposed control is then more effective to cope with the fuzzy control problem of nonlinear systems with large uncertainties. Under some mild conditions, the stability of the internal part of transformed system is guaranteed. The stability of the overall system is then verified by the Lyapunov theory so that the ultimately bounded tracking is accomplished. Simulations and experiments of velocity control of four-bar-linkage system confirm the usefulness of the proposed control.

II. Problem Formulation

Consider the following affine nonlinear single-input–single-output dynamic systems:

$$\dot{x}(t) = A(x) + B(x)u(t), \qquad y(t) = C^T x(t) \tag{1}$$

where $x(t) \in \Re^n$ denotes the system state, $u(t)$, $y(t) \in \Re$ represents the system input, system output, $C \in \Re^n$ is a known constant vector. Furthermore, $A(x) = \overline{A}(x) + \Delta A(x)$, $B(x) = \overline{B}(x) + \Delta B(x)$ are highly nonlinear functions, where $\overline{A}(x)$ and $\overline{B}(x)$ denote the nominal part of system matrices, $\Delta A(x)$ and $\Delta B(x)$ represent the unknown (or uncertain) part of system matrices, and are bounded and smooth. Define the following Lie derivatives of the scalar $\overline{C}(x) = C^T x(t)$ in the direction of the vector fields $A(x)$ and $B(x)$ [16], [17]:

$$L_A\overline{C}(x) = \sum_{i=1}^{n} \frac{\partial \overline{C}(x)}{\partial x_i} A_i(x)$$
$$L_B\overline{C}(x) = \sum_{i=1}^{n} \frac{\partial \overline{C}(x)}{\partial x_i} B_i(x). \tag{2}$$

Then the derivative of output with respect to time, i.e., $dy(t)/dt$ or $\dot{y}(t)$, is described as follows:

$$\dot{y}(t) = L_A\overline{C}(x) + L_B\overline{C}(x)u(t). \tag{3}$$

If $L_B\overline{C}(x) \neq 0$, then the system (1) has the relative degree one. Similarly, the system (1) with the relative degree r, is expressed as follows:

$$y^{(i)}(t) = L_A^i\overline{C}(x), \quad 0 \le i \le r-1, \; L_A^0\overline{C}(x) = \overline{C}(x) \tag{4a}$$
$$y^{(r)}(t) = L_A^r\overline{C}(x) + L_B L_A^{r-1}\overline{C}(x)u(t), \quad L_B L_A^{r-1}\overline{C}(x) \neq 0. \tag{4b}$$

It is assumed that:

A1: the system state $x(t)$ is available;

A2: the nominal system has the relative degree r, where $r \le n$.

Definition 1 [17]: The solutions of a dynamic system are said to be uniformly ultimately bound (UUB) if there exist positive constants υ and κ, and for every $\delta \in (0, \kappa)$ there is a positive constant $T = T(\delta)$, such that $\|x(t_0)\| < \delta \Rightarrow \|x(t)\| \le \upsilon$, $\forall t \ge t_0 + T$.

The problem is to develop an indirect-adaptive fuzzy sliding-mode control for a class of affine highly nonlinear dynamic systems subject to huge uncertainties (see Fig. 1). A

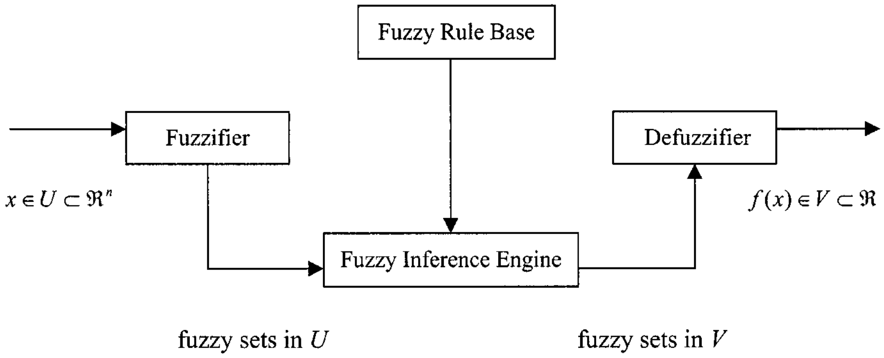


Fig. 2. Basic configuration of fuzzy logic system.

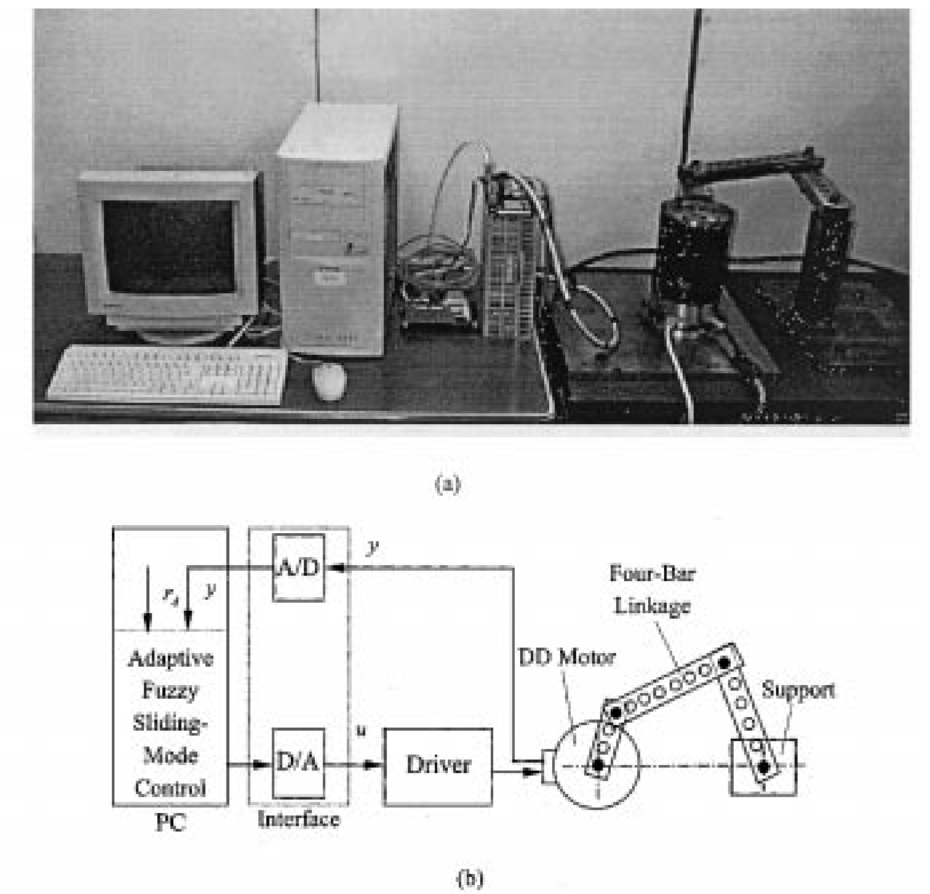


Fig. 3. Experimental setup. (a) Photograph. (b) Block diagram.

feedback linearizing control makes the external part of transformed system with relative degree r become a linear dynamic system with uncertainties. Then a reference model is designed to obtain a desired behavior including phase lag and amplitude relation. A fuzzy model is applied to model these huge uncertainties. A learning law with e-modification for the weight of a fuzzy model is constructed to ensure the boundedness of learning weight without the requirement of persistent excitation condition. Then, an equivalent control using the known part of system dynamics and the learning fuzzy model is designed to achieve the desired control behavior. Furthermore, a switching control is given to deal with the uncertainties caused by the approximation of the fuzzy model and the error of learning weight. Finally, the simulations and experiments of velocity control of four-bar-linkage system are presented to verify the usefulness of the proposed control.

Remark 1: If $\overline{A}(x) = Ax(t)$ and $\overline{B}(x) = B$, where A and B are constant matrices, then $\Delta A(x)$ and $\Delta B(x)$ are highly nonlinear. A feedback linearizing control is applied to the system (1) such that *the external part of transformed system becomes a linear dynamic system with reduced uncertainties*, and such that *a good nominal model for the controller design is achieved*.

