Team-Level Properties for Haptic Human-Swarm Interactions*

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Abstract— **This paper explores how haptic interfaces should be designed to enable effective human-swarm interactions. When a single operator is interacting with a team of mobile robots, there are certain properties of the team that may help the operator complete the task at hand if these properties were fed back via haptics. However, not all team-level properties may be particularly well-suited for haptic feedback. In this paper, characteristics that make a property of a multi-agent system appropriate for haptic feedback are defined. The focus here is on leader-follower networks, in which one robot, the so-called leader, is controlled via an operator with a haptic device, whereas the remaining robots, the so-called followers, are tasked with maintaining distances between one another. Multi-agent manipulability, a property which describes how effective the leader is at controlling the movement of the followers, is proposed as one such appropriate property for haptic feedback in a human-swarm interaction scenario. Manipulability feedback is implemented using a PHANTOM Omni haptic joystick and experiments in which a team of mobile robots is controlled via a human operator with access to this feedback show that this is viable in practice.**

I. INTRODUCTION

The area of multi-agent robotics has matured to the point where there is an extensive literature revolving around control laws for coordination in order to complete tasks including, but not limited to, rendezvous [1], area coverage [2], and formation control [3], [4]. Typically, these control laws are autonomous and do not include human input in the loop. In many applications, these multi-agent teams are operating in unknown environments where human intervention may be necessary, and thus it is important to understand how human input can be combined with autonomous control laws to effectively orchestrate the movement of a team of mobile robots. This growing field of research is known as human-swarm interaction.

One of the desirable properties in human-swarm interaction is that the human is able to receive information about the team of robots in a clear and simple manner. In much of the literature, the human is fed back information about the state of the robot team through the visual channel [5], [6], [7], [8], [9]. In [10], the visual feedback is combined with audio feedback in order to allow the user to gather more information about the group of robots at any instant.

One method of relaying this team-level information through a channel other than the visual and audio ones is through the use of haptics. Haptics are often used in virtual reality to provide the sense of touch to the user. Haptics also provide a promising way to aid in human-swarm interaction by relaying information about the swarm to the user via force feedback. Some prior work has been done in which the haptic channel was shown to be successful in helping a robot team navigate an environment filled with obstacles [11], [12], [13]. In much of the literature on haptic-enabled human-swarm interactions, the feedback given to the user contains information about the swarm's surrounding environment. For example, in [14] and [15], the feedback force generated by the haptic device comes from averaging the obstacle force across all the agents as to encourage the operator to avoid obstacles.

Although avoiding obstacles is important, the operator may be interested in knowing how effective the injected control inputs are at controlling the team of robots. Manipulability, a notion used classically in robotics to describe how effectively input joint angle velocities translate into end-effector velocities for robot-arm manipulators ([16], [17], [18]), was adapted in [19] for leader-follower networks of mobile agents. Manipulability describes how effectively the leader agent's velocity, which would be an input controlled by the human operator, translates to the velocities of the remaining agents, the so-called followers. In this paper, we will discuss why manipulability is a good choice for haptic feedback in human-swarm interaction and implement the feedback on a haptic device while the operator concurrently uses the device to control the leader of a team of mobile robots.

In Section II of this paper we discuss at a high level what it takes to turn a swarm-level property into a haptic feedback force and what makes a property well-suited for haptic feedback in a human-swarm interaction setting. In Section III, we give one such example of a well-suited property for haptic feedback, namely manipulability, and discuss how it can be mapped to a haptic feedback force. This was then implemented on a team of Khepera III Robots controlled by a human operator with a haptic device and the details of the implementation are in Section IV. The ideas presented in this paper are summarized in Section V.

II. HAPTIC SWARM CONTROL

In controlling a swarm with the assistance of a human operator, who provides inputs for the team, it would be useful for the operator to be aware of certain swarm-level properties that it can use to make decisions regarding the swarm's behavior. Haptic technology provides a way in which these properties can be relayed to the user via force feedback, while he or she is controlling the motion of the swarm. We

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investigate here what constitutes a haptic-appropriate swarm-level property and what would be needed in order to turn such a property into useful haptic forces.

