On equivalence classes in Iterative Learning Control

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Abstract

This paper advocates a new approach to study the relation between causal Iterative Learning Control (ILC) and conventional feedback control. Central to this approach is the introduction of the set of admissable pairs (of operators) defined with respect to a family of iterations. Considered are two problem settings: standard ILC, which does *not* include a current cycle feedback (CCF) term and CCF-ILC, which does. By defining an equivalence relation on the set of admissable pairs, it is shown that in the standard ILC problem there exists a bijective map between the induced equivalence classes and the set of all stabilizing controllers. This yields the well-known Youla parameterization as a corollary. These results do not extend in full generality to the case of CCF-ILC; though again every admissable pair defines a stabilizing equivalent controller, the converse is no longer true in general.

1 Introduction

Some twenty years ago, Arimoto and coworkers [1] were among the first to develop a theory of learning specifically for control applications. Upon observing the human tendency to learn from experience, the authors were led to the question whether it would be possible to implement a similar ability in the automatic operation of dynamical systems. With that, the field of Iterative Learning Control (ILC) was born.

Today we look back and we see that in two decades time, the field has evolved in many directions. For a recent overview we refer to the survey by Moore [6], which contains a topical classification of the most important developments up till 1997. See also [2]. More recent results can be found in the July 2000 edition of the International Journal of Control - special issue on Iterative Learning Control [7].

Inspection of the literature reveals a considerable interest in theoretical issues. Convergence, performance and robustness have all been discussed - seperatedly or jointly - in some detail [2]. A proper treatment of these issues can be seen to be of vital importance. On the synthesis part, a wide variety of algorithms has been proposed, many of them optimal in one sense or another - be it with respect to convergence speed or asymptotic performance.

Despite the focus on theoretical issues, the practical aspects of learning control have never been quite neglected. In fact, ILC has been an applied field of research from the very beginning with many applications, mainly in the field of robotics. Design issues have received a good deal of attention [5].

All in all, Iterative Learning has no doubt matured in the past twenty years. And with many succesful applications, it has earned its stand amongst other players in the field of control. In this respect it is remarkable that only few serious attempts have been made to study the relation between this- and other methods of control. Many papers nevertheless breathe the idea that ILC is quite distinct from conventional feedback or any other method of control for that matter. We believe - and the results in this paper confirm this - that such a viewpoint is misleading.

In this paper, we investigate the intimate connection between ILC and other control paradigms - particularly feedback control. Our basic aim is to extend some recent results obtained by Goldsmith [4] and Verwoerd and coworkers [8]. In the latter paper, the notion of the 'set of admissable pairs' was introduced for a particular family of iterations. This notion, which turned out to be useful for analysis purposes, will be the basis for most of the results formulated in this paper as well. We will focus on two families of iterations, sometimes distinctively refered to as *standard-* and *current cycle feedback* (CCF)-ILC.

The outline is as follows. Section 2 starts off with some preliminaries. Then in Section 3 the problem of ILC is reviewed. Sections 4 and 5 discuss several equivalence results for the standard- and the CCF-ILC problem respectively. Section 6 ends with some conclusions and future prospects.

2 Preliminaries

In this section we review some relevant notions in systems and signals and set theory and introduce a notation along the way.

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2.1 Signals and systems

In most of our analysis we will be concerned with $\mathcal{H}_2$ signal spaces. $\mathcal{H}_2$ is a Hardy space that is isomorphic to $\mathcal{L}_2(\mathbb{R}_+)$. An element $u(s) \in \mathcal{H}_2$ has associated norm

$$\|u(s)\|_{\mathcal{H}_2} = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty} \|u(j\omega)\|_2^2 \mathrm{d}\omega}$$

where $\|.\|_2$ denotes the standard euclidean vector norm. Of further interest is also the space of proper and real rational stable transfer matrices $\mathcal{RH}_\infty$ which has the well-known associated $\mathcal{H}_2$-induced norm

$$\|P\|_\infty = \sup_{\omega\in\mathbb{R}} \bar{\sigma}\left(P(j\omega)\right).$$