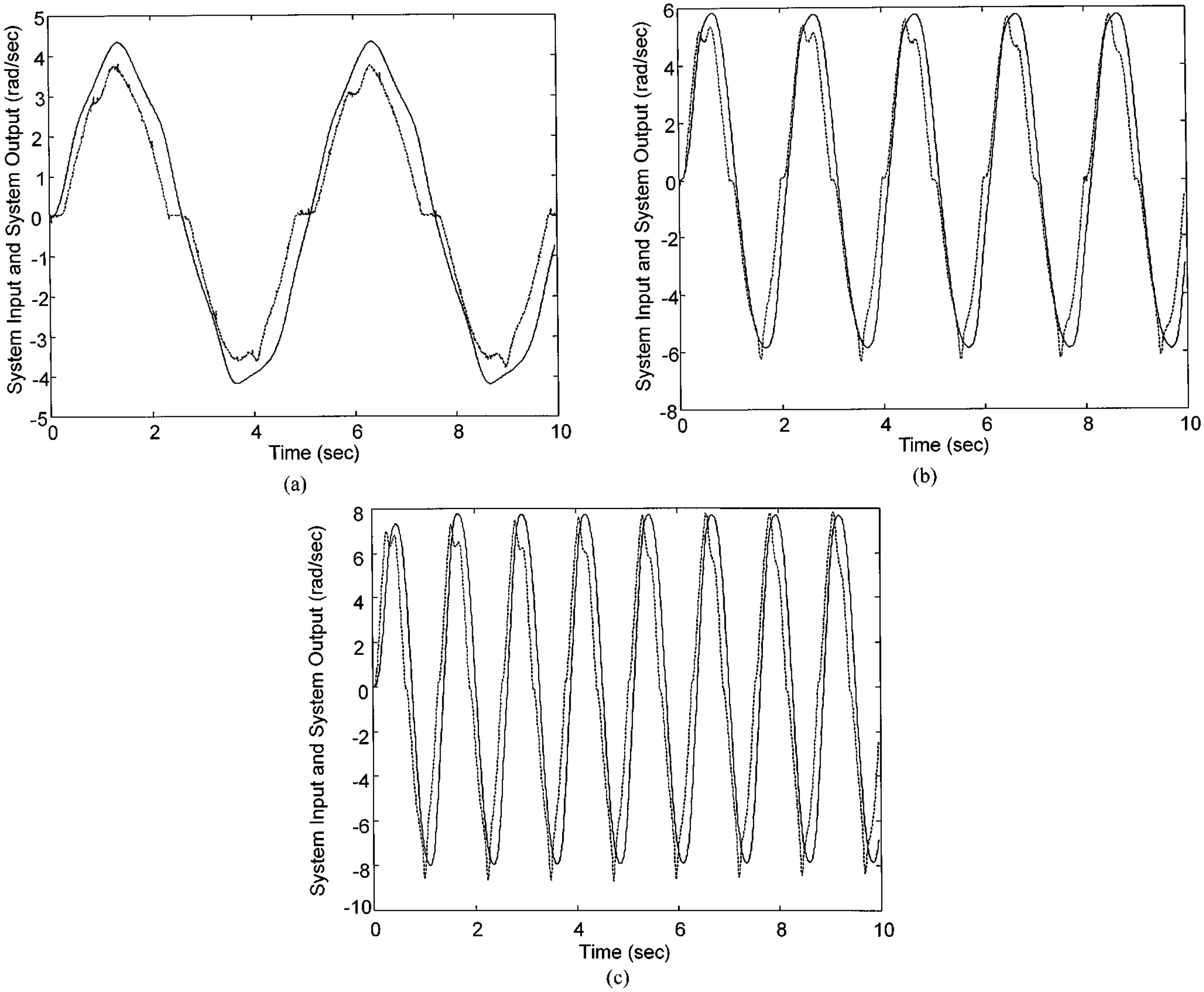


Fig. 4. Comparison of sinusoidal responses between physical system $(\cdots)$ and mathematical model (—) for system input $u(t) = u_m \sin(2\pi f t)$. (a) $u_m = 2.5$, $f = 0.2$. (b) $u_m = 3.5$, $f = 0.5$. (c) $u_m = 4.5$, $f = 0.8$.

Then the performance of proposed control can be better than that of traditional sliding-mode control.

III. Fuzzy Logic System

An important contribution of fuzzy system theory is to provide a systematic procedure for transforming a set of linguistic rules into a nonlinear mapping. The basic configuration of the fuzzy logic system is shown in Fig. 2. The fuzzy logic system performs a mapping from $U \in \Re^n$ to $\Re$. There are l fuzzy control rules and the upper script k denotes the kth rule from human experts in the following form:

$$\text{IF } x_1(t) \text{ is } F_1^k \text{ and } x_n(t) \text{ is } F_n^k, \quad \text{THEN } f(x) \text{ is } G^k \tag{5}$$

where $x(t) = [x_1(t) \quad x_2(t) \quad \ldots \quad x_n(t)]^T \in U \subset \Re^n$ and $f(x) \in V \subset \Re$ are the input and output of the fuzzy logic system, respectively, F_i^k $(1 \le i \le n, 1 \le k \le l)$ and G^k are labels of sets in U_i and V, respectively. The fuzzy inference engine performs a mapping from fuzzy sets in $U \subset \Re^n$ to fuzzy sets in $V \subset \Re$, based upon the fuzzy IF–THEN rules in the fuzzy rule base and the compositional rule of inference. Let A_x be an arbitrary fuzzy set in U. The fuzzifier maps a crisp point $x(t)$ into a fuzzy set A_x in U. The defuzzifier maps a fuzzy set in V to a crisp point in V. More information can be found in [21].

Let $\mu_{F_i^k}(x_i)$ and $\mu_{G^k}(\overline{w}_k)$ be membership functions. The fuzzy logic systems with center-average defuzzifier, product inference and singleton fuzzifier are in the following form [1]–[3], [5]–[9], [21], [22]:

$$f(x) = \sum_{k=1}^{l} \overline{w}_k \left(\prod_{i=1}^{n} \mu_{F_i^k}(x_i) \right) \Bigg/ \sum_{k=1}^{l} \left(\prod_{i=1}^{n} \mu_{F_i^k}(x_i) \right) \tag{6}$$

where $\overline{w}_i$ $(1 \le i \le l)$ denotes center of the ith fuzzy set and is the point at which μ_{G^k} achieves its maximum value and $\mu_{G^k}(\overline{w}_k) = 1$. Equation (6) can be rewritten as

$$f(x) = \overline{W}^T \Theta(x) \tag{7}$$

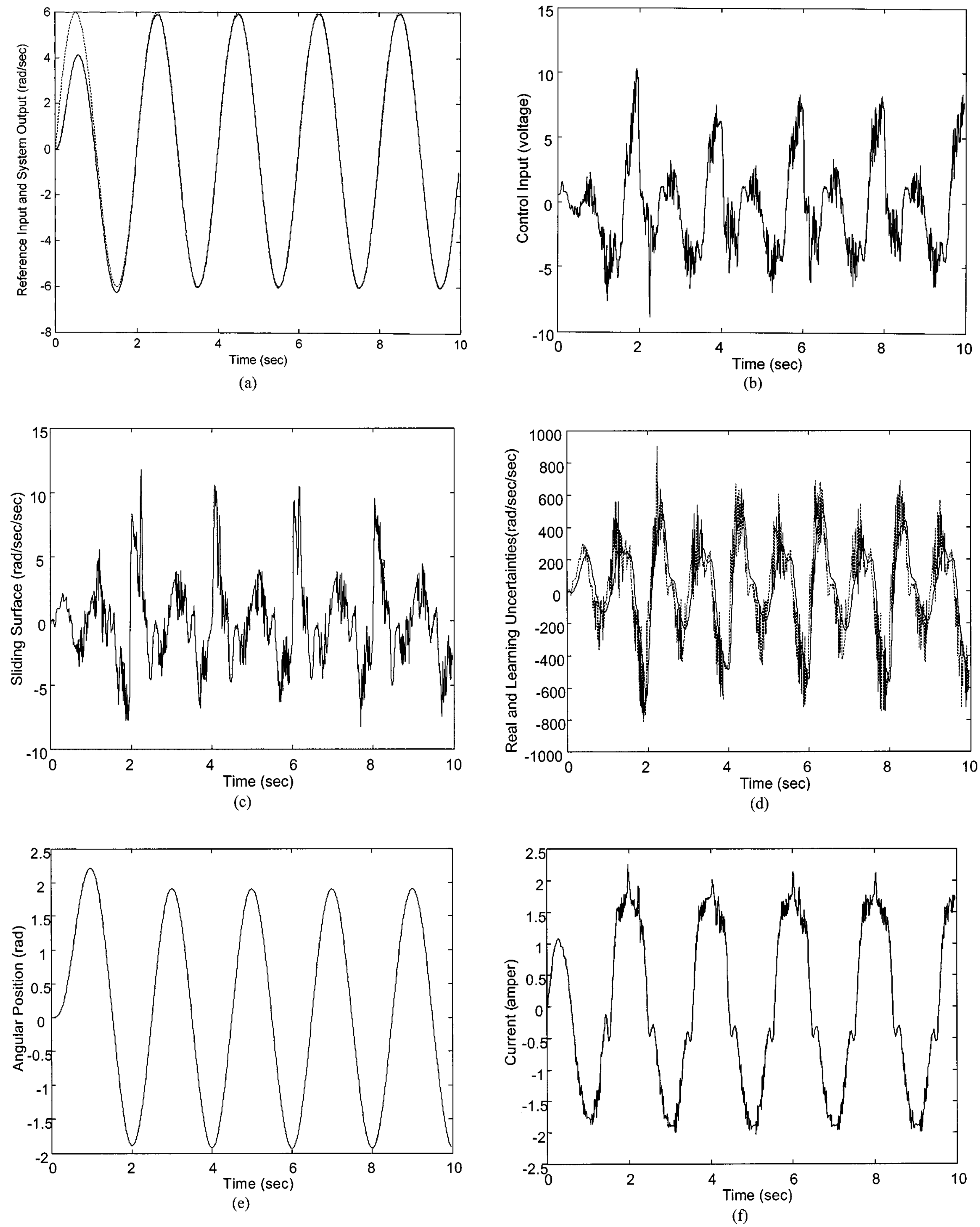


Fig. 5. The responses of $r_m = 6$ and $f = 0.5$ with $2\Delta a(x)$ and $2\Delta b(x)$. (a) $\dot{\theta}_d(t)(\cdots)$ and $\dot{\theta}(t)(—)$. (b) $u(t)$. (c) $s(t)$. (d) $f(\rho)(\cdots)$ and $\hat{W}^T(t)\Theta(x)(—)$. (e) $\theta(t)(—)$. (f) $i(t)$.

where $\overline{W} = [\overline{w}_1 \quad \overline{w}_2 \quad \ldots \quad \overline{w}_l]^T$ is a parameter vector, and $\Theta(x) = [\theta_1(x) \quad \theta_2(x) \quad \ldots \quad \theta_l(x)]^T$ is a fuzzy basis function defined as follows:

$$\theta_k(x) = \prod_{i=1}^{n} \mu_{F_i^k}(x_i) \Bigg/ \sum_{k=1}^{l} \left(\prod_{i=1}^{n} \mu_{F_i^k}(x_i) \right). \qquad (8)$$

If the fuzzy systems can approximate any nonlinear continuous functions to arbitrary accuracy, then they would be very useful in a wide variety of applications. The fuzzy logic systems in the form of (6) are proven in Wang [6] to be an universal approximator; i.e., for any given real continuous function f on the compact set U there exists a fuzzy logic system in the form of (6) such that it can uniformly approximate over U to arbitrary accuracy. The universal approximation theory is stated as follows (e.g., [2], [6], [7], [21], [23]):

Theorem 1 (Universal Approximation Theorem): Suppose that the input universe of discourse U is a compact set in $\Re^n$. Then, for any given real continuous function $g(x)$ on U and arbitrary $\varepsilon > 0$, there exists a fuzzy system $f(x)$ in the form of (6) such that

$$\sup_{x \in U} |f(x) - g(x)| < \varepsilon.$$

There are two main reasons for using the fuzzy logic systems. First, it was proven in [23] that the fuzzy logic systems in the form of (6) are universal approximators. Second, the fuzzy logic systems (6) are constructed from the fuzzy IF–THEN rules of (5) using some specific fuzzification, fuzzy inference, and defuzzification strategies; therefore, linguistic information from human experts [in the form of the fuzzy IF–THEN rules of (5)] can be directly incorporated into the controllers.

