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State-of-Charge Estimation for Batteries: A Multi-model Approach

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Abstract—Monitoring the state-of-charge (SoC) for batteries is challenging, especially when a battery has time-varying parameters. We propose to improve SoC estimation using an adaptive strategy and *multiple models* in this study, developing a unique algorithm called **MM-AdaSoC**. Specifically, two submodels in state-space form are generated from a modified Nernst battery model. Both are shown to be locally observable under mild conditions. The iterated extended Kalman filter (IEKF) is then applied to each submodel in parallel, estimating simultaneously the SoC variable and certain unknown parameters. The SoC estimates obtained from the two separately implemented IEKFs are fused to yield the final overall SoC estimates, which tend to have higher accuracy than those obtained from a single-model. Its effectiveness is demonstrated via experiments.

I. INTRODUCTION

State-of-charge estimation is a fundamental component of battery management systems to ensure the operational safety and performance of batteries [2]. It has remained an active research field during the past years, and the reader may refer to [3] for a survey. A notable trend in this area is the increasing emphasis on model-based SoC estimation methods. The dynamic models, derived from either equivalent circuits or electrochemical principles, facilitate the assimilation of the battery data and lead to real-time SoC estimation with bounded errors. Application of the Kalman filtering (KF) techniques has been remarkable in this respect. The classical linear KF and its extensions to nonlinear systems, including the extended KF (EKF), unscented KF (UKF), iterated extended KF (IEKF), have been used to deal with SoC estimation based on electrochemical and equivalent circuit models, see [4–14]. A variety of other state observers originating from control approaches have also played a role in constructing SoC estimators, including the sliding mode observer [15], adaptive model reference observer [16], Lyapunov-based observer [17] and PDE-based observer [18; 19].

A good battery dynamic model is a prerequisite for model-based SoC estimation. Unfortunately, accurate estimation of parameters is far from trivial due to time-varying operational

conditions and variability in battery manufacturing. Therefore, adaptive approaches are desirable, merging both model parameter estimation and SoC estimation in one step. An adaptive SoC estimator gives not only the SoC estimates but also the estimates of the model parameters in real time after assimilating the current-voltage data on the basis of a model. The parameter estimates will then be used to update the model to aid the next-step estimation. An adaptive EKF-based SoC estimator is designed in [9], which interacts with a parameter estimator. In [11], state augmentation is conducted to incorporate the SoC variable and model parameters, and then the UKF is applied to estimate the augmented state. However, the convergence, and as a result, the accuracy, are noted to be difficult to guarantee. In [13], an adaptive SoC estimator is developed using the IEKF, guided by an analysis of the observability/identifiability. A nonlinear geometric adaptive observer is studied in [20] for SoC estimation. Novel adaptive PDE observers for SoC estimation have also been reported in [21]. It should be noted that all these existing approaches are based on a single battery model, and we instead propose to exploit multiple models for better estimation performance.

Aiming to achieve *adaptive, high-fidelity and easy-to-implement SoC estimation*, we seamlessly link the notion of ‘multiple models’ and adaptive SoC estimation in this paper. A multitude of models, compared to a single one, can give better description of complicated uncertain dynamics [22–24], thus particularly suitable to deal with the tasks relevant to batteries. The design of the adaptive SoC estimator partially builds on our previous work [13; 14], where we propose an adaptive approach for SoC estimation via IEKF-based simultaneous state and parameter estimation. While credible estimation is observed, the accuracy is still limited by the mismatch between the model and the true system. This fact motivates the development of the **MM-AdaSoC** algorithm in this paper.

The rest of the paper is organized as follows. Section II presents a basic review of the multi-model estimation theory. Section III describes the model construction and gives observability analysis. Section IV incorporates adaptive SoC estimation and multi-model estimation to establish the **MM-AdaSoC** algorithm, the effectiveness of which is validated in Section V by experimental results. Finally, Section VI gathers our conclusions.

II. BASICS OF MULTI-MODEL ESTIMATION

The structure of a typical multi-model estimator is shown in Fig. 1. In this section, we give a review of the multi-model

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A complete version of this paper is available at [1].

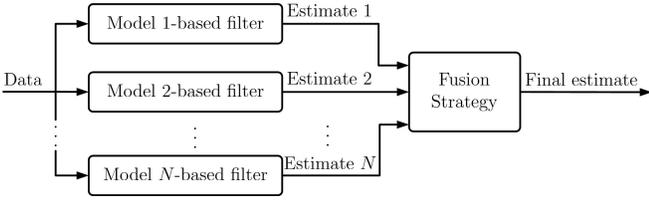


Fig. 1: The structure of a multi-model estimator.

estimation, with an emphasis on the estimate fusion strategy.

Its first part is composed of a bank of parallel filters based on different models. Each filter assimilates the data to produce its own estimate. All the estimates will then be fused to give the best estimate. Various options exist for the elemental filter, such as the KF for a linear model or the EKF for a nonlinear one. What is of particular interest here is the design of the fusion strategy.

