Proceedings of the American Control Conference
Anchorage, AK May 8-10, 2002

Credit-Risk Decision Model and Credit Rationing with Asymmetry Information*

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Abstract

We introduce theory and method of incentive mechanism to the research, and we establish credit-risk decision model with incentive effects. We consider that there are two types of borrowers in society: one is high-risk type and the other is low-risk type. Under the same rationing, we investigate proportions about the two types in borrowing group. In our model, the credit-risk decision policy the bank provides the same rationing will result in increase of the proportion of the high-risk applicants in borrowing group. Therefore, the bank must establish effective risky distinguishing mechanism so that they can avoid credit risk and ensure safe operation of credit money. In addition, we also give condition the bank provides the same rationing to the two types.

1 Introduction

In 1981, Stiglitz and Weiss [1] showed that banks might prefer to reject some borrowers because of negative adverse selection and incentive effects. In S-W's model, the distribution of returns to high-risk borrowers is a mean-preserving spread of the distribution of returns to low-risk borrowers, which implies that, as the loan interest rate increase, low-risk borrowers drop out of the market before high-risk borrowers. Therefore, when banks can't obtain perfect information about risky types of investors, the credit contracts may result in adverse selection. Hildegard Wette [2] (1983) showed that the increase of collateral requirements was likely to lead to adverse selection even though the investors are risk neutral. Bester [3] (1985) showed that adverse selection will be occurred when banks raise interest rate lonely or increase collateral lonely. The conclusion Gale [4] (1985) got is that the credit contracts will occur adverse selection when multi-borrowers are different risk types in the case of that banks have not perfect information of investors' risky types. Besanko and Thakor [5] (1987) studied the problem of credit decision mechanism from the point of view of maximal profits. In B-T's model, the distribution of returns to low-risk borrowers exhibits first-order stochastic dominance over the distribution of returns to the high-risk borrowers. This implies that, as the loan interest rate increase, high-risk borrowers drop out the market before low-risk borrowers. Meanwhile, they showed that low-risk borrowers choose

* This research is supported by NSSF of China, #10131030

contracts with low interest rates and high collateral requirements whereas high-risk borrowers choose contracts with high interest rates and low collateral requirements. Wu Ji, Huanchen Wang and Xiaohong Dong[6][7] (1996) analysed how to defend adverse selection when investors are risk averse. Glazer and McGuire [8] (2000) studied and established the model of adverse selection in the credit market. Sulin Pang, Rongzhou Li, Yongqing Liu and Jianmin Xu [9][10] (2001) showed that, under the equal collateral value, the results banks increase interest rate will occur adverse selection.

In this paper, we assume that there are two types of borrowers in society: one is high-risk type and the other is low-risk type. We establish credit-risk decision model and investigate proportions about the two types in borrowing group. We also give condition the bank provides the same rationing to the two types.

2 Credit-risk decision model

Assumed that there are two types of borrowers: one is high-risk type and the other is low-risk type. The amounts of their risky investment are all I. Their initial wealth are all $W(W < I)$, so they need to borrow funds B from a bank, here $B = I - W > 0$. Their expected returns are all R. Collaterals they must provide for the bank are C_1 and C_2 respectively. We assume that interesting rate the bank provides for the i th type is r_i ($i = 1$ represents the high-risk type, $i = 2$ represents the low-risk type). $q_i (0 \le q_i \le 1)$ is the possibility the i th type gets loan. q_i is also called credit rationing. If $q_i = 0$, it means that the bank rejects the i th type's loan application. If $0 < q_i < 1$, it means that the loan involves credit rationing to the i th type. If $q_i = 1$, it means that the loan does not involve credit rationing to the i th type. Let ρ ($0 < \rho < r_i$) denote rate of return of the safe investment, $k(0 < k < 1)$ denote realizable rate of collateral, $t(0 < t < 1)$ denote the proportion of high-risk type in borrowing group, $p_i(0 < p_1 < p_2 < 1)$ denote possibility the i th type invests success, R_i denote returns the projects are succeeded, otherwise, the return to entrepreneur is $R_f = 0$. That is,

$$p_i R_i + (1 - p_i) R_f = R$$

In general, in the case of information asymmetry, the borrowers always have more private information than banks. Therefore, when a borrower applies for a loan, the bank is often unable to judge risk of the project from his files. This is a direct reason of leading to risk. Once making a mistaken decision, the loan fund will be confronted with risk and the risk is very high. In this paper, we consider two losses of the funds: one is loss of fund and the other is loss of opportunity. The two losses all reduce the expected profits to the bank to a large extent. Generally, there are following several cases:

a. If a low-risk project is regarded as a high-risk project and the bank rejects the applicant (i.e., $q_i = 0$), but if the borrower can repay loan on time in the future (i.e., $W + kC_i - (1 + r_i)B \ge 0$), the loss of opportunity is:

$$\sigma = (1 + \rho)B - (1 + r_i)B = (\rho - r_i)B$$

b. If a high-risk project is regarded as a low-risk project and the bank offer loan to the applicant in non-rationing (i.e., $q_i = 1$), but if the borrower is unable to repayment because of bankruptcy or other causes (i.e., $(1 + r_i)B - (R_i + kC_i) > 0$), the loss of fund is:

$$\sigma = kC_i - (1 + r_i)B$$

c. When unable to judge the risk of investment project, the bank often offer a loan in the form of rationing (i.e., $0 < q_i < 1$)s, but if the rationing is unreasonable, it will bring about the loss of fund or the loss of opportunity. There are two cases as follows:

①. If the borrower can realize the expected return after investment (i.e., $kC-(1+r_i)B \geq 0$ and $(R_i+kC)-(1+r_i)B \geq 0$), but the loan involves rationing, therefore, for the bank, the loss of opportunity is:

$$\sigma=(1-q_i)[(1+\rho)B-(1+r_i)B]=(1-q_i)(\rho-r_i)B$$

②. If the investment fail so that the borrower has to declare bankruptcy (i.e., $(1+r_i)B-kC_i>0$ and $(1+r_i)B-(R_i+kC_i) \geq 0$), for the bank, the loss of fund is:

$$\sigma=q_i[kC_i-(1+r_i)B]$$

In the case of imperfect information, borrowers always have more private information about their risky investment projects than banks. In order to realize their own benefit, in general, borrowers may conceal their private information on purpose and even tell a lie in their loan application files. Therefore, when a borrower applies for a loan, it is very important how banks make a credit decision with defending risk. In order to be against adverse selection, the credit-risk decision policy for banks is to try to induce borrowers to tell their risky information truly by all means. The credit-risk decision mechanism for the bank should satisfy incentive compatibility constraints:

$$q_1[R-p_1(1+r_1)B-(1-p_1)(1+\rho)C_1-(1+\rho)W] \geq q_2[R-p_1(1+r_2)B-(1-p_1)(1+\rho)C_2-(1+\rho)\cdot W] \tag{1}$$

$$q_2[R-p_2(1+r_2)B-(1-p_2)(1+\rho)C_2-(1+\rho)\cdot W] \geq q_1[R-p_2(1+r_1)B-(1-p_2)(1+\rho)C_1-(1+\rho)W] \tag{2}$$

and satisfy individual rationality constraints:

$$q_1[p_1(1+r_1)B+(1-p_1)kC_1-(1+\rho)B] \geq 0 \tag{3}$$

$$q_2[p_2(1+r_2)B+(1-p_2)kC_2-(1+\rho)B] \geq 0 \tag{4}$$

$$q_1[R-p_1(1+r_1)B-(1-p_1)(1+\rho)C_1-(1+\rho)W] \geq 0 \tag{5}$$

$$q_2[R-p_2(1+r_2)B-(1-p_2)(1+\rho)C_2-(1+\rho)\cdot W] \geq 0 \tag{6}$$

There is a factor $(1+\rho)$ before C in (1)-(6), it means that collateral can give borrower interest.

We assume that the bank know information about the proportion t of the high-risk borrowers in borrowing group. Therefore, in the case of imperfect information, in order to avoid credit risk, the optimal credit-risk decision model should be below:

$$\min \quad \sigma=t\{q_1[kC_1-(1+r_1)B]+(1-q_1)(\rho-r_1)B\}+(1-t)\{q_2[kC_2-(1+r_2)B]+(1-q_2)(\rho-r_2)B\}$$

$$s.t. \begin{cases} q_1[R-p_1(1+r_1)B-(1-p_1)(1+\rho)C_1-(1+\rho)W] \geq q_2[R-p_1(1+r_2)B-(1-p_1)(1+\rho)C_2-(1+\rho)W] \\ q_2[R-p_2(1+r_2)B-(1-p_2)(1+\rho)C_2-(1+\rho)W] \geq q_1[R-p_2(1+r_1)B-(1-p_2)(1+\rho)C_1-(1+\rho)W] \\ q_1[p_1(1+r_1)B+(1-p_1)kC_1-(1+\rho)B] \geq 0 \\ q_2[p_2(1+r_2)B+(1-p_2)kC_2-(1+\rho)B] \geq 0 \\ q_1[R-p_1(1+r_1)B-(1-p_1)(1+\rho)C_1-(1+\rho)W] \geq 0 \\ q_2[R-p_2(1+r_2)B-(1-p_2)(1+\rho)C_2-(1+\rho)W] \geq 0 \end{cases}$$

When $q_i=0$ ($i=1,2$, So the below cases are), it means that the bank will reject the applicant. So in the following, we can further assume $q_i>0$.

