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Jitter Compensation in Digital Control Systems

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Abstract

A method to compensate for time-varying, possibly random, delays (*jitter*) in digital control systems is presented. Using time-stamps from the control system, a linear compensator is designed as an add-on to an existing control system. Stability and performance analysis is performed, and leads to frequency domain conditions. Therefore, loop-shaping can be used for the compensator design. The design of the compensator can be done without a full process model.

1 Introduction

Many real-time control systems are subject to random time-delays caused by computation time, propagation delay in networks, task switching, etcetera. Throughout this paper, this random delay will be denoted *jitter*. Whereas a constant time-delay can be accounted for in a linear process model during controller design, jitter compensation is harder. This paper presents a method to improve a control system's behavior in presence of jitter. The idea is to add a time-delay-aware compensator to the original controller, so as to improve stability and performance of the control loop. The design of the compensator can be done without a full process model. One reason to use such a setup is that the compensation can be added to a control system with bad jitter performance, even if the original control design knowledge is not available.

Previous work includes both model-based and model-free approaches. In both [6] and [4], LQ-optimal controllers are presented. These are based on time-stamps, just like our approach, but also on a perfect process model *and* delay statistics. At the other extreme, we have e.g. [5], where a linear filter is used as a predictor for the control output, to predict the next sample. No specific structure of the controller is assumed, and stability and performance are not discussed. Turning to specific controller structures, [2] presents the PIP-controller, a delay compensated PID controller using time-stamps, but essentially no
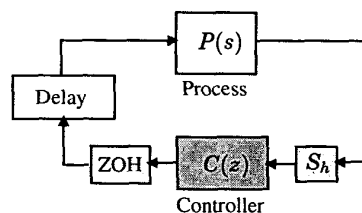

Figure 1: The control system with delay after the controller. Gray blocks are discrete-time, and white blocks are continuous-time. ZOH is a zero-order-hold (converting from discrete-time to continuous time) and S_h is a sample-and-hold with period h (converting from continuous-time to discrete-time).

process knowledge.

Much of the analysis in this paper is based on the stability criterion for varying time delays in [3], but the central results from that paper are repeated here.

2 Problem formulation

Many real-world control systems can be modeled as a linear continuous-time plant and a discrete-time controller (see Figure 1). We assume that all continuous-time parts of the model (including the upcoming compensator) are stable and *strictly proper*, so that sampled-data H_∞ theory can be used. This restriction is not very severe, as almost all real-world systems pass it. The plant is sampled with an exact period of h seconds, and the controller and actuator are completely event-driven, acting on new input. Delay in the system may occur both before and after the controller, but we will ignore any knowledge of the delay before the controller and act as if all delay is affecting the control signal after the controller. The delay for sample n is denoted $\tau(n)$, and is bounded by

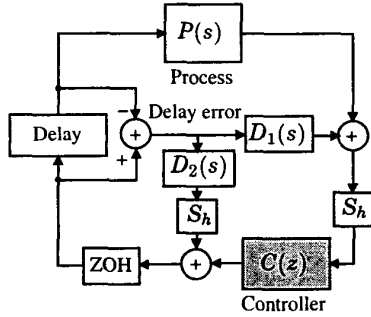$$0 \leq \tau(n) \leq Nh, \quad \forall n, \tag{1}$$

Figure 2: The control loop with add-on delay compensator, hiding high-frequency delay errors to the controller. The structure is exactly the same as for the Smith predictor, although the delay is time varying.

where $N \in \mathbf{Z}^+$. Throughout the paper, $N = 1$ will be assumed for a more clear presentation. It is straightforward to extend the results to any N, though, see [3]. One important property of our model is that the actuator sends **time-stamps** to the controller. This way, the controller may use knowledge of when former control signals actually appeared at the plant. In practice, time-stamps from the actuator may not be a standard feature. It can be implemented if the actuator contains a microcontroller, see e.g. [4] for some more details.