Let us consider a general system. Its unknown state at time instant k is denoted by $\mathbf{x}_k \in \mathbb{R}^{n_x}$ and its measurement by $\mathbf{z}_k \in \mathbb{R}^{n_z}$. Different models are available to describe the system, leading to a model set $\mathbb{M} = \{\mathcal{M}_1, \mathcal{M}_2, \dots, \mathcal{M}_N\}$. Suppose that \mathcal{M}_i is given by

$$\mathcal{M}_i : \begin{cases} \mathbf{x}_{k+1} = \mathbf{f}^i(\mathbf{x}_k) + \mathbf{w}_k^i, \\ \mathbf{z}_k = \mathbf{h}^i(\mathbf{x}_k) + \mathbf{v}_k^i, \end{cases} \quad (1)$$

where \mathbf{f}^i and \mathbf{h}^i are C^1 functions to represent the state transition and measurement, respectively, and $\{\mathbf{w}_k^i\}$ and $\{\mathbf{v}_k^i\}$ are uncorrelated, zero-mean, white Gaussian noise sequences with covariances $\mathbf{Q}_k^i \geq 0$ and $\mathbf{R}_k^i > 0$, respectively. While assuming that the true system coincides with one model at each time instant, we do not know which model matches the system at any time. Thus a probabilistic description is used. Let s_k denote the system running status at k . It may take any \mathcal{M}_i for $i = 1, 2, \dots, N$ to address the uncertainty of model matching. The probability of the event $s_k = \mathcal{M}_i$ is denoted as $p(s_k = \mathcal{M}_i)$, or simply, $p(s_k^i)$. In other words, $p(s_k^i)$ indicates the *a priori* probability that the true model is \mathcal{M}_i at time k . Obviously, $\sum_{i=1}^N p(s_k^i) = 1$.

From a statistical perspective, \mathbf{x}_k and \mathbf{z}_k are continuous random variables and s_k a discrete one. Without causing confusion, we use the symbol p to denote the probability density function (pdf), probability mass function (pmf) or mixed pdf-pmf in the sequel for convenience. We define the information set as $\mathbb{Z}_k = \{\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_k\}$ and intend to estimate \mathbf{x}_k from \mathbb{Z}_k , hence considering $p(\mathbf{x}_k|\mathbb{Z}_k)$. By the Bayes' theorem, we have

$$\begin{aligned} p(\mathbf{x}_k|\mathbb{Z}_k) &= \sum_{i=1}^N p(\mathbf{x}_k, s_k^i|\mathbb{Z}_k) \\ &= \sum_{i=1}^N p(\mathbf{x}_k|s_k^i, \mathbb{Z}_k)p(s_k^i|\mathbb{Z}_k). \end{aligned} \quad (2)$$

When $p(\mathbf{x}_k|\mathbb{Z}_k)$ becomes available, we can carry out minimum-mean-square-error (MMSE) estimation or Maxi-

mum a Posteriori (MAP) estimation of \mathbf{x}_k :

$$\text{MMSE: } \hat{\mathbf{x}}_{k|k} = \mathbb{E}(\mathbf{x}_k|\mathbb{Z}_k) = \int \mathbf{x}_k p(\mathbf{x}_k|\mathbb{Z}_k) d\mathbf{x}_k,$$

$$\text{MAP: } \hat{\mathbf{x}}_{k|k} = \arg \max_{\mathbf{x}_k} p(\mathbf{x}_k|\mathbb{Z}_k).$$

Independent of the method (MMSE or MAP) used, it follows from (2) that

$$\hat{\mathbf{x}}_{k|k} = \sum_{i=1}^N \hat{\mathbf{x}}_{k|k}^i p(s_k^i|\mathbb{Z}_k), \quad (3)$$

where $\hat{\mathbf{x}}_{k|k}^i$ is the estimate of \mathbf{x}_k based on the model \mathcal{M}_i . An observation from this analysis is that $p(s_k^i|\mathbb{Z}_k)$ turns out to be a probabilistic weight coefficient. The associated estimation error covariance is

$$\begin{aligned} \mathbf{P}_{k|k} &= \mathbb{E}[(\hat{\mathbf{x}}_k - \mathbf{x}_k)(\hat{\mathbf{x}}_k - \mathbf{x}_k)^\top | \mathbb{Z}_k] \\ &= \sum_{i=1}^N \left[\mathbf{P}_{k|k}^i + (\hat{\mathbf{x}}_k - \hat{\mathbf{x}}_k^i)(\hat{\mathbf{x}}_k - \hat{\mathbf{x}}_k^i)^\top \right] p(s_k^i|\mathbb{Z}_k). \end{aligned} \quad (4)$$

Let us take a closer look at $p(s_k^i|\mathbb{Z}_k)$:

$$\begin{aligned} p(s_k^i|\mathbb{Z}_k) &= \frac{p(s_k^i, \mathbb{Z}_k)}{p(\mathbb{Z}_k)} = \frac{p(\mathbf{z}_k | s_k^i, \mathbb{Z}_{k-1}) p(s_k^i | \mathbb{Z}_{k-1})}{p(\mathbf{z}_k | \mathbb{Z}_{k-1})} \\ &= \frac{p(\mathbf{z}_k | s_k^i, \mathbb{Z}_{k-1}) p(s_k^i | \mathbb{Z}_{k-1})}{\sum_{j=1}^N p(\mathbf{z}_k | s_k^j, \mathbb{Z}_{k-1}) p(s_k^j | \mathbb{Z}_{k-1})}. \end{aligned} \quad (5)$$

Furthermore, we have

$$\begin{aligned} p(\mathbf{z}_k | s_k^i, \mathbb{Z}_{k-1}) &= \int p(\mathbf{z}_k, \mathbf{x}_k | s_k^i, \mathbb{Z}_{k-1}) d\mathbf{x}_k \\ &= \int p(\mathbf{z}_k | \mathbf{x}_k, s_k^i) p(\mathbf{x}_k | s_k^i, \mathbb{Z}_{k-1}) d\mathbf{x}_k. \end{aligned}$$