2.2 Set theory

Let $\mathcal{S}$ denote a set and let $\mathcal{R}$ be a *relation* on $\mathcal{S}$, i.e. a collection of ordered pairs of elements of $\mathcal{S}$. Suppose $\mathcal{R}$ satisfies the following properties

1. $(a,a) \in \mathcal{R} \;\forall\; a \in \mathcal{S}$.
2. $(a,b) \in \mathcal{R} \Rightarrow (b,a) \in \mathcal{R} \;\forall\; a,b \in \mathcal{S}$.
3. $(a,b),(b,c) \in \mathcal{R} \Rightarrow (a,c) \in \mathcal{R} \;\forall\; a,b,c \in \mathcal{S}$.

then $\mathcal{R}$ is an *equivalence* relation. In that case two elements $a, b \in \mathcal{S}$, $(a,b) \in \mathcal{R}$ are said to be equivalent and we write $a \simeq b$. For each $a \in \mathcal{S}$, we define the *equivalence class* $[a]$ containing a to be the set of all elements in $\mathcal{S}$ equivalent to a, i.e.

$$[a] = \{b \in \mathcal{S} | b \simeq a\}$$

It is easy to see that the equivalence classes constitute a *partition* of $\mathcal{S}$ that is, a collection of nonempty disjoint subsets of $\mathcal{S}$ whose union is $\mathcal{S}$. In order to refer to a particular equivalence class, we can select one element from each equivalence class to represent the entire class. Such an element is called a *class representative.*

3 Iterative Learning Control

In the following text we set out to arrive at a formal problem definition for ILC. We will constrain ourselves to look at two problem settings in particular. In both settings, the plant is assumed to be linear and time-invariant (LTI) and the associated transfer function is assumed to be strictly proper.

Consider Figure 1. Given a plant $P : \mathcal{U} \mapsto \mathcal{Y}$, $y = Pu$, together with some desired output $y_d \in \mathcal{H}_2$. The problem of Iterative Learning Control now reads as follows. Find a recurrence relation (or *iteration*) on $\mathcal{U}$ that defines a convergent sequence $\{u_k\}_{k\in\mathbb{N}}$ such that the corresponding output sequence $\{y_k\}$ converges to an output that is close to y_d in some sense.

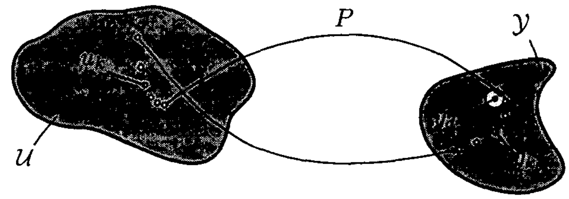

Figure 1: Iterative Learning Control

In the present text we will be concerned with a family of iterations $T(Q,L) : \mathcal{H}_2 \to \mathcal{H}_2$,

$$\begin{aligned} u_{k+1} &= Tu_k & (1)\\ &= Qu_k + L(y_d - Pu_k) + C(y_d - Pu_{k+1}) \end{aligned}$$

with $Q, L \in \mathcal{RH}_\infty$ and C a given stabilizing controller. It can be shown that this class of iterations is well-defined. Within this setting, the objective reduces to finding Q, L such that the asymptotic error

$$\bar{e} = \lim_{k\to\infty} (y_d - Pu_k) \quad (2)$$

is bounded and preferably small. In the *standard* ILC problem the plant is assumed to be stable and C is taken to be the zero controller (which in that case is indeed a stabilizing controller). In the *current cycle feedback* ILC problem the plant is no longer assumed to be stable and C is allowed to be any stabilizing controller - not necessarily stable by itself.

4 Equivalence in the standard ILC problem

In this section we will be concerned with the standard ILC problem, that is we will study the family of iterations defined by (1) for the special case that $P \in \mathcal{RH}_\infty$ and $C = 0$. We define $e_k := y_d - Pu_k$. The corresponding subset of iterations is then given by

$$u_{k+1} = Qu_k + Le_k \quad (3)$$

with $Q, L \in \mathcal{RH}_\infty$. Let us introduce the notion of admissability.

