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Data-Driven Robust Optimal Allocation of Shared Parking Spaces Strategy Considering Uncertainty of Public Users' and Owners' Arrival and Departure: An Agent-Based Approach

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ABSTRACT To alleviate the problem between parking demand and supply, as well as improving urban traffic environment, shared parking has attracted great interest among researchers, policymakers, and entrepreneurs. Naturally, it is a prerequisite of sharing private parking spaces with public users (P-users) that all of the owners (O-users) providing parking spaces have space to park whenever they come back, with a unified management of all parking resources. However, it remains a challenge both in theory and practice. To solve this problem, firstly, we introduced a management framework of shared parking resource in terms of time and spatial dimension. Under this framework, to control the access to parking spaces of P-users, four phases (preparatory phase, open phase, releasing phase, and reconstructive phase) are divided in time dimension, and two types of parking spaces (the prestored parking spaces, and the shared parking spaces) are classified in spatial dimension. Then, based on the proposed management framework, an intelligent parking management system (IPMS) was developed to simulate the operation of shared parking considering the uncertainties of P-users' and O-users' arrival and departure. Furthermore, detailed sensitivity analysis, based on real-world data and simulations, evaluated the proposed framework and the developed IPMS in a case study concerning parking lots in Beijing, China. The results show that the IPMS can not only realize that there will always be enough available parking spaces satisfying O-users' parking demand, but also bring about vast improvements in both utilization and turnover rate of parking spaces, comparing with the non-shared management strategy.

INDEX TERMS Agent-based simulation optimization, shared parking, parking space allocation, traffic demand management and control.

I. INTRODUCTION

With the increasingly high ownership and usage of automatics, parking has been an important issue for both government management departments and public travellers, especially in metropolitans, around the world [1]–[3]. It is reported that today's vehicles are parked nearly 95% of time [4]. Moreover, because of limited parking resources, cruising for parking spaces costs additional time, money, and fuel. It momentously contributes to traffic congestion and environmental pollution. Statistically, it spends an average of approximately 8 minutes finding an available parking space [37]. And an average of 30% of traffic congestion is caused by searching parking spaces in the road traffic network in the major cities around the world [4], [5]. In Chicago, cruising for parking can generate 48,000 tons of CO_2 every year, which has been a contributing factor to creating greenhouse effect [6]. In this context, much has been studied concerning how to alleviate

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parking problem. As a result, some innovative approaches have been proposed from both parking demand and parking supply.

From the demand-side of parking, the majority of research concerns parking behavior and its influencing factors, aiming to reduce private vehicle usage in metropolitans to satisfy parking demand. An important means with long-term for this purpose is to enhance the constructions of public transportation system and launch a relatively lower price of public transportation than private vehicles to attract motorists. For example, Washbrook *et al.* [7] established a model considering the parking factor to estimate commuter mode choice behaviour based on a survey conducted in a Greater Vancouver suburb. Another effective and auspicious alternative is implementing traffic congestion pricing, regarding the parking as an adjunctive part of trips [8]–[11]. Recently, a major body of researches paid much attention to model traveler response to such policies. For example, Simićević *et al.* [12] developed a model to predict the effects of introducing or changing the parking price and time limitation. Qian and Rajagopal [13] explored travelers' parking behaviors through real-time data collected by smartphones and GPS navigation, and then made efficient parking pricing policies. He *et al.* [14] established two parking space choice models based on stated preference survey data considering various factors that affect shared parking choice behaviours.

From the perspective of parking supply, the conventional methods are to expand the scale of urban parking facilities to meet the increasing parking demand [15], [16]. However, it is not practical in large cities, especially in the heart of many cities, within a short period due to the limitation of space and financing channels. Hence, one possible way is to maximize the utilization of the existing parking resources [17]. Geng *et al.* [18] constructed an integer programming model for commercial parking resource allocation considering the generalized parking cost. Lin *et al.* [20] established a smart allocation algorithm whit the aim of maximizing the management profit while ensuring parking service quality.