Under the mildly simplified assumption that $p(\mathbf{z}_k | \mathbf{x}_k, s_k^i) = \mathcal{N}(\mathbf{h}^i(\mathbf{x}_k), \mathbf{R}_k^i)$ and $p(\mathbf{x}_k | s_k^i, \mathbb{Z}_{k-1}) = \mathcal{N}(\hat{\mathbf{x}}_{k|k-1}^i, \mathbf{P}_{k|k-1}^i)$, $p(\mathbf{z}_k | s_k^i, \mathbb{Z}_{k-1})$ can be approximated as

$$p(\mathbf{z}_k | s_k^i, \mathbb{Z}_{k-1}) \approx (2\pi)^{-\frac{n_z}{2}} |\mathbf{S}_k^i|^{-\frac{1}{2}} \exp \left[-\frac{1}{2} (\tilde{\mathbf{z}}_k^i)^\top |\mathbf{S}_k^i|^{-1} \tilde{\mathbf{z}}_k^i \right],$$

where $\tilde{\mathbf{z}}_k^i = \mathbf{z}_k - \mathbf{h}^i(\hat{\mathbf{x}}_{k|k-1}^i)$, $\mathbf{S}_k^i = \mathbf{H}_k^i \mathbf{P}_{k|k-1}^i (\mathbf{H}_k^i)^\top + \mathbf{R}_k^i$ and $\mathbf{H}_k^i = \frac{\partial \mathbf{h}^i}{\partial \mathbf{x}}(\hat{\mathbf{x}}_{k|k-1}^i)$. Furthermore,

$$p(s_k^i | \mathbb{Z}_{k-1}) = \frac{p(\mathbb{Z}_{k-1} | s_k^i) p(s_k^i)}{p(\mathbb{Z}_{k-1})} = p(s_k^i),$$

since $p(\mathbb{Z}_{k-1} | s_k^i) = 1$ and $p(\mathbb{Z}_{k-1}) = 1$ because \mathbb{Z}_{k-1} is an event with probability 1 at time k . If we define $\mu_k^i = p(s_k^i | \mathbb{Z}_k)$ and $w_k^i = p(\mathbf{z}_k | s_k^i, \mathbb{Z}_{k-1})$ and suppose $\pi_k^i = p(s_k^i)$, (5) becomes

$$\mu_k^i = \frac{w_k^i \pi_k^i}{\sum_{j=1}^N w_k^j \pi_k^j}. \quad (6)$$

Hence, by (3)-(4), the fusion strategy is given by

$$\hat{\mathbf{x}}_{k|k} = \sum_{i=1}^N \hat{\mathbf{x}}_{k|k}^i \mu_k^i, \quad (7)$$

$$\mathbf{P}_{k|k} = \sum_{i=1}^N \left[\mathbf{P}_{k|k}^i + (\hat{\mathbf{x}}_k - \hat{\mathbf{x}}_k^i)(\hat{\mathbf{x}}_k - \hat{\mathbf{x}}_k^i)^\top \right] \mu_k^i. \quad (8)$$

The final conclusion drawn from this analysis is as follows: the fused estimate (covariance) is a linear weighted combination of the estimates from the elemental filters. It can be noted that

- The estimation is based on a series of elemental filters and the fusion. The process is similar to a ‘weight-based reconciliation’, which balances the role that different models potentially play in the estimation task.
- The residuals of the elemental filter based on the ‘correct’ model that best matches the true system should be remarkably smaller than those of the others [22]. As a result, its the probabilistic weight will tend to increase and downplay the others. The fused estimate will approach the estimate based on the correct model.

III. BATTERY MODELS AND OBSERVABILITY ANALYSIS

We investigate the battery modeling in this section. We first develop two submodels from a slightly modified Nernst model and then analyze each one’s observability.

A. Construction of Multiple Battery Models

A battery model consists of a set of equations that relate the input u_k (charging/discharging current), the state variables (e.g., SoC) and the output y_k (terminal voltage) sampled at discrete-time instants indicated by the subscript k . Various models have been proposed and used, depending on the specific purposes. For SoC estimation, we consider the Nernst model here [5]:

$$y_k = K_1 + K_2 \ln(\text{SoC}_k) + K_3 \ln(1 - \text{SoC}_k) - Ru_k, \quad (9)$$

where y_k is the terminal voltage, u_k is the applied current ($u > 0$ for discharging and $u < 0$ for charging), R is the internal resistance, and K_i for $i = 1, 2, 3$ are constants. To make (9) more capable of grasping the dynamics of certain batteries, we propose the following modification:

$$y_k = K_1 + K_2 \ln(\tau_1 + \text{SoC}_k) + K_3 \ln(\tau_2 + 1 - \text{SoC}_k) - Ru_k, \quad (10)$$

where two additional constants τ_1 and τ_2 are added. In above, $K_1 + K_2 \ln(\tau_1 + \text{SoC}_k) + K_3 \ln(\tau_2 + 1 - \text{SoC}_k)$ in (10) can be regarded as the open-circuit voltage (OCV) term. The dynamic change of the SoC is described by the integration of the current over time. In the discrete time, it is given by

$$\text{SoC}_k = \text{SoC}_0 - \sum_{i=0}^{k-1} \frac{\eta \cdot \Delta T}{C_0} u_i,$$

where η is the Coulombic efficiency, C_0 the nominal capacity in ampere-hour (Ah), and ΔT is the sampling period. An equivalent difference equation is

$$\text{SoC}_{k+1} = \text{SoC}_k - K_0 u_k, \quad (11)$$

where $K_0 = \eta \cdot \Delta T / C_0$. We then obtain a state-space model for batteries by putting together (10)-(11). The model state is SoC_k and the parameters are K_i for $i = 0, \dots, 3$ and R .