Remark 2: The more complex of nonlinear function is to be approximated, the more number of rule is required for the specified accuracy (i.e., ε). The minimum number of rule for odd and symmetric distribution of input signal is n_i^3, where n_i denotes the number of input signal. The reason for using the odd and symmetric distribution of input signal is that the input signal often can be zero, and that the learning uncertainties are probably known in a compact set only. Based on the previous studies (e.g., [5]–[10]), the Gaussian membership function is suitable for many function approximations. The other types of membership function have similar result.

IV. Feedback Linearizing Control

In this section, the feedback linearizing control for matched and unmatched uncertainties are discussed. Before transforming system (1) into another coordinate, the following definition about diffeomorphism is given.

Definition 2 [16], [17]: Suppose X, Z are open subsets of $\Re^n$ and $T: X \to Z$ is C^1 (i.e., T is continuously differentiable with respect to each of its arguments). Then T is a diffeomorphism of X onto Z if: i) $T(X) = Z$; ii) T is one-to-one; iii) $T^{-1}: Z \to X$ is also C^1. T is said to be a global diffeomorphism if and only if: i) $\partial T/\partial x$ is nonsingular for all $x(t) \in \Re^n$; ii) T is proper (i.e., $\lim_{\|x\| \to \infty} \|T(x)\| \to \infty$).

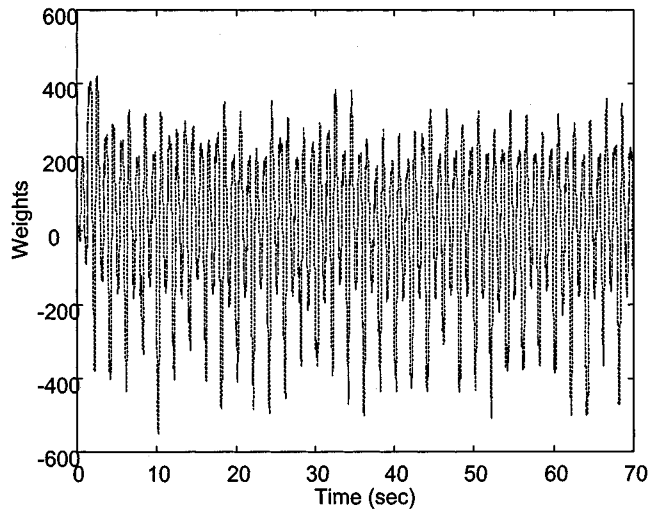


Fig. 6. The responses of typical weights $\hat{w}_3(t)(\cdots)$, $\hat{w}_5(t)(---)$, and $\hat{w}_{10}(t)(—)$ for Fig. 5 case with the exception of high gain $\gamma_1 = 83$ and $d_2 = 85$.

A. Matched Uncertainties

If the uncertainties satisfy the following matching condition:

$$\Delta A(x) = \overline{B}(x)\Delta a(x) \quad \text{and} \quad \Delta B(x) = \overline{B}(x)\Delta b(x) \qquad (9)$$

it is called "matched uncertainties." The following lemma is given to discuss a transformation for the system (1) into "triangular" form.

Lemma 1: Consider the nonlinear system (1) with the following global diffeomorphism $z(t) = T(x) = [\Psi^T(x) \quad \Phi^T(x)]^T$ and the satisfaction of matching condition (9). Then the following dynamic system is achieved:

$$\begin{aligned} \dot{\Psi}(x) &= A_c\Psi(x) + B_c\beta_0^{-1}(\Psi, \Phi) \\ &\quad \cdot [u(t) - \alpha_0(\Psi, \Phi) + \Delta a_0(\Psi, \Phi) \\ &\quad + \Delta b_0(\Psi, \Phi)u(t)] \qquad (10a) \\ \dot{\Phi}(x) &= A_0(\Psi, \Phi) \qquad (10b) \end{aligned}$$

where $\Psi(x) \in \Re^r$, $\Phi(x) \in \Re^{n-r}$, the functions $\alpha_0(\Psi, \Phi)$, $\beta_0(\Psi, \Phi)$, $\Delta a_0(\Psi, \Phi)$, $\Delta b_0(\Psi, \Phi)$ are the functions $\alpha(x) = -\{\partial\psi_r(x)/\partial x\overline{A}(x)\}/\{\partial\psi_r(x)/\partial x\overline{B}(x)\}$, $\beta(x) = 1/\{\partial\psi_r(x)/\partial x\overline{B}(x)\}$, $\Delta a(x)$, $\Delta b(x)$ evaluated at $x(t) = T^{-1}(z)$, respectively. $A_0(\Psi, \Phi)$ is the representation in the transformed coordinate $x(t) = T^{-1}(z)$ of $\partial\Phi(x)/\partial x\overline{A}(x)$. The matrices A_c, B_c are described as follows:

$$A_c = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ & & \ddots & & \\ & & & & \\ & & & & 1 \\ & & & & 0 \end{bmatrix}, \qquad B_c = \begin{bmatrix} 0 \\ \\ \vdots \\ 0 \\ 1 \end{bmatrix}. \qquad (11)$$

Proof: See Appendix A.

Equation (10) is said to be in the normal form [16], [17]. This form decomposes the system into an external part $\Psi(x)$ and an internal part $\Phi(x)$. The external part is linearized by the (14),

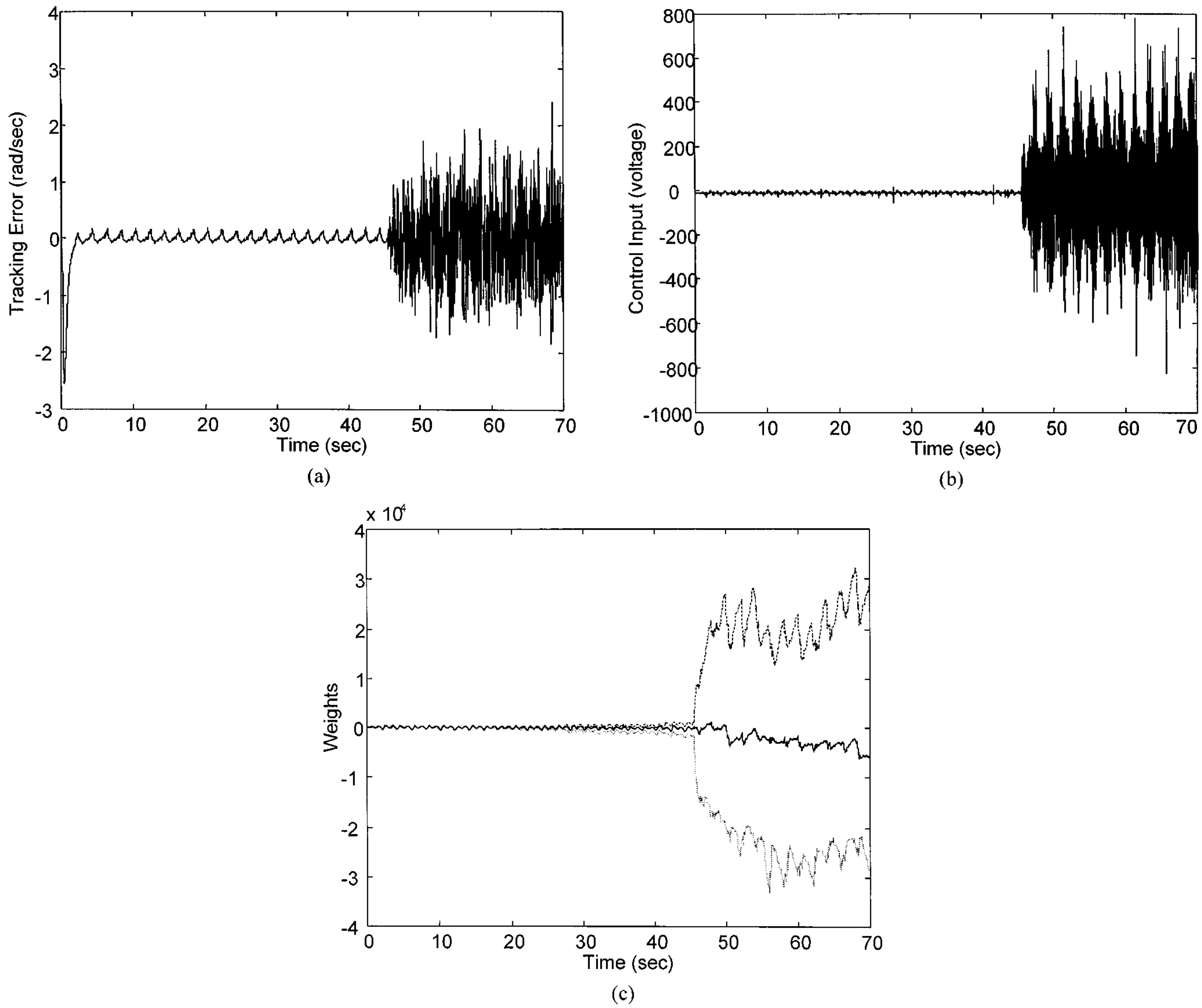


Fig. 7. The responses of Fig. 6 case with $\eta = 0$. (a) $\dot{\theta}_d(t)(\cdots) - \dot{\theta}(t)(—)$. (b) $u(t)$. (c) $\hat{w}_3(t)(\cdots)$, $\hat{w}_5(t)(---)$, and $\hat{w}_{10}(t)(—)$.

while the internal part is unobservable by the same control. For a bounded smooth trajectory $r_d(t)$, the following reference model is considered.

