Source Seeking via Collaborative Measurements by a Circular Formation of Agents

Brandon Moore, Carlos Canudas de Wit

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***Abstract*—** This paper presents a multi-agent algorithm to address the source-seeking problem in which the task is to locate the source of some signal (e.g., a radio transmitter, a location of chemical contamination, etc.). This algorithm is based on the formation control work of another researcher in which they designed a control structure to stabilize a group of non-holonomic vehicles to a circular formation and also to move that formation by changing the location of its center. The source-seeking algorithm builds on these results by providing an outer-loop control law to move this circular formation towards a source. The resulting control law depends only on direct measurements of the signal to calculate an approximate gradient direction which is then used to steer the formation. Under certain assumptions about the spatial propagation of the signal this algorithm causes the center of the agents' formation to asymptotically converge to the location of the source.

I. Introduction

This work addresses one possible solution to the *source-seeking* problem in which an autonomous vehicle (or group of vehicles) must locate the source of some signal based on measurements of the signal's strength at different positions (where these measurements are usually weaker the further one moves from the source). For example the source could be a radio transmitter and the signal would be a radio frequency transmission. Alternatively, the source could be a point of chemical contamination and the signal would be that chemical's concentration in the environment. As opposed to techniques such as triangulation in which no vehicle actually visits the source's location, source-seeking algorithms are designed to steer the vehicle to the physical location of the source (or at least to the vicinity thereof).

There exist many different approaches to this problem in the current literature. If it is available, the gradient of the signal strength can be used to produce a gradient-decent algorithm for a vehicle or group of vehicles [2], but this information may not be available in reality. Alternatively, spatially distributed measurements of the signal strength can be used to approximate its gradient. There are two strategies for this method. In one, all the measurements would come from a single vehicle as it changes position over time [2], [3]. In the other, multiple vehicles would collaborate to take the measurements at different locations simultaneously [8].

There exist two particularly elegant solutions for the source-seeking problem using a single nonholonomic vehicle. In [7], [6] a hybrid controller is developed to implement an optimization method with successive line minimizations and heading changes based on conjugate vectors, and the resulting system is shown to be practically stable under perturbation for a certain class of signal strength distributions. In [5], [4], the techniques of extremum-seeking control are used by adding an excitatory input to the vehicle's steering control, using a special filter on the signal strength measurement to approximate its gradient, and using this information to direct the vehicle towards the source. The main drawback to both of these approaches is that the vehicle may have to travel over large distances in order to collect sufficient information about the signal, thus delaying the vehicle's convergence to the source. In this work we seek to make use of multiple, spatially distributed vehicles to collect this sort of information in the hopes of more efficiently guiding the vehicles' motion.

The work in this paper is based on the formation control work presented in [1] in which a control structure was designed to stabilize a group of non-holonomic vehicles to a circular formation and also to move that formation by changing the location of its center (subject to certain dynamic restrictions). This paper builds on these results by providing an outer loop control to steer this formation towards a source. The resulting control law depends only on direct signal strength measurements and is analytically shown to drive the formation to the source under certain assumptions about the spatial propagation of the signal.

II. Background

The source-seeking algorithm of this paper builds on the formation control work of [1]. That work modeled a group of N agents using a kinematic unicycle vehicle model of the following form for each vehicle k:

$$\begin{aligned} \dot{\mathbf{r}}_k &= v_k e^{\hat{\imath}\theta_k} \\ \dot{\theta}_k &= u_k \end{aligned} \tag{1}$$

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where $\mathbf{r}_k$ is the position vector (a two dimensional value expressed by a complex number), θ_k is the heading angle, and the control inputs are the vehicle's forward velocity $v_k > 0$ and turning rate u_k. With appropriate limits on the control inputs, this model can provide a reasonable approximation for many air and underwater vehicles.

Stabilization of the N agents to a circular formation around a fixed point was accomplished in [10]. In [1], the authors were able to develop a control law that asymptotically stabilizes the vehicles to a circular formation around a dynamic center point $\mathbf{c}_d(t)$ with a uniform distribution (i.e., with the vehicles evenly separated on the circle by $2\pi/N$ radians each) provided certain conditions are met. With a desired forward velocity v_0, a desired rotational velocity ω_0, and an inner product defined by $< \mathbf{z}_1, \mathbf{z}_2 >= \mathrm{Re}\{\mathbf{z}_1^H \mathbf{z}_2\}$ (where the superscript H denotes the conjugate transpose), the control law of [1] is given as

$$\begin{aligned} v_k &= |v_0 e^{\hat{\imath}\psi_k} + \dot{\mathbf{c}}_d| &(2)\\ u_k &= \left(1 - \frac{< \dot{\mathbf{r}}_k, \dot{\mathbf{c}}_d >}{< \dot{\mathbf{r}}_k, \dot{\mathbf{r}}_k >}\right)\dot{\psi}_k - \frac{< \dot{\mathbf{r}}_k, \hat{\imath}\ddot{\mathbf{c}}_d >}{< \dot{\mathbf{r}}_k, \dot{\mathbf{r}}_k >} &(3)\\ \dot{\psi}_k &= \omega_0(1 + \kappa < \mathbf{r}_k - \mathbf{c}_d, v_0 e^{\hat{\imath}\psi_k} >) - \frac{\partial U}{\partial \psi_k} &(4)\\ U(\psi) &= -\frac{K}{N}\sum_{m=1}^{N/2} \frac{1}{2m^2} < e^{\hat{\imath}m\psi}, \mathbf{L}e^{\hat{\imath}m\psi} > &(5) \end{aligned}$$

where $\kappa > 0$, $\mathbf{L}$ is the Laplacian matrix associated with the communication network of the vehicles, and $\psi_k(t)$ has the initial condition

$$\psi_k(0) = \arctan \frac{< \dot{\mathbf{r}}_k(0) - \dot{\mathbf{c}}_d(0), \hat{\imath} >}{< \dot{\mathbf{r}}_k(0) - \dot{\mathbf{c}}_d(0), 1 >} + \epsilon_k \pi \qquad (6)$$

where $\epsilon = 0$ if $< \dot{\mathbf{r}}_k - \dot{\mathbf{c}}_d, 1 > \geq 0$ and 1 otherwise.