Definition 1 (Admissability: standard ILC) *Given $P \in \mathcal{RH}_\infty$. Consider the family of iterations $T(Q,L)$ defined by Equation 3. We say that the pair $(Q,L) \in \mathcal{RH}_\infty \times \mathcal{RH}_\infty$ is* admissible *if for every $y_d \in \mathcal{H}_2$, $\exists \bar{u}(y_d) \in \mathcal{H}_2$ s.t. $\lim_{k\to\infty} u_k = \bar{u}(y_d)$ for every $u_0 \in \mathcal{H}_2$.*

Admissability is a somewhat arbitrary concept that can be used to single out "bad" pairs of operators that lack certain desirable properties. Admissability in the sense of Definition 1 guarantees convergence of the sequence $\{u_k\}$ induced on $\mathcal{U}$ by a given pair (Q,L) for every initial condition and every desired output. The set of all admissable pairs will be denoted by $\mathcal{A}$. Following [8], we define an equivalence relation on $\mathcal{A}$.

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Definition 2 (Equivalence on $\mathcal{A}$) *Two elements (Q_1, L_1), $(Q_2, L_2) \in \mathcal{A}$ are said to be equivalent if*

$$(I - Q_1)^{-1} L_1 = (I - Q_2)^{-1} L_2 \tag{4}$$

Verwoerd [8, Lemma 9] showed that this relation is well-defined for all $(Q, A) \in \mathcal{A}$ provided P is strictly proper. Equivalence in the sense of Definition 2 has the following interpretation. Under assumption of admissability, the sequence $\{u_k\}$ induced on $\mathcal{U}$ has a limit point $\bar{u} \in \mathcal{U}$

$$\bar{u} = (I - Q)^{-1} L\bar{e} \tag{5}$$

From the above equation and Figure 2 it is clear that the quantity $(I - Q)^{-1} L$ has the interpretation of a feedback controller. This was first recognized by Goldsmith [3] who subsequently coined the term *equivalent controller*. Note that the equivalent controller is completely determined by the free parameters Q and L. Hence, coming back to Definition 2 we can alternatively say that two admissable pairs $(Q_1, L_1), (Q_2, L_2)$ are equivalent if they yield the same equivalent controller.

In [8], based on a result by Goldsmith [4], it is

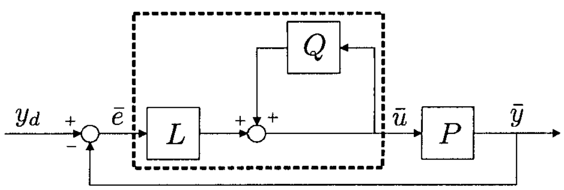

Figure 2: Equivalent Feedback Controller (dashed) for standard ILC.

shown that for every $(Q, L) \in \mathcal{A}$ with the restriction that Q and L are both causal, the equivalent controller is internally stabilizing. Conversely it was shown that given any stabilizing controller K, we can always find an admissable pair that matches the given controller. The corresponding theorems are written out below.

Theorem 3 *Given $P \in \mathcal{RH}_\infty$ strictly proper and $(Q, L) \in \mathcal{A}$. Then the controller $K = (I - Q)^{-1} L$ is proper and internally stabilizing.*

Theorem 4 *Suppose K is a stabilizing controller for P. Then there exists an admissable pair $(Q, L) \in \mathcal{A}$ satisfying $(I - Q)^{-1} L = K$.*

Remark 5 *With respect to Theorem 4 we remark that there is not just one admissable pair that satisfies the given equality, but there are in fact infinitely many.*

Let $\mathcal{K}$ denote the set of all stabilizing controllers for P. Theorem 3 says that every admissable pair defines one and only one internally stabilizing equivalent controller $K \in \mathcal{K}$. On the other hand, Theorem 4 and Remark 5 together constitute a result that says that to every given stabilizing controller there corresponds a multitude of admissable pairs. In other words, the mapping $\phi : \mathcal{A} \mapsto \mathcal{K}$,

$$K = (I - Q)^{-1} L \tag{6}$$

that maps an admissable pair (Q, L) to a stabilizing controller K is surjective but not injective. See Figure 3 for a graphical interpretation.