For adaptive SoC estimation, we will perform simultaneous estimation of the SoC and the parameters. To obtain a locally observable model, one or several parameters usually need to be fixed in order to estimate the others and the SoC. While a few options may exist regarding which parameters are assumed fixed or unknown, we separate the parameters into two sets, fix one set and augment the state vector to incorporate the SoC and the other set. Accordingly, two submodels will be constructed.

Letting K_0 and K_1 be fixed, the first one can be obtained:

$$\mathcal{M}_1 : \begin{cases} \mathbf{x}_{k+1}^1 = \mathbf{f}^1(\mathbf{x}_k^1, u_k), \\ y_k = h^1(\mathbf{x}_k^1, u_k), \end{cases} \quad (12)$$

where

$$\begin{aligned} \mathbf{x}_k^1 &= [\text{SoC}_k \quad K_2 \quad K_3 \quad R]^\top, \\ \mathbf{f}_1(\mathbf{x}_k^1, u_k) &= \mathbf{x}_k^1 - [K_0 \quad 0 \quad 0 \quad 0]^\top u_k, \\ h_1(\mathbf{x}_k^1, u_k) &= K_1 + \mathbf{x}_{k,2}^1 \ln(\tau_1 + \mathbf{x}_{k,1}^1) \\ &\quad + \mathbf{x}_{k,3}^1 \ln(\tau_2 + 1 - \mathbf{x}_{k,1}^1) - \mathbf{x}_{k,4}^1 u_k. \end{aligned}$$

Analogously, by fixing K_i for $i = 1, 2, 3$, we have

$$\mathcal{M}_2 : \begin{cases} \mathbf{x}_{k+1}^2 = \mathbf{f}^2(\mathbf{x}_k^2, u_k), \\ y_k = h^2(\mathbf{x}_k^2, u_k), \end{cases} \quad (13)$$

where

$$\begin{aligned} \mathbf{x}_k^2 &= [\text{SoC}_k \quad K_0 \quad R]^\top, \\ \mathbf{f}^2(\mathbf{x}_k^2, u_k) &= [\mathbf{x}_{k,1}^2 - \mathbf{x}_{k,2}^2 u_k \quad 0 \quad 0]^\top, \\ h^2(\mathbf{x}_k^2, u_k) &= K_1 + K_2 \ln(\tau_1 + \mathbf{x}_{k,1}^2) \\ &\quad + K_3 \ln(\tau_2 + 1 - \mathbf{x}_{k,1}^2) - \mathbf{x}_{k,3}^2 u_k. \end{aligned}$$

Remark 1: In an implicit manner, \mathcal{M}_1 places more confidence on the state equation (11), assuming that K_0 is accurate, while the belief in the measurement equation (10) is emphasized in \mathcal{M}_2 similarly. Nevertheless, it is noteworthy that the confidence level on each submodel during the estimation process is dynamically determined by the fusion strategy outlined earlier in Section II.

Remark 2: An extended series can be constructed on the basis of each submodel if we let the parameters take different values that are believed to be close or equal to the truth. For instance, the Coulombic efficiency may be 100%, 90% or even 80% depending on the operating conditions. Then \mathcal{M}_1 will give birth to three more submodels if K_0 assumes $\Delta T / C_0$, $0.9 \Delta T / C_0$ and $0.8 \Delta T / C_0$, respectively. This allows considerable flexibility for us to describe the battery dynamics and brings improvements to the single-model case.

B. Observability Analysis

Here, we will analyze the observability properties of \mathcal{M}_1 and \mathcal{M}_2 before proceeding to SoC estimation. Consider a general single-input-single-output system

$$\mathcal{S} : \begin{cases} \mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k, u_k), \\ y_k = h(\mathbf{x}_k, u_k), \end{cases} \quad (14)$$

where $\mathbf{x} \in \mathbb{X}$ of dimension n , $y \in \mathbb{Y}$ and $u \in \mathbb{U}$. We assume that 1) \mathbb{X} and \mathbb{Y} connected, second countable, Hausdorff, differentiable manifolds of class C^q with $q \in \mathbb{N}$, 2) \mathbb{U} is an open interval of \mathbb{R} , and 3) $\mathbf{f} : \mathbb{X} \times \mathbb{U} \rightarrow \mathbb{X}$ and $h : \mathbb{X} \rightarrow \mathbb{Y}$ are of class C^q . For convenience, $\mathbf{f}(\mathbf{x}, u)$ is denoted as $\mathbf{f}^u(\mathbf{x})$, and $h(\mathbf{f}(\mathbf{x}, u_0), u_1) = h^{u_1} \circ \mathbf{f}^{u_0}(\mathbf{x})$. Following [25; 26], the local observability for \mathcal{S} is defined as follows:

Definition 1: (Distinguishability) Two states \mathbf{x} and \mathbf{x}^* are said to be indistinguishable, written as $\mathbf{x} \rightleftharpoons \mathbf{x}^*$, if for each $l \neq 0$ and for each input sequence, $\{u_0, \dots, u_l\} \in \mathbb{U}^l$, we have

$$h^{u_l} \circ \mathbf{f}^{u_{l-1}} \circ \dots \circ \mathbf{f}^{u_0}(\mathbf{x}) = h^{u_l} \circ \mathbf{f}^{u_{l-1}} \circ \dots \circ \mathbf{f}^{u_0}(\mathbf{x}^*).$$

Otherwise, they are distinguishable.

Definition 2: (Local observability) The system \mathcal{S} is locally observable if for any state $\mathbf{x}^o \in \mathbb{X}$, there exists a neighborhood \mathbb{D} of \mathbf{x}^o such that, $\mathbf{x} \rightleftharpoons \mathbf{x}^*$ implies $\mathbf{x} = \mathbf{x}^*$ for each $\mathbf{x}, \mathbf{x}^* \in \mathbb{D}$.