Parking demand modes, like other transport demand modes, operate on a peak and off-peak schedule depending on related land use. Distinct but complementary patterns, such as ''residential parking'' that is generally empty in the daytime and on workdays, and ''office parking'', ''hospital parking'' that are generally fuller in the daytime, offer an opportunity for cities to better satisfy residents and commuters without increasing supply. Thus, shared parking was proposed [21], [22], [38], which uses existing gaps or spaces intended for parking vehicles when the owners are not using it, especially with the development of communication and information technology [2] and the rise of the sharing economy [23]. To this end, Guo *et al.* [24] regarded the process of sharing parking spaces as the repurchase and proposed a management revenue maximization model based on queue theory. It is worth noting that once the owners that provide shared parking spaces can get an amount of compensation payment if their parking spaces are still occupied by other users. But, in practice, owners of parking spaces are not willing to participate in sharing project because they might be confronted with the risk of cruising for parking spaces. Considering the utility loss of public travelers who failed to obtain shared parking spaces by reservation, Shao *et al.* [1] proposed a framework for sharing the use of residential parking spaces between residents and public users, assuming all the owners of parking spaces and public users arrive and depart on time. A binary integer linear model was proposed to calculate the maximum profit, which was tested by hypothetical data. However, the proposed shared parking model will be inapplicable with unpunctual users. Xiao and Xu [25] proposed a fair recurrent double VCG auction mechanism to avoid the potential owners of parking spaces and public uses opting out during sharing process. Babić *et al.* [26] evaluated three parking policies based on real-world data and simulations to meet electric vehicle charging demand and maximize parking resource management revenue.

With the development of computer simulation technology, Agent simulation has been succeeded in simulating the travellers' behaviour, such as routing choice [27], parking lot choice [19], [28] and so on. Zhao *et al.* [19] developed a parking management platform for multi-class users and multiclass parking spaces. Ni and Sun [28] analyzed the influence of intelligent parking reservation system on travel time. Therefore, Agent simulation technology provides us with an effective tool to reproduce complex behaviour.

Although there is much meaningful research on the allocation and revenue management of shared parking, to ensure that all owners sharing their parking spaces with public users have space to park remains an open problem.. It is quite important to make them be willing to participate in shared parking. In addition, the uncertainty of paring duration or other factors, in real life, makes it more difficult to achieve the above goal [29].

To solve this problem, we developed an intelligent parking management system (donated as IPMS) that can depict parking behaviours with the uncertainty of arrival and departure of both public users and the owners of shared parking resources. The IPMS can control the access to parking spaces both in time dimension by setting multiple phases and in spatial dimension by defining shared parking spaces and prestored parking spaces. Further, through a sufficient rounds of training based on historical parking behavior data, the optimal assignment of shared parking spaces, as well as prestored parking spaces, can be obtained considering stochastic parking demand.

The paper identifies three main categories of contribution:

1) A novel parking resource management framework is introduced in the context of shared parking in time and spatial dimension, with the aim of controlling access to parking spaces and ensuring that all of the owners' parking demand can be met.
2) An intelligent parking management system that automatically processes collected parking data and derives the optimal management strategy by iterative trainings