For the purpose of this discussion, we assume that the human operator is controlling the velocity of the leader of the swarm and the velocities of the remaining agents, or the followers, are defined through pairwise distance-based interactions between neighboring agents, as is standard in much of the multi-robot literature, e.g., [2], [20]. What this means is that the follower agents are tasked with maintaining pairwise inter-robot distances. That is, if agents i and j are adjacent in the information-exchange network, they are tasked with holding the distance between them, $||x_i - x_j||$, to a desired, pre-specified, positive value d_{ij}. If a follower is adjacent to the leader, only the follower's dynamics will strive to maintain the distance between the two agents. This type of network is known as "leader-follower". The operator controls the leader's velocity through the use of a haptic device, which is the same device that generates the feedback forces acting upon the operator. This way only one device is being used by the operator and it eliminates the need for any intermediary senses. The operator should be able to apply the information given by the haptic device without having to think about it.

We first investigate what characteristics are needed by a swarm-level property for it to be an appropriate haptic feedback signal in this setting. It has been shown, for example, in [21] and [22], that when haptic delays are present, the person using the haptic device perceives the force feedback to be weaker than it is in actuality. In order for the operator to feel the forces with the strength that they were intended to have, the delay caused by the computation of the force feedback should be minimized. In order to minimize delay, the swarm-level property used for haptic feedback should be an instantaneous notion. That is, it should address instantaneous effects that the input, given by the human operator, has on the swarm of agents. This way, as soon as the operator changes the state of the swarm by moving the haptic device, a new haptic feedback force can be computed instantaneously, giving the operator instant feedback about whether that input motion was "good" or "bad".

All haptic devices are limited by the amount of force they can produce, so we need to map the value of the swarm-level property to an appropriate amount of force in the range that the device can produce. In order to generate forces that are easily distinguishable by the operator, it seems desirable to map the full range of the swarm-level property to the full range of the haptic device. In order to do so, the swarm-level property needs to have both a maximum and minimum value, known a priori so that the mapping can be be defined ahead of time and remain constant throughout the human-swarm interaction task. The swarm-level property should also be continuous as to not cause discontinuities in the haptic feedback force and so that the mapping from the property to the feedback force is straight-forward.

In order for the haptic feedback to be useful, the operator needs to know *how* to use the information being relayed to him or her. Therefore, the properties being used for haptic feedback should also be beneficial to the user in completing the task at hand. For example, haptic feedback indicating obstacles in an environment would be useful to an operator who is tasked with moving a swarm through an environment without colliding with obstacles. If the task were different, this type of feedback may not be as useful.

In addition to being useful, the haptic feedback needs to be forceful enough so that it can successfully influence the user's decisions. If we wish to impede the user from moving the leader of the swarm in a certain direction, the force needs to be strong enough to overcome the force that the operator is applying to the device, or at the very least be strong enough for the operator to notice the resistance. Device limitations aside, this is a matter of picking an appropriate mapping between the swarm-level property and the haptic force.

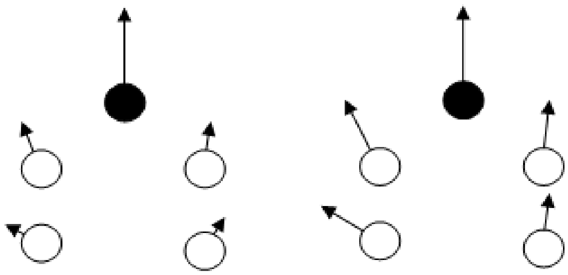
If the properties discussed in this section are met, haptics can be effectively used to assist a human operator in controlling a swarm of mobile agents, by allowing the operator to be informed about the state of the swarm as a whole. In the next section we will give an example of a swarm-level property that fits our needs.