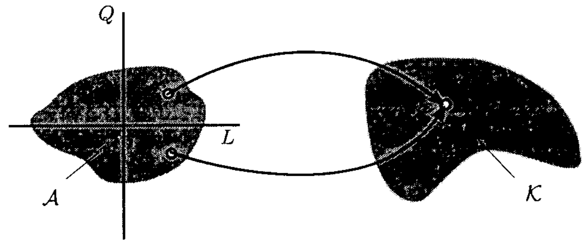

Figure 3: The mapping $\phi : \mathcal{A} \mapsto \mathcal{K}$ is surjective but not injective.

By considering equivalence classes instead, we can uniquely identify one stabilizing controller with one equivalence class. This is immediate from the fact that $\phi(Q_1, L_1) = \phi(Q_2, L_2)$ if and only if $[(Q_1, L_1)] = [(Q_2, L_2)]$ with $[.]$ defined in Section 2.2. This is depicted in Figure 4.

The following lemma says that every equivalence

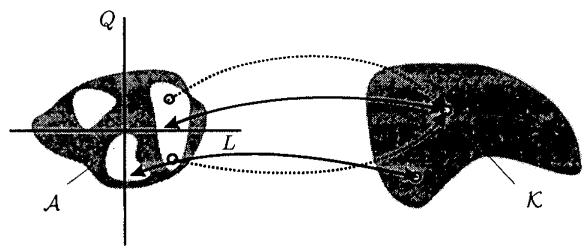

Figure 4: Every equivalence class in $\mathcal{A}$ can be uniquely identified with a particular stabilizing controller.

class $[(Q, L)] \subset \mathcal{A}$ has precisely one member (Q_0, L_0) that satisfies a particular additional constraint. We will take this element to be our class representative.

Lemma 6 *Every equivalence class induced on $\mathcal{A}$ by the equivalence relation (4) has one and only one member (Q_0, L_0) satisfying $Q_0 - L_0 P = 0$.*

Proof: Let $[(\hat{Q}, \hat{L})]$ be a given equivalence class on $\mathcal{A}$. The claim is that there exists a unique element $(Q_0, L_0) \in [(\hat{Q}, \hat{L})]$ satisfying the given constraint. Define $K = \left(I - \hat{Q}\right)^{-1} \hat{L}$. It is easy to see that if there at

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all exists a solution (Q_0, L_0) then it has to satisfy the following set of equations.

$$\begin{cases} (I - Q_0)^{-1} L_0 &= K \\ Q_0 - L_0 P &= 0 \end{cases} \tag{7}$$

The unique solution of (7) is given by

$$\begin{cases} Q_0 &= K(I + PK)^{-1} P \\ L_0 &= K(I + PK)^{-1} \end{cases}$$

Hence $Q_0, L_0 \in \mathcal{RH}_\infty$ by internal stability of the closed loop (K is stabilizing, see Theorem 3). Admissability follows from [8, Theorem 2]. This completes the proof. ■

Define $\mathcal{A}_0$ to be the set of all class representatives or equivalently the set of all $(Q, L) \in \mathcal{A}$ satisfying $Q - LP = 0$. Note that by definition every member of this set has the special form (LP, L). Note furthermore that $\mathcal{A}_0$ cannot contain any equivalent pairs because the representatives were taken from disjoint sets. As a consequence, the restriction of ϕ to $\mathcal{A}_0$, hereafter denoted by ϕ_0, is injective. Clearly ϕ_0 is also surjective and hence we conclude that $\phi_0 : \mathcal{A}_0 \mapsto \mathcal{K}$ is a bijection.