$$\dot{\overline{\Psi}}(t) = A_c\overline{\Psi}(t) + B_c\big[K^T\overline{\Psi}(t) + k_{r+1}r_d\big(t+\overline{\theta}\big)\big] \tag{12}$$

where constant $K \in \Re^r$ is selected to obtain the desired response, $k_{r+1} \in \Re$ is chosen to accomplish the desired amplitude relation between input and output, and $\overline{\theta}$ denotes the phase lag of reference model. The state tracking error of system can be written as follows:

$$\begin{aligned}\dot{\tilde{\Psi}}(t) = {} & A_c\tilde{\Psi}(t) + B_c\big\{\beta_0^{-1}(\Psi, \Phi)[u(t) - \alpha_0(\Psi, \Phi) \\ & + \Delta a_0(\Psi, \Phi) + \Delta b_0(\Psi, \Phi u(t)] \\ & - K^T\overline{\Psi}(t) - k_{r+1}r_d(t+\overline{\theta})\big\}\end{aligned} \tag{13}$$

where $\tilde{\Psi}(t) = \Psi(x) - \overline{\Psi}(t)$. The following linearizing feedback control is designed for the system (13)

$$u(t) = \alpha_0(\Psi, \Phi) + \beta_0(\Psi, \Phi)v(t) \tag{14}$$

where $v(t)$ is the stable adaptive fuzzy sliding-mode control discussed in the next section. Then the external part of system by using the linearizing feedback control (14) becomes the following linear error system with uncertainties:

$$\begin{aligned}\dot{\tilde{\Psi}}(t) = {} & A_c\tilde{\Psi}(t) + B_c\{[1 + \Delta b_0(\Psi, \Phi)]v(t) \\ & - K^T\overline{\Psi}(t) - k_{r+1}r_d(t+\overline{\theta}) + \beta_0^{-1}(\Psi, \Phi) \\ & \cdot [\Delta a_0(\Psi, \Phi) + \Delta b_0(\Psi, \Phi)\alpha_0(\Psi, \Phi)]\}.\end{aligned} \tag{15}$$

In short, (10b) and (15) represent the stability of closed-loop system. If the control $v(t)$ asymptotically stabilizes the dynamics $\tilde{\Psi}(t)$ in (15) and the dynamics $\Phi(x)$ is input-to-state stable in (10b), then the asymptotic tracking of the closed-loop system is guaranteed. Setting $\Psi(x) = 0$ in (10b) results in

$$\dot{\Phi}(x) = A_0(0, \Phi) \tag{16}$$

which is called the zero dynamics. For input-to-state stability of (10b), the origin of (16) must be exponentially stable and $A_0(\Psi, \Phi)$ is Lipschitz in (Ψ, Φ), i.e., $\|A_0(\Psi_2, \Phi_2) - A_0(\Psi_1, \Phi_1)\| \le L\|[(\Psi_2 - \Psi_1)^T \quad (\Phi_2 - \Phi_1)^T]\|$, where L is a constant.

Furthermore, the following assumption about the uncertainty of control gain is made.

A3: $|\Delta b_0(\Psi, \Phi)| \le \gamma_0 < 1$, $z(t) \in \Omega$, where $\Omega = \{z(t) \in \Re^n | \|z(t)\| < c_b\}$is a compact set.

If the uncertainties $\beta_0^{-1}(\Psi, \Phi)[\Delta a_0(\Psi, \Phi) + \Delta b_0(\Psi, \Phi)\alpha_0(\Psi, \Phi)]$are large, a robust control for the system (15) will be poor (i.e., see [20] and Fig. 10). Based on the approximation theory of Theorem 1, the following fact exists

$$f(\rho) = \overline{W}^T \Theta(\rho) + \varepsilon(\rho) \tag{17a}$$

where

$$\rho(t) = [v_{\text{eq}}(t) \quad \Psi^T(t) \quad \Phi^T(t)]^T, \qquad |\varepsilon(\rho)| < \varepsilon_0 \tag{17b}$$

$$\begin{aligned} f(\rho) = &\Delta b_0(\Psi, \Phi) v_{\text{eq}}(t) + \beta_0^{-1}(\Psi, \Phi) \\ &\cdot [\Delta a_0(\Psi, \Phi) + \Delta b_0(\Psi, \Phi)\alpha_0(\Psi, \Phi)] \end{aligned} \tag{17c}$$

where $v_{\text{eq}}(t)$ is described in (25) and $\rho(t) \in \Omega_v \times \Omega$ which is a compact set [i.e., $v_{\text{eq}}(t) \in \Omega_v = \{v_{\text{eq}}(t) \in \Re | |v_{\text{eq}}(t)| < c_{\text{eq}}\}$]. Furthermore, the dimension and upper bound of weight are described as follows:

$$\overline{W} \in \Re^l, \qquad \|\overline{W}\|_F \le w_{\max} \tag{18}$$

where $\|\cdot\|_F$ denotes the Frobenius' norm (i.e., $\|\overline{W}\|_F^2 = \text{tr}\{\overline{W}^T \overline{W}\} = \text{tr}\{\overline{W}\,\overline{W}^T\}$) and l, $w_{\max}$ are known. The fact that the dimension of $\overline{W}$, the upper bound of $\varepsilon(\rho)$ and the fuzzy basis function $\Theta(\rho)$ is known, implies that the function $\overline{W}^T\Theta(\rho) + \varepsilon(\rho)$ can represent a class of uncertainties $f(\rho)$. Because the uncertainty $f(\rho)$ is assumed to be completely unknown, the value of l must be guessed from low value to high value. Fortunately, if the uncertainty $f(\rho)$ is partially known based on the system analysis, the suitable value of l can be attained. Furthermore, the fuzzy model is not applied to the whole nonlinear system, the resolution of the fuzzy model increases or a good description of system uncertainties is accomplished.

B. Unmatched Uncertainties

If the matching condition (9) does not satisfy, the stability of closed-loop system is discussed as follows. The uncertainties are assumed to be the following form:

$$\Delta A(x) = \overline{B}(x)\Delta a(x) + \Delta A_u(x)$$

and

$$\Delta B(x) = \overline{B}(x)\Delta b(x) + \Delta B_u(x) \tag{19}$$

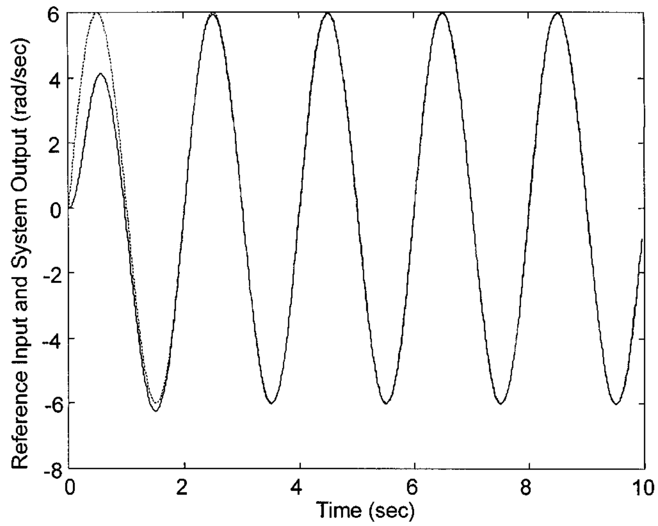


Fig. 8. The response $\dot{\theta}_d(t)(\cdots)$ and $\dot{\theta}(t)$(—) of $v_m = 6$ and $f = 0.5$ for matched uncertainties and unmatched uncertainties.

where $\Delta A_u(x)$ and $\Delta B_u(x)$ denote the uncertainties those do not satisfy the matching condition. Then (10b) and (15) become

$$\begin{aligned} \dot{\Phi}(x) = &A_0(\Psi, \Phi) + \partial\Phi/\partial x\{\Delta A_{u0}(\Psi, \Phi) + \Delta B_{u0}(\Psi, \Phi) \\ &\cdot [\alpha_0(\Psi, \Phi) + \beta_0(\Psi, \Phi)v(t)]\} \end{aligned} \tag{20a}$$

$$\begin{aligned} \dot{\tilde{\Psi}}(t) = &A_c\tilde{\Psi}(t) + B_c\{[1 + \Delta b_0(\Psi, \Phi) \\ &+ B_c^T \partial\Psi/\partial x \Delta B_{u0}(\Psi, \Phi)]v(t) \\ &- K^T\overline{\Psi}(t) - k_{r+1}r_d(t + \overline{\theta}) + \beta_0^{-1}(\Psi, \Phi) \\ &\cdot [\Delta a_0(\Psi, \Phi) + \Delta b_0(\Psi, \Phi)\alpha_0(\Psi, \Phi)] \\ &+ B_c^T \partial\Psi/\partial x \Delta A_{u0}(\Psi, \Phi)\} \end{aligned} \tag{20b}$$

where the functions $\Delta A_{u0}(\Psi, \Phi)$, $\Delta B_{u0}(\Psi, \Phi)$ are the functions $\Delta A_u(x)$, $\Delta B_u(x)$ evaluated at $x(t) = T^{-1}(z)$, respectively.

Fortunately, many physical systems (e.g., four-bar-linkage system, robot systems, frictional system) can be expressed as an affine nonlinear system with constant nominal control matrix gain, i.e., $\overline{B}(x)$ in (1) is a constant matrix. Under the circumstances, the unmatched uncertainty probably does not exist if it is within the range space of $\overline{B}(x)$, i.e., $\Delta B_{u0}(\Psi, \Phi) = 0$. Under the circumstances, (20) becomes

$$\dot{\Phi}(x) = A_0(\Psi, \Phi) + \Delta A_0(\Psi, \Phi) \tag{21a}$$

$$\begin{aligned} \dot{\tilde{\Psi}}(t) = &A_c\tilde{\Psi}(t) + B_c \\ &\cdot \{[1 + \Delta b_0(\Psi, \Phi)]v(t) - K^T\overline{\Psi}(t) \\ &- k_{r+1}r_d(t + \overline{\theta}) + \beta_0^{-1}(\Psi, \Phi) \\ &\cdot [\Delta a_0(\Psi, \Phi) + \Delta b_0(\Psi, \Phi)\alpha_0(\Psi, \Phi)] \\ &+ B_c^T \partial\Psi/\partial x \Delta A_{u0}(\Psi, \Phi)\} \end{aligned} \tag{21b}$$

where $\Delta A_0(\Psi, \Phi) = \partial\Phi/\partial x \Delta A_{u0}(\Psi, \Phi)$. For input-to-state stability of (21a), the origin of the following system (22a) must