III. HAPTIC MANIPULABILITY

When controlling a team of robots via external inputs, it is desirable to know how the inputs are affecting the team's behavior. In some situations, it is desirable to have all of the agents moving as one collective unit, with each robot having the same velocity. Instead of broadcasting the signal to all robots, we control the movement of one robot directly and use local interaction laws to control the motion of the followers, as described in the previous section as leader-follower control. Manipulability describes how effectively the follower agents are being controlled by the leaders at any point in time. As opposed to controllability, which is a property that describes between which states the system can be in, manipulability is an instantaneous notion, and thus fits our needs for haptic feedback.

In order to define manipulability explicitly, we first need to introduce some notation. Consider a swarm of N mobile robots, consisting of N_f followers and N_l leaders, where $N_f + N_l = N$. At time t, robot i's position is given by $x_i(t) \in \mathbb{R}^d,\ i = 1, \dots, N$, where d is the spatial dimension of the network, e.g., $d = 2$ corresponds to the case of planar robots, $d = 3$ represents agents that move in a three dimensional space, and so forth. The positions are aggregated together to give the overall position of the robot team at time t, $x(t) = [x_1^T(t), \dots, x_N^T(t)]^T \in \mathbb{R}^{Nd}$. We assume that the indexing of the agents is such that the first N_f agents are the followers, and the last N_l agents are the leaders. Under this indexing scheme, $x(t) = [x_f^T(t), x_l^T(t)]^T$, where $x_f(t) = [x_1^T(t), \dots, x_{N_f}^T(t)]^T \in \mathbb{R}^{N_f d}$ and $x_l(t) = [x_{N_f+1}^T(t), \dots, x_N^T(t)]^T \in \mathbb{R}^{N_l d}$.

Now we can introduce the formal definition of manipulability, which is formulated as the ratio between the leaders' and the followers' velocities, i.e.,



(a) Lower Manipulability (b) Higher Manipulability

Fig. 1. Manipulability comparison between two leader-follower multi-robot networks ($N_l = 1$). The filled circle in each network is the leader and the arrows represent the agents' velocities. Here, the network on the left has a lower manipulability than the network on the right due to the follower velocities being smaller in magnitude. In this case, the difference in the follower velocities between the two networks and thus the difference in manipulability would be caused by a difference in interaction topology (not shown).

$$M = \frac{\|\dot{x}_f\|^2}{\|\dot{x}_l\|^2}. \tag{1}$$

For example, if you have two multi-agent networks with the same velocity applied to the leader in each, the larger the magnitude of the followers' velocities, the higher the manipulability index. This is illustrated in Figure 1, where the robot configuration in 1(a) has a lower manipulability than that in 1(b). Here, since the leaders in each network have the same velocity, the difference in the follower velocities is due to a difference in interaction topology between the two networks. If the two networks had the same interaction topology, differences in manipulability would be caused by different leader velocities.

Since we want to use this manipulability index for haptic feedback, it is important to understand what all it depends on. It is clearly a function of $\dot{x}_l$, because that is the control input directly specified by the user. It also depends on where each of the robots are in space, x, as well as the structure of the multi-agent network, i.e., which robot pairs would like to maintain inter-robot distances. In this leader-follower network, the control law of the followers is designed so that adjacent agents maintain desired distances between each other. If we let $V = 1,\ldots,N$ denote the set of agents, then we can define the unordered set $E \subset V \times V$ to contain the robot pairs that are adjacent in the underlying network. By combining the *vertex* set V with the *edge* set E, we form the undirected graph $G = (V,E)$, which defines the information exchange network of the multi-agent team of robots. The manipulability index given in Equation (1) also depends on this graph.

We know that M in Equation (1) depends on $\dot{x}_l$, x, and G, but in order to compute it, we also need to know $\dot{x}_f$. However, the followers' velocities depend on the choice of interaction dynamics. In [23], this was remedied by developing an approximate manipulability measure that does not depend on the interaction dynamics, i.e.,

$$\tilde{M} \approx \frac{\|\dot{x}_f\|^2}{\|\dot{x}_l\|^2}.$$

Here we recall the construction of this approximate manipulability index from [23].

