Lean and Steering Motorcycle Dynamics Reconstruction : An Unknown Input HOSMO Approach

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L. Nehaoua[1], D. Ichalal[1], H. Arioui[1], S. Mammar[1] and L. Fridman[2]

***Abstract*— This paper deals with state estimation of Powered Two Wheeled (PTW) vehicle and robust reconstruction of related unknown inputs. For this purpose, we consider a unknown input high order sliding mode observer (UIHOSMO). First, a motorcycle dynamic model is derived using JourdainŠs principle. In a second time, we consider both the observation of the PTW dynamic states, the reconstruction of the lean dynamics (roll angle $\phi(t)$) and the rider's torque applied on the handlebar. Finlay, several simulation cases are provided to illustrate the efficiency of the proposed observer.**

I. INTRODUCTION

In recent years, the use of powered two wheeled (PTW) vehicles is constantly growing, upsetting driving practices and road traffic. Unfortunately, this expansion is also inflected by an important increase of motorcycle's fatalities. Recent statistics confirm this fact and consider riders as the most vulnerable road users. Several programs are launched to answer this issue and to find solutions for enhancing safety, [3].

The success, of proposed safety systems, depends primarily on the knowledge of: 1) the dynamics of motorcycle and 2) the evolution of its states strongly involved by the rider's action and / or the infrastructure geometry. Regarding the first issue, several studies were carried out in order to understand the motorcycle dynamics [16], [17], the stability analysis (eigenmodes) of PTW, optimal and safe trajectories [2] and the proposal of risk functions [5] to detect borderline cases of loss-of-control. These research are very few sustainable if they are not propped by a system estimating the dynamic states of the PTW.

The direct measurement, by sensors, of all the PTW states is not conceivable for two reasons: 1) instrumentation can be very expensive leading inevitably to expensive new PTW, and 2) according to used technologies, the measurement noise can seriously compromise the future safety systems. Thus, we propose to use observation techniques to overcome previous shortcomings. Within this context, including all methodologies (Luenberger, Takagi-Seguno, Extended Kalman Filter and Sliding Mode observers), very few studies exist [1], [4]. The present work proposes a robust (UIHOSMO) [12], helping in states observation of motorcycle model and the reconstruction of rider's action.

*This work was not supported by any organization

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Sliding mode based observer (SMO) for system states estimation in the presence and/or absence of unknown inputs has been the subject of several work from the control community. Today, one can notice that observation theory has matured and has succeed to deal with many technical issues where some restrictive conditions related to the observability and the reconstruction of unknown inputs were released even suppressed. In its simplest version, a SMO uses the same structure as a traditional Luenberger observer with an injection term related to the system's output. However, it is necessary that the system's unknown inputs and outputs satisfy the so-called matching condition [6]. To overcome this limitation, system coordinates transformation are introduced and the use of sliding mode differentiator (SMD) for the auxiliary output generation is generalized [7], [8]. This allows to use a powerful optimization technique, like linear matrix inequality (LMI), to offer a systematic design procedure of the observer gain [9]. Nevertheless, the chattering of these observers requires filtering [10].

To avoid filtering, the discontinuous output injection is replaced by a continuous super-twisting algorithm (STA) [11]. In this new version, the relative degree of the system's outputs with respect to the unknown inputs must be equal to the system order. This restriction is resolved by the introduction of the high-order sliding mode observers (HOSMO) [12] based on the high-order robust exact sliding mode differentiator [13] where the notion of strong observability and strong detectability were presented [12]. It remains at least that the outputs relative degree must exist which brings a novel restriction treated by the development of the concept of weakly observable subspaces detailed in [14].

This paper is organized as follows: a succinct problem statement is provided in section II. Sections III and IV are dedicated respectively to motorcycle modelling and the synthesis of the UIHOSMO. Simulation results are given is section V. The conclusion in section VI wrap up the paper.

II. PROBLEM STATEMENT

Our study concerns the identification of all relevant inputs and dynamic states helping in a next stage to quantify the risk of loss-of-control when cornering. Indeed, poor cornering is responsible for most PTW accidents (single-vehicle motorcycle crashes).

To perform a safe cornering, riders should consider : 1) the appropriate speed when starting the corner, 2) the road condition (under weak friction) and 3) weather conditions do not allow optimal visibility for driving.

Early warning systems are based generally on related work carried out for standard cars [21]. The goal is the synthesis of a function giving the maximum safe speed at which a vehicle can be kept on the road while moving at a constant speed on a circular section. This speed depends, among other factor, on the lateral friction which its computing involves all the dynamic states of the bike and a good modelling of the tire-road contact. This makes the success of such warning system strongly dependent on the availability of dynamic states of the motorcycle. This challenge constitutes the contribution of our ongoing work.

III. MOTORCYCLE DYNAMICS

A. Modeling Assumptions

The study of the dynamics of motorcycle vehicles highlights two main modes of motion: in-plane mode representing the motorcycle movements in its plane of symmetry including the longitudinal motion and that of suspensions and the out-of-plane mode which describes the lateral dynamics when cornering [15], [16]. The last mode involves the roll inclination, the yaw rotation, the steering and the lateral motions of the bike. We consider here only the out-of-mode dynamics of the two-wheeled vehicle. The coupling between the two modes is materialized, when neccesery, by considering a variable longitudinal velocity that appears in the lateral dynamics.

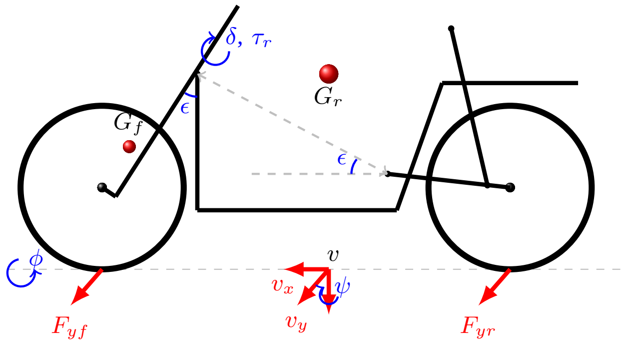

Fig. 1. Geometrical representation of the Sharp's motorcycle model

In this study, the motorcycle is represented as two linked bodies: the rear one G_r which includes the chassis, engine and the rear wheel and the front body G_f which represents the steering assembly and the front wheel. In that case, we recover the well known two bodies Sharp's model which allows to simulate 4 degrees of freedom (DoF): the lateral displacement v_y, the yaw rotation ψ, the roll inclination φ and the handlebar steer angle δ with respect to the rider torque input τ_r applied on the motorcycle's handlebar (Fig.1).