Assuming that $\mathbf{c}_d(t)$ is twice differentiable, has bounded first and second time-derivatives, and satisfies $\sup_{t\geq 0} |\dot{\mathbf{c}}_d(t)| < v_0$, then the control law above drives the vehicles to trajectories that lie on the circle with radius $R = v_0/|\omega_0|$ and time-varying center $\mathbf{c}_d(t)$. Moreover, if $K > 0$ and the communication network is complete (i.e., every agent talks to every other agent), then the vehicles will be uniformly distributed on that circle.

III. System Model and Control Law

In this section we will assume that we are given a stable circular formation of N mobile agents in the plane that is described by a center point $\mathbf{c}_d \in \mathbb{R}^2$, a radius $R > 0$, and an angle θ which is linearly increasing with time (i.e., $\theta = \omega t$ for some angular speed $\omega > 0$). In this formation, the position of each agent k is given by the following equation:

$$\mathbf{r}_k = \mathbf{c}_d + R\mathbf{e}\big(\theta + \tfrac{2\pi}{N}k\big) \qquad (7)$$

where $\mathbf{e}(\phi)$ is the unit vector at an angle ϕ from the horizontal axis, i.e.,

$$\mathbf{e}(\phi) = \begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix} \qquad (8)$$

and so (7) describes a formation in which the agents are uniformly distributed on a circle of radius R.

In this paper we will provide an outer-loop control that steers the formation of agents by determining $\dot{\mathbf{c}}_d$. That is to say that we view our system as the two dimensional single integrator

$$\dot{\mathbf{c}}_d = \mathbf{u} \qquad (9)$$

The control signal u will be based on measurements of signal strength taken by the individual agents. The distribution of the signal strength in the environment will be described by an unknown positive spatial mapping $\rho : \mathbb{R} \to \mathbb{R}^+$, and so agent k measures the signal strength at its position as $\rho(\mathbf{r}_k)$.

In what follows we will use the following control law

$$\mathbf{u} = \lambda \sum_{k=1}^{N} \rho(\mathbf{r}_k)\mathbf{e}\big(\theta + \tfrac{2\pi}{N}k\big) \qquad (10)$$

which is a sum of the agents' current normalized displacement vectors from the center of the formation, $\mathbf{e}\big(\theta + \tfrac{2\pi}{N}k\big)$, weighted by their individual signal strength measurements, $\rho(\mathbf{r}_k)$, and a common possibly time-varying gain factor, $\lambda > 0$. This control law steers the formation in the direction of an estimate of the gradient of ρ at the point $\mathbf{c}_d$ based on the signal strength measurements taken by the agents distributed uniformly about $\mathbf{c}_d$. Letting the gain factor λ vary with time (so long as it remains positive) allows one some design flexibility to improve the performance of the system in specific scenarios and it is used to this effect in the simulations we include in Section V.

IV. Stability Analysis

We now prove the stability of the system (9) under the source seeking control law (10) for two special cases.

A. Signal Distributions with Circular Level Sets

Theorem 1: Assume that the signal strength is a continuously differentiable mapping and satisfies the following property:

$$\|\mathbf{z}_1 - \mathbf{z}^\star\|_2 > \|\mathbf{z}_2 - \mathbf{z}^\star\|_2 \Rightarrow \rho(\mathbf{z}_1) < \rho(\mathbf{z}_2) \qquad (11)$$

which is to say that the signal strength has a maximum at some point $\mathbf{z}^\star$, is strictly decreasing as the Euclidean distance from $\mathbf{z}^\star$ increases, and has circular level sets. Under the control input of (10) the point $\mathbf{c}_d = \mathbf{z}^\star$ is an asymptotically stable equilibrium. ■

Proof: We use the following Lyapunov function

$$V(\mathbf{c}_d) = \rho(\mathbf{z}^\star) - \rho(\mathbf{c}_d) \qquad (12)$$

which is zero at $\mathbf{c}_d = \mathbf{z}^\star$ and positive otherwise. This Lyapunov function has the time derivative

$$\dot{V}(\mathbf{c}_d) = -\nabla\rho(\mathbf{c}_d)^\top \dot{\mathbf{c}}_d \qquad (13)$$

Substituting the control formula (10) for $\dot{\mathbf{c}}_d$ in (13) yields

$$\dot{V}(\mathbf{c}_d) = -\nabla\rho(\mathbf{c}_d)^\top \lambda \sum_{k=1}^{N} \rho(\mathbf{r}_k)\mathbf{e}\big(\theta+\tfrac{2\pi}{N}k\big) \quad (14)$$

$$= -\lambda \sum_{k=1}^{N} \rho(\mathbf{r}_k)\nabla\rho(\mathbf{c}_d)^\top \mathbf{e}\big(\theta+\tfrac{2\pi}{N}k\big) \quad (15)$$