Using these time-stamps, an add-on compensator for a given system will be designed. The compensator has two main objectives:

1. Maintain stability of the system in presence of jitter.

2. Maintain control performance in presence of jitter.

These two objectives are of course strongly connected. Without objective 2, though, a trivial way of stabilizing the control loop would be to turn off the controller as the plant $P(s)$ is assumed to be stable.

Our proposed compensator structure is shown in Figure 2. It is based on measuring the difference between the actual control signal and the wanted control signal from the controller. This difference signal (from now on *delay error*) is piecewise constant, as the controller is discrete-time with a zero-order hold at the output. The idea of the controller is to let the delay error pass two continuous-time linear systems ($D_1(s)$ and $D_2(s)$) and add these signals before and after the compensator, respectively.

Naturally, $D_1(s)$ and $D_2(s)$ are implemented in discrete-time in the digital controller using time-stamps from the actuator. Using the continuous-time model, though, *stability and performance* of the system can be analyzed using *standard linear systems theory*.

The compensator structure is very similar to the well-known Smith predictor with imperfect process model. In this paper, its effects on *time-varying* delays are discussed, as are the effects of choice of $D_1(s)$ and $D_2(s)$. Ideally, we would like to require as little process knowledge as possible and still create a useful compensator. The following sections describe a stability criterion and an approximate performance condition, which both lead to loop-shaping design of $D_1(s)$ and $D_2(s)$.

3 Stability analysis

If the controller is designed without jitter consideration (no matter which design method), it assumes the plant to behave in a certain way. Intuitively, bad performance of the control loop in presence of jitter is due to the unexpected behavior caused by the control signal not appearing in time at the plant. The idea of our compensator is to hide the delay-error of the plant to the controller for a short period of time. Thus, the compensator is a delay-error cancellation.

As both discrete-time and continuous-time transfer functions are used in the following sections, the notation $\tilde{S}(z)$ is introduced for the system $S(s)$ sampled at period h with zero-order-hold input.

If the delay $\tau(n)$ fulfills (1), where h is the sample period of the controller, then the delay can be equivalently replaced by the structure in Figure 3 (as described in detail in [3]). The output from the delay is

$$u_\tau(t) = \begin{cases} u(n-1) & t_n \leq t < t_n + \tau(n) \\ u(n) & t_n + \tau(n) \leq t < t_{n+1}. \end{cases} \quad (2)$$

Therefore, it can be rewritten as

$$u_\tau(t) = u(n) + u_{\text{err}}(t) =$$
$$u(n) + (-u(n) + u(n-1))\Delta(t) \quad (3)$$

where

$$\Delta(t) = \begin{cases} 1 & t_n \leq t < t_n + \tau(n) \\ 0 & t_n + \tau(n) \leq t < t_{n+1}. \end{cases} \quad (4)$$

The delay is split in a direct-feedthrough path and an error path, where the latter can be written as a discrete-time differentiator
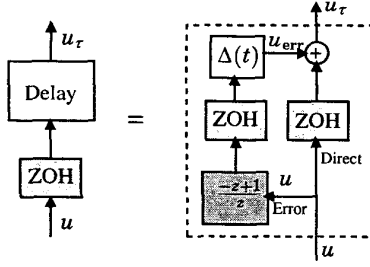
$$B_1(z) = \frac{-z + 1}{z}$$

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Figure 3: The delay can be split in a direct-feedthrough path and an error path.

in series with the "gate"-function $\Delta(t)$.

Remark: If part of the delay is expected (e.g. mean-time-delay), this delay can be inserted as the "error-free-path", with delay error added just as described above. The $\Delta(t)$ can then take both positive and negative values, but this does not change the analysis below.