B. Motorcycle Motion Equations

Let $\Re_v$ be a reference frame attached to the motorcycle at point v in which the motion of the overall mechanical system will be expressed. With respect to the inertial reference frame $\Re_o$, $\Re_v$ is referred by the velocity vector components v_x, v_y of point v, and the yaw rotation ψ around the $\Re_o$ z-axis (Fig.2).

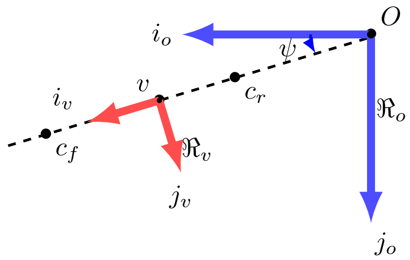

Fig. 2. Configuration of system axis $\Re_v$ w.r.t the inertial frame $\Re_o$

In $\Re_v$, the linear velocity vector of the two bodies are respectively defined by:

$$
\begin{aligned}
\boldsymbol{v}_{oG_r} &= \boldsymbol{v}_{ov} + \boldsymbol{\omega}_{oG_r} \times \boldsymbol{r}_{vG_r} \\
\boldsymbol{v}_{oG_f} &= \boldsymbol{v}_{ov} + \boldsymbol{\omega}_{oG_r} \times \boldsymbol{r}_{vB} + \boldsymbol{\omega}_{oG_f} \times \boldsymbol{r}_{BG_f}
\end{aligned} \tag{1}
$$

where $\boldsymbol{\omega}_{oG_r}$ and $\boldsymbol{\omega}_{oG_f}$ are the angular velocity vector of each body, expressed in $\Re_v$ by:

$$
\begin{aligned}
\boldsymbol{\omega}_{oG_r} &= \dot{\psi}\boldsymbol{k} + \dot{\varphi}\boldsymbol{i} \\
\boldsymbol{\omega}_{oG_f} &= \boldsymbol{\omega}_{oG_r} + \dot{\delta}\boldsymbol{k}_\delta
\end{aligned} \tag{2}
$$

By differentiating (1) and (2), the linear and the angular acceleration of G_r and G_f can be written for $i = r, f$ as following:

$$
\begin{aligned}
\boldsymbol{a}_{oG_i} &= \frac{\partial \boldsymbol{v}_{oG_i}}{\partial \boldsymbol{\vartheta}}\dot{\boldsymbol{\vartheta}} + \boldsymbol{a}_{R,G_i} \\
\boldsymbol{\epsilon}_{oG_i} &= \frac{\partial \boldsymbol{\omega}_{oG_i}}{\partial \boldsymbol{\vartheta}}\dot{\boldsymbol{\vartheta}} + \boldsymbol{\epsilon}_{R,G_i}
\end{aligned} \tag{3}
$$

where $\boldsymbol{\vartheta} = [v_y, \dot{\psi}, \dot{\varphi}, \dot{\delta}]^T$ denotes the vector of generalized velocities. From the Jourdain's principle of dynamics [18], the dynamic model of the motorcycle vehicle can be expressed by:

$$
\boldsymbol{M}\dot{\boldsymbol{\vartheta}} = \boldsymbol{Q} \tag{4}
$$

where the mass matrix $\boldsymbol{M}$ is symmetric and positive definite and obtained directly from the Jacobian matrices by:

$$
\boldsymbol{M} = \sum_{i=r,f} \left\{ m_i \frac{\partial \boldsymbol{v}_{oG_i}^T}{\partial \boldsymbol{\vartheta}} \frac{\partial \boldsymbol{v}_{oG_i}}{\partial \boldsymbol{\vartheta}} + \frac{\partial \boldsymbol{\omega}_{oG_i}^T}{\partial \boldsymbol{\vartheta}} \boldsymbol{I}_i \frac{\partial \boldsymbol{\omega}_{oG_i}}{\partial \boldsymbol{\vartheta}} \right\} \tag{5}
$$

and $\boldsymbol{Q}$ is the vector of the generalized efforts, given by:

$$
\begin{aligned}
\boldsymbol{Q} = \boldsymbol{Q}_\delta + \boldsymbol{Q}_g + \boldsymbol{Q}_T + \sum_{i=r,f} \frac{\partial \boldsymbol{v}_{oG_i}^T}{\partial \boldsymbol{\vartheta}} m_i \left(\boldsymbol{g} - \boldsymbol{a}_{R,G_i}\right) - \\
\sum_{i=r,f} \frac{\partial \boldsymbol{\omega}_{oG_i}^T}{\partial \boldsymbol{\vartheta}} \left(\boldsymbol{I}_i \boldsymbol{\epsilon}_{R,G_i} + \boldsymbol{\omega}_{oG_i} \times \boldsymbol{I}_i \boldsymbol{\omega}_{oG_i}\right)
\end{aligned} \tag{6}
$$

In (4) and (5), $\boldsymbol{I}_i$ is the inertia tensor matrix, $\boldsymbol{Q}_\delta$ includes the effect of the rider's torque and also the handlebar damping, $\boldsymbol{Q}_g$ represents the gyroscopic effect resulting from the tires' spin, $\boldsymbol{Q}_T$ includes the effect of the tire's sideslip force and $\boldsymbol{g}$ is the gravity force vector.

C. Tire/Road Interaction and Wheels Gyroscopic Effect

To describe the tire motion, a new reference frame $\Re_T(c, \boldsymbol{i}_T, \boldsymbol{j}_T, \boldsymbol{k}_T)$ is introduced at the contact point c of each wheel's tire. $\boldsymbol{k}_T$ is the normal vector to the road surface along the F_z force.

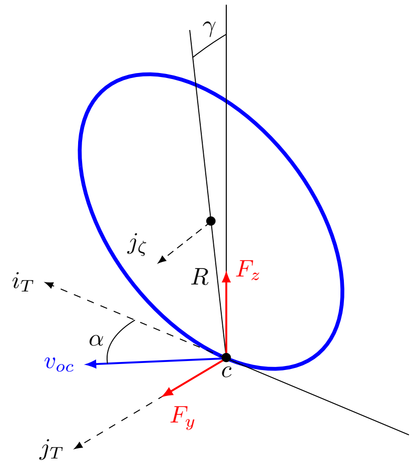

Fig. 3. Tire reference frame

The linear velocity vector at point location c_i is expressed in $\Re_v$ by:

$$\boldsymbol{v}_{oc_r} = \boldsymbol{v}_{ov} + \boldsymbol{\omega}_{oG_r} \times \boldsymbol{r}_{vc_r} \qquad (7)$$
$$\boldsymbol{v}_{oc_f} = \boldsymbol{v}_{ov} + \boldsymbol{\omega}_{oG_r} \times \boldsymbol{r}_{vc_f} + \dot{\delta}\boldsymbol{e}_\delta \times \boldsymbol{r}_{c_\eta c_f}$$