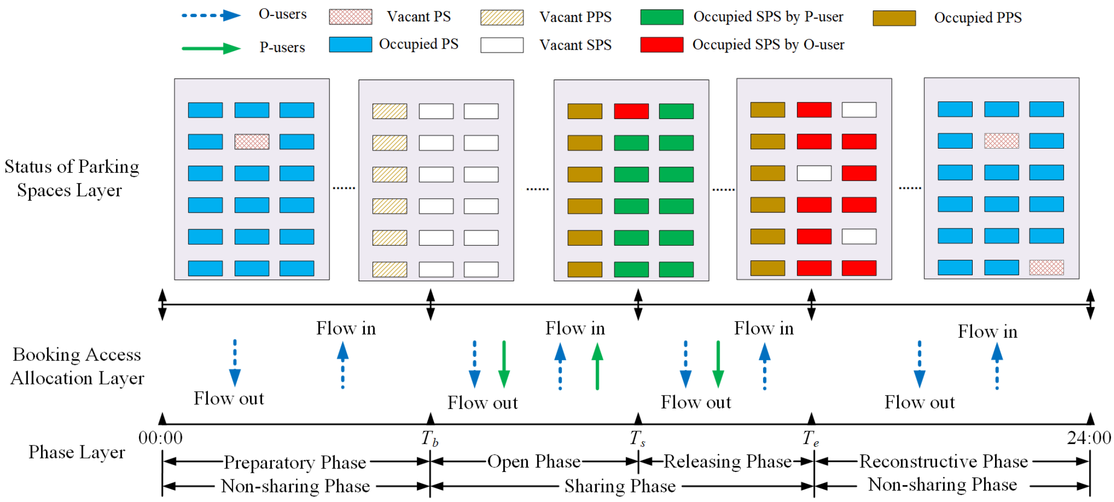


FIGURE 1. Evolution process of shared parking spaces within day.

is developed based on Agent simulation technology, in which the stochastic and dynamic characteristics of arrival and departure of O-users and P-users can be represented with the help of historical parking behavior data.

3) The optimal management strategies were obtained through a case study. We observed an interesting inverted S-shaped relationship between the number of optimal required prestored parking spaces and the number of P-users, which will be beneficial to efficiently manage and utilize parking resources.

The rest of this paper is organized as follows. Section II describes the operation mechanism of shared parking. Section III explains components of the IPMS which mainly includes four modules, namely data storage and update module, training data sampling module, training module, and optimal strategy output module. In Section IV, the validity and feasibility of the IPMS is verified by the case of sharing parking space in a residential area to parking demand from a hospital. Finally, conclusions and future research directions are in Section V.

II. DESIGNING OF SHARING EVOLUTION PROCESS

Before the implementation of shared parking, in general, only the owners have access to their private parking spaces. Hence, the fact is that when these residents drive their cars out to work or do other activities, their parking spaces will be idle. In this context, if (some of) the residents (denoted as O-users) are willing to share their parking spaces with other motorists (denoted as P-users) who need parking spaces, this can be a win-win strategy for both O-users and P-users, because O-users will benefit from the sharing, and P-suers can avoid cruising for parking [1], [30].

In order to effectively manage these shared parking resources, the use right of O-users' parking spaces is managed by the IPMS that processes parking space allocation and management based on the relevant information from O-users, P-users, and parking spaces. The IPMS works based on a staged and hierarchical framework of shared parking space, see Figure 1.

The proposed framework includes three layers and four phases. From the bottom to up, three layers are phase layer, booking access and allocation layer, and status of parking space layer, respectively. In the phase layer, the period of 24 hours within a day is divided into four phases: preparatory phase, open phase, releasing phase, and reconstructive phase, respectively. T_b, T_s, and T_e respectively present the beginning time point of the shared parking spaces, the timing point that ceases P-users entering the shared parking lot and the deadline of permissible parking for P-users. And the combination of open phase and releasing phase is named sharing phase. During the sharing phase, the P-users will have access to parking resources. During open phase, P-users can both enter and leave the shared parking lots. But during releasing phase, P-users can only leave the shared parking lots, namely entering is prohibited. For all O-users, they have access to parking spaces at any time within a day, which means that they can enter and leave at any time. This is the so-called control strategy in the time dimension. The access for O-users and P-users are shown in the booking access and allocation layer in Figure 1.

Besides, to meet O-users' stochastic parking demand, some prestored parking spaces (denoted as PPSs) among all parking spaces are defined in case parking spaces are occupied by excessive P-users, and these PPSs can only be used by O-users. Parking Spaces except PPSs are called the shared parking spaces (denoted as SPSs). This is the so-called control strategy in the spatial dimension since parking spaces are spatially independent of each other.