To address the observability condition, the following sets of functions are defined:

$$\begin{aligned} \Omega_0 &= \{h(\cdot)\}, \\ \Omega_l &= \{h^{u_j} \circ \mathbf{f}^{u_{j-1}} \circ \dots \circ \mathbf{f}^{u_0}(\cdot) : \\ &\quad u_i \in \mathbb{U} \forall i = 1, \dots, j \text{ and } 1 \leq j \leq l\}, \\ \Omega &= \cup_{j \geq 0} \Omega_j. \end{aligned}$$

An observability criterion is presented in the following theorem, please see [25] for the proof.

Theorem 1: If $\dim d\Omega(\mathbf{x}) = n \forall \mathbf{x} \in \mathbb{X}$, then the system \mathcal{S} is locally observable.

The results and proof of Theorem 1 can be found in [25]. Theorem 1 gives a sufficient condition to determine the local observability by relating it to the full dimensionality of the codistribution $d\Omega$. Now the local observability of \mathcal{M}_1 and \mathcal{M}_2 can be analyzed using Theorem 1. Let us take \mathcal{M}_1 for an example since the analysis for both follows similar lines.

Note that \mathbf{f}^1 and h^1 are of class C^∞ . Suppose that the initial state is \mathbf{x}_0^1 for \mathcal{M}_1 and that there are L measurements $\{y_1, \dots, y_L\}$. By (12), \mathbf{x}_k^1 is given by

$$\mathbf{x}_k^1 = \mathbf{x}_0^1 - [K_0 \quad 0 \quad 0 \quad 0]^\top \sum_{i=0}^{k-1} u_i.$$

Hence, we have

$$\begin{aligned} \bar{h}_k^1(\mathbf{x}_0^1) &= h^{1u_k} \circ \mathbf{f}^{1u_{k-1}} \circ \dots \circ \mathbf{f}^{1u_0}(\mathbf{x}_0^1) \\ &= K_1 + \mathbf{x}_{0,2}^1 \ln \left(\tau_1 + \mathbf{x}_{0,1}^1 - K_0 \sum_{i=0}^{k-1} u_i \right) \\ &\quad + \mathbf{x}_{0,3}^1 \ln \left(\tau_2 + 1 - \mathbf{x}_{0,1}^1 + K_0 \sum_{i=0}^{k-1} u_i \right) - \mathbf{x}_{0,4}^1 u_k, \end{aligned}$$

where $\bar{h}_k^1 \in \Omega$. Define a matrix \mathbf{J} with dimensions $L \times 4$:

$$\mathbf{J} = \begin{bmatrix} \frac{d\bar{h}_1^1}{d\mathbf{x}_0^1} & \dots & \frac{d\bar{h}_k^1}{d\mathbf{x}_0^1} & \dots & \frac{d\bar{h}_L^1}{d\mathbf{x}_0^1} \end{bmatrix}^\top.$$

The elements in the k -th row of \mathbf{J} are

$$\begin{aligned} \mathbf{J}_{k,1} &= \frac{\partial \bar{h}_k^1}{\partial \mathbf{x}_{0,1}^1} = \frac{\mathbf{x}_{0,2}^1}{\tau_1 + \mathbf{x}_{0,1}^1 - K_0 \sum_{i=0}^{k-1} u_i} \\ &\quad - \frac{\mathbf{x}_{0,3}^1}{\tau_2 + 1 - \mathbf{x}_{0,1}^1 + K_0 \sum_{i=0}^{k-1} u_i}, \\ \mathbf{J}_{k,2} &= \frac{\partial \bar{h}_k^1}{\partial \mathbf{x}_{0,2}^1} = \ln \left(\tau_1 + \mathbf{x}_{0,1}^1 - K_0 \sum_{i=0}^{k-1} u_i \right), \\ \mathbf{J}_{k,3} &= \frac{\partial \bar{h}_k^1}{\partial \mathbf{x}_{0,3}^1} = \ln \left(\tau_2 + 1 - \mathbf{x}_{0,1}^1 + K_0 \sum_{i=0}^{k-1} u_i \right), \\ \mathbf{J}_{k,4} &= \frac{\partial \bar{h}_k^1}{\partial \mathbf{x}_{0,4}^1} = -u_k. \end{aligned}$$

By observation, we have the following conclusions:

- The submodel \mathcal{M}_1 is locally observable if a suitable input sequence $\{u_k\}$ is applied. By 'suitable', we mean that u_k varies sufficiently in magnitude over time, or in other words, $\{u_k\}$ contains a rich mix of frequency contents. In this case, \mathbf{J} will have full column rank, and as a result, $\dim d\Omega$ has a dimension of 4, satisfying the condition in Theorem 1. It should be emphasized such a condition imposed on the input is a mild constraint that can be easily satisfied when a battery is in use.
- We can analogously determine that \mathcal{M}_2 is also locally observable if a suitable $\{u_k\}$ is used to excite the system.

IV. MULTI-MODEL ADAPTIVE SOC ESTIMATION

In this section, an IEKF-based elemental filter will be applied to \mathcal{M}_1 and \mathcal{M}_2 , respectively, for adaptive SoC estimation. The overall estimate will be obtained by fusing all the estimates for the elemental filters, leading to the MM-AdaSoC algorithm.

Adaptive SoC estimation can be attained via state estimation, because the state vector of each consists of both the SoC variable and the parameters. Following [13], we use the IEKF. As an improved version of the EKF, it is capable of giving more accurate state estimates even for highly nonlinear systems by iteratively refining the estimate around the current point at each time instant.