From (7), the tires sideslip angles are defined by:

$$\alpha_r = -\text{atan}\left(\frac{\boldsymbol{j}_v \cdot \boldsymbol{v}_{oc_r}}{\boldsymbol{i}_v \cdot \boldsymbol{v}_{oc_r}}\right) \qquad (8)$$
$$\alpha_f = -\text{atan}\left(\frac{\boldsymbol{j}_v \cdot \boldsymbol{v}_{oc_f}}{\boldsymbol{i}_v \cdot \boldsymbol{v}_{oc_f}}\right) + \delta \cos\epsilon$$

The equivalent tire effort at the center of each wheel is governed by:

$$\boldsymbol{F}_T = F_y \boldsymbol{j}_T + F_z \boldsymbol{k}_T \qquad (9)$$

where $F_y = F_y(\alpha, \gamma)$ is obtained from the sideslip angle α and the camber angle γ computed by $\sin\gamma = \boldsymbol{j}_\zeta \cdot \boldsymbol{k}_T$. Consequently, the contribution $\boldsymbol{Q}_T$ of the tire/road contact forces in the vector of the generalized efforts $\boldsymbol{Q}$ is given by:

$$\boldsymbol{Q}_T = \sum_{i=f,r} \left(\frac{\partial \boldsymbol{v}_{oc_i}}{\partial \boldsymbol{\vartheta}}\right)^T \boldsymbol{F}_{T,i} \qquad (10)$$

To compute the contribution of the gyroscopic effect $\boldsymbol{Q}_g$, let first write the spin velocity vector $\omega_{s,i}$ equation of each tire in $\Re_v$ reference frame:

$$\boldsymbol{\omega}_{os_r} = \boldsymbol{\omega}_{oG_r} + \dot{\xi}_r \boldsymbol{j}_v \qquad (11)$$
$$\boldsymbol{\omega}_{os_f} = \boldsymbol{\omega}_{oG_r} + \dot{\delta}\boldsymbol{k}_\delta + \dot{\xi}_f \boldsymbol{j}_\xi$$

From (3), the contribution of the gyroscopic effect in the vector of the generalized efforts $\boldsymbol{Q}$ is :

$$\boldsymbol{Q}_g = -\sum_{i=r,f} i_{iy} \frac{\partial \boldsymbol{\omega}_{s_i}^T}{\partial \boldsymbol{\vartheta}} \boldsymbol{\epsilon}_{R,s_i} \qquad (12)$$

D. Linearized Model

The motorcycle dynamic model (4) is linearized around the straight-running trim trajectory and can be expressed by the following state-space:

$$\dot{\boldsymbol{x}}_v = \boldsymbol{A}_v \boldsymbol{x}_v + \boldsymbol{B}_v \tau_r \qquad (13)$$

Here, $\boldsymbol{x}_v = [v_y, \dot{\psi}, \dot{\varphi}, \dot{\delta}, \varphi, \delta, F_{yr}, F_{yf}]^T$ denotes the state vector. $\boldsymbol{A}_v$ is a time-varying matrix related to the forward velocity v_x while $\boldsymbol{B}_v$ is a time-invariant vector. F_{yr} and F_{yf} represent respectively the tires sideslip forces introduced in the state space representing the tire relaxation.

IV. States and Unknown Inputs Estimation

In this section, we aim to estimate the motorcycle states and reconstruct both roll angle φ and rider's torque τ_r by using a HOSMO [12].

At first, we recall some important definitions about observability and detectability of linear systems (for proofs see [12], [19]). Consider the following SISO system, where $\boldsymbol{x} \in \mathbb{R}^n$ and $\zeta \in \mathbb{R}$ is the unknown input:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}u + \boldsymbol{D}\zeta \qquad (14)$$
$$y = \boldsymbol{C}\boldsymbol{x}$$

Definition 4.1: In the absence of unknown input ($\zeta = 0$), system (14) is observable if and only if the observability matrix $\boldsymbol{P}$ such that:

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{C} \\ \vdots \\ \boldsymbol{C}\boldsymbol{A}^{n-1} \end{bmatrix} \qquad (15)$$

has the full rank. Otherwise, it is detectable if the system's invariant zeros are stable.

Definition 4.2: The relative degree of the output y with respect to the unknown input ζ is the number r such that:

$$\boldsymbol{C}\boldsymbol{A}^j\boldsymbol{D} = 0 \qquad j = 1, \cdots, r-2$$
$$\boldsymbol{C}\boldsymbol{A}^{r-1}\boldsymbol{D} \neq 0$$

Definition 4.3: In the presence of an unknown input, system (14) is strongly observable if and only if the relative degree r satisfies: $r = \text{rank}(\boldsymbol{P})$. Otherwise, it is strongly detectable if and only if the relative degree r exists and (14) is minimum phase system. In that case, $r \leq \text{rank}(\boldsymbol{P})$.

As previous, consider the MIMO system of the form (14), where $\boldsymbol{x} \in \mathbb{R}^n$, $\boldsymbol{y} \in \mathbb{R}^m$ is the output vector and $\boldsymbol{\zeta} \in \mathbb{R}^m$ is the unknown input vector.

Definition 4.4: In the absence of unknown inputs ($\boldsymbol{\zeta} = 0$), system (14) is observable if and only if the observability matrix $\boldsymbol{P}$ such that:

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{C}_1 \\ \vdots \\ \boldsymbol{C}_1\boldsymbol{A}^{n-1} \\ \vdots \\ \boldsymbol{C}_m \\ \vdots \\ \boldsymbol{C}_m\boldsymbol{A}^{n-1} \end{bmatrix} \tag{16}$$

has the full rank, where $\boldsymbol{C}_i$, $i = 1, \cdots, m$ is the i-th row of the matrix $\boldsymbol{C}$.

Definition 4.5: The relative degree of the output $\boldsymbol{y}$ with respect to the unknown input $\boldsymbol{\zeta}$ is the vector $\boldsymbol{r} = [r_1, \cdots, r_m]$ such that:

$$\begin{aligned} &\boldsymbol{C}_i\boldsymbol{A}^s\boldsymbol{D}_j = 0 \quad i, j = 1, \cdots, m, \quad s = 1, \cdots, r_i - 2 \\ &\boldsymbol{C}_i\boldsymbol{A}^{r_i-1}\boldsymbol{D}_j \neq 0 \end{aligned}$$

and:

$$\det \begin{bmatrix} \boldsymbol{C}_1\boldsymbol{A}^{r_1-1}\boldsymbol{D}_1 & \cdots & \boldsymbol{C}_1\boldsymbol{A}^{r_1-1}\boldsymbol{D}_m \\ \vdots & \vdots & \vdots \\ \boldsymbol{C}_m\boldsymbol{A}^{r_m-1}\boldsymbol{D}_1 & \cdots & \boldsymbol{C}_m\boldsymbol{A}^{r_m-1}\boldsymbol{D}_m \end{bmatrix} \neq 0$$

Definition 4.6: In the presence of an unknown input, system (14) is strongly observable if and only if the total relative degree $r_T = r_1 + \cdots + r_m$ satisfies: $r_T = \text{rank}(\boldsymbol{P})$. Otherwise, it is strongly detectable if and only if the relative degree vector $\boldsymbol{r} = [r_1, \cdots, r_m]$ exists, and (14) is minimum phase.