The assumption (11) about ρ means that its gradient can be expressed as follows

$$\nabla\rho(\mathbf{c}_d) = \alpha(\|\mathbf{c}_d - \mathbf{z}^\star\|_2)\frac{\mathbf{c}_d - \mathbf{z}^\star}{\|\mathbf{c}_d - \mathbf{z}^\star\|_2} \quad (16)$$

which is to say that $\nabla\rho(\mathbf{c}_d)$ points from $\mathbf{c}_d$ towards $\mathbf{z}^\star$ with a magnitude determined by a function of the distance from $\mathbf{c}_d$ to $\mathbf{z}^\star$. Because of our assumptions about ρ, this magnitude function α is continuous and satisfies

$$\alpha(0) = 0 \text{ and } \alpha(d) > 0 \;\forall\; d > 0 \quad (17)$$

Substituting the expression for $\nabla\rho(\mathbf{c}_d)$ into $\dot{V}(\mathbf{c}_d)$,

$$\dot{V}(\mathbf{c}_d) = -\lambda\frac{\alpha(\|\mathbf{c}_d - \mathbf{z}^\star\|_2)}{\|\mathbf{c}_d - \mathbf{z}^\star\|_2}\sum_{k=1}^{N}\rho(\mathbf{r}_k)(\mathbf{c}_d - \mathbf{z}^\star)^\top\mathbf{e}\big(\theta+\tfrac{2\pi}{N}k\big) \quad (18)$$

In order to determine a bound on $\dot{V}(\mathbf{c}_d)$, define a time-varying set of agents $\mathcal{M}$ to be those agents whose displacement from the formation center, $\mathbf{r}_k - \mathbf{c}_d = R\mathbf{e}\big(\theta+\frac{2\pi}{N}k\big)$, has a positive projection onto the vector $\mathbf{c}_d - \mathbf{z}^\star$, i.e.,

$$\mathcal{M} = \{k : (\mathbf{c}_d - \mathbf{z}^\star)^\top\mathbf{e}\big(\theta+\tfrac{2\pi}{N}k\big) > 0\} \quad (19)$$

Now separate the sum in $\dot{V}(\mathbf{c}_d)$ as follows,

$$\dot{V}(\mathbf{c}_d) = -\lambda\frac{\alpha(\|\mathbf{c}_d - \mathbf{z}^\star\|_2)}{\|\mathbf{c}_d - \mathbf{z}^\star\|_2}\cdot \left(\sum_{k\in\mathcal{M}}\rho(\mathbf{r}_k)(\mathbf{c}_d - \mathbf{z}^\star)^\top\mathbf{e}\big(\theta+\tfrac{2\pi}{N}k\big) + \sum_{k\notin\mathcal{M}}\rho(\mathbf{r}_k)(\mathbf{c}_d - \mathbf{z}^\star)^\top\mathbf{e}\big(\theta+\tfrac{2\pi}{N}k\big)\right) \quad (20)$$

Due to the geometry of the situation (circular level sets of the signal strength mapping and a circular formation of the agents) we know that if $\mathbf{c}_d \neq \mathbf{z}^\star$ then the agents in $\mathcal{M}$ are all closer to the source $\mathbf{z}^\star$ than those agents not in $\mathcal{M}$ (see Figure 1). Hence any agent from $\mathcal{M}$ has a higher signal measurement than any agent from $\mathcal{M}$ and there must be some middle value between them both. Mathematically speaking

$$\begin{gathered}\text{for all } k \in \mathcal{M}, m \notin \mathcal{M},\ \exists\, \delta > 0 \text{ such that} \\ \rho(\mathbf{r}_k) > \delta > \rho(\mathbf{r}_m)\end{gathered} \quad (21)$$

Applying this inequality to the sumation terms in (20)

$$\begin{gathered} k \in \mathcal{M} \Rightarrow \\ \underbrace{\rho(\mathbf{r}_k)}_{>\delta}\underbrace{(\mathbf{c}_d - \mathbf{z}^\star)^\top\mathbf{e}\big(\theta+\tfrac{2\pi}{N}k\big)}_{>0} > \delta(\mathbf{c}_d - \mathbf{z}^\star)^\top\mathbf{e}\big(\theta+\tfrac{2\pi}{N}k\big)\end{gathered}$$

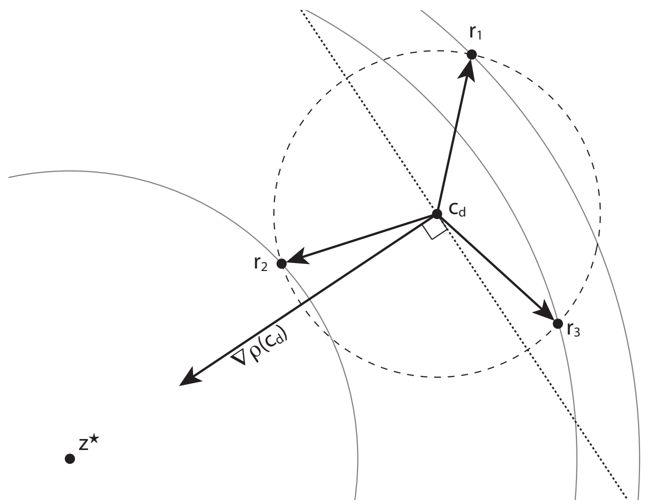

Fig. 1. Illustration of vectors used in the proof. Level curves of ρ are shown in light gray. In this situation the set $\mathcal{M}$ contains only agent 2.