The approximate manipulability measure was derived by using the rigid-link approximation. The rigid-link approximation assumes that the desired distances $\{d_{ij}\}_{(v_i,v_j)\in E}$ between connected agents are perfectly maintained by the followers at all times. In other words, $\|x_i(t) - x_j(t)\| = d_{ij}, \forall (v_i,v_j) \in E, t \geq 0$. Under the rigid link approximation, the distances between connected agents do not change over time. Given smooth and differentiable trajectories of $x_i(t)$, this means that

$$\frac{d}{dt}\|x_i(t) - x_j(t)\|^2 = 0,\ \forall (v_i,v_j) \in E,\ t \geq 0,$$

which expands to

$$(x_i - x_j)^T(\dot{x}_i - \dot{x}_j) = 0,\ \forall (v_i,v_j) \in E, \tag{2}$$

where, for the sake of notational simplicity, we have dropped the dependence on t.

Using (2), the rigid-link approximation condition can be written in matrix form as

$$R(x)\dot{x} = 0,$$

where $R(x) \in \mathbb{R}^{|E|\times Nd}$ is the so-called *rigidity matrix* of the system, and where $|E|$ is the cardinality of the edge set. Or, if we split this into the parts contributed by the leaders and the followers,

$$R(x,G)\begin{bmatrix} \dot{x}_f \\ \dot{x}_l \end{bmatrix} = \left[R_f(x,G) | R_l(x,G)\right]\begin{bmatrix} \dot{x}_f \\ \dot{x}_l \end{bmatrix} = 0,$$

where $R_f \in \mathbb{R}^{|E|\times N_f d}$ and $R_l \in \mathbb{R}^{|E|\times N_l d}$. This allows the follower velocities to be directly expressed as a function of the leader velocities, as

$$\dot{x}_f = -R_f^{\dagger}(x,G)R_l(x,G)\dot{x}_l, \tag{3}$$

where $R_f^{\dagger}$ is the Moore-Penrose pseudoinverse of R_f.

The approximate manipulability measure was derived using this relation in Equation (3) and is given by

$$\tilde{M}(x,\dot{x}_l,G) = \frac{\dot{x}_l^T J^T(x,G)J(x,G)\dot{x}_l}{\|\dot{x}_l\|^2}, \tag{4}$$

where $J(x,G) = -R_f^{\dagger}(x,G)R_l(x,G)$. This expression for approximate manipulability can now be used to provide a human operator with haptic feedback about the instantaneous effectiveness of his or her control inputs.

This approximate manipulability measure fits all of the criteria for a well-suited haptic force discussed in the previous section. First of all, when $N_l = 1$, it was shown in [23] that

$$0 \leq \tilde{M} \leq N_f.$$

Because $\tilde{M}$ is the ratio of squared norms, it is clear that $\tilde{M} \geq 0$. As for the other side of the inequality, when $N_l = 1$, R_l can be expressed in terms of R_f as

$$R_l = -R_f \tilde{I}_f,$$

where $\tilde{I}_f = \mathbf{1}_{N_f} \otimes I_d$, where $\mathbf{1}_{N_f}$ is an N_f-dimensional column vector with 1s in all of its entries, $\otimes$ denotes the Kronecker product, and I_d denotes the $d \times d$ identity matrix. By substituting this R_l into Equation (3), we get

$$\dot{x}_f = -R_f^{\dagger} R_l \dot{x}_l = R_f^{\dagger} R_f (\mathbf{1}_{N_f} \otimes I_d)\dot{x}_l = R_f^{\dagger} R_f (\mathbf{1}_{N_f} \otimes \dot{x}_l).$$

Since $R_f^{\dagger} R_f$ is a projection matrix, we get $\|\dot{x}_f\|^2 \leq N_f \|\dot{x}_l\|^2$. Thus, the desired result,

$$\tilde{M} = \frac{\|\dot{x}_f\|^2}{\|\dot{x}_l\|^2} \leq N_f,$$

follows.