From definition (4.1), the motorcycle dynamics (13) is neither observable nor detectable. Indeed, for all v_x in the allowable velocities range, the observability index is equal to 6 which is less than the system order ($n = 8$), in addition, the motorcycle dynamics has one unstable invariant zero which makes the motorcycle dynamics to be a non-minimum phase.

A. Estimation of the Roll Angle and Rider's Torque

In order to make the system observable, the motorcycle model (13) is rewritten such that the roll angle φ and the steering torque τ_r are considered as unknown inputs. In fact, the unstable invariant zero is a direct consequence of the counter-steering phenomena generated by the motorcycle roll. In this case, the new system equation is written as:

$$\begin{aligned} &\dot{\boldsymbol{x}}_p = \boldsymbol{A}_p\boldsymbol{x}_p + \boldsymbol{D}_{pp}\boldsymbol{\zeta} \\ &\boldsymbol{y}_{pp} = \begin{bmatrix} \delta \\ \dot{\psi} \end{bmatrix} = \boldsymbol{C}_{pp}\boldsymbol{x}_p \\ &\boldsymbol{D}_{pp} = \begin{bmatrix} \boldsymbol{D}_p & \boldsymbol{B}_p \end{bmatrix} \quad \boldsymbol{\zeta}^T = \begin{bmatrix} \varphi & \tau_r \end{bmatrix} \end{aligned} \tag{17}$$

where $\boldsymbol{x}_p = [v_y, \dot{\psi}, \dot{\varphi}, \dot{\delta}, \delta, F_{yr}, F_{yf}]^T$ denotes the state vector and matrices $\boldsymbol{A}_p$, $\boldsymbol{B}_p$ and $\boldsymbol{D}_p$ are given by:

$$\boldsymbol{B}_p = \begin{bmatrix} 0.016 \\ -0.006 \\ -0.019 \\ 0.008 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \boldsymbol{D}_p = \begin{bmatrix} -26.176 \\ 0.826 \\ 42.796 \\ 261.877 \\ 0 \\ 5437.244v_x \\ 3849.877v_x \end{bmatrix}$$

From definition (4.5), the output $\boldsymbol{y}_{pp}$ in (17) has a relative degree vector $\boldsymbol{r} = [2, 1]$ with respect to the unknown input vector $\boldsymbol{\zeta}$. In addition, system (17) has 3 stable invariant zeros for all v_x in the allowable velocities range. It results from (4.6), that (17) is also strongly detectable. This definition implies that only $r_T = r_1 + r_2$ system's states can be estimated exactly while the observation of the remaining states are asymptotically exact.

As before, to estimate the system's state $\boldsymbol{x}_p$ and the unknown input vector $\boldsymbol{\zeta}$, it is necessary to separate the clean states from those contaminated by the unknown inputs. To achieve this, system (17) is transformed to a new coordinates system $\boldsymbol{\xi}_p = \boldsymbol{T}\boldsymbol{x}_p$ such that the closed-loop system dynamics $(\boldsymbol{A}_p - \boldsymbol{C}_{pp}\boldsymbol{L}_{pp}, \boldsymbol{D}_{pp}, \boldsymbol{C}_{pp})$ is expressed by:

$$\begin{aligned} &\dot{\xi}_{11} = \xi_{12} \\ &\dot{\xi}_{12} = a_{11,11}\xi_{11} + a_{11,12}\xi_{12} + a_{11,13}\xi_{13} + a_{12,11}\xi_{21} + \\ &\qquad \cdots + a_{12,14}\xi_{24} + d_{11}\varphi + d_{12}\tau_r \\ &\dot{\xi}_{13} = a_{11,21}\xi_{11} + a_{11,22}\xi_{12} + a_{11,23}\xi_{13} + a_{12,21}\xi_{21} + \\ &\qquad \cdots + a_{12,24}\xi_{24} + d_{21}\varphi + d_{22}\tau_r \\ &\dot{\boldsymbol{\xi}}_2 = \boldsymbol{A}_{21}\boldsymbol{\xi}_1 + \boldsymbol{A}_{22}\boldsymbol{\xi}_2 \\ &\boldsymbol{y}_{pp,new}^T = \begin{bmatrix} \xi_{11} & \xi_{13} \end{bmatrix} \end{aligned} \tag{18}$$

where $\boldsymbol{\xi}_p^T = [\boldsymbol{\xi}_1^T, \boldsymbol{\xi}_2^T] \in \mathbb{R}^n$ and $\boldsymbol{\xi}_1^T = [\xi_{11}, \xi_{12}, \xi_{12}] \in \mathbb{R}^{r_T}$ and $\boldsymbol{\xi}_2^T = [\xi_{21}, \cdots, \xi_{24}] \in \mathbb{R}^{n-r_T}$. Next, the state observer is designed as:

$$\begin{aligned} &\dot{\boldsymbol{z}} = \boldsymbol{A}_p\boldsymbol{z} + \boldsymbol{L}_{pp}(\boldsymbol{y}_{pp} - \boldsymbol{C}_{pp}\boldsymbol{z}) \\ &\dot{\boldsymbol{\vartheta}}_2 = \boldsymbol{A}_{21}\boldsymbol{\vartheta}_1 + \boldsymbol{A}_{22}\boldsymbol{\vartheta}_2 \\ &\hat{\boldsymbol{x}}_p = \boldsymbol{z} + \boldsymbol{T}^{-1}\boldsymbol{\vartheta} \end{aligned} \tag{19}$$

in which $\hat{\boldsymbol{x}}_p$ is the vector of estimated states, $\boldsymbol{z} \in \mathbb{R}^n$ and $\boldsymbol{\vartheta} \in \mathbb{R}^n$ is given by (recall that $n = 7$, $r = [2, 1]$ and $r_T = 3$):

$$\boldsymbol{\vartheta} = \begin{bmatrix} \boldsymbol{\vartheta}_1 \\ \boldsymbol{\vartheta}_2 \end{bmatrix} \begin{matrix} \updownarrow \mathbb{R}^{r_T} \\ \updownarrow \mathbb{R}^{n-r_T} \end{matrix} = \begin{bmatrix} v_{1,1} \\ v_{1,2} \\ v_{2,1} \\ \hline \boldsymbol{\vartheta}_2 \end{bmatrix} \tag{20}$$

