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Intervalwise Receding Horizon H_∞ Tracking Controls for Linear Continuous Time-Varying Systems

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Abstract

In this paper, a finite horizon H_∞ tracking control (HTC) for continuous time-varying systems is obtained in a state-feedback form. From the HTC, an intervalwise receding horizon H_∞ tracking control (IHTC) is obtained for continuous time-varying systems. It is shown that the proposed IHTC guarantees the closed-loop stability and an H_∞ norm bound for continuous time-varying systems. Conditions are proposed under which the IHTC with integral action provides zero offset for a constant reference signal and time-invariant systems. The performance of the IHTC is illustrated via simulation studies. The results in this paper are also applicable to periodic and time-invariant systems which belong to the class of time-varying systems.

1 Introduction

The receding horizon control has been widely used for real applications. There are two kinds of receding horizon controls, pointwise and intervalwise. In the pointwise receding horizon control, the terminal point of a fixed-length finite cost horizon recedes continuously. In the intervalwise receding horizon control, the terminal point is kept fixed for a period of a finite cost horizon and, after one period, the terminal point moves by one period and is fixed for the next period.

The intervalwise receding horizon control requires more memory but has a much lower computational cost than the pointwise receding horizon control as it requires calculation of control gains once in every period, while the pointwise strategy requires the same calculation at every time instant. The tracking performance of the intervalwise receding horizon control also seems to be similar to that of the pointwise one as shown in the simulation example later in this paper.

The pointwise receding horizon linear quadratic (LQ) control [1]-[4], and its extension to H_∞ control [5], [6] have been developed for time-invariant and time-varying systems. The intervalwise receding horizon LQ control has only been developed for time-invariant and periodic systems [7]-[9]. Recently, its extension to H_∞ control [10] has been developed for discrete periodic systems. If the stabilizing intervalwise receding horizon control can be considered for time-varying systems, then it can handle the stabilizing pointwise receding horizon control as a special case for time-varying systems including periodic and time-invariant systems. However, the intervalwise receding horizon H_∞ control including the tracking problem for continuous and/or time-

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varying systems has not been investigated elsewhere.

In order to obtain a receding horizon H_∞ tracking control (HTC), a finite horizon HTC must be obtained first. A finite horizon HTC was proposed for continuous systems in [11]. Even though the finite horizon HTC in [11] can be used for obtaining a receding horizon HTC, we derive a different finite horizon HTC for a different system model from that of [11] which is based on a different approach from that of [11]. Based on the derived HTC, an intervalwise receding horizon HTC (IHTC) is proposed for continuous time-varying systems in this paper. It is shown that the proposed IHTC guarantees the closed-loop stability and the infinite horizon H_∞ norm bound for continuous time-varying systems including periodic and time-invariant systems. Conditions are proposed under which the IHTC with integral action provides zero offset for a constant reference signal and time-invariant systems where the zero offset means that the tracking error goes to zero as time goes on. The performance of the IHTC is illustrated via a simulation example.

2 H_∞ tracking control for continuous time-varying systems

In this section, we derive a finite horizon HTC by using existing results [12] which investigate only the regulation problem. We consider the following continuous time-varying system:

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B_1(t)w(t) + B_2(t)u(t) \quad (1)\\ y(t) &= C(t)x(t), \quad x(t_0) = x_0 \\ z(t) &= \begin{bmatrix} y(t) - y_r(t) \\ u(t) \end{bmatrix}, \quad (2) \end{aligned}$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ the control, $w(t) \in R^l$ the disturbance, $y(t) \in R^p$ the system output, $y_r(t) \in R^p$ the given reference signal, and $z(t) \in R^{p+m}$ the controlled variable. For tracking problems, $C(t)x(t)$ is expected to approach $y_r(t)$. It is well known that for a given $p \times n$ $(p \leq n)$ full rank matrix $C(t)$, there always exist some $n \times p$ matrices $L(t)$ such that $C(t)L(t) = I_{p \times p}$. Then, we consider the following dynamic game problem based on a finite horizon cost function with the finite terminal weighting matrix Q_f:

$$\min_u \max_w \quad J(x(t), t, t+T) \quad (3)$$

where

$$\begin{aligned} J(x(t), t, t+T) = \int_t^{t+T} & [\|z(\tau)\|_2^2 - \gamma^2 \|w(\tau)\|_2^2] d\tau \\ & + [x(t+T) - L(t+T)y_r(t+T)]^T \\ & Q_f [x(t+T) - L(t+T)y_r(t+T)]. \end{aligned}$$

Here, $Q_f \geq 0$, γ is the disturbance attenuation level, and $y_r(\tau)$s are reference signals which are assumed to be available over the future horizon $\tau \in [t, t+T]$. $Q_f \geq 0$ is a critical design parameter for the closed-loop stability and thus, discussed in the next section.