This means that approximate manipulability has a known minimum and maximum when there is one leader in the group. Since the case we are interested in is a single human operator controlling a single leader via a haptic device, this assumption is fine.

Manipulability describes how effectively the followers are being controlled by the leader, and a higher value of manipulability is better, so we want to use a haptic force that encourages the human operator to move the network of agents in directions that produce a high manipulability value. The question that remains is: how do we create a mapping from manipulability to haptic force? One option is to apply the force feedback in the direction opposite the one that the user is trying to move the leader in. In this case, the force would impede the motion of the user, so we map high manipulability to low force feedback and low manipulability to high force feedback. This way, if the user is moving the leader in a direction that produces a high manipulability of the swarm, little feedback force would resist the user's motion. Alternatively, if the user is moving the leader in a direction that produces low manipulability, the resistive force from the haptic device would encourage the user to choose a different direction in which to move the leader.

Using this type of force feedback, the approximate manipulability of the swarm and the haptic feedback are inversely related. There are many ways we can form an equation for the mapping. It seems reasonable that the maximum manipulability value should map to zero haptic feedback force, so the user does not experience any resistance when moving the leader in this desirable direction. Similarly, the minimum value of the approximate manipulability measure should map to the maximum force of the haptic device. Thus we need to map the range $[0, N_f]$ to the range $[0, H]$, where H is the maximum applicable force of the haptic device being used.

There are many ways that this mapping can be done and the simplest such way is through a linear mapping, given by

$$F_l(\tilde{M}) = H\left(1 - \frac{\tilde{M}}{N_f}\right), \tag{5}$$

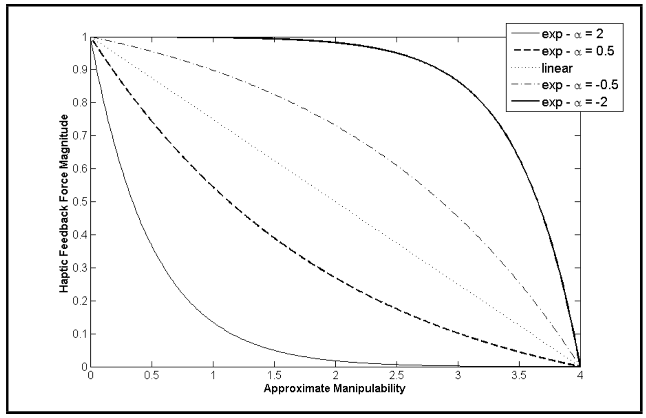


Fig. 2. Magnitude of the haptic feedback force vs approximate manipulability for five different mappings: One linear mapping and four exponential mappings (each with a different α parameter). Here, $H = 1$ and $N_f = 4$

where F_l is the magnitude of the haptic force meant to resist the user's motion, $\tilde{M}$ is the approximate manipulability of the team, and N_f is the number of followers in the swarm.

This linear map does not encourage high values of manipulability in a particularly powerful way. Another option is an inverse exponential map, given by

$$F_e(\tilde{M}) = H\,\frac{e^{-\alpha\tilde{M}} - e^{-\alpha N_f}}{1 - e^{-\alpha N_f}}. \tag{6}$$

Here, α is a parameter that can be changed to adjust the rate of change of the force as a function of manipulability. Figure 2 contains a plot showing the linear mapping and the exponential mapping for α values of -2, -0.5, 0.5, and 2. The plot was made using $N_f = 4$ and $H = 1$. It can be seen that as α becomes more negative, the resistive feedback force remains high for a larger range of manipulability values, indicating that the mapping should be more forceful in encouraging high values of manipulability.