Rewriting Figure 2 using this delay formulation, Figure 4 is obtained. If $P(s)$, $D_1(s)$ and $D_2(s)$, as well as the closed loop of $\tilde{P}(z)$ and $C(z)$ are stable, it suffices to show stability of the "outer" loop. This can be done using the Small Gain Theorem as described in [3]. Define

$$G(z) = \left[\frac{C(z)}{1-C(z)\tilde{P}(z)} \quad \frac{1}{1-C(z)\tilde{P}(z)} \right],$$

$$Q(s) = \begin{bmatrix} P(s) - D_1(s) \\ D_2(s) \end{bmatrix},$$

and the H_∞ gain of the linear system is the maximum of

$$\|\Psi(\omega)\| = \|B_1(e^{i\omega h})\| \cdot$$
$$\underbrace{\sqrt{\sum_{k=-\infty}^{\infty} \|G(e^{i\omega h})Q(i(\omega + k\omega_h))\|^2}}_{Q_{\mathrm{stab}}(\omega)}. \quad (5)$$

As the gain of $\Delta(t) \leq 1$, the Small Gain Theorem gives stability if

$$\|\Psi(\omega)\| < 1, \ \forall\omega. \quad (6)$$

Remark: Approximating the criterion (6) in discrete time with compensation gives

$$\left\| \frac{C\tilde{P} - \tilde{D}_1 C - \tilde{D}_2}{1 - C\tilde{P}} \right\| \leq \left\| \frac{1}{B_1} \right\|. \quad (7)$$

It can be seen from this criterion that without compensation, i.e. with $D_1(s) = D_2(s) = 0$, high gain of the complementary sensitivity function at *high frequencies* (i.e high bandwidth) will destroy our stability guarantee, and, empirically, stability.
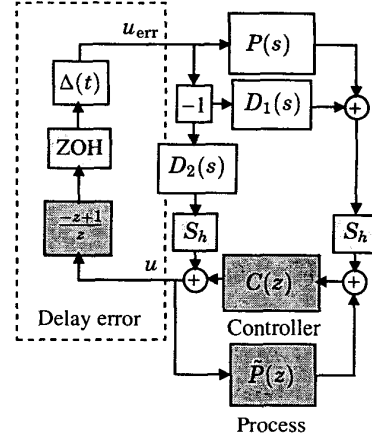

Figure 4: The control loop rewritten. Stability can now be proven using Small Gain Theorem.

4 Performance analysis

The stability compensation in the previous section gives a hard stability guarantee if $D_1(s)$ and/or $D_2(s)$ is designed so that (6) holds. This section presents an approximate method to determine how performance is affected by jitter and our compensation. It will show how to choose $D_1(s)$ and $D_2(s)$ so that performance in presence of jitter is improved.

Throughout this section, performance is based on plant output error. Naturally, control signal variance can also be included in the performance measure, but this is in some sense handled by the stability analysis in Section 3. As the effect of jitter is studied, our performance measure should only be affected by the *increased error* induced by jitter. Good performance means low impact from disturbance to plant output and control signal.

4.1 Performance measure

A disturbance v is introduced before the plant in Figure 5. An approximate operator from v to output y will now be derived, and from this a frequency domain performance measure. To simplify matters, $\Delta(t)$ will be treated as an LTI gain, and discrete-time versions of all continuous-time blocks will be used. The relation between y and v in Figure 5 then becomes

$$\|y\| = \left\| \frac{\tilde{P} + (\tilde{P}C\tilde{D}_1 + \tilde{P}\tilde{D}_2)\Delta B_1}{1 - C\tilde{P} + (C\tilde{D}_1 + \tilde{D}_2 - C\tilde{P})\Delta B_1} \right\| \|v\|$$

$$\approx \left\| \frac{\tilde{P} + (\tilde{P}C\tilde{D}_1 + \tilde{P}\tilde{D}_2)\Delta B_1}{1 - C\tilde{P}} \right\| \|v\|. \quad (8)$$
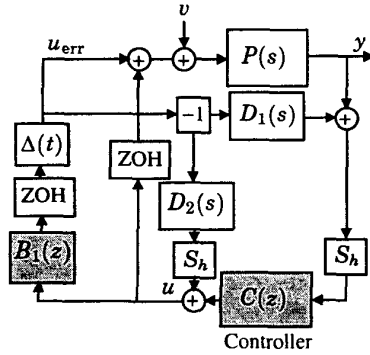
Figure 5: The control loop with a disturbance at the plant input.