The matrices $A(t)$, $B_1(t)$, $B_2(t)$, and $C(t)$ are assumed to be bounded. Define $Q(t)$ as $Q(t) = C^T(t)C(t) \geq 0$, $B_\gamma(t)$ as $B_\gamma(t) = \gamma^{-1}B_1(t)$, $\bar{Q}(t)$ as $\bar{Q}(t) = B_2(t)\ B_2^T(t) - B_\gamma(t)\ B_\gamma^T(t)$, and the threshold value $\hat{\gamma}^{CL}$ as $\hat{\gamma}^{CL} = inf\ \{\gamma > 0 : K(\tau, t+T)$ does not have a conjugate point for all $\tau \in [t, t+T]\}$ where $K(\tau, t+T)$ is calculated backward from

$$\begin{aligned} -\frac{\partial K(\tau, t+T)}{\partial \tau} &= A^T(\tau)K(\tau, t+T) + K(\tau, t+T) \\ & A(\tau) + Q(\tau) - K(\tau, t+T)\bar{Q}(\tau)K(\tau, t+T) \quad (4) \end{aligned}$$

with the terminal condition

$$K(t+T, t+T) \;=\; Q_f. \quad (5)$$

In order to find a solution of the above problem (3), we introduce an existing result of the finite horizon H_∞ regulation problem where $y_r(t)=0$ for $\forall\ t$. In this case, the dynamic game theory described by (1) and (3) [12] admits a bounded unique feedback saddle-point solution, if and only if $\gamma > \hat{\gamma}^{CL}$. The unique feedback saddle point solution is given for $\tau \in [t, t+T]$ by

$$u^*(\tau) = -B_2^T(\tau)K(\tau, t+T)x(\tau) \quad (6)$$

$$w^*(\tau) = \gamma^{-1}B_\gamma^T(\tau)K(\tau, t+T)x(\tau). \quad (7)$$

Then, the saddle-point value of the dynamic game of (3) is given for $\tau \in [t, t+T]$ by

$$J^*(x(\tau), \tau, t+T) \;=\; x^T(\tau)K(\tau, t+T)x(\tau) \quad (8)$$

THEOREM 1 *If $\gamma > \hat{\gamma}^{CL}$, then the dynamic game problem (3) for the system (1) admits a unique feedback saddle-point solution which is given by*

$$u^*(\tau) = -B_2^T(\tau)[K(\tau, t+T)x(\tau) + g(\tau, t+T)] \quad (9)$$

$$w^*(\tau) = \gamma^{-2} B_1^T(\tau)[K(\tau, t+T)x(\tau) + g(\tau, t+T)] \quad (10)$$

where $g(\tau, t+T)$ is calculated backward from

$$-\frac{\partial g(\tau, t+T)}{\partial \tau} = A^T(\tau)g(\tau, t+T) - K(\tau, t+T) \bar{Q}(\tau)g(\tau, t+T) - C^T(\tau)y_r(\tau) \quad (11)$$

with the boundary condition $g(t+T, t+T) = -Q_f L(t+T)y_r(t+T)$.

Proof: The detailed proof is omitted because of the page limit. For the full version, refer to the website http://cisl.snu.ac.kr/~kkb. □

Now, we will compare the proposed HTC with that of [11]. Under the assumption that $B_3 = 0$, $C_1 = C$, $D_{12}^T C_1 = 0$, $D_{12}^T D_{12} = I$, $D_{13} = -I$, $D_{12}^T D_{13} = 0$, $X(\tau) = K(\tau, t+T)$, and $\theta(\tau) = g(\tau, t+T)$ in [11], the solutions (9) and (10) are the same as (3.15) and (3.5) of [11]. However, we can easily know that the above assumption is impossible. Moreover, $X(t+T)$ in [11], which corresponds to Q_f, is assumed to be zero while Q_f in this paper is non-negative definite matrix including zero. Thus, the proposed HTC in this paper is not the same as that in [11] and cannot be derived from the result in [11].