and

$$\begin{gathered} k \notin \mathcal{M} \Rightarrow \\ \underbrace{\rho(\mathbf{r}_k)}_{<\delta}\underbrace{(\mathbf{c}_d - \mathbf{z}^\star)^\top\mathbf{e}\big(\theta+\tfrac{2\pi}{N}k\big)}_{\leq 0} \geq \delta(\mathbf{c}_d - \mathbf{z}^\star)^\top\mathbf{e}\big(\theta+\tfrac{2\pi}{N}k\big)\end{gathered}$$

Assuming that $\mathcal{M}$ is not empty (which is guaranteed if $N \geq 3$), the sum in (20) can be bounded from below as

$$\begin{aligned}&\sum_{k\in\mathcal{M}}\rho(\mathbf{r}_k)(\mathbf{c}_d - \mathbf{z}^\star)^\top\mathbf{e}\big(\theta+\tfrac{2\pi}{N}k\big) + \\ &\sum_{k\notin\mathcal{M}}\rho(\mathbf{r}_k)(\mathbf{c}_d - \mathbf{z}^\star)^\top\mathbf{e}\big(\theta+\tfrac{2\pi}{N}k\big) \\ &> \delta(\mathbf{c}_d - \mathbf{z}^\star)^\top\sum_{k=1}^{N}\mathbf{e}\big(\theta+\tfrac{2\pi}{N}k\big)\end{aligned} \quad (22)$$

Due to the uniform distribution of the agents, the sum in (22) is equal to zero and since λ is positive and $\frac{\alpha(\|\mathbf{c}_d - \mathbf{z}^\star\|_2)}{\|\mathbf{c}_d - \mathbf{z}^\star\|_2}$ is non-negative we thus have that $\dot{V}(\mathbf{c}_d) < 0$ for all $\mathbf{c}_d \neq \mathbf{z}^\star$ whenever $\mathcal{M}$ is not empty. The only situation where $\mathcal{M}$ is empty occurs when $N = 2$ and the agents' displacement vectors from the center of the formation, $\mathbf{r}_1 - \mathbf{c}_d$ and $\mathbf{r}_2 - \mathbf{c}_d$, are orthogonal to $\mathbf{c}_d - \mathbf{z}^\star$. In this instance, due to the symmetry of ρ it must be the case that $\rho(\mathbf{r}_1) = \rho(\mathbf{r}_2)$ and thus $\dot{\mathbf{c}}_d = 0$. Since θ keeps increasing, $\dot{V}(\mathbf{c}_d)$ will immediately become negative again so these situations do not constitute an invariant set. Thus by LaSalle's principle [9], the point $\mathbf{c}_d = \mathbf{z}^\star$ is an asymptotically stable equilibrium of the system (9) with control law (10). □

B. Signal Distributions with Elliptical Level Sets

Theorem 2: Assume that the signal strength is a continuously differentiable mapping and satisfies the following property,

$$\begin{array}{c}(\mathbf{z}_2-\mathbf{z}^\star)^\top\mathbf{A}(\mathbf{z}_2-\mathbf{z}^\star)>(\mathbf{z}_1-\mathbf{z}^\star)^\top\mathbf{A}(\mathbf{z}_1-\mathbf{z}^\star)\\ \Downarrow \\ \rho(\mathbf{z}_1)<\rho(\mathbf{z}_2)\end{array} \tag{23}$$

for some positive definite matrix $\mathbf{A}$. This is to say that the signal strength has a maximum at some point $\mathbf{z}^\star$ and has compact elliptical level sets. If the number of agents N is even, then under the control input of (10) the point $\mathbf{c}_d = \mathbf{z}^\star$ is an asymptotically stable equilibrium. ■

Proof: We will use the same Lyapunov function (12) and we start the analysis from the expression for $\dot{V}(\mathbf{c}_d)$ in (15).

$$\dot{V}(\mathbf{c}_d) \quad = \quad -\nabla\rho(\mathbf{c}_d)^\top \sum_{k=1}^{N} \rho(\mathbf{r}_k)\mathbf{e}\big(\theta+\tfrac{2\pi}{N}k\big) \tag{24}$$

At every time t at least half of the agents will have displacement vectors $\mathbf{r}_k - \mathbf{c}_d = R\mathbf{e}\big(\theta+\frac{2\pi}{N}k\big)$ that have a non-negative projection along the direction of the signal gradient. Without loss of generality assume that these agents are numbered from 1 to $\frac{N}{2}$ so that the following condition holds,

$$\nabla\rho(\mathbf{c}_d)^\top\mathbf{e}\big(\theta + \tfrac{2\pi}{N}k\big) \geq 0 \;\forall\; 1 \leq k \leq \tfrac{N}{2} \tag{25}$$

The expression for $\dot{V}(\mathbf{c}_d)$ can then be written as

$$\begin{aligned}\dot{V}(\mathbf{c}_d) =& -\nabla\rho(\mathbf{c}_d)^\top \sum_{k=1}^{\frac{N}{2}} \big(\rho(\mathbf{r}_k)\mathbf{e}\big(\theta+\tfrac{2\pi}{N}k\big) \\ &\qquad + \rho(\mathbf{r}_{k+\frac{N}{2}})\mathbf{e}\big(\theta + \tfrac{2\pi}{N}k+\pi\big)\Big) & (26)\\ =& -\nabla\rho(\mathbf{c}_d)^\top \sum_{k=1}^{\frac{N}{2}} \Big(\rho(\mathbf{r}_k)-\rho(\mathbf{r}_{k+\frac{N}{2}})\Big)\,\mathbf{e}\big(\theta+\tfrac{2\pi}{N}k\big) & (27)\\ =& -\sum_{k=1}^{\frac{N}{2}} \Big(\rho(\mathbf{r}_k)-\rho(\mathbf{r}_{k+\frac{N}{2}})\Big)\,\nabla\rho(\mathbf{c}_d)^\top\mathbf{e}\big(\theta+\tfrac{2\pi}{N}k\big) & (28)\end{aligned}$$