The choice of mapping here goes back to the discussion in the previous section about the haptic force being powerful enough. Clearly, different choices of mappings will produce different levels of force feedback, and picking the "best" choice of mapping is not our intent of this paper.

Now that we know that approximate manipulability is an appropriate team-level property for use with haptic feedback, we will describe the experiments used as proof of concept for this method.

IV. EXPERIMENTS

We implemented haptic manipulability feedback as described previously using a haptic device known as the PHANTOM Omni by SensAble Technologies. The force feedback was rendered and applied to the device while a user controlled a swarm of robots under the leader-follower configuration. The user controlled the velocity of the single leader robot with the haptic device at the same time that the manipulability forces were being fed back through the device.

A network of five mobile agents was created using Khepera III differential drive robots. The user was instructed to

Fig. 3. Initial setup of the robot team, where the leader agent is the right-most robot. The two target locations are circled.

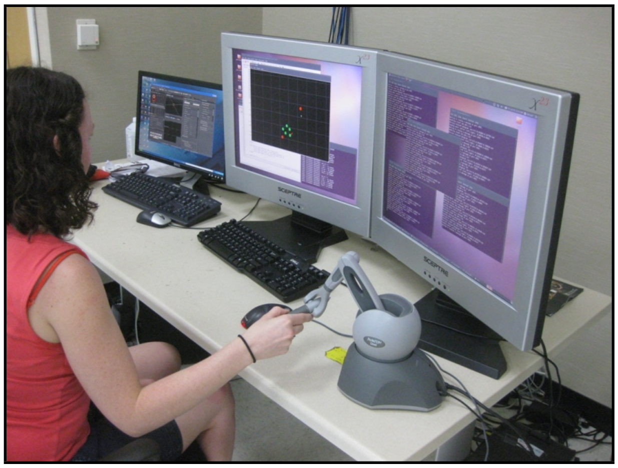


Fig. 4. Depicted is a student utilizing haptic device while looking at the virtual environment (middle screen).

move the group of robots to one target location and then to the other, in either order, using the haptic device to control the leader robot. Each target location was marked with an 'X' on the floor. The initial setup of the robots, along with the target locations, can be seen in Figure 3. In addition to the physical robots, a virtual environment was provided to the user to show the robots' locations, along with the target locations. Completion was defined by the leader being on top of the target locations. Figure 4 shows one of the students using the haptic device to control the swarm while looking at the virtual environment.

The network configuration chosen for this team of robots was a triangle formation with two extra robots on the ends, as shown in Figure 5. The lines in the diagram represent the edges, or the links that represent which agents are maintaining distances with each other. Here, if the agents are numbered 1 through 5 from left to right, agent 1 is only connected to agent 2 and agent 5 is only connected to agent 4. Therefore, the network is not rigid, meaning it does not necessarily stay in the same shape as shown in Figure 5 while it is being moved by the user.

The haptic force magnitude given by F_e and F_l in the previous section, after being computed using the instantaneous approximate manipulability of the system, is converted into a value that can be applied to the haptic device. Only the x and z degrees of freedom of the haptic device are used for

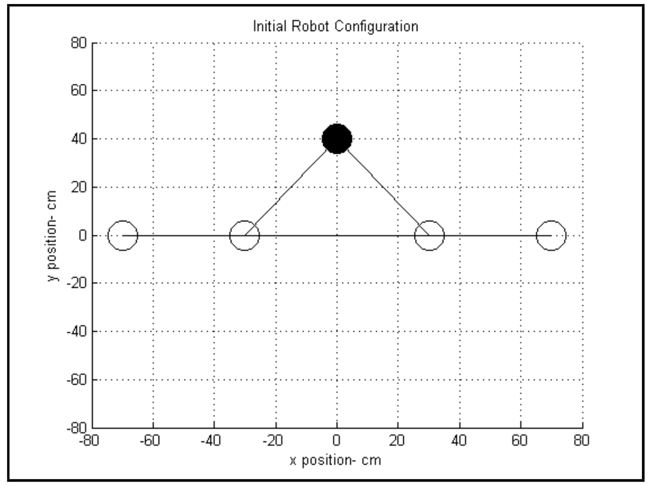


Fig. 5. Initial configuration of multi-robot team, showing the connections between agents. The filled circle represents the leader.