For all O-users, they need to submit their unique codes of the parking space that had been occupied by them to the IPMS when they leave. Of course, with the help of intelligent parking locks, the submission can be automatically processed. After receiving these codes, the IPMS changes the statue of corresponding parking spaces from "Occupied" to "Vacant" instantly. And when O-users come back, they need to request the codes of currently vacant parking spaces from the IPMS. To maximize the utilization of parking spaces, O-users are preferred to be assigned to PPSs when PPSs are available. By doing this, P-users can obtain as many opportunities as possible to utilize the SPSs. Otherwise, O-users are assigned to SPSs.

For P-users, only with successful reservations can they obtain the unique codes of vacant parking spaces during the Open Phase. The IPMS will automatically change the statue of parking spaces from "Vacant" to "Occupied". When leaving the parking spaces, P-users also need to submit the unique codes of parking spaces they used. After receiving these codes, the IPMS automatically change the statue of parking spaces from "Occupied" to "Vacant" instantly.

The main designed functions of IPMS are listed as follows:

1) The IPMS save the status data of parking spaces and control P-suers' access to parking spaces during different periods.
2) The IPMS can reproduce previous parking behaviors based on historical data, and train the allocation model, and obtain the optimal management strategy (more details in the next section).
3) The IPMS can decide to whether or not allocate parking spaces to P-users, according to real-time dynamic information about parking spaces.
4) The IPMS can guide all users, namely P-users and O-users, their assigned parking spaces by online guidance precisely according to their unique coding.

III. SYSTEM MODULE

In this section, we present an agent-based simulation approach for reproducing dynamic and stochastic characteristics of arrival and departure from both O-users and P-users the shared parking lot based on MATLAB environment [31].

The IPMS consists of four modules, which are illustrated in Figure 2. The first module is the data storage and update module by which the collected parking behaviour data will be preprocessed and stored. The second module is the training data sampling module, which is basic for the model training in the next step. The third module is the training module. By setting a sufficiently large number of training iterations, an optimal allocation strategy can be obtained. The last module is named output module, which outputs the optimal strategy generated in the previous training module. Because this function is relatively simple to implement, here we only explain functions of first three modules of the IPMS in detail.

A. DATA STORAGE AND UPDATE MODULE

As the first part of the IPMS, the data storage and update module has three important functions: collecting, preprocessing, and storing parking behaviour data. As a result, we can

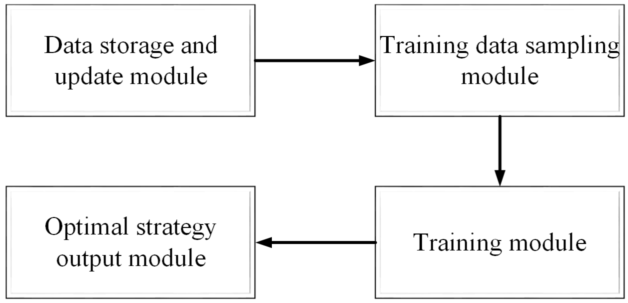


FIGURE 2. **Modules of the proposed IPMS.**

effectively reproduce both O-users' and P-users' stochastic characteristics of parking using random sampling technique, rather than defining the parking behaviors by standard deterministic mathematical models [1]. Thanks to intelligent parking locks, the status of parking spaces can be effectively monitored. And then, parking users' behaviour data and status of parking spaces data can be transmitted to the IPMS by advanced communication technology. The collected data items include Users' Types (namely, P-users, or O-users), Users' IDs (which is a desensitized information given the private-preserving scheme [32]), reservation time, parking start time, parking departure time, and so on.

However, due to system errors of sensors and other uncontrolled factors during data collection and transmission, the raw collected data can not be directly applied by the subsequent processes. It is necessary to clean and filter some invalid data.