Removing $(C\tilde{D}_1 + \tilde{D}_2 - C\tilde{P})\Delta B_1$ in the denominator does not change much if (6) holds with some margin. (As, at high frequencies $\|(C\tilde{D}_1 + \tilde{D}_2 - C\tilde{P})\| \ll 1$, and at low frequencies $\|B_1\| \ll 1$.)

If there is no delay, i.e. $\Delta = 0$, the relation becomes

$$\|y\| \approx \left\|\frac{\tilde{P}}{1 - C\tilde{P}}\right\| \|v\|. \tag{9}$$

On the other hand, using "worst-case" delay, i.e $\Delta = 1$ it can be written as

$$\|y\| \approx \left\|\frac{\frac{\tilde{P}}{z} + (-\tilde{P} + \tilde{P}C\tilde{D}_1 + \tilde{P}\tilde{D}_2)B_1}{1 - C\tilde{P}}\right\| \|v\|$$

$$\leq \underbrace{\left\|\frac{\tilde{P}}{1 - C\tilde{P}}\right\|}_{\text{Cost for no jitter}} \|v\| + \left\|\frac{\tilde{P} - \tilde{P}C\tilde{D}_1 - \tilde{P}\tilde{D}_2}{1 - C\tilde{P}}\right\| \|B_1\| \cdot \|v\|$$

$$\tag{10}$$

The relation from disturbance to *increased output error* (due to jitter) can thus be approximated by

$$\|y_{\text{jitter}}\| \approx \underbrace{\left\|\frac{\tilde{P} - \tilde{P}C\tilde{D}_1 - \tilde{P}\tilde{D}_2}{1 - C\tilde{P}}\right\|}_{Q_{\text{perf}}(\omega)} \|B_1\| \cdot \|v\| \tag{11}$$

Thus, for good performance, this should be as low as possible. Note that this measure is approximate but useful when comparing two alternative compensator (or controller) designs. Comparing with exactly computed results for many examples indicates that the performance plot gives good information on which controller is best.

5 Compensator Design

This section will describe illustrate how the compensators $D_1(s)$ and $D_2(s)$ can be chosen to increase stability and performance. The results are obtained by

comparing (7) and (11). First, consider two extreme cases:

Error feedback: Hypothetically, by letting $D_1(s) = 0$ and $D_2(s) \approx 1$, essentially all delay error is fed back to the next control signal (although this is not possible from the requirement of strictly proper D_1 and D_2). The approximate performance measure (11) shows that not much increase in output error due to jitter is to be expected. Turning to stability, though, (7) becomes

$$\left\|\frac{1}{1 - C(z)\tilde{P}(z)}B_1(z)\right\| \leq 1 \tag{12}$$

for high frequencies. As $\|B_1(e^{i\omega h})\| \to 2$ for high ω, (12) is not expected to hold.

Perfect cancellation: Another extreme approach is to let $D_1(z) = P(s)$ and $D_2(s) = 0$, i.e. to hide all delay error from the controller (this is the classic Smith predictor). This means that (7) holds with infinite margin. On the other hand, (11) becomes

$$\|y_{\text{jitter}}\| \approx \|P\| \cdot \|B_1\| \cdot \|v\|, \tag{13}$$

and thus a P with e.g. a double integrator will lead to very bad performance – as expected, as this approach essentially hides all delay error. Note that for fixed delays (and a perfect process model), the Smith predictor does not introduce performance problems. With varying delays, though, this delay error hiding can lead to large plant errors.