To see that the term $\rho(\mathbf{r}_k) - \rho(\mathbf{r}_{k+\frac{N}{2}})$ is non-negative, note that $\mathbf{r}_k$ and $\mathbf{c}_d$ and $\mathbf{r}_{k+\frac{N}{2}}$ all lie along the same line. The value of $\mathbf{z}^\top\mathbf{A}\mathbf{z}$ evaluated along this line is a parabola with its minimum corresponding to a point in the direction of $\mathbf{r}_k$ from $\mathbf{c}_d$ (see Figure 2). Due to the symmetry of this parabola, since $\mathbf{r}_k$ is closer to its minimum (or at least as close as $\mathbf{r}_{k+\frac{N}{2}}$ if that minimum corresponds to $\mathbf{c}_d$) it must be the case that

$$\mathbf{r}_k^\top\mathbf{A}\mathbf{r}_k \leq \mathbf{r}_{k+\frac{N}{2}}^\top\mathbf{A}\mathbf{r}_{k+\frac{N}{2}} \tag{29}$$

which implies that $\rho(\mathbf{r}_k) \geq \rho(\mathbf{r}_{k+\frac{N}{2}})$. Thus $\rho(\mathbf{r}_k) - \rho(\mathbf{r}_{k+\frac{N}{2}}) \geq 0$ and $\dot{V}(\mathbf{c}_d)$ is non-positive. In fact, the only situation in which $\dot{V}(\mathbf{c}_d) = 0$ for $\mathbf{c}_d \neq \mathbf{z}^\star$ is the case where $N = 2$ and the agents' displacement vectors from the formation center, $R\mathbf{e}(\theta)$ and $R\mathbf{e}(\theta + \pi)$, are orthogonal to $\nabla\rho(\mathbf{c}_d)$. As before, $\dot{\mathbf{c}}_d = 0$ in this case and as θ continues to increase $\dot{V}(\mathbf{c}_d)$ immediately becomes negative again. Thus these situations do not constitute an invariant set and the point of maximum signal strength $\mathbf{z}^\star$ is an asymptotically stable equilibrium of system (9) with control law (10). □

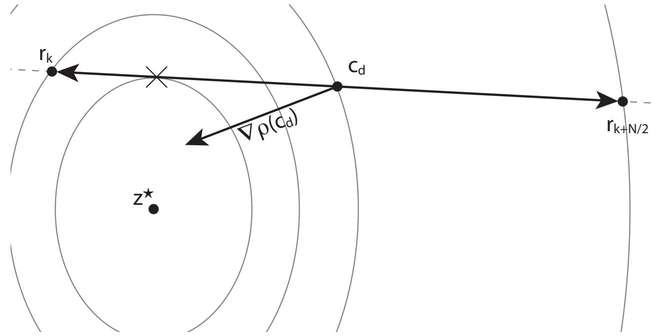

Fig. 2. Illustration of the concept used in the proof of Theorem 2. Level curves of ρ are shown in light gray and location of minimum value of $\mathbf{z}^\top\mathbf{A}\mathbf{z}$ along the line connecting $\mathbf{r}_k$, $\mathbf{c}_d$, and $\mathbf{r}_{k+\frac{N}{2}}$ is marked with an $\times$.

C. Variations on the System and Control Law

In this subsection we present two changes that may be made to the system (9) and the control law (10) without affecting the basic formulation of the proofs of Theorems 1 and 2. In regards to the system, note that except for agent groups with $N = 2$, neither the value of θ nor the fact that it is changing over time plays any role in the above proofs, so it is not necessary to assume that $\theta = \omega t$ or any other dynamical model for that matter. This is important because the circular formation control of [1] may not be able to maintain a linearly increasing value of θ when $\mathbf{c}_d$ is moving quickly.

In regards to the control law, note that the main technique used in the proofs was to segregate the vehicle measurements by their *relative* value and not their *absolute* value. This means that the results of Theorems 1 and 2 will still hold if a weighting value w_k is substituted for each measurement $\rho(\mathbf{r}_k)$ in the control law (10) so that we have

$$\mathbf{u} = \lambda \sum_{k=1}^{N} w_k\mathbf{e}\big(\theta+\tfrac{2\pi}{N}k\big) \tag{30}$$

so long as the weights w_k have the same relative order as the measurements $\rho(\mathbf{r}_k)$, i.e.,

$$\rho(\mathbf{r}_k) > \rho(\mathbf{r}_m) \Rightarrow w_k > w_m \tag{31}$$

The flexibility of using weighting values in (30) may allow one to design additional rules that improve the performance of the control law for certain scenarios without impacting the analytical stability properties.

V. Simulations

In this section we present the results of a few simulations of the source-seeking algorithm. One potential drawback to the control law is that when the signal strength is low and fairly uniform over the measurement points (e.g., far away from the source), the magnitude of $\dot{\mathbf{c}}_d$ can become very small and formation will move very slowly. For all the following simulations we used the following time-varying gain factor in the control law (10)

$$\lambda(t) = \frac{N}{\sum_{k=1}^{N} \rho(\mathbf{r}_k(t))} \tag{32}$$

which is simply the reciprocal of the current average measurement. Using (32) acts to boost the magnitude of $\dot{\mathbf{c}}_d$ when the measurement values are low. While (32) worked well for our simulations, design of a useful weighting factor will in general depend the nature of the signal.