Through the analysis of collected data, it is found that the invalid data can be divided into two categories: unreasonable parking duration time data and incomplete parking behavior data. For the first category of invalid data, it can be further divided into two subcategories: too short parking time data and too long parking duration time data. Because the collection time interval is 30 seconds, so we deservedly assorted the data of parking duration time less than 30 seconds as the first subcategory of invalid data. Since shared parking takes place on a daily basis, parking duration that exceeds 24 hours is defined here as the second subcategory of invalid data. In the follow-up analysis, parking user types, parking start time, and parking end time are three essential elements for a data item, therefore, we define the data item that missing any one of the three items as the second category of invalid data.

After deleting the invalid collected data, the valid data will be uploaded and used as the basic original sampling dataset for the sub-sequential sampling module. Details on the function of sampling module is discussed in later sections.

B. TRAINING DATA SAMPLING MODULE

In the basic original sampling dataset, each piece of information contains the stochastic behaviours of previous parking users that are naturally random and authentic. Thus effectively utilizing these samples is a feasible way of representing

Algorithm 1 Random Sampling Based on Mersenne Twister Algorithm

Input: $N_{P-users}$-Number of P-users' sampling;
$N_{O-users}$ -Number of O-users' sampling;
T_s - Start time of sampling time period;
T_e - End time of sampling time period;
$\Omega(P-users)$-Original dataset of P-users;
$\Omega(O-users)$-Original dataset of O-users.
1 Generate original sampling dataset of P-users: $\Omega'(P-users)$ from truncated $\Omega(P-users)$ during T_s to T_e;
2 Calculate the size of P-users' samples: $N(\Omega'(P-users))$;
3 Generate N($\Omega'(P-users)$) unequal random integers based on Mersenne Twister Algorithm;
4 Assign $N(\Omega'(P-users))$ unequal random integers to every sample in $\Omega'(P-users)$ as their IDs;
5 Sort $\Omega'(P-users)$ by increasing order of their IDs;
6 Truncate the first $N_{P-users}$ samples as training dataset of P-users: $\Omega''(P-users)$;
7 Generate original sampling dataset of O-users $\Omega'(O-users)$from truncated $\Omega(O-users)$ during during 00:00 to 24:00;
8 Calculate the size of O-users' samples: $N(\Omega'(O-users))$;
9 Generate $N(\Omega'(O-users))$ unequal random integers based on Mersenne Twister Algorithm;
10 Assign $N(\Omega'(O-users))$ unequal random integers to every sample in $\Omega'(O-users)$ as their IDs;
11 Sort $\Omega'(O-users)$ by increasing order of their IDs;
12 Truncate the first $N_{O-users}$ samples as training dataset of O-users: $\Omega''(O-users)$;
13 Combine $\Omega''(O-users)$ and $\Omega''(P-users)$ as training dataset: $\Omega''(O-users, P-users)$;
14 Update $\Omega''(O-users, P-users)$ by the increasing order of parking starting time of P-users and O-users.
Output: $\Omega''(O-users, P-users)$

the parking behaviours of both P-users and O-users. To this end, the key is to effectively sampling from the valid collected dataset.

Random sampling is a common technique that culls a smaller sample size from a larger population and use it to research and make generalizations about the larger group in real life. Because its fast generation of high-quality pseudorandom integer passes numerous statistical randomness tests, Mersenne Twister algorithm [33] has been widely used in many commercial software systems, such as Maple, MATLAB, Python, R, and so on.

To prevent over-sampling or under-sampling problems, we respectively sampled O-users' behavior data and P-users' behavior data, based on Mersenne Twister algorithm. After that, the combination of P-users' sample dataset and O-users' sample dataset is used as train data. The procedures of random sampling based on mersenne twister algorithm are as following **Algorithm 1**.