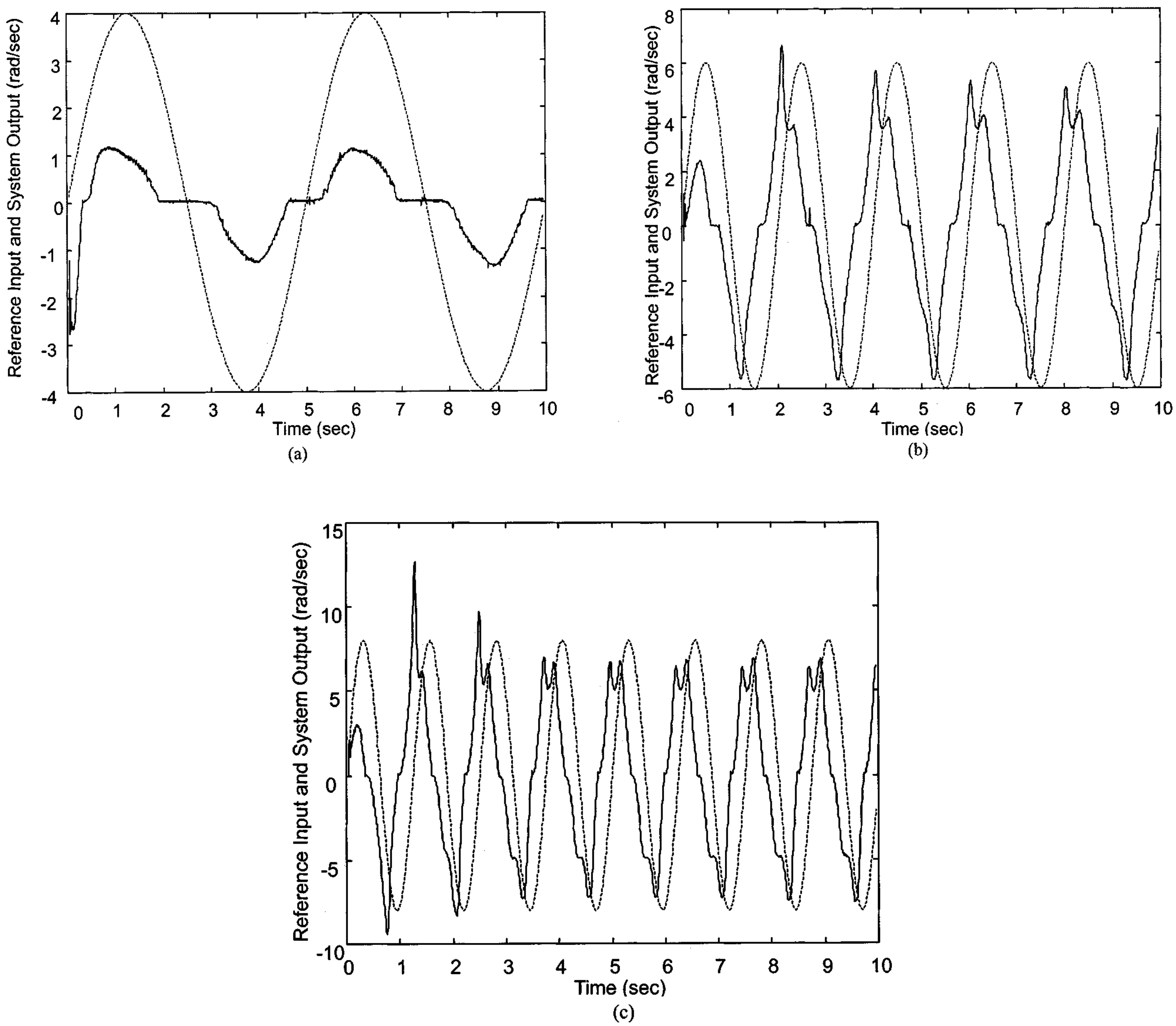


Fig. 9. The responses of experiment without using fuzzy adaptive law (i.e., robust control). (a) $v_m = 4$, $f = 0.2$. (b) $v_m = 6$, $f = 0.5$. (c) $v_m = 8$, $f = 0.8$.

be exponentially stable and $A_0(\Psi, \Phi) + \Delta A_0(\Psi, \Phi)$ is Lipschitz in (Ψ, Φ)

$$\dot{\Phi}(x) = A_0(0, \Phi) + \Delta A_0(0, \Phi). \tag{22a}$$

In addition, the approximation of uncertainties becomes

$$\begin{aligned} f(\rho) = &\Delta b_0(\Psi, \Phi) v_{\text{eq}}(t) + \beta_0^{-1}(\Psi, \Phi) \\ &\cdot [\Delta a_0(\Psi, \Phi) + \Delta b_0(\Psi, \Phi) \alpha_0(\Psi, \Phi)] \\ &+ B_c^T \partial\Psi/\partial x \Delta A_{u0}(\Psi, \Phi). \end{aligned} \tag{22b}$$

As compared with (17c) and (22b), an extra term caused by the unmatched uncertainties [i.e., $B_c^T \partial\Psi/\partial x \Delta A_{u0}(\Psi, \Phi)$] is approximated by the fuzzy model. Similarly, an extra term caused by the unmatched uncertainties [i.e., $\Delta A_0(0, \Phi)$] is added into the (16). As compared with the case in Section IV-A, the margin of input-to-state stability of internal system decreases.

V. Stable Adaptive Fuzzy Sliding-Mode Control

First, a sliding surface is defined as follows:

$$s(t) = D^T \tilde{\Psi}(t) \tag{23}$$

where $\tilde{\Psi}(t) = [\tilde{\psi}_1(t) \quad \dot{\tilde{\psi}}_1(t) \quad \ldots \quad \tilde{\psi}_1^{(r-1)}(t)]^T$ and $D = [d_r \quad \ldots \quad d_2 \quad 1]^T$. The coefficients d_i $(2 \le i \le r)$ are chosen such that the sliding surface $s(t) = 0$ is Hurwitz. Furthermore, the following updating law for the weight is considered

$$\dot{\hat{W}}(t) = s(t)\Lambda\Theta(\rho) - \eta\Lambda|s(t)|\hat{W}(t) \tag{24}$$

where $\Lambda = \text{diag}\{\lambda_{ii}\}$, $\lambda_{ii} > 0$, and $\eta > 0$ denote the learning rate and the e-modification rate, respectively. Because the fuzzy basis function $\Theta(\rho)$ in (8) is small as compared with the radical basis function in neural-network control (e.g., [10], [20]), the learning rate of (24) is chosen large enough to accomplish an effective learning of uncertainties. The selection of $\eta\Lambda|s(t)|\hat{W}(t)$

in (24) is the reason for the boundedness of learning weight matrix without the requirement of persistent excitation [18]–[20]. In general, η is small to allow a possibility of effective learning of $\hat{W}(t)$. Too large value of η will force $\hat{W}(t)$ converge into the neighborhood of zero. Under the circumstances, a poor learning of $\hat{W}(t)$ occurs if $\overline{W}$ is not small. The following theorem discusses the stable adaptive fuzzy sliding-mode control for the system (21).

Theorem 2: Consider (21) and the following stable adaptive fuzzy sliding-mode control:

$$v(t) = v_{\text{eq}}(t) + v_{\text{sw}}(t) \tag{25}$$

where

$$\begin{aligned} v_{\text{eq}}(t) = &\sum_{i=1}^{r-1} d_{r-i+1}\tilde{\psi}_{i+1}(t) + K^T\overline{\Psi}(t) \\ &+ k_{r+1}r_d(t+\overline{\theta}) + \hat{W}^T(t)\Theta(\rho) \end{aligned} \tag{26}$$

$$\begin{aligned} v_{\text{sw}}(t) = &-\left[\gamma_1 s(t) + \frac{\gamma_2 s(t)}{|s(t)|+\xi}\right] \Big/ (1-\gamma_0), \\ &\gamma_1,\, \gamma_2 > 0,\; \xi \geq 0. \end{aligned} \tag{27}$$

The overall system satisfies the following conditions: i) a stable sliding surface (23); ii) the assumptions A1–A3; iii) the satisfaction of input-to-state stability; and iv) $\rho(t) \in \Omega_v \times \Omega$. Then $s(t)$, $\hat{W}(t)$, $v(t)$, $u(t)$, and $x(t)$ are UUB, and the system performance satisfying $|s(t)| \leq g$, where

$$\begin{aligned} g &= \sqrt{g_1^2 + g_2} - g_1 \\ g_1 &= \left\{\xi + \frac{\gamma_2}{\gamma_1} - \frac{\eta w_{\max}^2}{4\gamma_1} - \frac{\varepsilon_0}{\gamma_1}\right\} \Big/ 2 \\ g_2 &= \frac{\eta w_{\max}^2 \xi}{4\gamma_1} + \frac{\varepsilon_0 \xi}{\gamma_1}. \end{aligned} \tag{28}$$

Proof: See Appendix B.

Remark 3: The first term of the proposed control, i.e., $v_{\text{eq}}(t)$, is to assign the desired linear dynamic behavior and to cancel the effect of uncertainties by using the learning uncertainties. It is much improved as compared to traditional fuzzy (or adaptive) control because the uncertainties are attenuated by the equivalent control. In addition, the uncertainties caused by the approximation of the fuzzy model and the error of learning weight are tackled by the switching control, i.e., $v_{\text{sw}}(t)$. The proposed control is then more effective to cope with the fuzzy control problems of nonlinear system in the presence of large uncertainties.