The first simulation we present compared the relative efficiency of different sized agent groups. For this simulation we used a radius of $R = 200$m for the formation and a rotational velocity of $\omega = 0.01$ rad/s. The signal used was a gaussian function

$$\rho(\mathbf{z}) = e^{-5\times10^{-7}\mathbf{z}^\top\mathbf{z}} \tag{33}$$

Figure 3 show a plot of the trajectories $\mathbf{c}_d$ for formations of two, three, and four agents each. It is fairly clear from this figure and the plots of the signal strength at the formation center (Figure 4) and it's derivative (Figure 5) that using more agents produces a smoother and more accurate estimation of the gradient over time, and hence the formation makes faster progress to the source.

The second simulation was similar to the first except that it used an elliptical gaussian for the signal function given as

$$\rho(\mathbf{z}) = e^{\mathbf{z}^\top\mathbf{A}\mathbf{z}}, \;\mathbf{A} = 10^{-7}\begin{bmatrix} 6 & 8 \\ 8 & 20 \end{bmatrix} \tag{34}$$

and used a larger circle with radius $R = 1000$m and rotational velocity $\omega = 0.002$ rad/s. The trajectory plots appear in Figures 6 and plots of the signal strength at the formation center and its time-derivative in Figure 7. Given the oscillations in these figures, it would seem to be the case that the gradient approximation generated by (10) is less accurate for signal functions with elliptical level sets than for those with circular level sets. Figure 6 also clearly shows how the group of three agents fails to maintain a positive rate of change for $\rho(\mathbf{c}_d)$ and how $\mathbf{c}_d$ appears to approach a limit cycle around the source.

The last simulation heuristically applies the control law (10) to a situation where the signal distribution function ρ does *not* satisfy the conditions of either Theorem 1 or 2. In this case, ρ was a sum of two elliptical gaussian functions and it's level sets are not elliptical and are not even always convex. This simulation used the values $N = 4$, $R = 200$m, and $\omega = 0.01$ rad/s. Figures 8 and 9 clearly show that the agent group follows a somewhat contorted trajectory and fails to maintain a positive rate of change in $\rho(\mathbf{c}_d)$ due to an inaccurate gradient estimate. However, in this case at least, the agent group is still able to eventually find the location of the source.

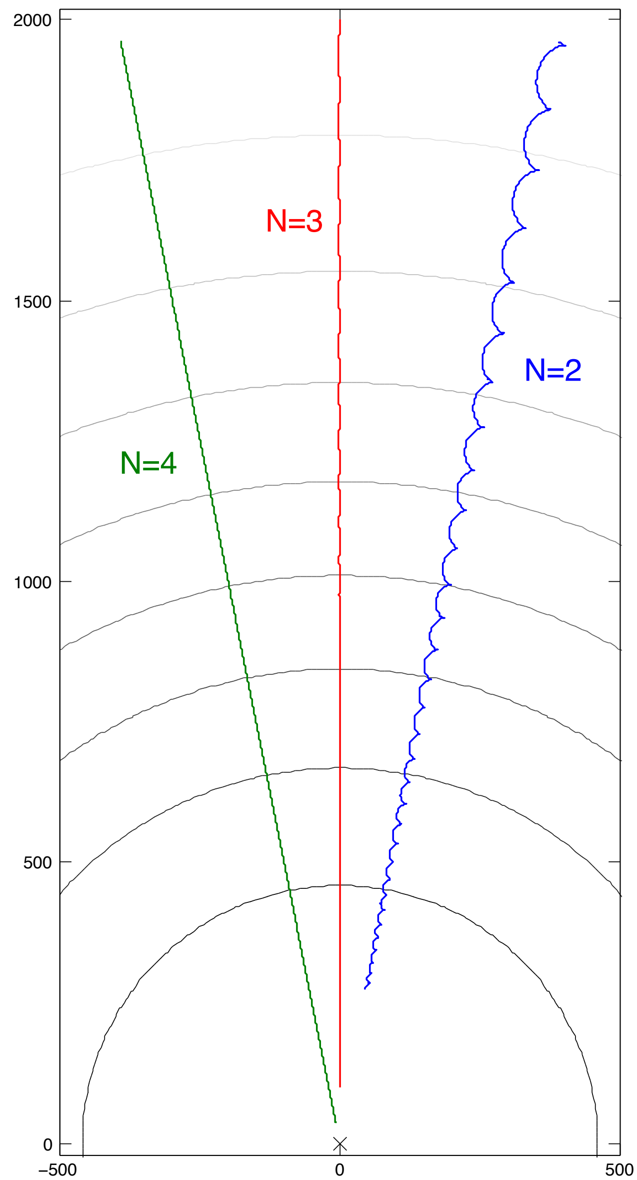

Fig. 3. Trajectories of the formation center $\mathbf{c}_d$ for different numbers of agents N when ρ has circular level sets. For this simulation $\rho(\mathbf{z}) = e^{-5\times10^{-7}\mathbf{z}^\top\mathbf{z}}$, $R = 200$m, and $\omega = 0.01$ rad/s. Location of source is denoted by the $\times$.