3 IHTC for continuous time-varying systems and its stability

In this section, using the HTC derived in the previous section, we propose an intervalwise receding horizon HTC (IHTC) which guarantees the closed-loop stability and H_∞ norm bound for continuous time-varying systems. For the closed-loop stability, we introduce the following sufficient condition based on the result in [5]:

$$A^T(\sigma)Q_f + Q_f A(\sigma) + Q(\sigma) - Q_f \bar{Q}(\sigma) Q_f \leq 0 \text{for all } \sigma \geq t_0 + T. \quad (12)$$

Note that there exists a finite Q_f satisfying (12) if the pair $(A(\cdot), B_2(\cdot))$ is uniformly stabilizable. Also note that in many systems like one included in a polytope, the set of all pairs $(A(\sigma), \bar{Q}(\sigma))$ can be represented by using a finite number of pairs. In these cases, we have only to solve finite number of linear inequalities by using the well-known Linear Matrix Inequality (LMI) Toolbox as in [13].

Assume that $T \geq \Delta > 0$ for an intervalwise horizon Δ. Then, the IHTC strategy is as follows.

(a) Let $k = 0$.
(b) At the present time $t_0 + k\Delta$, obtain $u^*(\tau)$ over $\tau \in [t_0 + k\Delta, t_0 + k\Delta + T]$ by solving the problem (3) with Q_f satisfying (12).
(c) Implement the control inputs $u^*(\tau)$ for $\tau \in [t_0 + k\Delta, t_0 + (k+1)\Delta)$.
(d) Let $k = k + 1$ and repeat procedure (b)-(c).

The resulting IHTC $u^*(\tau)$ for $\tau \in [t_0 + k\Delta, t_0 + (k+1)\Delta)$ is given by

$$\begin{aligned} u^*(\tau) \;\; &= \;\; -B_2^T(\tau)[K(\tau, t_0 + k\Delta + T)x(\tau) \\ &\quad + g(\tau, t_0 + k\Delta + T)] \end{aligned} \quad (13)$$

where $k = 0, 1, \cdots, \infty$ and Q_f satisfies (12).

Now, we are ready to state the closed-loop stability of the proposed IHTC (13) with $y_r(\cdot) = 0$ and $w(\cdot) = 0$.

THEOREM 2 *If the pair $(A(t), C(t))$ is uniformly observable (or uniformly detectable), the system (1) with (13) is uniformly asymptotically stable for $\Delta \leq T < \infty$.*

Proof: By optimality, for all $t_0 + k\Delta \leq \tau_1 \leq \tau_2 \leq t_0 + (k+1)\Delta$ and integer $k \geq 0$, $J^*(x(\tau_1), \tau_1, t_0 + k\Delta + T) = \int_{\tau_1}^{\tau_2} [x^T Q x + u^{*T} u^* - \gamma^2 w^{*T} w^*] d\tau + J^*(x(\tau_2), \tau_2, t_0 + k\Delta + T) \geq \int_{\tau_1}^{\tau_2} [x_2^T Q x_2 + u^{*T} u^*] d\tau + J^*(x_2(\tau_2), \tau_2, t_0 + k\Delta + T)$ where $x_2(\cdot)$ is the state trajectory from $x_2(\tau_1) = x(\tau_1)$ when $u(\tau) = u^*(\tau)$ and $w(\tau) = 0$ for $\tau \in [\tau_1, \tau_2)$, and $u(\tau) = u_2^*(\tau)$ and $w(\tau) = w_2^*(\tau)$ for $\tau \in [\tau_2, t_0 + k\Delta + T)$. Here, $u_2^*(\cdot)$ and $w_2^*(\cdot)$ are the saddle-point solution for $J^*(x_2(\tau_2), \tau_2, t_0 + k\Delta + T)$.

From Theorem 1 in [5], we can easily know that under the condition (12), $J^*(t_0+(k+1)\Delta, t_0+k\Delta+T) \geq J^*(t_0+(k+1)\Delta, t_0+(k+1)\Delta+T)$

Thus, for any $t_0+k_1\Delta \leq \tau_1 < t_0+(k_1+1)\Delta$ and $t_0+k_2\Delta \leq \tau_2 < t_0+(k_2+1)\Delta$, and any integers $k_2 \geq k_1 \geq 0$ where $\tau_2 \geq \tau_1$, $J^*(x(\tau_1), \tau_1, t_0+k_1\Delta+T) \geq \int_{\tau_1}^{\tau_2}[x^TQx+u^{*T}u^*]d\tau$ $+\ J^*(x(\tau_2), \tau_2, t_0+k_2\Delta+T)$.