VI. Simulations and Experiments

A. Simulations

The four-bar-linkage driven by a direct-driven motor through a rigid coupling in horizontal plane is expressed as follows (e.g., [24]):

$$L_m \dot{i}_m(t) + R_m i_m(t) + K_b\dot{\theta}_m(t) = E_a(t) \tag{29}$$

$$\begin{aligned} M_e(\theta_2)\ddot{\theta}_2(t) + (B_m+B_l)\dot{\theta}_2(t) + C_b(\theta_2)\dot{\theta}_2^2(t) &= T_m(t) \\ &= K_t i_m(t) \end{aligned} \tag{30}$$

$$\theta_2(t) = \theta_m(t), \quad \dot{\theta}_2(t) = \dot{\theta}_m(t), \quad \ddot{\theta}_2(t) = \ddot{\theta}_m(t) \tag{31}$$

where the symbols $M_e(\theta_2)$, $B_m + B_l$, and $C_b(\theta_2)$ denote effective inertia, linear damping of motor and load, centrifugal and Coriolis, respectively; the symbols L_m, R_m, K_b, $i_m(t)$, $\theta_m(t)$, and $T_m(t)$ represent motor inductance, resistance, back-emf constant, current, angular position, and torque, respectively. More details of the proposed four-bar-linkage systems can refer to Appendix C. Rewrite the above four-bar-linkage system as the form of (1) with the following definitions:

$$\begin{aligned} [x_1(t) \quad x_2(t) \quad x_3(t)] &= [\theta_2(t) \quad \dot{\theta}_2(t) \quad i_m(t)] \\ U(t) &= E_a(t) \end{aligned}$$

and

$$y(t) = x_2(t). \tag{32}$$

The system (29)–(32) has relative degree 2. The modeling check is shown in Fig. 4. It indicates that the dynamics of mathematical model captures the dominant dynamics of real four-bar linkage system. Beside the parameters of mathematical model in Appendix C, the other parameters are described as follows: $B_l = B_m = 0.01$ kgms/rad, $J_m = 0.3$ kgm. In this paper, not the signal $A(x)$ or $B(x)$, but the uncertainty caused by $\Delta A(x)$ or $\Delta B(x)$, i.e., $f(\rho)$, is approximated by the fuzzy model. Suppose the following coordinate transformation:

$$\begin{aligned} \psi_1(x) &= t_1 x_1(t) + t_2 x_2(t) \\ \psi_2(x) &= t_1 \overline{A}_1(x) + t_2 \overline{A}_2(x) \\ \phi(x) &= t_3 x_1(t) + t_4 x_2(t) \end{aligned} \tag{33}$$

where $\overline{A}_i(x)$ denotes the ith component of $\overline{A}(x)$, $t_2 = t_4$, and $t_1 \neq t_3$. The values of $t_1 = 2$, $t_2 = t_3 = t_4 = 1$ are selected because as $\psi_1(x) = \psi_2(x) = 0$, $\dot{\phi}(t) = -\phi(t)t_1/t_2 = -2\phi(t)$ is exponentially stable. Furthermore, $\partial T(x)/\partial x$ is nonsingular for all $x(t)$. In short, the input-to-state stability is satisfied. One cannot let $t_1 = t_4 = 0$, $t_2 = t_3 = 1$ because $\dot{\phi}(t) = 0$ is not exponentially stable [16], [17].

The desired velocity is set as $x_{2d}(t) = v_m \sin(2\pi f t + \overline{\theta})$ rad/s, where $\overline{\theta} = \tan^{-1}[2\pi f k_2/(k_1 - 4\pi^2 f^2)]$, then $x_{1d}(t) = \int x_{2d}(\tau)\,d\tau = -v_m\cos(2\pi f t + \overline{\theta})/(2\pi f)$ rad. The parameters of reference model are chosen as $k_1 = 50$, $k_2 = 10$, and $k_3 = \sqrt{(2\pi f)^4 + (k_2^2 - 2k_1)(2\pi f)^2 + k_1^2}$. Hence, the reference input for (12) becomes $r_d(t) = t_1 x_{1d}(t) + t_2 x_{2d}(t)$. According to the explanation of Remark 1, three fuzzy sets F^k $(k = 1, 2, 3)$ have the following Gaussian membership functions:

$$\begin{aligned} \mu_i(x_1) &= e^{-((x_1 - c_{1i})/\sigma_1)}, \quad \text{where } c_{1i} = [-10 \;\; 0 \;\; 10], \\ &\qquad i = 1, 2, 3 \text{ and } \sigma_1 = 80 \\ \mu_i(x_2) &= e^{-((x_2 - c_{2i})/\sigma_2)}, \quad \text{where } c_{2i} = [-25 \;\; 0 \;\; 25], \\ &\qquad i = 1, 2, 3 \text{ and } \sigma_2 = 200 \\ \mu_i(x_3) &= e^{-((x_3 - c_{3i})/\sigma_3)}, \quad \text{where } c_{3i} = [-10 \;\; 0 \;\; 10], \\ &\qquad i = 1, 2, 3 \text{ and } \sigma_3 = 80 \\ \mu_i(v_{\text{eq}}) &= e^{-((v_{\text{eq}} - c_{4i})/\sigma_4)}, \quad \text{where } c_{4i} = [-100 \;\; 0 \;\; 100], \\ &\qquad i = 1, 2, 3 \text{ and } \sigma_4 = 800. \end{aligned} \tag{34}$$

The total number of fuzzy rule is $l = 81$. The control parameters are assigned as follows:

$$\begin{aligned} d_2 &= 55, \qquad \gamma_0 = 0.1, \qquad \gamma_1 = 45, \qquad \gamma_2 = 0.2, \\ \xi &= 0.5, \qquad \Lambda = 5 \times 10^4 \quad \text{and} \quad \eta = 10^{-5}. \end{aligned} \tag{35}$$

The matched uncertainties are supposed to be as follows:

$$\begin{aligned} \Delta a(x) &= -0.5x_1(t)x_2(t)x_3(t)\sin(40x_3 - 0.2\pi) \\ &\quad + 0.04x_2^2(t) + 3x_3(t)\sin(0.25x_2) \end{aligned} \tag{36a}$$

$$\Delta b(x) = 0.25\sin(10x_3) + 0.5\sin(0.5x_1). \tag{36b}$$

Because $\overline{B}(x) = [0 \quad 0 \quad 1/L_m]^T$, the unmatched uncertainties have the following form:

$$\begin{aligned} \Delta A_u(x) &= [\Delta A_{u1}(x) \quad \Delta A_{u2}(x) \quad 0]^T \\ \Delta B_u(x) &= [0 \quad 0 \quad 0]^T \end{aligned} \tag{37a}$$

where

$$\begin{aligned} \Delta A_{u1}(x) &= \phi(x)\{0.15\sin(35(\varphi_1 + \phi)) \\ &\quad - 0.25\sin(0.5\varphi_2) + 0.1\} \end{aligned} \tag{37b}$$

$$\begin{aligned} \Delta A_{u2}(x) &= \phi(x)\{0.1\sin(40(\varphi_1 + \phi)) \\ &\quad + 0.2\sin(0.25\varphi_2) - 0.05\}. \end{aligned} \tag{37c}$$

The responses for matched uncertainty (36) are shown in Fig. 5. The tracking performance is excellent. The maximum steady-state tracking error is 0.214 rad/s or 3.56% of amplitude of reference input. The responses of system states shown in Fig. 5(a), (e), and (f) are smooth enough. Because the high-frequency uncertainties occur, the control signal in Fig. 5(b) has high-frequency components. Owing to the existence of uncertainties and the feature of time-varying reference input, the response of sliding surface in Fig. 5(c) is only in the neighborhood of zero. The responses of real and learning uncertainties are quiet matched in the sense of low-frequency dominant trend [refer to Fig. 5(d)]. The proposed fuzzy logic system (6) and the updating law for weight (24) demonstrate an effective tool for the learning of uncertainties. Similarly, the responses for the reference inputs $v_m = 4$, $f = 0.2$ and $v_m = 8$, $f = 0.8$ can be achieved. The maximum steady-state tracking errors for $v_m = 4$, $f = 0.2$ and $v_m = 8$, $f = 0.8$ are 0.044, 0.681 rad/s (or 1.09%, 8.51% of amplitude of reference input), respectively. For brevity, those are left out. In summary, the tracking error for the reference input with small amplitude and low frequency is smallest; on the contrary, the tracking error for the reference input with large amplitude and high frequency is largest. To demonstrate the effectiveness of the updating law (24), the responses of typical weights of Fig. 5 case with the exception of control parameters: $\gamma_1 = 83$, $d_2 = 85$ (which is high gain) are shown in Fig. 6. Its maximum steady-state tracking error is 3.32% that is a little smaller than that of Fig. 5. The responses of typical weight for updating law without e-modification [i.e., $\eta = 0$ in (24)] are shown in Fig. 7. The response of weight in tradition updating law (e.g., [2], [5], [9]) cannot guarantee its boundedness. The drift of weight eventually makes the overall system unstable (see Fig. 7). Its maximum steady-state tracking error for the 70-s interval is 31.71%. As compared with Figs. 6 and 7, the proposed updating law (24) guarantees the boundedness of weight. Similarly, the system with unmatched uncertainties (37a)–(37c) and matched uncertainties (36), which still satisfies the input-to-state stability, is considered. The corresponding response is presented in Fig. 8. Its maximum steady-state tracking error 0.324 rad/s is a little larger than that of Fig. 5 (i.e., 0.214 rad/s). It seems that the robustness of the proposed control is excellent.

B. Experiments

1) Experiment Setup: The hardware of the four-bar-linkage system mainly consists of five parts: a direct-driven motor, a driver, a four-bar-linkage, an AD/DA card, and a personal computer (refer to Fig. 3). The direct-driven motor and the driver in this study are a Model No. DM1075B and a Model No. SD1075B-2 from the Yokowaga Co. The specifications of this direct-driven motor system are briefly introduced as follows: rate speed 12.56 rad/s, maximum output torque 7.653 kgm, power consumption 1.6 KVA, and stiffness 9.8×10^{-7} rad/kgm. After sampling by the 12-b A/D card (PCL-1800), the resolution of velocity and current is 2.686×10^{-3} rad/s and 5.63×10^{-5} amp, respectively. The conversion factor of velocity and current for voltage are 0.55 rad/s/V and 1.153×10^{-2} amp/V, respectively. The control cycle time of the current paper is 0.007 s.