Once again, $\boldsymbol{v}_i \in \mathbb{R}^{r_M+k+1}$ is the nonlinear part of the observer where $i = 1, \cdots, m$ and $r_M = \max(r_i)$. Each unknown input ζ_i is bounded with $|\zeta_i| \leq \zeta_{i,\max}$ and the $(r_M - r_i + k)$ successive derivatives of ζ_i are bounded by the same constant $\zeta'_{i,\max}$, consequently the auxiliary variable $\boldsymbol{v}_i$

$$
\boldsymbol{A}_p = \begin{bmatrix} 0 & -0.899v_x - 0.015v_x & 0.033v_x & -0.057 & -1.862 & 0.018 & 0.009 \\ 0 & 0.028v_x & 0.033v_x & 0.016v_x + 0.374 & -2.443 & -0.018 & 0.024 \\ 0 & -0.163v_x & -0.009v_x & 0.109 - 0.059v_x & 2.949 & -0.021 & -0.012 \\ 0 & -2.373v_x & 4.516v_x & -0.025v_x - 15.611 & 127.454 & 0.035 & -0.309 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -64934.37 & 31155.51 & 0 & 0 & 0 & -4.102v_x & 0 \\ -45832.65 & -42841.04 & 0 & 5307.421 & 42580.46v_x & 0 & -4.101v_x \end{bmatrix}
$$

is a solution of the discontinuous vector differential equation by considering $k = 1$ as:

$$
\begin{aligned}
\dot{v}_{i,1} &= -3\lambda^{\frac{1}{4}} \left| v_{i,1} - y_p + \boldsymbol{C}_p \boldsymbol{z} \right|^{\frac{3}{4}} \operatorname{sign}(v_{i,1} - y_p + \boldsymbol{C}_p \boldsymbol{z}) + v_{i,2} \\
\dot{v}_{i,2} &= -2\lambda^{\frac{1}{3}} \left| v_{i,2} - \dot{v}_{i,1} \right|^{\frac{2}{3}} \operatorname{sign}(v_{i,3} - \dot{v}_{i,1}) + v_{i,3} \\
\dot{v}_{i,3} &= -1.5\lambda^{\frac{1}{2}} \left| v_{i,3} - \dot{v}_2 \right|^{\frac{1}{2}} \operatorname{sign}(v_{i,3} - \dot{v}_{i,2}) + v_{i,4} \\
\dot{v}_{i,4} &= -1.1\lambda \operatorname{sign}(v_{i,4} - \dot{v}_{i,3})
\end{aligned} \tag{21}
$$

Finally, the reconstruction of the unknown input vector is possible by using:

$$
\begin{aligned}
\begin{bmatrix} \hat{\varphi} \\ \hat{\tau}_r \end{bmatrix} &= \bar{\boldsymbol{D}}^{-1} \left(\begin{bmatrix} v_{1,3} \\ v_{2,2} \end{bmatrix} - \begin{bmatrix} a_{11,11} & \cdots & a_{12,14} \\ a_{11,21} & \cdots & a_{12,24} \end{bmatrix} \boldsymbol{\vartheta} \right) \\
\bar{\boldsymbol{D}} &= \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}
\end{aligned} \tag{22}
$$

Remark 4.1: In the strong observability case, all system's states are exactly estimated and the coordinates transformation matrix $\boldsymbol{\xi} = \boldsymbol{T}\boldsymbol{x}$ is none than the observability matrix $\boldsymbol{P}$. However, in the strong detectability case, only r system's states are exactly estimated where r is the relative degree. Consequently, it is simple to show that the r first lines of the coordinates transformation matrix $\boldsymbol{T}$ are the same of those of the observability matrix $\boldsymbol{P}$. The expression of the remaining $n - r$ lines are explicitly described and proven in [20].

V. SIMULATION RESULTS

In this section, the unknown input high order sliding mode observer is constructed for the presented motorcycle model. Some simulations and discussions are provided to illustrate the effectiveness and the ability of the UIHOSMO in estimating simultaneously the dynamic states and both roll angle and the applied torque by the rider on the handlebar. The observer is designed in such a way to estimate all the dynamic states and unknown inputs from only the knowledge of steering angle $\delta(t)$ and the yaw rate $\dot{\psi}(t)$. The parameters $\lambda_i,\ i = 1, 2$ of the differentiator (21) are chosen as follows $\lambda_1 = \lambda_2 = 10000$. The Luenberger gain $\boldsymbol{L}_{pp}$ of (25) is computed by pole placement at: $-15, -30, -45, -60, -150, -165, -180$, and the obtained matrix is:

$$
\boldsymbol{L}_{pp} = \begin{pmatrix} -254.15 & -155,92 \\ 99,80 & 297,89 \\ 1097,82 & 1537,85 \\ 5932,04 & 7017,05 \\ 208,44 & 51,40 \\ 2,58 \times 10^5 & 9,16 \times 10^4 \\ 1,35 \times 10^6 & 6,69 \times 10^5 \end{pmatrix}
$$

The change of coordinates matrix $\boldsymbol{T}$ is given in equation (23). With these parameters and matrices, the UIHOSMO is implemented with initial conditions $\hat{x}(0) = [0.1745 \;\; 0.1745 \;\; -0.2618 \;\; -0.0873 \;\; 50 \;\; 50 \;\; 0]$ and $\hat{\phi}(0) = 0$. These initial conditions correspond to a early cornering case with longitudinal speed at $v_x(t) = 15m/s$.