C. TRAINING MODULE

Using training dataset $\Omega''(O-users, P-users)$, training module aims to obtain the optimal control strategy $\pi^* = \{N_{O-users}, N_{O-users}, N^*_{PPSs}\}$ that includes through enough rounds of simulation for the shared parking spaces reservation and allocation process. During simulations, parking users' arrival and parking behaviour can be regarded as a queuing system [24], [34] in which parking spaces can be considered to be server agents; O-users and P-users can be viewed as customer agents. In addition, for real-time (first-come-first-serve) parking resource management, the IPMS only needs to update idle and occupied information on parking spaces for specific time periods, a P-user can park his/her car if available upon arrival, without advanced booking through the platform.

Due to the stochastic scenario or behavior of O-users' and P-users' arriving and departure, it is somewhat difficult to solve analytical results [24]. Here, multi-agent-based shared parking spaces simulation system is developed based on discrete-event simulation toolbox of MATLAB R2019b [35], which is widely used in data science, computational mathematics, engineering, and so on, to present discrete dynamic parking behaviours. The interface of multi-agent-based simulation is shown in Figure 3. The system consists of six types of blocks. The unidirectional arrows represent the transitive direction of data flow.

Using multi-agent-based shared parking spaces simulation system, the optimal control strategy $\pi^* = \{N_{O-users}, N_{O-users}, N^*_{PPSs}\}$ that means the N^*_{PPSs} with a given number of P-users and O-users can be obtained by a large enough number of rounds of training simulations, similar to the method in Reference [36].

Figure 4 illustrates the iterative processes for obtaining the optimal control strategies. At the beginning of simulation, several initial parameters should be set:

- N^*_{PPSs}: The optimal required PPSs;
- $N_{PPSs} = 0$:The number of PPSs;
- $N_{ite} = 0$:The number of current iterations;
- $i = 0$:The number of samples;
- Ω''(O-users, P-users) :The sample dataset for training;
- N_{max_ite} :Max number of iteration for training tests with the optimal strategy;
- $N_{Re-O-users} = 0$:The number of O-users without parking spaces after their returns.

IV. CASE STUDY

In this section, we present some simulation results to illustrate the essential ideas in the paper.

A. SCOPE OF STUDY

The empirical data we used here was collected by smart parking systems of ETCP company that is the largest intelligent parking company. The P-users' parking behaviour data was collected in Second Artillery General Hospital in Beijing, China, where physicians and patients usually cruise for parking spaces. The O-users' parking bahaviours data was