Since the saddle-point value is nonnegative, as $t \to \infty$ $u^*(t) \to 0$. Then, by uniform observability, as $t \to \infty$ $x(t) \to 0$, i.e., the closed-loop system is uniformly attractive when $w(t) = 0$ for all t. Since the closed-loop matrix $[A(\tau) - B_2(\tau)B_2^T(\tau)K(\tau, t_0+k\Delta+T)]$ for all $\tau \in [t_0+k\Delta, t_0+(k+1)\Delta)$ is bounded, the closed-loop system when $w(t)=0$ for all t is uniformly asymptotically stable from Lemma 1 in [5]. □

Remark 1 *The above results are also applicable to periodic and time-invariant systems which belong to the class of time-varying systems. For periodic systems, by selecting Δ and T appropriately, we can get a very simple stabilizing IHTC as follows. Assume that the system matrices $A(\cdot)$, $B_1(\cdot)$, $B_2(\cdot)$, and $C(\cdot)$ are N-periodic. Then, let $\Delta = N$ and Q_f satisfy*

$$A^T(\sigma)Q_f + Q_fA(\sigma) + Q(\sigma) - Q_f\bar{Q}(\sigma)Q_f \leq 0 \textit{for all} \quad \sigma \in [t_0+T, t_0+T+\Delta]. \quad (14)$$

In this case, the solutions of $K(\tau, t_0+T)$ over $[t_0, t_0+\Delta]$ from (4) at the initial point t_0 are repeated since the systems are continuous Δ-periodic. Thus, we have only to solve the Riccati equation (4) only once at the initial time. It is the same for time-invariant systems.

In the following, we show that the stabilizing IHTC guarantees the H_∞-norm bound for continuous time-varying systems.

THEOREM 3 *For any 2-norm bounded disturbances, the stabilizing IHTC $u^*(t)$ guarantees the H_∞-norm bound of the closed-loop system when $x_0 = 0$, i.e.,*

$$\|T_{zw}\|_\infty \quad \leq \quad \gamma \textit{ where } \|T_{zw}\|_\infty. \quad (15)$$

Proof. From Theorem 2, $J^*(t_0, t_0+T) \geq \int_{t_0}^{t_0+k\Delta} (\|z(t)\|_2^2 - \gamma^2\|w(t)\|_2^2)dt + J^*(t_0+k\Delta, t_0+k\Delta+T)$. Since the closed-loop system is uniformly asymptotically stable, $J^*(t_0+k\Delta, t_0+k\Delta+T) \to 0$ as $k \to \infty$. Since $J^*(t_0, t_0+T) = 0$ with $x_0 = 0$, we have (15). □

In the following section, we show how to guarantee zero-offset with the IHTC.

4 Stabilizing IHTC with integral action

In this section, we investigate the zero offset property of the proposed stabilizing IHTC with integral action when the tracking command is constant and the system is time-invariant. In order to derive IHTC minimizing the cost function with integral action, we transform the model (1) and the cost function (3) as follows.

$$\dot{x}_e(t) = A_e x_e(t) + B_{1e}\dot{w}(t) + B_{2e}\dot{u}(t)$$
$$x_e(t_0) = x_{e0} \quad (16)$$
$$y(t) = C_e x_e(t), \qquad z_e(t) = \begin{bmatrix} y(t) - y_r \\ \dot{u}(t) \end{bmatrix}$$

$\min_{\dot{u}} \max_{\dot{w}} \int_t^{t+T} [\|z_e(\tau)\|_2^2 - \gamma^2\|\dot{w}(\tau)\|_2^2]d\tau + [x_e(t+T) - L_e y_r]^T Q_{fe}[x_e(t+T) - L_e y_r]$. where

$$x_e(t) = \begin{bmatrix} y(t) \\ \dot{x}(t) \end{bmatrix}, \ A_e = \begin{bmatrix} 0 & C \\ 0 & A \end{bmatrix}, \ B_{1e} = \begin{bmatrix} 0 \\ B_1 \end{bmatrix}$$
$$B_{2e} = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}, \ C_e = \begin{bmatrix} I & 0 \end{bmatrix}, \ L_e = \begin{bmatrix} I \\ 0 \end{bmatrix}.$$

In this case, the finite horizon H_∞ tracking control for $\tau \in [t, t+T)$ is given by

$$\dot{u}^*(\tau) = -B_{2e}^T[K_e(\tau, t+T)x_e(\tau) + g_e(\tau, t+T)] \quad (17)$$
$$\dot{w}^*(\tau) = \gamma^{-2}B_{1e}^T[K_e(\tau, t+T)x_e(\tau) + g_e(\tau, t+T)] \quad (18)$$

where $K_e(\tau, t+T)$ and $g_e(\tau, t+T)$ are obtained from (4) and (11) with $A(\cdot)$, $B_1(\cdot)$, $B_2(\cdot)$, $C(\cdot)$, $Q(\cdot)$, $\bar{Q}(\cdot)$, $L(\cdot)$, and $Q_f(\cdot)$ replaced by A_e, B_{1e}, B_{2e}, C_e, Q_e, $\bar{Q}_e$, L_e, and Q_{fe}, respectively.