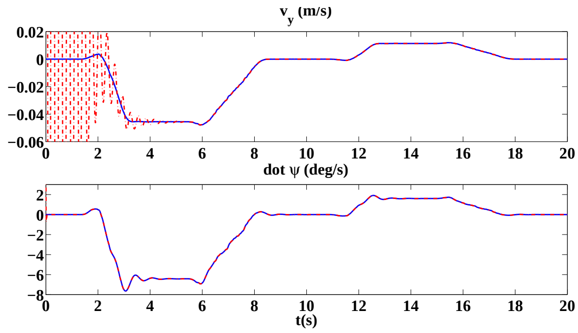

Fig. 4. Actual state (solid blue line) and estimated state (dashed red line)

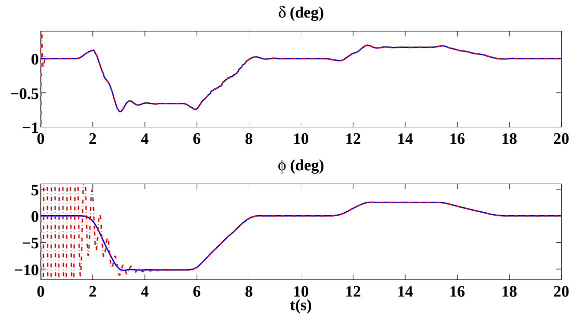

Fig. 5. Actual state (solid blue line) and estimated states (dashed red line)

A. Initial Conditions

A first simulation is performed with the same initial conditions $\hat{x}(0) = x(0)$. For different initial conditions, let

$$
\boldsymbol{T} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -51,4 & 0 & 1 & -208,44 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 6,28 & -1 & 0,14 & -4,08 & 0 & 0 \\ -1 & -3,82 & 0 & -0.08 & 4,3 & 0 & 0 \\ 0 & -1.14\times10^4 & 0 & -275,19 & 1.3\times10^3 & 1 & 0 \\ 0 & -8,13\times10^3 & 0 & -194,85 & -4,3\times10^3 & 0 & 1 \end{bmatrix} \tag{23}
$$

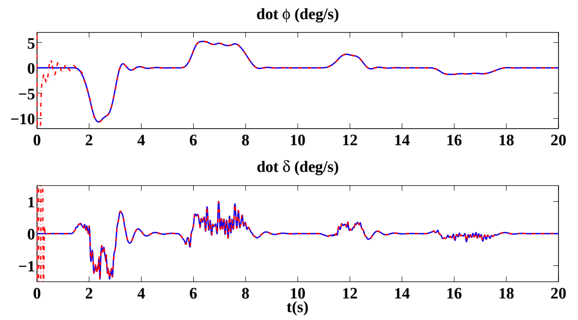

Fig. 6. Actual state (solid blue line) and estimated states (dashed red line)

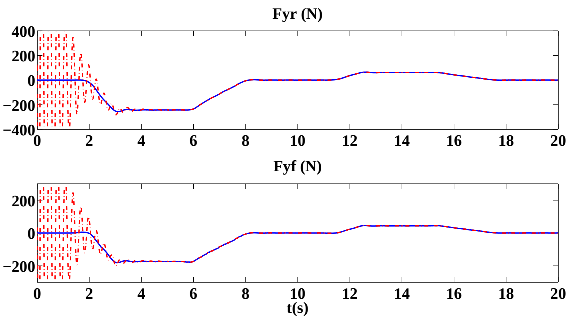

Fig. 7. Actual state (solid blue line) and estimated states (dashed red line)

us consider the initial conditions of the system as $x(0) = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0.1]$ and $\phi(0) = 0$. The simulation results concerning the dynamics states and their estimations are depicted in figures 5 (top), 6 and 4. The lateral forces on each wheel (rear and front) are illustrated in figure 7. Finally, the unknown inputs (roll angle $\phi(t)$ and rider torque $\tau(t)$ are shown in figures 5 (bottom) and 8. One can conclude that the UIHOSMO provides satisfactory results. In this work, only state and unknown input estimation is considered, but it is possible to deal with the problem of oscillations by parameter adjustment of the observer (λ).

B. Simulation With Uncertain Parameters

In order to illustrate the performances of the proposed observer in the presence of modeling uncertainties, the observer is designed by using the motorcycle nominal model and applied to the system with 20% parameters variation with respect to nominal values. As shown in figures 9, 10 and 11 the observer provides accurate estimations for almost the state variables. The effect of the uncertainties can be seen

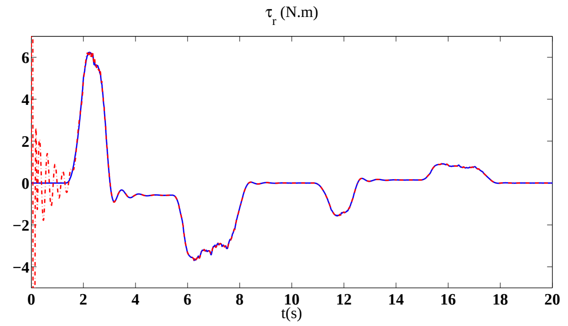

Fig. 8. Actual states (solid blue line) and estimated states (dashed red line)

only in the estimation of the lateral forces and rider torque. Nevertheless, the obtained estimations are acceptable. As

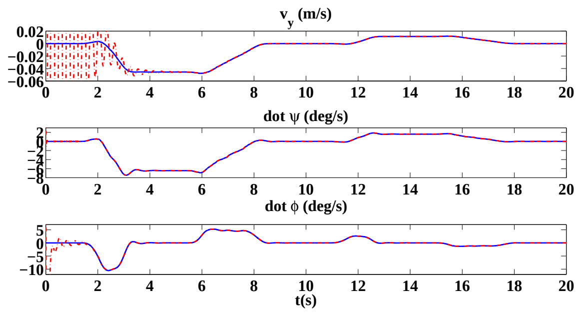

Fig. 9. Actual states (solid blue line) and estimated states (dashed red line) in the presence of 20% variations in the parameters

explained previously, the longitudinal velocity v_x is considered constant, but in practice, it is not. The proposed observer is subjected to a time-varying $v_x \in [13.8, 15.3](m/s)$ figure 12. The estimated states in this case are illustrated in the figure 13. Notice that the observer provides an acceptable state estimation for almost all states, only the estimations of $\dot{\phi}(t)$ and $\tau(t)$ are faintly affected, in particular, in the range time $[3, 5.5](s)$ where $v_x(t) = 13.8m/s$ which is far from the nominal value of $15m/s$.