Consider applying the IEKF to the system in (14). At $k-1$, prediction can be made about the next time instant. The formulas are as follows:

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{f}(\hat{\mathbf{x}}_{k-1|k-1}, u_{k-1}), \quad (15)$$

$$\mathbf{P}_{k|k-1} = \mathbf{F}_{k-1} \mathbf{P}_{k-1|k-1} \mathbf{F}_{k-1}^\top + \mathbf{Q}, \quad (16)$$

where $\hat{\mathbf{x}}$ is the estimate of \mathbf{x} , \mathbf{P} is the error covariance, $\mathbf{Q} \geq 0$ is an adjustable matrix to account for the process noise, and \mathbf{F} is given by

$$\mathbf{F}_{k-1} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}(\hat{\mathbf{x}}_{k-1|k-1}, u_{k-1}).$$

When the measurement y_k arrives, $\hat{\mathbf{x}}_{k|k-1}$ can be updated by the new information y_k brings. The procedure is based

on iteration. Let ℓ denote the iteration number and $\hat{\mathbf{x}}_{k|k}^{(\ell)} = \hat{\mathbf{x}}_{k|k-1}$ for $\ell = 0$. The update formulas are

$$\mathbf{K}_k^{(\ell)} = \mathbf{P}_{k|k-1} \mathbf{H}_k^{(\ell-1)} \left[\mathbf{H}_k^{(\ell-1)} \mathbf{P}_{k|k-1} \mathbf{H}_k^{(\ell-1)\top} + R \right]^{-1}, \quad (17)$$

$$\hat{y}_k^{(\ell)} = h \left(\hat{\mathbf{x}}_{k|k}^{(\ell-1)}, u_k \right), \quad (18)$$

$$\hat{\mathbf{x}}_{k|k}^{(\ell)} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k^{(\ell)} \left[y_k - \hat{y}_k^{(\ell)} - \mathbf{H}_k^{(\ell-1)} \left(\hat{\mathbf{x}}_{k|k-1} - \hat{\mathbf{x}}_{k|k}^{(\ell-1)} \right) \right], \quad (19)$$

where $R > 0$ accounts for the measurement noise and

$$\mathbf{H}_k^{(\ell)} = \frac{\partial h}{\partial \mathbf{x}} \left(\hat{\mathbf{x}}_{k|k}^{(\ell)}, u_k \right).$$

The iteration process stops when ℓ achieves the pre-specified maximum iteration number ℓ_{\max} or when the error between two consecutive iterations is less than the pre-selected tolerance level. Then $\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k}^{(\ell_{\max})}$, and the associated error covariance is given by

$$\mathbf{P}_{k|k} = \left[\mathbf{I} - \mathbf{K}_k^{(\ell_{\max})} \mathbf{H}_k^{(\ell_{\max})} \right] \mathbf{P}_{k|k-1}.$$

Following the above description, the IEKF can be applied as an elemental filter to \mathcal{M}_1 and \mathcal{M}_2 . The resultant state estimates are $\hat{\mathbf{x}}_{k|k}^1$ and $\hat{\mathbf{x}}_{k|k}^2$, respectively. Accordingly, the SoC estimates are denoted as $\widehat{\text{SoC}}_k^1 = \hat{\mathbf{x}}_{k|k,1}^1$ and $\widehat{\text{SoC}}_k^2 = \hat{\mathbf{x}}_{k|k,1}^2$, respectively. A weighted combination of them forms the overall estimate $\widehat{\text{SoC}}_k$. In the light of the fusion strategy in (7)-(8), we have

$$\widehat{\text{SoC}}_k = \sum_{i=1}^2 \widehat{\text{SoC}}_k^i \mu_i, \quad (20)$$

where the weight coefficient μ_i for $i = 1, 2$ can be determined using (6). Putting together the results, we obtain the MM-AdaSoC algorithm, which is summarized in Table I.

V. EXPERIMENTAL RESULTS

In this section, we present one example using experiment data to evaluate the MM-AdaSoC algorithm.

For the experimental evaluation of the MM-AdaSoC algorithm, data was collected from a Li⁺ battery in the Advanced Technology R&D Center, Mitsubishi Electric Corporation¹. The current input was a PRBS signal stretched by 10 times over the time axis with a magnitude of 5A. Despite many other options, we chose the PRBS because it has white-noise-like properties and is admissible for observability. The profile of the input current and the output voltage is shown in Fig. 2. The battery has a nominal capacity of 4.93Ah. The sampling period was 1s. During the experiment, the ambient temperature in the chamber was maintained at 25.8°C.

We consider the model in (10)-(11). The Coulombic efficiency constant $K_0 = 5.6342 \times 10^{-5}$ when $u_k > 0$ (100% for discharging) and $K_0 = 4.7891 \times 10^{-5}$ when $u_k < 0$

¹Technical specifications about the battery system are not available currently due to intellectual property protection status.