The conditions of Kuhn-Tucker on the non-linear programming problem are below:

$$t+\lambda_1 q_1 p_1-\lambda_2 q_1 p_2-\lambda_3 q_1 p_1+\lambda_5 q_1 p_1=0 \tag{7}$$

$$(1-t)-\lambda_1 q_2 p_1+\lambda_2 q_2 p_2-\lambda_4 q_2 p_2+\lambda_6 q_2 \cdot p_2=0 \tag{8}$$

$$-tk+\lambda_1(1-p_1)(1+\rho)-\lambda_2(1-p_2)(1+\rho)$$
$$-\lambda_3(1-p_1)k+\lambda_5(1-p_1)(1+\rho)=0$$

(9)

$$-(1-t)k-\lambda_1(1-p_1)(1+\rho)+\lambda_2(1-p_2)(1+$$
$$\rho)-\lambda_4(1-p_2)k+\lambda_6(1-p_2)(1+\rho)=0$$

(10)

$$-t[(1+\rho)B-kC_1]-\lambda_1[R-p_1(1+r_1)B-(1-p_1)(1+$$
$$\rho)C_1-(1+\rho)W]+\lambda_2[R-p_2(1+r_1)B-(1-p_2)(1+$$
$$\rho)C_1-(1+\rho)W]-\lambda_3[p_1(1+r_1)B+(1-p_1)(1+\rho)$$
$$kC_1-(1+\rho)B]-\lambda_5[R-p_1(1+r_1)B-(1-p_1)(1+\rho)$$
$$C_1-(1+\rho)W]=0$$

(11)

$$-(1-t)[(1+\rho)B-kC_2]+\lambda_1[R-p_1(1+r_2)B-(1-$$
$$p_1)(1+\rho)C_2-(1+\rho)W]-\lambda_2[R-p_2(1+r_2)B-(1-$$
$$p_2)(1+\rho)C_2-(1+\rho)W]-\lambda_4[p_2(1+r_2)B+(1-p_2)$$
$$\cdot(1+\rho)kC_2-(1+\rho)B]-\lambda_6[R-p_2(1+r_2)B-(1-$$
$$p_2)(1+\rho)C_2-(1+\rho)W]=0$$

(12)

$$\lambda_1\{q_1[R-p_1(1+r_1)B-(1-p_1)(1+\rho)C_1-(1+\rho)$$
$$\cdot W]-q_2[R-p_1(1+r_2)B-(1-p_1)(1+\rho)C_2-(1+$$
$$\rho)W]\}=0$$

(13)

$$\lambda_2\{q_2[R-p_2(1+r_2)B-(1-p_2)(1+\rho)C_2-(1+\rho)$$
$$\cdot W]-q_1[R-p_2(1+r_1)B-(1-p_2)(1+\rho)C_1-(1+$$
$$\rho)W]\}=0$$

(14)

$$\lambda_3 q_1[p_1(1+r_1)B+(1-p_1)kC_1-(1+\rho)B]=0$$

(15)

$$\lambda_4 q_2[p_2(1+r_2)B+(1-p_2)kC_2-(1+\rho)B]=0$$

(16)

$$\lambda_5 q_1[R-p_1(1+r_1)B-(1-p_1)(1+\rho)C_1-(1+\rho)$$
$$\cdot W]=0$$

(17)

$$\lambda_6 q_2[R-p_2(1+r_2)B-(1-p_2)(1+\rho)C_2-(1+$$
$$\rho)W]=0$$

(18)

Where λ_1, λ_2, λ_3, λ_4, λ_5, $\lambda_6\geq 0$ are generalized Lagrangian multiplier. It is very easy to proof that $\lambda_4\neq 0$, $\lambda_6=0$, $\lambda_5\neq 0$, $\lambda_3=0$, $\lambda_2\neq 0$, λ_1 can be random non-negative number.

3 The same credit rationing

Theorem 3.1 If the bank provides the same rationing to borrowers, that is, $q_1=q_2$, the proportion of the high-risk borrowers in borrowing group is larger than the low-risk borrowers'.