It is appropriate to elaborate on the significance of the above result since it reveals an intimate connection between standard ILC and feedback control. To this end, let us have another look at Figure 2. The figure shows us a layout of how the respective signals of interest relate to one another. More important however is that under hypothesis of convergence, i.e. admissability of (Q, L), it suggests a possible parameterization of all stabilizing controllers. This parameterization however lacks certain desirable properties. For one, it is not easy - if at all possible - to give a complete characterization of the set of admissable pairs. That is to say it is hard to derive a necessary and sufficient condition for a given pair (Q, L) to be admissable. Moreover, the given parameterization would not be minimal. On top of that $(I - Q)$ and L need not be left comprime. This poses a possible threat to the stability of the closed loop in case an unstable pole-zero cancelation occurs within the equivalent controller. This problem can be resolved by considering only minimal realizations of K. In any case, all problems are solved by imposing the additional constraint $Q = LP$ on the set of admissable pairs. This enables an easy and exact characterization and the resulting parameterization is easily shown to be minimal. The corresponding block diagram is depicted in Figure 5. Those who are familiar with the theory of stabilization will immediately recognize the Youla parameterization of all stabilizing controllers. Here we state this well-known result as a corollary.

Corollary 7 (Youla) *The set $\mathcal{K}$ of all (proper real-rational) K's stabilizing P can be parameterized as follows*

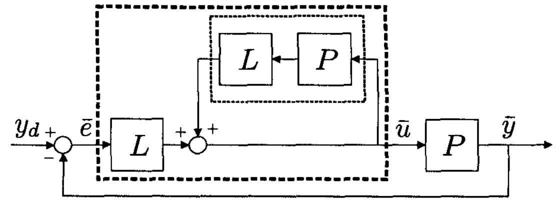

Figure 5: Minimal parameterization of all stabilizing controllers. Shown is the equivalent controller (fat dashed box) under the constraint $Q = LP$ (thin dashed box). Compare with Figure 2.

$$\mathcal{K} = \left\{ K : K = (I - LP)^{-1} L \, ; \; L \in \mathcal{RH}_\infty \right\}$$

Proof: Note that the set of class representatives $\mathcal{A}_0$ has a trivial parameterization

$$\mathcal{A}_0 = \{(LP, L) : L \in \mathcal{RH}_\infty\} \tag{8}$$

and recall that $\mathcal{A}_0$ and $\mathcal{K}$ were shown to be bijective under ϕ_0. Hence any given parameterization of $\mathcal{A}_0$ induces a parameterization of $\mathcal{K}$. In particular:

$$\begin{aligned} \mathcal{K} &= \{\phi_0(LP, L) : L \in \mathcal{RH}_\infty\} \\ &= \left\{(I - LP)^{-1} L : L \in \mathcal{RH}_\infty\right\} \end{aligned}$$

This completes the proof. ■

5 Equivalence in the CCF-ILC problem

This section deals with the CCF-ILC problem. The main distinction between this - and the standard ILC problem is the presence of the current cycle feedback term Ce_{k+1} in the iteration (1). The plant is no longer assumed to be stable and hence for reasons of well-posedness C is assumed to be stabilizing.

We will consider the following two problems. Given a plant P together with a stabilizing controller C.

1. Let (Q, L) be a given admissable pair for the family of iterations defined by (1) and define the *equivalent controller* $K = (I - Q)^{-1} (L + C)$. Is K always stabilizing?
2. Given any stabilizing controller K. Does there always exist an admissable pair (Q, L) for which K is an equivalent controller?

As we will see, the respective answers to the questions posed above will be "Yes" and "No". Note that in the context of the standard ILC problem both answers would have been affirmative (compare Theorems

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3 and 4). To be complete it is good to remark that the answer to the second question critically depends on C.

The CCF-ILC problem requires a slightly more involved definition of admissability. As in the standard case, we will require convergence of the input (and output) sequence. In addition we will also impose a kind of continuity or robustness constraint. To this end we introduce a class of perturbed iterations ('p' for 'perturbed')

$$u_{k+1}^p \quad = \quad Qu_k^p + Le_k^p + Ce_{k+1}^p + w_k \qquad (9)$$

which, apart from the disturbance term w_k, is identical to the one introduced in Eqn. 1 We will demand u_k^p to approach u_k continuously as $\|w_k\|$ tends to 0.