VI. Conclusions

This paper has presented a control law which a circular vehicle formation can employ to steer itself towards the source of a signal using only direct measurements of that signal at the vehicles' individual locations. Asymptotic convergence of the formation's rotational center to the location of the source was proven analytically for signal distributions that have circular level sets for two or more agents, and for those that have elliptical level sets for groups with an

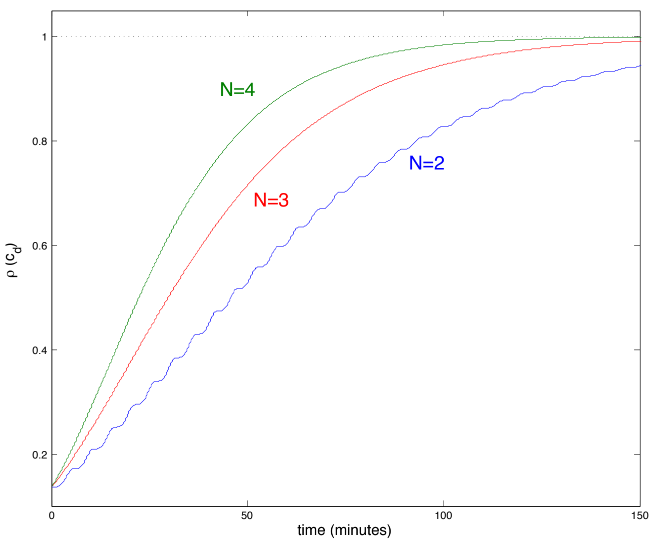

Fig. 4. Signal strength at the formation center $\mathbf{c}_d$ for the simulation shown in Figure 3.

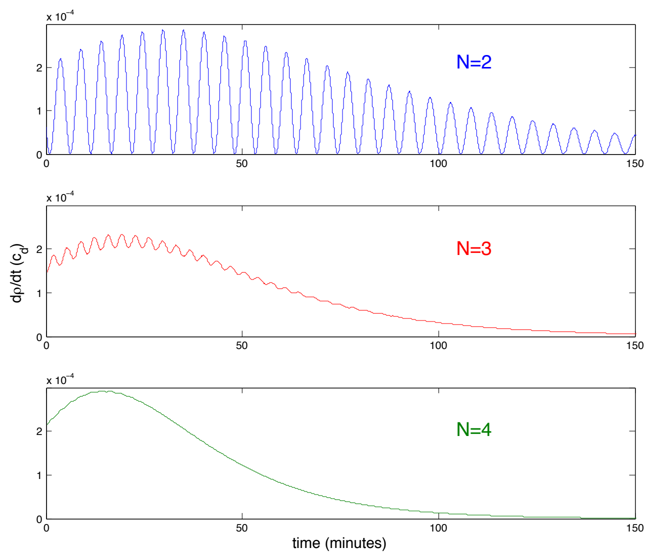

Fig. 5. Time derivative of signal strength at the formation center $\mathbf{c}_d$ for the simulation shown in Figure 3.

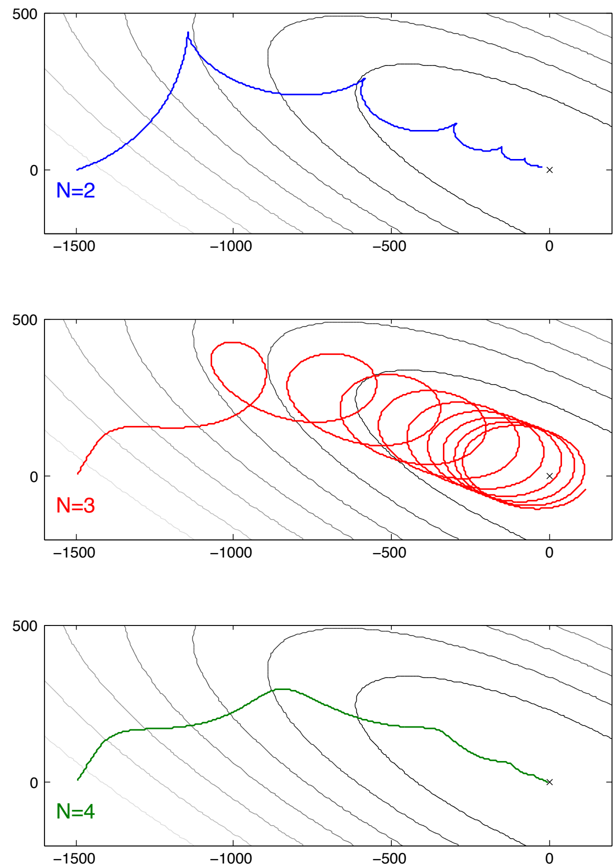

Fig. 6. Trajectories of the formation center $\mathbf{c}_d$ for different numbers of agents N when ρ has elliptical level sets. For this simulation the signal function was given by (34), $R = 1000$m, and $\omega = 0.002$ rad/s. Location of source is denoted by the $\times$.

even number of agents. Future work in this area will either involve finding ways to improve the gradient estimate in order to expand the class of signal distribution functions that guarantee asymptotic convergence or applying the concepts of this paper to a discrete-time control law and to situations in which the agents do not have all-to-all communications.

REFERENCES

[1] Lara Briñón Arranz, Alexandre Seuret, and Carlos Canudas de Wit. Translation control of a fleet circular formation of AUVs under finite communication range. In *48th IEEE Conference on Decision and Control & 28th Chinese Control Conference*, page ?, Shanghai, China, December 2009.

[2] Ralf Bachmayer and Naomi Ehrich Leonard. Vehicle networks for gradient descent in a sampled environment. In *41st IEEE Conference on Decision and Control*, pages 112–117, Las Vega, NV, December 2002.