A compromise: The two examples above enhance either performance or stability, but not both. Using the fact that the two approaches may be separated in frequency, one can use the combination of the two:

- For $D_1(s)$, a compensator which cancels $P(s)$ at high frequencies should be used. The high-frequency characteristics of $P(s)$ is mainly determined by the roll-off $RO(P(s))$, defined as

$$RO(P(s)) = \# \text{ of poles} - \# \text{ of zeros.}$$

As the compensator should also not affect the performance at low frequencies, the following structure for $D_1(s)$ is suggested:

$$D_1(s) = \frac{Ks}{(s + \omega_P)^{RO(P(s))+1}}, \tag{14}$$

where ω_P is the frequency from which the compensator is active and K is the gain.

- If low-frequency disturbance-rejection is important, let $D_2(s)$ be a low pass filter with

$D_2(0) = 1$. This will increase performance at low frequencies, but not destroy stability at high frequencies.

Using this compensator, only high frequency knowledge of $P(s)$ is required to tune $D_1(s)$ and $D_2(s)$. For low frequencies, the "built-in" knowledge in the controller $C(z)$ is reused by feeding back delay errors through $D_2(s)$.

For complex control systems, it may not be feasible to use the structure in (14) to cancel $P(s)$ at high frequencies. Naturally, even $D_2(s)$ may be used to accomplish this if it can be done with a less complex compensator. This is done in the DVD control example in Section 6.2.

6 Evaluation

Two examples of the proposed jitter compensation are presented in this section. The first is a classic, theoretical, jitter-sensitive control loop, whereas the other is a DVD focus process model from the real world. For performance calculation, standard LQ cost have been used in all cases.

The cost evaluation is done using the the author's MATLAB toolbox JITTERBUG (available upon request), which calculates steady-state variance (and thus cost) for mixed continuous/discrete-time systems with random delays.

6.1 Example 1: A simple system

The following simple system is taken from [1]. It is controlled by a LQG controller with $h = 0.05$ s. The choice of cost function J below makes it very sensitive to jitter.

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 35 \\ -61 \end{bmatrix} \xi$$

$$y = \begin{bmatrix} 2 & 1 \end{bmatrix} x + \eta$$

$$Q = 80 \begin{bmatrix} 35 & \sqrt{35} \\ \sqrt{35} & 1 \end{bmatrix}$$

$$R = 1$$

$$h = 0.05$$

The cost function for which the LQG controller is designed, and which is used in the evaluation is the standard

$$J = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)^T Q x(t) + u(t)^T R u(t) dt. \quad (15)$$

As can be seen in Figure 6, stability cannot be guaranteed for the system if jitter with $\tau \leq h$ is intro-
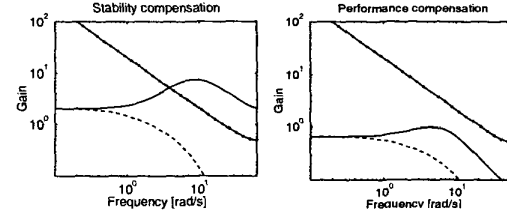

Figure 6: $Q_{\mathrm{stab}}(\omega)$ (left) and $Q_{\mathrm{perf}}(\omega)$ (right) for the uncompensated (full line) and compensated (dashed) Example 1. The closer to $\frac{1}{B_1(e^{i\omega h})}$ (dash-dotted), the less stability margin and performance. As $Q_{\mathrm{stab}}(\omega) < \frac{1}{B_1(e^{i\omega h})}$, the compensated system is guaranteed to be stable for all delays $\tau < h$. Also, the compensated system has a better $Q_{\mathrm{perf}}(\omega)$, even though $D_2(s) = 0$.
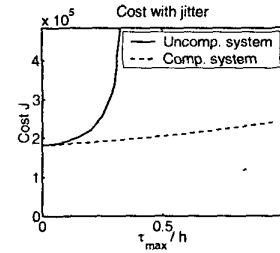

Figure 7: Cost J for different amounts of jitter in Example 1. The jitter (delay probability) is uniformly distributed in $[0, \tau_{\max}]$, and $\tau_{\max}$ varies from 0 to h in the plot. The uncompensated system is destabilized for $\tau_{\max} \geq 0.4h$.

duced. Therefore, $D_1(s)$ is chosen as in (14) to reduce the effects of jitter:

$$D_1(s) = \frac{s}{(s+1)^2},$$

i.e to mimic the behavior of $P(s)$ at high frequencies. In this case, the choice of $D_1(s)$ does not only increase stability, but also improves $Q_{\mathrm{perf}}(\omega)$, as can be seen in Figure 6.