Definition 8 (Admissability: CCF-ILC) *Consider the family of CCF-iterations with, and without perturbation term. (Eqns. (9) and (1) respectively). Let C be a stabilizing controller for P. We say that the pair (Q, L) is* admissable *if $Q, L \in \mathcal{RH}_\infty$ and*

1. *(with respect to Eqn. (1):)*

   $\forall y_d \in \mathcal{H}_2, \exists \bar{u}(y_d) \in \mathcal{H}_2\ ;\ \bar{y}(y_d) \in \mathcal{H}_2$ *s.t.*

   (a) $\lim_{k\to\infty} u_k = \bar{u}$

   (b) $\lim_{k\to\infty} y_k = \bar{y}$

   for every $\hat{u}_0 \in \mathcal{H}_2$ (Figure 6).

2. *(with respect to both Eqns. (1) and (9):)*

   $\forall \varepsilon > 0,\ \exists \delta > 0$ *s.t.*

   $\|w_k\|_{\mathcal{H}_2} < \delta \Rightarrow \|u_k - u_k^p\|_{\mathcal{H}_2} < \varepsilon$

   for all k.

Note that the condition "for all u_0" in the original definition (Def. 1) has been replaced by a condition on the auxilary variable $\hat{u}_0$ which is defined in Figure 6. Condition 2 in Definition 8 requires bounded difference between the solution of the perturbed- (Eq. 9) and the original equation. If we ignore this condition then it is clear that both definitions coincide in case $P \in \mathcal{RH}_\infty$ and $C = 0$. In other words, Definition 8 provides a natural generalization of the concept of admissability that was first introduced in Definition 1.

We are now ready to formulate an answer to the first question as to whether or not the equivalent controller is internally stabilizing.

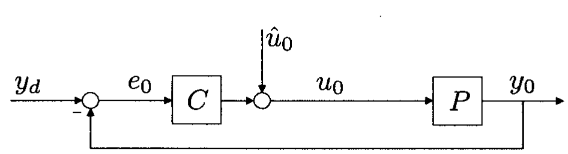

Figure 6: Setting the initial condition in CCF-ILC

Theorem 9 *Given P strictly proper and let (Q, L) be an admissable pair in the sense of Definition 8. Then the equivalent controller, defined by $K = (I-Q)^{-1}(L+C)$ is well-defined (proper) and yields the closed loop well-posed and internally stable.*

Proof: First we prove properness of K. Admissability implies that a bounded $\bar{u}$ results for every $y_d \in \mathcal{H}_2$. This implies that the input sensitivity matrix $U := (I - Q + (L+C)P)^{-1}(L+C)$ is a stable transfer matrix. In particular, U is bounded at infinity. Strict properness of P implies that $U(\infty) = K(\infty)$ and hence K is also bounded at infinity (proper). To prove well-posedness, we need to show that $(I + K(\infty)P(\infty))$ is invertible, which is an immediate consequence of the above. To prove internal stability, consider the block diagram depicted in Figure 7. The dashed box represents the equivalent controller. The shaded box represent the ILC part of the overall system, which we will denote by G_1. The remaining, non-ILC part of the system is denoted by G_2. The respective systems are given by

$$\begin{aligned} G_1 &= \begin{bmatrix} Q & L \end{bmatrix} \\ G_2 &= \begin{bmatrix} (I+CP)^{-1} \\ -P(I+CP)^{-1} \end{bmatrix} \end{aligned} \qquad (10)$$

Note that G_1 and G_2 are both stable transfer matrices. The overall system can be represented as the feedback interconnection of the subsystems (see Figure 8). Under these conditions the overall system is internally stable if and only if [9, Theorem 5.7]

$$(I - G_1G_2)^{-1} \in \mathcal{RH}_\infty$$

where

$$(I - G_1G_2)^{-1} \quad = \quad \left[I - (Q - LP)(I+CP)^{-1}\right]^{-1}$$

We will show that this condition holds by assumption of admissability. Let $w_k = w$ with $w \in \mathcal{H}_2$ be a disturbance that is independent of k. For ease of exposition, suppose furthermore that $y_d \equiv 0$. Admissability implies that the perturbed system (9) converges to a bounded solution.