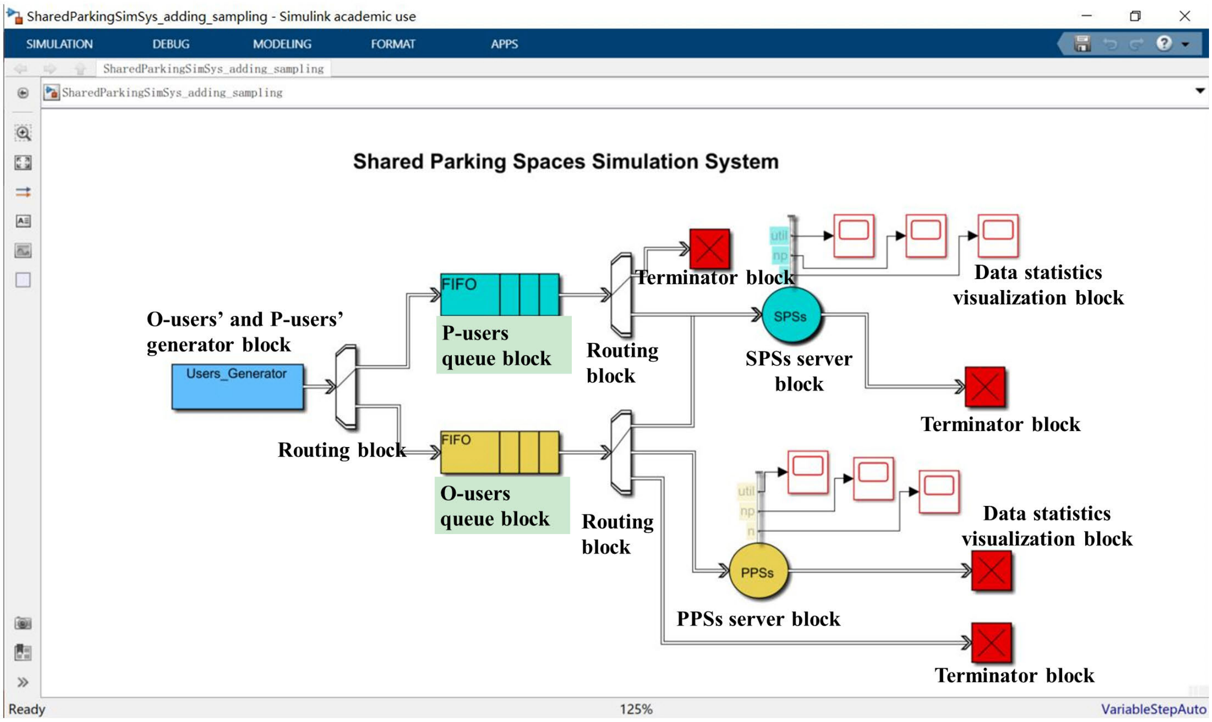


FIGURE 3. Interface of multi-agent-based shared parking spaces simulation system.

collected in a community located at Xinjiekou Outer street, A No.25, Beijing, China. The distance between the hospital and the community is 300 meters, which is within acceptable range of walking distance. The totally valid sample sizes of P-users and O-users are 1,958 and 1,640 respectively, accounting for 96.8% and 98.1% of the collected raw data. Figure 5 presents the characteristics of idle parking spaces of O-users. Figure 6 shows the characteristics of parking demand from P-users.

The distributions of probability of P-users and O-users at different times are shown at the top and right of each graph. The upper bar graph represents the proportion of users whose start time corresponds to the horizontal axis at the bottom. The right bar graph represents the proportion of users whose end time corresponds to the left vertical axis.

From the top bar graph in Figure 5, it can be seen that the parking supply period concentrated between 6 a.m. and 24 p.m. By the same way, we can find the parking demand concentrated between 6 a.m. and 23 p.m in Figure 6. This implies that it is feasible to satisfy some of the parking demand by sharing these idle parking spaces in the residential areas. Furthermore, no P-users arrived after 9 p.m., and the end parking time of P-users was no later than 23 p.m.

In order to quantify distributions of probability of P-users and O-users at different times, the mixed Gaussian distribution models are adopted. The fitting results are shown in Figure 7. The probability shown by solid blue line equals to the sum of the probability represented by solid red line and solid green line. The R-squares of four fitting models are greater than 0.96, which indicated that the fitting results are reliable.

B. PARAMETER DESIGN

To evaluate the proposed framework and simulation model, numerical experiments are conducted based on the empirical supply and demand data. Table 1 summaries the parameters' design. Totally 120 residential parking spaces are used to share with P-users in the tested simulation environment. When P-users and O-users swipe their smart cards for both check-in and check-out, all necessary information, which will be applied in the simulation, such as user types, arrival time, and departure time, can be recorded through the ETCP parking information collection system.

Given that residents go out regularly during working days, and can provide effective parking resources for sharing, we here only focused on the shared parking during working days. The collected data from Monday, April 2, 2018, to Friday, April 6, 2018. Based on the previous analysis of Figure 5 and Figure 6, the simulation parameters are set as in Table 1.