To end the simulation part, let us consider the abilities of the proposed observer in noisy measurement case. A centered random noise in the range $[-0.0001, 0.0001]$ is added to the measurements $\delta(t)$ and $\dot{\psi}(t)$. The obtained results are depicted in the figure 14. The states are correctly estimated expect the steering angle rate $\dot{\delta}(t)$. This problem can be solved by reducing the value of the parameters $\lambda_i, i = 1, 2$, for example, with the values $\lambda_1 = \lambda_2 = 10$, the obtained

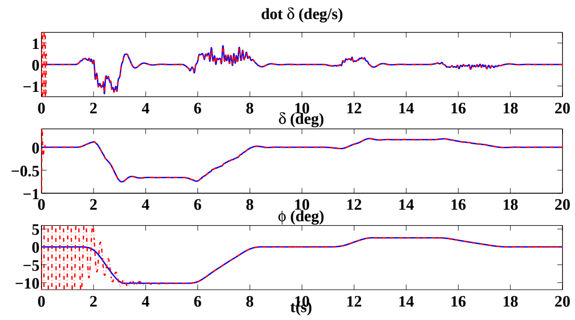

Fig. 10. Actual states (solid blue line) and estimated states (dashed red line) in the presence of 20% variations in the parameters

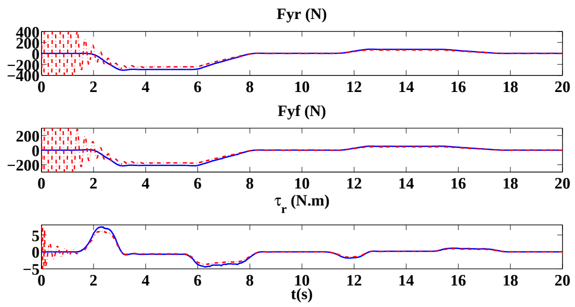

Fig. 11. Actual states (solid blue line) and estimated states (dashed red line) in the presence of 20% variations in the parameters

results in figure 15 are less affected by the measurement noises and more accurate estimations are obtained.

VI. CONCLUSION

In this article, the problem of observer design for simultaneously estimating the dynamics states of a motorcycle, an unknown inputs (rider's action) and the lateral forces on each wheel (front and rear) is considered. For that purpose, an Unknown Inputs High Order Sliding Mode observer is proposed. A motorcycle model similar to the well-known Sharp model is derived using JourdainŠs principle. The observability of the initial model is not guaranteed, then a transformation of this last into a model with two inputs by considering the unobservable state $\phi(t)$ as an unknown input as well as the rider torque applied on the handlebar. The obtained model is then exploited to construct the Unknown Inputs High Order sliding mode observer. Some simulation results are provided in order to illustrate the efficiency of the High Order Sliding Mode Observer.

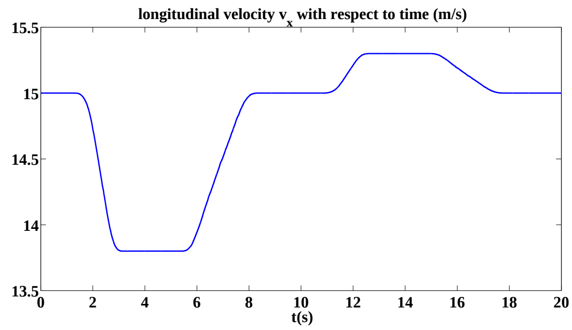

Fig. 12. Longitudinal velocity with respect to time

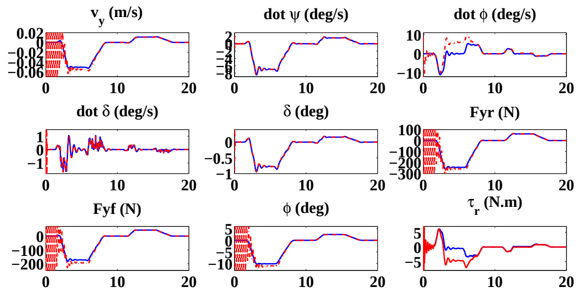

Fig. 13. Actual states (blue line) and estimated states (red line) in time varying longitudinal velocity situation

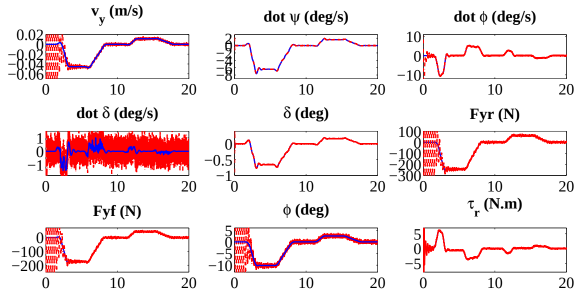

Fig. 14. Actual states (blue line) and estimated states (red line) in noised measurement situation

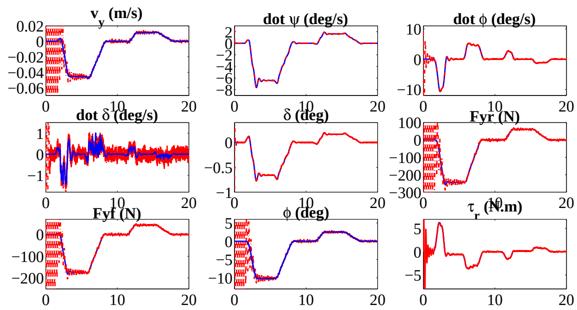

Fig. 15. State estimation with noised measurements ($\lambda_1 = \lambda_2 = 10$)

VII. APPENDIX

motorcycle	
v_x, v_y:	longitudinal and lateral velocity
φ, ψ, δ:	roll, yaw and steer rotations
τ_r:	rider torque
F_{yf}, F_{yr}:	lateral force
$\boldsymbol{A}_v$	state matrix $\boldsymbol{A}_v = \boldsymbol{M}^{-1}\boldsymbol{E}$
$\boldsymbol{B}_v$	input matrix $\boldsymbol{B}_v = \boldsymbol{M}^{-1}[0,0,0,1,0,0,0]^T$
$\boldsymbol{M}$:	motorcycle mass matrix
$\boldsymbol{E}$:	motorcycle generalized effort vector
notations	
$\dot{x}$, $\ddot{x}$:	derivatives of a variable x w.r.t time
$\hat{x}$:	estimate of a variable x
x^T:	transpose of vector or matrix x
x_f, x_r	denotes front and rear
motorcycle [15]	
m_f, m_r	front and rear mass 30.65, 217.45 [kg]
Z_f	front tire normal force -1005.3 [N]
I_{fx}, I_{fz}	front body inertia 1.23, 0.44 [kg.m^2]
I_{rx}, I_{rz}, C_{rxz}	rear body inertia 31.18, 21.07, 1.73 [kg.m^2]
i_{fy}, i_{ry}	wheels spin inertia 0.718, 1.051 [kg.m^2]
R_f, R_r	wheels radius 0.304, $R_r = 0.304$ [m]
σ_f, σ_r	tire's relaxation [m]
a, b, h, e, f	0.948, 0.479, 0.615, 0.024, 0.028 [m]
η	pneumatic trail 0.116 [m]
ϵ	castor angle 0.4715 [rad]
C_δ	handlebar damping [N.m^{-1}.s]
g	gravity force 9.81 [N]
l	vehicle wheelbase [m]

$$\boldsymbol{M} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & 0 & 0 & 0 & 0 \\ & a_{22} & a_{23} & a_{24} & 0 & 0 & 0 & 0 \\ & & a_{33} & a_{34} & 0 & 0 & 0 & 0 \\ & & & a_{44} & 0 & 0 & 0 & 0 \\ & \star & & & -1 & 0 & 0 & 0 \\ & & & & & -1 & 0 & 0 \\ & & & & & 0 & \sigma_r & 0 \\ & & & & & & & \sigma_f \end{bmatrix}$$