$$\bar{u}^p \quad = \quad \left[I - (Q - LP)(I+CP)^{-1}\right]^{-1} w$$

Since $w \in \mathcal{H}_2$ is free, this is equivalent to saying that $\left[I - (Q - LP)(I+CP)^{-1}\right]^{-1} \in \mathcal{RH}_\infty$, which proves internal stability. ■

The remaining part of this section is dedicated to answering the second question: Is it true that - like in the case of the standard ILC problem - for every stabilizing controller K we can find a corresponding admissable pair (Q, L)? The following theorem shows that in general this is not the case.

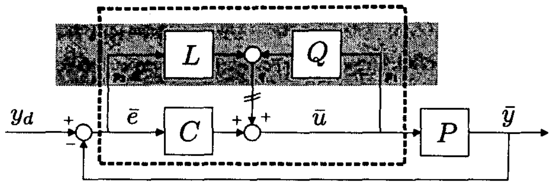

Figure 7: Equivalent Feedback Controller (dashed) and the ILC-subsystem (shaded) for CCF-ILC.

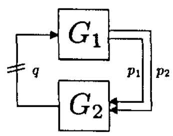

Figure 8: The overall system as the interconnection of two stable subsystems G_1 and G_2.

Theorem 10 *Suppose $K \in \mathcal{RH}_\infty$ is a strongly stabilizing controller for P (assuming it exists). Then there exists an admissable pair (Q, L) satisfying $(I-Q)^{-1}(L+C) = K$ if and only if $C \in \mathcal{RH}_\infty$*

Proof: To prove necessity, suppose $C \notin \mathcal{RH}_\infty$ and (Q, L) admissable. Rewrite $K = (I-Q)^{-1}(L+C)$ to get $L = (I-Q)K - C$. We conclude $L \notin \mathcal{RH}_\infty$ which contradicts the assumption of admissability. To prove sufficiency, suppose $C \in \mathcal{RH}_\infty$. Select $Q = (K-C)(I+PK)^{-1}P$ and $L = (K-C)(I+PK)^{-1}$. It is easy to verify that under the given conditions. $Q, L \in \mathcal{RH}_\infty$. Moreover, $(Q-LP)(I+CP)^{-1} = 0$, which is sufficient for admissability. ∎

Theorem 10 says that if C happens to be a controller that is not stable by itself then the set of equivalent controllers does not contain a single stable element. It is however very well possible that the set of *all* stabilizing controllers does contain a stable subset. The theorem does not tell us whether or not a given stabilizing controller is within the set of equivalent controllers. This may be an interesting question for future research. The purpose of the theorem however was to show that some results obtained for the standard ILC case do not extend to the CCF-ILC case.

6 Conclusion

In this paper we studied the relation between ILC and conventional feedback control. Contrary to what has been suggested before [6] we found that both methods are strongly related, if not equivalent. More specifically, we established a connection between the standard ILC problem and a stabilization problem in controller design. We were not the first at that. In recent work, Goldsmith [3, 4] had already shown that to every converging sequence there corresponds an equivalent stabilizing controller. The results in this paper however show that the converse is also true, which shows that both problems are truly equivalent - at least within the given framework. Similar results were obtained for the CCF-ILC case, with the exception that for CCF-ILC, the set of equivalent controllers was generally found to be just a subset of *all* stabilizing controllers. This implies that inclusion of the current cycle term affects the structure of the problem; a fact that should be taken into account in the design of the ILC scheme.

As the results in this paper show that ILC and feedback control are very much akin, we believe that future research in ILC should be directed towards the exploitation of the distinguishing features of ILC. One of these features is the possibility to allow for noncausal signal processing. In a recent paper [8] on the use of noncausal operators in ILC it was shown that the 'equivalent' controller in noncausal ILC is generally destabilizing. This proves that ILC can be a competitive player in controlling non-minimum phase plants, for instance. In any case, it would be worthwhile to initiate or continue an open discussion on the use of ILC in various control situations.

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