2) Experimental Results: The initial state and weight are the same as Fig. 5. The control parameters for experiment are assigned as follows:

$$\begin{aligned} d_2 &= 25, \qquad \gamma_0 = 0.1, \qquad \gamma_1 = 5, \qquad \gamma_2 = 0.1, \\ \xi &= 0.5, \qquad \Lambda = 5 \times 10^4, \qquad \eta = 10^{-5} \qquad \sigma_1 = 100, \\ \sigma_2 &= 300, \qquad \sigma_3 = 100 \quad \text{and} \quad \sigma_4 = 800. \end{aligned} \tag{38}$$

To avoid the saturated input and the drift of weight, the control gains in experiment are smaller than those in simulation [compare (34), (35), and (38)]. The responses of experiment without using fuzzy adaptive law [i.e., the equivalent control without the term $\hat{W}^T(t)\Theta(x)$, or call it as “robust control”] are shown in Fig. 9. Then the responses of experiment using the proposed control are shown in Figs. 10 and 11. The maximum steady-state tracking errors for $v_m = 6$, $f = 0.5$, $v_m = 4$, $f = 0.2$ and $v_m = 8$, $f = 0.8$ are 0.79542, 0.32688, 1.5552 rad/s (or 13.257%, 8.172%, and 18.44% of amplitude of reference input), respectively. As compared with Figs. 5, 9, and 10, the following conclusions are drawn: i) The responses of proposed control indeed better than those of “robust control.” It reveals that the learning uncertainties can be used to cancel the real uncertainties and then the system performances are improved. ii) The maximum tracking error for experiment is larger than that of corresponding simulation. The main reasons are that the dynamics of physical system is more complex than the dynamics of mathematical model (see Fig. 3) and that a smaller control gain for the experiment is used to prevent practical instability (e.g., saturated input, drift of weight). iii) For

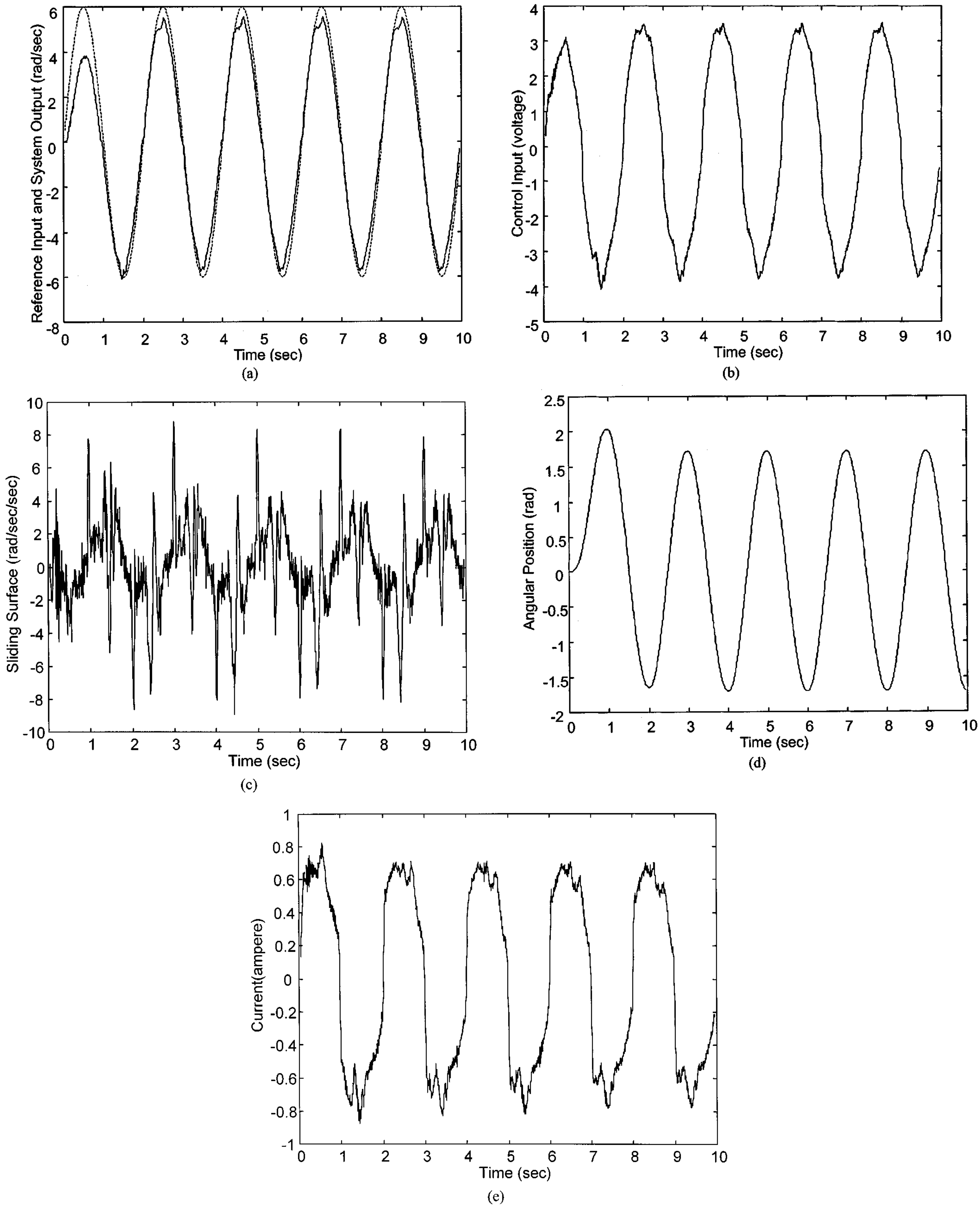
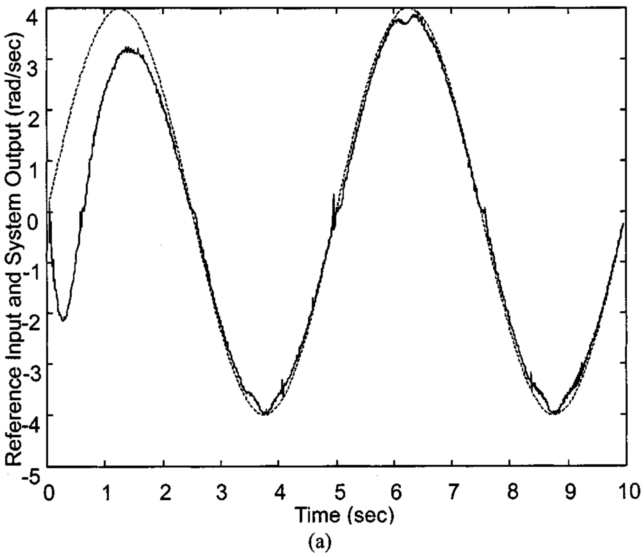


Fig. 10. The responses of experiment for $v_m = 6$ and $f = 0.5$. (a) $\dot{\theta}_d(t)(\cdots)$ and $\dot{\theta}(t)$(—). (b) $u(t)$. (c) $s(t)$. (d) $\theta(t)$(—). (e) $i(t)$.

improving the system performance, the more accurate mathematical model (e.g., the flexible coupling of linkage, friction phenomenon of joint) must be considered (or derived). Then a higher control gain can be used to achieve an excellent perfor-



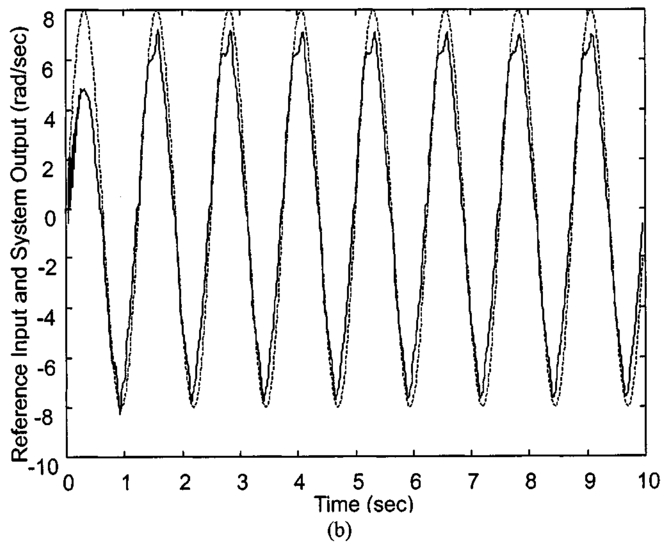


Fig. 11. The responses $\dot{\theta}_d(t)(\cdots)$ and $\dot{\theta}(t)(—)$ of experiments. (a) $v_m = 4$ and $f = 0.2$. (b) $v_m = 8$ and $f = 0.8$.

mance. iv) Without the requirement of persistent excitation condition, the updating law (24) ensures the boundedness of weight. Then the stability of closed loop is assured.

VII. Conclusion

In the beginning, a feedback-linearizing control with a desired reference model makes the external part of transformed system become a linear-error dynamic system with uncertainties. Without the requirement of persistent excitation condition, a fuzzy model with an e-modification weight updating law is employed to on-line model these uncertainties. Then, an equivalent control using the known part of system dynamics and the learning fuzzy model is applied to achieve the desired control behavior. Since the fuzzy model is not applied to the whole nonlinear system, the resolution of the fuzzy model increases or a good description of system uncertainties is accomplished. Because the uncertainties are reduced by the equivalent control and because the uncertainties caused by the approximation of fuzzy-model and the error of learning weight are tackled by the switching control, the system performance is much improved as compared with traditional (adaptive) fuzzy controls. Under mild conditions, the stability of the internal part of transformed system is guaranteed. Simulations and experiments of velocity control of four-bar-linkage system confirm the usefulness of the proposed control. The authors believe that the proposed scheme can be applied to many control problems.

Appendix A
The Proof of Lemma 1

In the sequence, the arguments of variables are omitted if there are not vague. With the matching condition (9), the nonlinear system (1) becomes

$$\dot{x} = \overline{A} + \overline{B}u + \overline{B}(\Delta a + \Delta bu). \tag{A1}$$

The derivative of the transformation

$$\dot{z} = [\,\dot{\Psi}^T \quad \dot{\Phi}^T\,]^T = \frac{\partial T}{\partial x}\,\dot{x}$$

is described as follows:

$$\dot{\Psi} = \frac{\partial \Psi}{\partial x}\,\overline{A} + \frac{\partial \Psi}{\partial x}\,\overline{B}u + \frac{\partial \Psi}{\partial x}\,\overline{B}(\Delta a + \Delta bu) \tag{A2}$$

and

$$\dot{\Phi} = \frac{\partial \Phi}{\partial x}\,\overline{A} + \frac{\partial \Phi}{\partial x}\,\overline{B}u + \frac{\partial \Phi}{\partial x}\,\overline{B}(\Delta a + \Delta bu). \tag{A3}$$

Rewrite (A2) as the following form:

$$\dot{\Psi} = A_c\Psi + B_c\beta^{-1}(u - \alpha) + \frac{\partial \Psi}{\partial x}\,\overline{B}(\Delta a + \Delta bu) \tag{A4}$$

where the pair $(A_c,\, B_c)$ is described in (11) and is controllable, $\alpha,\, \beta$ is expressed in (10) and $\beta(x)$ nonsingular for all $x \in \Re^n$. The function Ψ that transforms (A2) into the form (A4) must satisfy the following partial differential equations:

$$\frac{\partial \Psi}{\partial x}\,\overline{A} = A_c\Psi - B_c\beta^{-1}\alpha, \qquad \frac{\partial \Psi}{\partial x}\,\overline{B} = B_c\beta^{-1} \tag{A5}$$

or

$$\psi_{i+1} = \frac{\partial \psi_i}{\partial x}\,\overline{A}, \qquad \frac{\partial \psi_i}{\partial x}\,\overline{B} = 0, \qquad 1 \le i \le r-1, \\ \frac{\partial \psi_r}{\partial x}\,\overline{B} = \beta^{-1} \ne 0. \tag{A6}$$

Then $\psi_i\ (1 \le i \le r)$ can be found. To transform the system into the normal form, a function Φ is chosen such that $\partial\Phi/\partial x\overline{B} = 0$

and $\Phi(0) = 0$. Then ϕ_i $(1 \le i \le n-r)$ can be found and (A3) is simplified into the following equation:

$$\dot{\Phi} = \partial\Phi/\partial x \overline{A}. \tag{A7}$$

Finally, (10) is achieved. Q.E.D.