Corollary 1 *Assume that Q_{fe} satisfies the following inequality condition:*

$$A_e^T Q_{fe} + Q_{fe}A_e + Q_e - Q_{fe}\bar{Q}_e Q_{fe} \leq 0. \quad (19)$$

Then, if the pair (A, C) is observable (or detectable), the system (16) with the resulting IHTC is asymptotically stable for $\Delta \leq T < \infty$.

Proof: We can easily know that if the pair (A, C) is observable, then the pair (A_e, C_e) is observable. The remaining proof here follows that of Theorem 2. □

Now, we are ready to introduce the zero offset property of the proposed stabilizing IHTC with integral action.

THEOREM 4 *The stabilizing IHTC with integral action provides zero offset.*

Proof: The detailed proof is omitted because of the page limit. For the full version, refer to the website http://cisl.snu.ac.kr/~kkb. □

Note that the proof method of Theorem 4 cannot be applied to periodic or time-varying systems.

5 Simulation Studies

We compare the tracking performance of the proposed IHTC with those of the pointwise receding horizon H_∞ tracking control (PHTC) and the pointwise receding horizon LQ tracking control (PLQTC). Here, the PHTC and the PLQTC are obtained by using a finite horizon HTC derived in this paper.

We consider the following time-invariant system matrices:

$$A = \begin{bmatrix} 0.5 & -0.3 \\ 0.2 & -1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.5 \\ 0.7 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix}$$

$$C = \begin{bmatrix} 30 & 2 \end{bmatrix}, \quad L = \begin{bmatrix} \frac{1}{30} \\ 0 \end{bmatrix}.$$

For this example, $Q_f = \begin{bmatrix} 99.4630 & 5.5002 \\ 5.5002 & 0.6610 \end{bmatrix}$, which is obtained by using "feasp()" function in LMI Toolbox [13]. In this simulation, we assume that $T = 0.2$ sec, $\gamma = 1$, $\Delta = 0.1$, the sampling time $T_s = 0.02$ sec, and $x(t_0) = [0 \ \ 0]^T$. Since the simulation time is 2 sec, we have 100 steps as shown in x-axis of Fig. 1 and Fig. 2.

Using these values, we obtain a stabilizing IHTC. We make the disturbance by multiplying 20% of the tracking command value by a random signal which has a normal distribution between -0.5 and 0.5.

Fig. 1 shows the given reference signal ($y_r(t)$) in (a) and tracking errors ($y(t) - y_r(t)$) of the IHTC, PHTC, and PLQTC in (b), (c), and (d), respectively.

The online computation times of the PHTC and PLQTC are about 3.8 times longer than that of the IHTC, while the IHTC, PHTC and PLQTC have similar performances as shown in Fig. 1. This is the same when there is no disturbance.

6 Conclusion

In this paper, a finite horizon H_∞ tracking control (HTC) is derived for continuous time-varying systems. Using the derived HTC, an intervalwise receding horizon HTC (IHTC) is proposed for continuous time-varying systems. It is shown that the proposed IHTC guarantees the closed-loop stability and the infinite horizon H_∞-norm bound for continuous time-varying systems including periodic and time-invariant systems. Conditions are proposed under which the IHTC with integral action provides zero offset for a constant reference signal and time-invariant systems. Through simulation, it is shown that the proposed IHTC has a much less computational burden than those of existing pointwise receding horizon LQ tracking control (PLQTC) and pointwise receding horizon HTC (PHTC), while the performance of the IHTC is similar to those of the PLQTC and the PHTC.

The advantage of the proposed IHTC is that computational burdens are less than those for the PHTC and the PLQTC, although more memory for control gains may be required. Therefore, the proposed stabilizing IHTC will be useful for real-

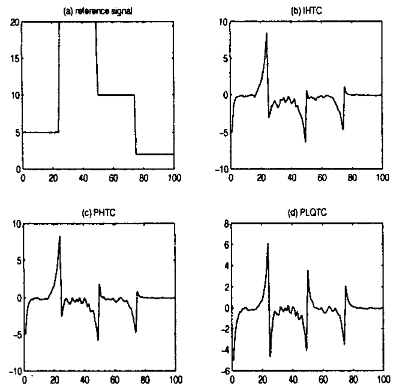

Figure 1: Reference signal and tracking errors

time tracking control. The IHTC with integral action will be useful for an accurate control because of the zero-offset property.

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