this experiment. When the subject uses the device to control the velocity of the leader, the x and z positions are captured by the program and converted into a velocity by computing the norm and the direction angle. Let x_{pos} and z_{pos} be the positions obtained from the haptic device. The magnitude and angle are computed as in Equations (7) and (8).

$$mag = \sqrt{z_{pos}^2 + x_{pos}^2} \tag{7}$$

$$\theta = \arctan(\frac{z_{pos}}{x_{pos}}) \tag{8}$$

After the magnitude of the haptic feedback force is computed, which is a value between 0 and H, it is used to scale the input velocity given by the user. Here we take H to be 1. This is a slight modification to the forces described in the previous section of the paper. This is done so that the magnitude of the feedback force generated by the haptic device can be at most the magnitude of the force exerted by the user onto the device. In other words, the haptic feedback force will only be as strong as the force exerted by the user onto the device while he or she is controlling the motion of the leader. In practice, this allows the force feedback to come on gradually and helps keep the haptic device from becoming unstable.

In addition, both the x and z components are negated so that the output force is in the opposite direction of the input velocity. Thus, as described previously, the force that the haptic device generates is a repulsive force that is intended to impede the motion of the haptic device in certain directions. Below are the equations used for the force rendering, where F represents the function that maps the approximate manipulability, $\tilde{M}$, of the swarm to the haptic force (either F_l or F_e from the previous section), and mag and θ come from Equations (7) and (8), respectively. The x and z haptic forces that are published to the PHANTOM Omni after being computed are given below as F_x and F_z, while the y component is always set to zero.

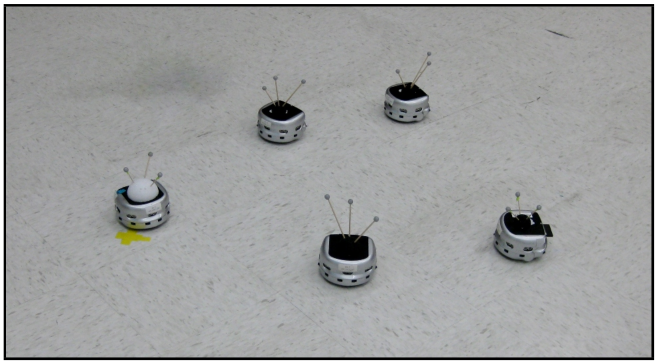


Fig. 6. Depicted is a team of Khepera III robots nearing a target location.

$$F_x = -\operatorname{sgn}(x_{pos})\,F(\tilde{M})\,mag\,|\cos(\theta)| \tag{9}$$

$$F_z = -\operatorname{sgn}(z_{pos})\,F(\tilde{M})\,mag\,|\sin(\theta)| \tag{10}$$

The haptic feedback described here was successfully implemented and the user was able to move the swarm of robots while feeling the resistive forces of the haptic device. The team of robots moving during one of the experiments can be seen in Figure 6, where the leader is the robot with the white styrofoam object on top of it and the 'X' on the floor marks one of the target locations.

V. CONCLUSIONS

In this paper, we discussed what characteristics are needed in a swarm-level property or metric in order for it to be appropriate for haptic force feedback in a human-swarm interaction scenario. The metric should have a maximum and minimum that are known a priori, it should be an instantaneous notion, and it should be useful to the human operator in completing the multi-robot task at hand. Manipulability was proposed as one such appropriate swarm-level property and mappings between approximate manipulability and haptic feedback force were defined. Experiments were done in which an operator used a haptic device to control the movement of a swarm while also experiencing haptic manipulability force feedback through the device, showing that this is viable in practice.

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