1:	initialize the implementation: $k = 0$, $\hat{\mathbf{x}}_{0 0}^i = \mathbf{x}_0^i$, $\mathbf{P}_{0 0}^i = \delta^i \mathbf{I}$, where $\delta^i \gg 0$, for $i = 1, 2$
2:	repeat
3:	$k \leftarrow k + 1$
	<u>IEKF based adaptive SoC estimation:</u>
4:	for $i = 1$ to 2 do
5:	import the submodel \mathcal{M}_i
	<u>\mathcal{M}_i-based prediction (time-update):</u>
6:	project the state ahead to obtain $\hat{\mathbf{x}}_{k k-1}^i$
	$\hat{\mathbf{x}}_{k k-1}^i = \mathbf{f}^i(\hat{\mathbf{x}}_{k-1 k-1}^i, u_k)$
7:	project the error covariance ahead to obtain $\mathbf{P}_{k k-1}^i$
	$\mathbf{F}_{k-1}^i = \frac{\partial \mathbf{f}^i}{\partial \mathbf{x}^i}(\hat{\mathbf{x}}_{k-1 k-1}^i, u_k)$
	$\mathbf{P}_{k k-1}^i = \mathbf{F}_{k-1}^i \mathbf{P}_{k-1 k-1}^i \mathbf{F}_{k-1}^{i\top} + \mathbf{Q}^i$
	<u>\mathcal{M}_i-based update (measurement-update):</u>
8:	initialize the iteration procedure: $\ell = 0$, $\hat{\mathbf{x}}_{k k}^i = \hat{\mathbf{x}}_{k k-1}^i$
9:	while $\ell < \ell_{\max}$ do
10:	$\ell \leftarrow \ell + 1$
11:	compute the Kalman gain matrix
	$\mathbf{H}_k^{i(\ell)} = \frac{\partial h^i}{\partial \mathbf{x}^i}(\hat{\mathbf{x}}_{k k}^{i(\ell)}, u_k)$
	$\mathbf{K}_k^{i(\ell)} = \mathbf{P}_{k k-1}^i \mathbf{H}_k^{i(\ell-1)} \left[\mathbf{H}_k^{i(\ell-1)} \mathbf{P}_{k k-1}^i \mathbf{H}_k^{i(\ell-1)\top} + R^i \right]^{-1}$
12:	update the state estimate
	$\hat{\mathbf{x}}_{k k}^{i(\ell)} = \hat{\mathbf{x}}_{k k-1}^i + \mathbf{K}_k^{i(\ell)} \left[y_k - h^i(\hat{\mathbf{x}}_{k k}^{i(\ell-1)}, u_k) - \mathbf{H}_k^{i(\ell-1)} (\hat{\mathbf{x}}_{k k-1}^i - \hat{\mathbf{x}}_{k k}^{i(\ell-1)}) \right]$
13:	end while
14:	assign $\hat{\mathbf{x}}_{k k}^i = \hat{\mathbf{x}}_{k k}^{i(\ell_{\max})}$
15:	update the error covariance
	$\mathbf{P}_{k k} = \left[\mathbf{I} - \mathbf{K}_k^{(\ell_{\max})} \mathbf{H}_k^{(\ell_{\max})} \right] \mathbf{P}_{k k-1}$
16:	export \mathcal{M}_i -based SoC estimate $\widehat{\text{SoC}}_k^i = \hat{\mathbf{x}}_{k k,1}^i$
17:	end for
	<u>Estimation fusion</u>
18:	determine the probability π_k^i that the battery runs on \mathcal{M}_i for $i = 1, 2$ with $\sum_{i=1}^2 \pi_k^i = 1$
19:	for $i = 1$ to 2 do
20:	compute the initial weights
	$\mathbf{H}_k^i = \frac{\partial h^i}{\partial \mathbf{x}^i}(\hat{\mathbf{x}}_{k k-1}^i, u_k)$
	$S_k^i = \mathbf{H}_k^i \mathbf{P}_{k k-1}^i (\mathbf{H}_k^i)^\top + R^i$
	$\hat{y}_{k k-1}^i = h^i(\hat{\mathbf{x}}_{k k-1}^i, u_k)$
	$\tilde{y}_{k k-1}^i = y_k - \hat{y}_{k k-1}^i$
	$w_k^i = (2\pi)^{-\frac{n}{2}} (S_k^i)^{-\frac{1}{2}} \exp \left[-\frac{(\tilde{y}_{k k-1}^i)^2}{2S_k^i} \right]$
21:	end for
22:	compute the normalized weights
	$\mu_k^i = \frac{w_k^i \pi_k^i}{\sum_{j=1}^N w_k^j \pi_k^j}$ for $i = 1, 2$
23:	fuse the SoC estimates from \mathcal{M}_1 and \mathcal{M}_2
	$\widehat{\text{SoC}}_k = \sum_{i=1}^2 \widehat{\text{SoC}}_k^i \mu_i$
24:	until SoC estimation task ends

TABLE I: The MM-AdaSoC algorithm: Adaptive SoC estimation using multiple models.

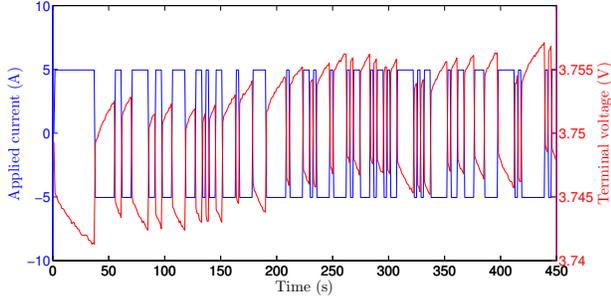
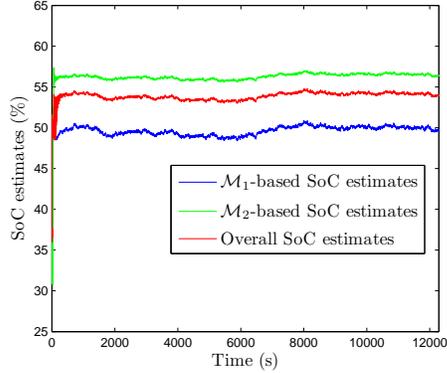
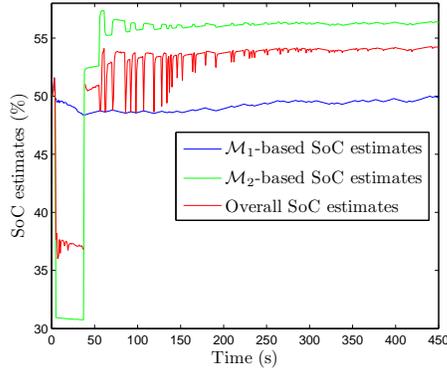