Calculating the exact steady-state cost J using JITTERBUG, using a delay $\tau(n)$ uniformly distributed in $[0, \tau_{\max}]$, the cost in Figure 7 is obtained.

6.2 Example 2: A DVD player

In a DVD (or CD) player, a track on the disk has to be followed with high precision. This is done by controlling a lens so that the read laser spot is centered on the track, both in radial and focus position (sideways and in height). The following is a sixth order linear model of the focus servo in a common
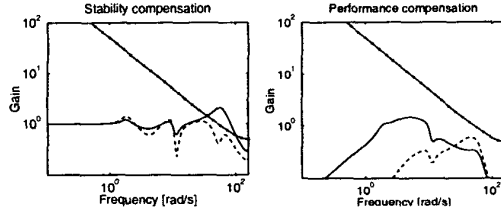
Figure 8: $Q_{stab}(\omega)$ (left) and $Q_{perf}(\omega)$ (right) for the uncompensated (full line) and compensated (dashed) focus loop in Example 2. Again, the compensated system is guaranteed to be stable for all delays $\tau < h$, and has a better $Q_{perf}(\omega)$ for most ω.

DVD player:

$$P(s) =$$
$$\frac{426(s^2 + 1.1s + 3.5)(s^2 + 1.9s + 119)}{(s^2 + 0.26s + 0.84)(s^2 + 0.96s + 45)(s^2 + 3.3s + 228)},$$

where the time has been scaled with a factor of 1000 to avoid numeric problems. The controller runs at $1/h = 50$ Hz (or 50 kHz for the real system), which is an appropriate sampling frequency. The system is very resonant, and therefore hard to control and sensitive to jitter. The focus controller in this example is a discrete-time PID:

$$C(z) = K\left(1 + \frac{1}{T_i(z-1)} + \frac{T_d(z-1)}{z}\right)$$
$$K = 0.55, \quad T_d = 10, \quad T_i = 12$$
$$x^T Q x = 30\|y\|^2, \quad u^T R u = \|u\|^2$$

with (15) as cost function (with our Q and R). In Figure 8, it can be seen that a resonance around $\omega = 50$ rad/s may cause stability problems. The resonance also makes it hard to design $D_1(s)$ in the form (14) with good cancellation at $\omega \approx 50$. Leaving $D_1(s) = 0$ and designing $D_2(s)$ as

$$D_2(s) = \frac{1764}{s^2 + 38.14s + 1764}$$

i.e. with $D_2(0) = 1$ and $D_2(i\omega) \approx C(e^{i\omega h})\tilde{P}(e^{i\omega h})$ at $\omega \approx 50$, Figure 8 shows that the stability margin, i.e. the distance from $Q_{stab}(\omega)$ to $\frac{1}{B_1(e^{i\omega h})}$ is improved. The performance, as indicated by $Q_{perf}(\omega)$ is improved over most frequencies.

Calculating the exact cost under the same assumptions as in Example 1 gives the plot in Figure 9. Again, using loop shaping and partial process knowledge, an add-on jitter compensator was designed, improving both control performance and stability.
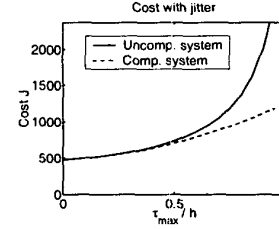

Figure 9: Cost J for different amounts of jitter in Example 2.

7 Conclusions

In this paper, a method to do compensation for time-varying delays (jitter) in control loops, using time-stamps is presented. Stability and performance analysis leads to frequency domain conditions, which implies that loop-shaping can be used to design the compensator. The method can be used even in cases where a full process model is not available.

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