Proof: From (11), we get:

$$\lambda_2=\frac{t[(1+\rho)B-kC_1]}{R-p_2(1+r_1)B-(1-p_2)(1+\rho)C_1-(1+\rho)W} \tag{19}$$

From (12), we can get:

$$\lambda_2=\frac{(1-t)[kC_2-(1+\rho)B]}{R-p_2(1+r_2)B-(1-p_2)(1+\rho)C_2-(1+\rho)W} \tag{20}$$

From (19) and (20), we get:

$$\frac{(1-t)[kC_2-(1+\rho)B]}{R-p_2(1+r_2)B-(1-p_2)(1+\rho)C_2-(1+\rho)W}=\frac{t[(1+\rho)B-kC_1]}{R-p_2(1+r_1)B-(1-p_2)(1+\rho)C_1-(1+\rho)W} \tag{21}$$

Because $\lambda_2\neq 0$, From (14), we get:

$$\frac{q_1}{q_2}=\frac{R-p_2(1+r_2)B-(1-p_2)(1+\rho)C_2-(1+\rho)W}{R-p_2(1+r_1)B-(1-p_2)(1+\rho)C_1-(1+\rho)W} \tag{22}$$

From (21) and (22), we get:

$$\frac{q_1}{q_2}=\frac{(1-t)[kC_2-(1+\rho)B]}{t[(1+\rho)B-kC_1]} \tag{23}$$

If $q_1=q_2$, from (23), we have:

$$\frac{(1-t)[kC_2-(1+\rho)B]}{t[(1+\rho)B-kC_1]}=1 \tag{24}$$

That is,

$$\frac{t}{1-t}=\frac{kC_2-(1+\rho)B}{(1+\rho)B-kC_1}=\left|\frac{kC_2-(1+\rho)B}{kC_1-(1+\rho)B}\right|>1$$

Therefore, $t>1-t$.

Theorem 3.1 shows that the proportion of the high-risk borrowers in borrowing group is larger than the low-risk borrowers' if the bank provides the same rationing to them. This is because the low-risk applicants are always encouraged and supported. However, the high-risk applicants are not the case, and they are usually rejected. But after all, the high-risk applicants can expect to gain higher returns by investing high-risk projects. Therefore, for investors, the high-risk projects have larger attraction, especially for those who leave no stone unturned to earn opportunity benefits. Therefore, when the bank provides the same rationing to the different risky types, it is very naturally that most investors invest high-risk projects.

On the other hand, theorem 3.1 also tells us if wanting to decrease the proportion of the high-risk borrowers in borrowing group, the bank must provide the different credit rationing to different risky types (That is, $q_1 \neq q_2$). Therefore, the bank can distinguish risky types of the borrowers and reveal their private information by designing rationing q_1 and q_2.

We also know from theorem 3.1, if a bank provides the same rationing to the two types, it will result in increase of the proportion of the high-risk applicants in borrowing group. Therefore, under the same rationing, banks must establish effective risky distinguishing mechanism so that they can avoid credit risk and ensure safe operation of credit money.

Theorem 3.2: When C_1, C_2 and B satisfy:

$$tkC_1 + (1-t)kC_2 = (1+\rho)B$$

the bank will provides the same rationing to the two types, that is, $q_1 = q_2$.

Proof: From (12), we get:

$$\lambda_2 = \frac{(1-t)[kC_2 - (1+\rho)B]}{R - p_2(1+r_2)B - (1-p_2)(1+\rho)C_2 - (1+\rho)W} \tag{25}$$

From (11), we get:

$$\lambda_2 = \frac{t[(1+\rho)B - kC_1]}{R - p_2(1+r_1)B - (1-p_2)(1+\rho)C_1 - (1+\rho)W} \tag{26}$$

From (25) and (26), we have:

$$\frac{(1-t)[kC_2 - (1+\rho)B]}{R - p_2(1+r_2)B - (1-p_2)(1+\rho)C_2 - (1+\rho)W} = \frac{t[(1+\rho)B - kC_1]}{R - p_2(1+r_1)B - (1-p_2)(1+\rho)C_1 - (1+\rho)W} \tag{27}$$

Because $\lambda_2 \neq 0$, from (14), we obtain:

$$\frac{q_1}{q_2} = \frac{R - p_2(1+r_2)B - (1-p_2)(1+\rho)C_2 - (1+\rho)W}{R - p_2(1+r_1)B - (1-p_2)(1+\rho)C_1 - (1+\rho)W} \tag{28}$$

From (27) and (28), we get:

$$\frac{q_1}{q_2} = \frac{(1-t)[kC_2 - (1+\rho)B]}{t[(1+\rho)B - kC_1]}$$

We have known $tkC_1 + (1-t)kC_2 = (1+\rho)B$, therefore, $q_1 = q_2$.

4 Conclusion

In this paper, we assume that there are two types of borrowers in society: one is high-risk type and the other is low-risk type. In the case of imperfect information, we establish credit-risk decision model with incentive effects. The study shows that if a bank provides the same rationing to the borrowers, the proportion of the high-risk borrowers in borrowing group is larger than the low-risk borrowers'. It implies that if a bank provides the same rationing to the two types, it will result in increase of proportion of the high-risk applicants in borrowing group. Therefore, under the same rationing, banks must establish effective risky distinguishing mechanism so that they can avoid credit risk and ensure safe operation of credit money. In addition, we also give condition the bank provides the same rationing to the two types.

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