[3] Emrah Biyik and Murat Arcak. Gradient climbing in formation via extremum seeking and passivity-based coordination rules. In *46th IEEE Conference on Decision and Control*, pages 3133–3138, New Orleans, LA, December 2007.

[4] Jennie Cochran and Miroslav Krstic. Nonholonomic source seeking with tuning of angular velocity. *IEEE Transactions on Automatic Control*, 54(4):717–731, 2009.

[5] Jennie Cochran, Antranik Siranosian, Nima Ghods, and Miroslav Krstic. 3-D source seeking for underactuated vehicles without position measurement. *IEEE Transactions on Robotics*, 25(1):117–129, 2009.

[6] Christopher G. Mayhew, Ricardo G. Sanfelice, and Andrew R. Teel. Robust hybrid source-seeking algorithms based on directional directional derivatives and their approximations. In *47th IEEE Conference on Decision and Control*, pages 1735–1740, Cancun, Mexico, December 2008.

[7] Christopher G. Mayhew, Ricardo G. Sanfelice, and Andrew R. Teel. Robust source-seeking hybrid controllers for nonholonomic vehicles. In *2008 American Control Conference*, pages 2722–2727, Seattle, WA, June 2008.

[8] Petter Ögren, Edward Fiorelli, and Naomi Ehrich Leonard. Cooperative control of mobile sensor networks: adaptive gradient climbing in

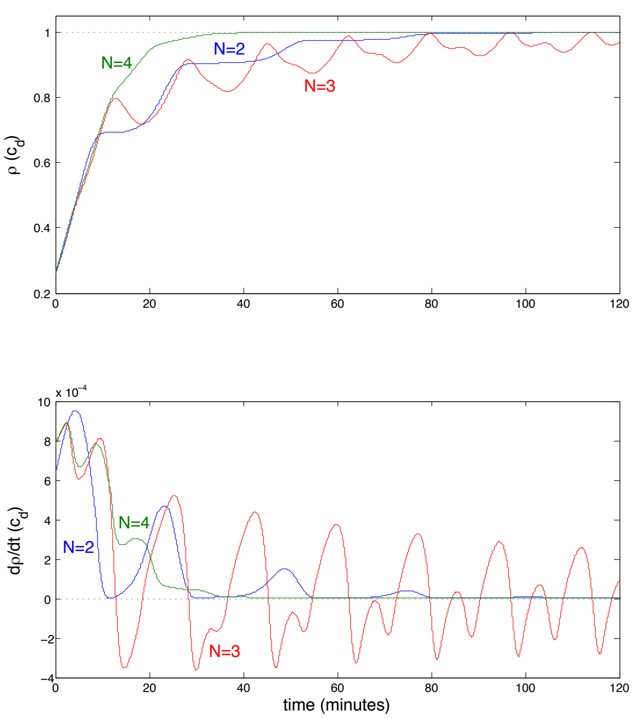

Fig. 7. Signal strength at the formation center $\mathbf{c}_d$ and its time derivative for the simulation shown in Figure 6.

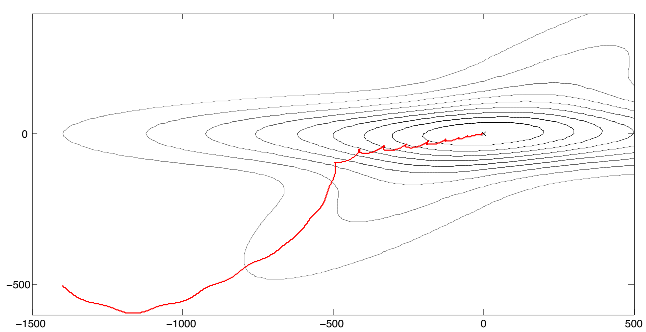

Fig. 8. Trajectories of the formation center $\mathbf{c}_d$ when ρ is the sum of two gaussian functions (and does not meet the conditions of the theorems). For this simulation $N = 4$, $R = 200$m, and $\omega = 0.01$ rad/s. Location of source is denoted by the $\times$.

a distributed environment. *IEEE Transactions on Automatic Control*, 49(8):1292–1302, 2004.

[9] Shankar Sastry. *Nonlinear systems: analysis, stability, and control*. Springer-Verlag, New York, NY, 1999.

[10] Rodolphe Sepulchre, Derek Paley, and Naomi Ehrich Leonard. Stabilization of planar collective motion: All-to-all communication. *IEEE Transactions on Automatic Control*, 52(5):811–824, 2007.

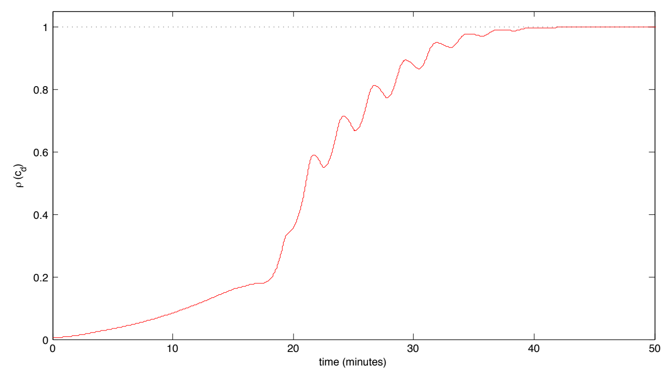

Fig. 9. Signal strength at the formation center $\mathbf{c}_d$ for the simulation shown in Figure 8.