APPENDIX B
THE PROOF OF THEOREM 2

First, the matched uncertainties are considered. Define a Lyapunov function candidate for the closed-loop system as follows:

$$\begin{aligned} V &= s^2/2 + \tilde{W}^T\Lambda^{-1}\tilde{W}/2 \\ &= \sigma^T P \sigma > 0, \qquad \text{as } s \ne 0,\ \tilde{W} \ne 0 \end{aligned} \tag{B1}$$

where

$$\tilde{W} = W - \hat{W}, \quad \sigma = [\,s \quad \tilde{W}^T\,]^T$$

and

$$P = \mathrm{diag}\{\,1 \quad \lambda_{11}^{-1} \quad \cdots \quad \lambda_{ll}^{-1}\,\}/2.$$

The derivative $\dot{V}$ is given

$$\dot{V} = s\dot{s} + \tilde{W}^T\Lambda^{-1}\dot{\tilde{W}}. \tag{B2}$$

Similarly, the derivative of s using (15) and (17) is given as follows:

$$\begin{aligned} \dot{s} &= D^T\{A_c\tilde{\Psi} + B_c[(1+\Delta b_0)v - K^T\overline{\Psi} - k_{r+1}r_d \\ &\qquad + \beta_0^{-1}(\Delta a_0 + \Delta b_0\alpha_0)]\} \\ &= \sum_{i=1}^{r-1} d_{r-i+1}\tilde{\psi}_{i+1} + [(1+\Delta b_0)(v_{\mathrm{eq}} + v_{\mathrm{sw}}) - K^T\overline{\Psi} \\ &\qquad - k_{r+1}r_d + \beta_0^{-1}(\Delta a_0 + \Delta b_0\alpha_0)] \\ &= (1+\Delta b_0)v_{\mathrm{sw}} + \tilde{W}^T\Theta + \varepsilon. \end{aligned} \tag{B3}$$

Substituting (24), (26), (27), and (B3) into (B2) yields

$$\begin{aligned} \dot{V} &= s\{(1+\Delta b_0)v_{\mathrm{sw}} + \tilde{W}^T\Theta + \varepsilon\} - s\tilde{W}^T\Theta + \eta|s|\tilde{W}^T\hat{W} \\ &\le |s|\left\{\varepsilon_0 - \left[\gamma_1|s| + \frac{\gamma_2|s|}{|s|+\xi}\right] + \eta\tilde{W}^T(\overline{W} - \tilde{W})\right\} \\ &= \frac{-\gamma_1|s|}{|s|+\xi}\left\{-\frac{|s|+\xi}{\gamma_1}\,\varepsilon_0 + |s|(|s|+\xi) + \frac{\gamma_2}{\gamma_1}\,|s| \right. \\ &\qquad \left. + \frac{\eta(|s|+\xi)}{\gamma_1}\,\tilde{W}^T(\tilde{W} - \overline{W})\right\} \\ &\le \frac{-\gamma_1|s|}{|s|+\xi}\{G(|s|) + H(\|\tilde{W}\|)\} \end{aligned} \tag{B4}$$

where

$$G(|s|) = |s|^2 + 2g_1|s| - g_2 \tag{B5}$$

$$H(\|\tilde{W}\|) = \eta(|s|+\xi)[\|\tilde{W}\| - w_{\max}/2]^2/\gamma_1. \tag{B6}$$

Because $g_2 > 0$, $g > 0$ is achieved. Hence, if

$$|s| > g, \qquad \text{then } \dot{V} \le -\zeta_1\left(|s|,\ \left\|\tilde{W}\right\|\right),$$

where

$$\zeta_1\left(|s|,\ \left\|\tilde{W}\right\|\right) > 0.$$

Similarly, if

$$\left\|\tilde{W}\right\| > w_{\max}/2 + \sqrt{\frac{w_{\max}^2}{4} + \frac{\varepsilon_0}{\eta}} = w_{\mathrm{in}}$$

then

$$\dot{V} \le -\zeta_2\left(|s|,\ \left\|\tilde{W}\right\|\right)$$

where

$$\zeta_2\left(|s|,\ \left\|\tilde{W}\right\|\right) > 0.$$

Hence, if

$$|s| > g \quad \text{and} \quad \left\|\tilde{W}\right\| > w_{\mathrm{in}}$$

then

$$\dot{V} \le -\min\left\{\zeta_1\left(|s|,\ \left\|\tilde{W}\right\|\right),\ \zeta_2\left(|s|,\ \left\|\tilde{W}\right\|\right)\right\}. \tag{B7}$$

Hence, outside of the following domain $\overline{\Omega}$ makes (B7) exist.

$$\overline{\Omega} = \left\{\sigma \in \Re^{l+1} | 0 \le \left\|\tilde{W}\right\| \le w_{\mathrm{in}},\ 0 \le |s| \le g\right\}. \tag{B8}$$

Finally, from (25)–(27), v is UUB. Because the dynamics (10b) is input-to-state stable, Ψ, Φ, or x is UUB. From Lemma 1, α and β are also UUB. Then u is UUB.

Similarly, the system with unmatched uncertainties can be achieved. For simplicity, those are omitted. Q.E.D.

APPENDIX C
THE PARAMETER VALUES OF FOUR-BAR-LINKAGE SYSTEM

$$\begin{aligned} M_e(\theta_2) &= J_m + G_1 + r_1^2G_2 + r_1\cos(\theta_2 - \theta_3)G_3 \\ &\quad + r_2^2G_4 \end{aligned} \tag{C1}$$

$$\begin{aligned} C_b(\theta_2) &= r_1q_1G_2 + [q_1\cos(\theta_2 - \theta_3) \\ &\quad - r_1(1 - r_1)\sin(\theta_2 - \theta_3)]G_3/2 + r_2q_2G_4 \end{aligned} \tag{C2}$$

$$\begin{aligned} G_1 &= m_2z_2^2 + I_2 + m_3l_2^2, \quad G_2 = m_3l_3^2/4 + I_3, \\ G_3 &= m_3l_2l_3, \quad G_4 = m_4l_4^2/4 + I_4 \end{aligned} \tag{C3}$$

$$\begin{aligned} q_1 &= [-l_4r_2^2 + l_2\cos(\theta_2 - \theta_4) + l_3r_1^2\cos(\theta_3 - \theta_4)]/ \\ &\quad [l_4\sin(\theta_4 - \theta_3)] \end{aligned} \tag{C4}$$

$$\begin{aligned} q_2 &= [-l_3r_1^2 - l_2\cos(\theta_2 - \theta_3) + l_4r_2^2\cos(\theta_4 - \theta_3)]/ \\ &\quad [l_4\sin(\theta_3 - \theta_4)] \end{aligned} \tag{C5}$$

$$\begin{aligned} r_1 &= l_2\sin(\theta_2 - \theta_4)/[l_3\sin(\theta_4 - \theta_3)] \\ r_2 &= l_2\sin(\theta_2 - \theta_3)/[l_4\sin(\theta_4 - \theta_3)] \end{aligned} \tag{C6}$$

$$\begin{aligned} \theta_i &= 2\tan^{-1}\left[-\left(b_i + \sqrt{b_i - 4a_ic_i}\right)\Big/(2a_i)\right] \\ a_i &= [1 + (-1)^{i+1}k_{i2}]\cos(\theta_2) + k_{i3} - k_{i1}, \\ &\qquad i = 3, 4 \end{aligned} \tag{C7}$$

$$\begin{aligned} b_i &= -2\sin(\theta_2) \\ c_i &= -[1 + (-1)^ik_{i2}]\cos(\theta_2) + k_{i3} + k_{i1}, \\ &\qquad i = 3, 4 \end{aligned} \tag{C8}$$

$$\begin{aligned} k_{i1} &= l_1/l_2, \qquad k_{i2} = l_1/l_i \\ k_{i3} &= [l_4^2 - l_3^2 + (-1)^i(l_2^2 + l_1^2)]/(2l_2l_i), \\ &\qquad i = 3, 4. \end{aligned} \tag{C9}$$

Furthermore, four-bar-linkage has the following length, mass and inertia: $l_1 = 0.31$ m, $l_2 = 0.1$ m, $l_3 = 0.35$ m, $l_4 = 0.25$ m, $m_2 = 1.55$ kg, $m_3 = 4.3$ kg, $m_4 = 3.55$ kg, $I_2 =$

5.8125×10^{-4} kgm, $I_3 = 1.3125 \times 10^{-3}$ kgm, and $I_4 = 1.3313 \times 10^{-3}$ kgm. Because the first linkage is fixed, the information of m_1 and I_1 are not required. The torque constant is $K_t = 3.5$ kgm/amp achieved from the maximum torque and the maximum current. The following system parameters are also assigned $K_b = 0.7$ V/rad/s, $R_m = 1.0\ \Omega$, $L_m = 0.1$ H.

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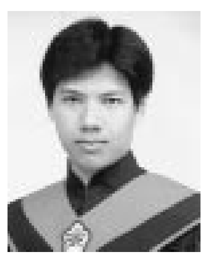

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