$$\boldsymbol{E} = \begin{bmatrix} 0 & b_{12} & 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & b_{22} & b_{23} & b_{24} & 0 & 0 & -b & -l \\ 0 & b_{32} & 0 & b_{34} & b_{35} & b_{36} & 0 & 0 \\ 0 & b_{42} & b_{43} & C_\delta & b_{45} & b_{46} & 0 & \eta \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

parameters a_{ij}
$a_{11} = m_f + m_r \quad a_{12} = m_f k \quad a_{13} = m_r h + m_f j$
$a_{14} = m_f e \quad a_{22} = m_f k^2 + I_{rz} + I_{fz}\cos^2\epsilon + I_{fx}\sin^2\epsilon$
$a_{23} = m_f kj - C_{rxz} + (I_{fz} - I_{fx})\cos\epsilon\sin\epsilon$
$a_{24} = m_f ek + I_{fz}\cos\epsilon$
$a_{33} = m_r h^2 + m_f j^2 + I_{rx} + I_{fx}\cos^2\epsilon + I_{fz}\sin^2\epsilon$
$a_{34} = m_f ej + I_{fz}\sin\epsilon \qquad a_{44} = m_f e^2 + I_{fz}$
parameters b_{ij}
$b_{12} = (m_f + m_r)v_x \quad b_{22} = m_f k v_x$
$b_{23} = -(i_{fy}/R_f + i_{Ry}/R_r)v_x \qquad b_{24} = -i_{fy}/R_f \sin\epsilon v_x$
$b_{32} = (m_f j + m_r h + i_{fy}/R_f + i_{Ry}/R_r)v_x$
$b_{34} = i_{fy}/R_f \cos\epsilon v_x$
$b_{35} = -(m_f j + m_r h)g \qquad b_{36} = Z_f\eta - m_f eg$
$b_{42} = (m_f e + i_{fy}/R_f \sin\epsilon)v_x \qquad b_{43} = -i_{fy}/R_f \cos\epsilon v_x$
$b_{45} = Z_f\eta - m_f eg \qquad b_{46} = (Z_f\eta - m_f eg)\sin\epsilon$

REFERENCES

[1] B.A.M. van Daal, "Design and automatic tuning of a motorcycle state estimator". Master thesis, TNO Science & Industry, Eindhoven University of Technology, Eindhoven, Nederland, 2009.

[2] S. Bobbo and V. Cossalter and M. Massaro and M. Peretto, "Application of the ŞOptimal Maneuver MethodŤ for Enhancing Racing Motorcycle Performance". SAE International Journal of Passenger Cars - Mechanical Systems, pp. 1311-1318, 2009.

[3] Evangelos Bekiaris, "SAFERIDER Project". Final Report nř RIDER200503-10, French National Agency of Reaserch, 2010.

[4] Ichalal, D. and Arioui, H. and Mammar, S., "Observer design for two-wheeled vehicle: A Takagi-Sugeno approach with unmeasurable premise variables". 19th Mediterranean Conference on Control & Automation (MED'11), Corfu, Greece, 2011.

[5] H. Slimi and H. Arioui and S. Mammar, "Motorcycle Speed Profile in Cornering Situation". American Control Conference (ACC'10), Baltimore, Maryland, USA, pp. 1172-1177, 2010.

[6] C. Edwards, S.K. Spurgeon and C.P. Tan, "On development and applications of sliding mode observers". In Variable Structure Systems: Towards 21st Century, Lecture Notes in Control and Information Science, Vol. 274, Berlin, 2002.

[7] A. Levant, "Robust Exact Differentiation via Sliding Mode Technique". Automatica, Vol. 34(3), pp. 379-384, 1998.

[8] T. Floquet and J.P. Barbot, "A canonical form for the design of unknown input sliding mode observers". In Advances in variable structure and sliding mode control, Lecture Notes in Control and Information Science, Vol. 334, Springer, Berlin, 2006.

[9] C.P. Tan and C. Edwards, "An LMI Approach for Designing Sliding Mode Observers". In Proc. of the IEEE CDC, Sydney, Australia, 2000.

[10] L. Fridman, A. Levant and J. Davila, "Observation and Identification Via High-Order Sliding Modes". Modern Sliding Mode Control Theory, Lecture Notes in Control and Information Science, Springer, Berlin, 2008.

[11] J. Davila, L. Fridman and A. Levant, "Second-order sliding-mode observer for mechanical systems", IEEE Trans. on Automatic Control, Vol. 50, pp. 1785-1789, 2005.

[12] L. Fridman, A. Levant and J. Davila, "Observation of linear systems with unknown inputs via high-order sliding-modes ". Int. J. of Systems Science, Vol. 38(10), 773-791, 2007.

[13] A. Levant, "Higher-order sliding modes, differentiation and output-feedback control". Int. J. of Control, Vol. 76, 924-941, 2003.

[14] L. Fridman, J. Davila and A. Levant, "Higher-order sliding modes observation for linear systems with unknown inputs". Nonlinear Analysis: Hybrid Systems, Vol. 5, pp. 189-205, 2011.

[15] R.S. Sharp, "The stability and control of motorcycles", J.of Mechanical Engineering Science, Vol. 13, 316-329, 1971.

[16] R.S. Sharp and D.J.N. Limebeer, "A Motorcycle Model for Stability and Control Analysis", Multibody System Dynamics, vol. 6, pp 123-142, 2001.

[17] S. Hima, L. Nehaoua, N. Seguy and H. Arioui, "Suitable Two Wheeled Vehicle Dynamics Synthesis for Interactive Motorcycle Simulator", In Proc. of the IFAC World Congress, pp 96-101, Seoul, Korea, July 6-11, 2008.

[18] G. Rill, "Simulation von Kraft-fahrzeugen", Vieweg, Braunschweig, Germany, 1994.

[19] A. Isidori, "Nonlinear Control Systems", London: Springer-Verlag, 1996.

[20] A. Ilchmann, E.P. Ryan and P. Townsend, "Tracking with prescribed transient behaviour for nonlinear systems of known relative degree". SIAM J. of Control and Optimization, Vol. 46, 210-230, 2007.

[21] H. Slimi, H. Arioui, L. Nouveliere and S. Mammar, "Advanced Motorcycle-Infrastructure-Driver Roll Angle Profile for Loss Control Prevention", 12th International IEEE Conference on Intelligent Transportation Systems., St. Louis, Missouri, U.S.A., October 3-7, 2009