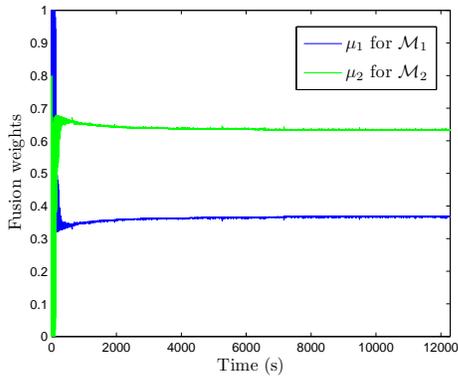
Fig. 2: The input-output (current-voltage) profile.



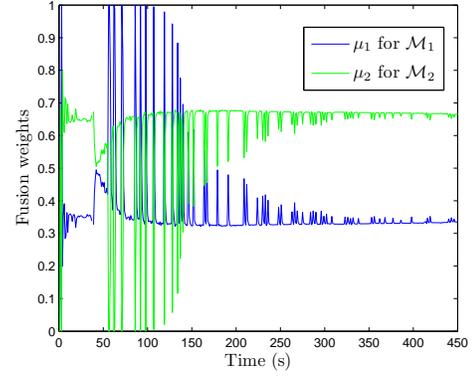
(a)



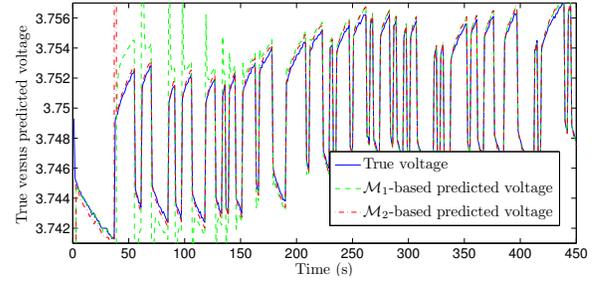
(b)



(c)



(d)



(e)

Fig. 2: (a) SoC estimates versus time; (b) SoC estimates during the initial 450s; (c) fusion weights for \mathcal{M}_1 and \mathcal{M}_2 versus time; (d) fusion weights during the initial 450s; (e) comparison between the true and the one-step-forward predicted voltage.

(85% for charging). From the SoC-OCV data collected from this type of batteries, it can be determined that $K_1 = 1.294$, $K_2 = 0.0984$, $K_3 = 3.972$, $\tau_1 = \tau_2 = 0.3$.

As aforementioned, the actual values of the parameters K_i for $i = 0, \dots, 3$ can change as a result of the operating conditions. Hence, rather than depending fully on their nominal values, we perform multi-model adaptive SoC estimation by applying the MM-AdaSoC algorithm. The construction of two submodels from (10)-(11) is described in Section III-A.

The SoC estimation results are shown in Fig. 2. The full view over the available experimental data is given in Fig. 3(a). The initial SoC of the battery is known to be approximately 50%. It is seen that there is a difference of approximately 5% between the \mathcal{M}_1 -based and \mathcal{M}_2 -based estimates. Based on our experience, \mathcal{M}_1 tends to yield conservative estimates in this case and \mathcal{M}_2 does the opposite. The MM-AdaSoC algorithm, through the fusion strategy, makes adjustment to give neutralized overall estimates. Although the true SoC data are not available, we still judge that the estimates are close to the truth, based on our *a priori* knowledge about the battery behavior. Fig. 3(b) illustrates what happens during the initial 450s. It is seen from Figs. 3(a)-3(b) that the overall estimates are closer to those based on \mathcal{M}_2 . This is verified in Figs. 3(c)-2(d), where the weight μ_1 for \mathcal{M}_1 fluctuates slightly around 0.63 and μ_2 around 0.37. Thus, with a larger weight, \mathcal{M}_2 is given

more confidence than \mathcal{M}_1 by the MM-AdaSoC algorithm during the implementation. It is understood that the fusion depends on the performance of one-step-forward prediction of the terminal voltage. Fig. 2(e) compares the measured data with the prediction based on \mathcal{M}_1 and \mathcal{M}_2 , respectively. The prediction is satisfactory for both submodels, but \mathcal{M}_2 is observed to lead to the better predicted voltage.

From the above results, we believe that the MM-AdaSoC algorithm is quite effective, supported by the findings that the obtained SoC estimates exhibit considerable accuracy and that the voltage prediction approximates the truth well.

VI. CONCLUSIONS

Development of adaptive approaches for SoC estimation is of practical significance, because battery dynamics are often hard to fully determine and are time-varying. Adaptive SoC estimation proposed in this paper uses a multi-model strategy, motivated by the proven success of multi-model estimation in addressing problems involving structural and parameter changes.

The main contribution of this paper is the development and validation of the MM-AdaSoC algorithm. It is built to estimate a battery's SoC in real time through carrying out simultaneous state and parameter estimation on a set of (sub)models. We first construct two submodels from a general state-space battery model by fixing different parameters, with both shown to be locally observable with admissible inputs. The well-known IEKF is then applied to each submodel to produce the SoC and parameter estimates. The final overall estimates are generated by fusing the submodel-based estimates, and it is shown that the fusion is a linear weighted combination of the estimates. An experiment is presented to demonstrate and validate the effectiveness of the algorithm.

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