Sensitivity analysis is the study to measure the impacts of fluctuations in parameters of the IPMS on the optimal control strategies. To this end, ten different levels of $N_{P-users}$, which means the total daily parking demand of P-users, are defined. An estimated maximum total daily parking demand of this hospital is 1,800, which includes medical staff, patients, visiting groups, according to "Beijing's trip generation manual". Some of them can use the parking spaces in the hospital

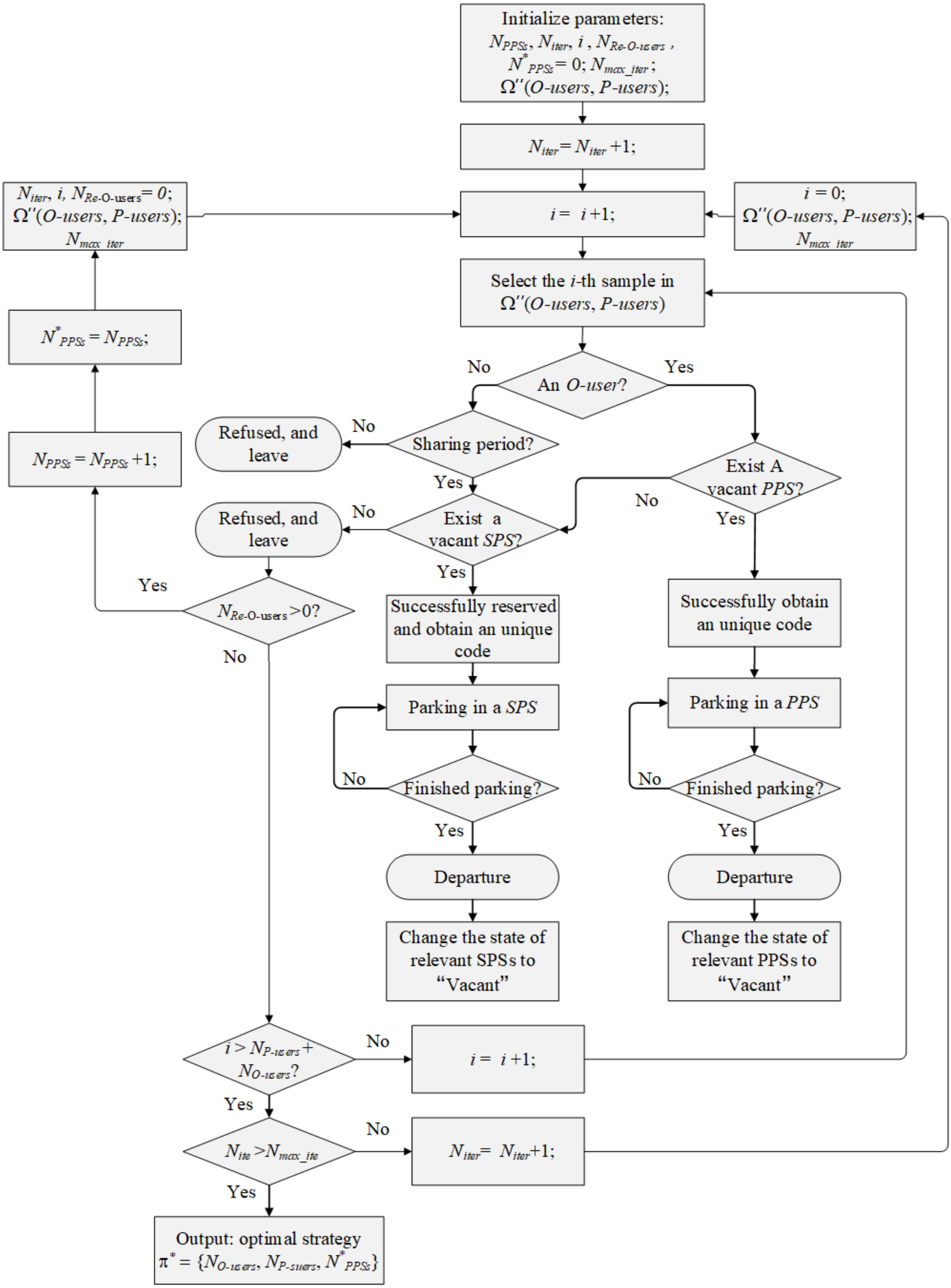


FIGURE 4. Logic flowchart of shared parking spaces simulation system operation.

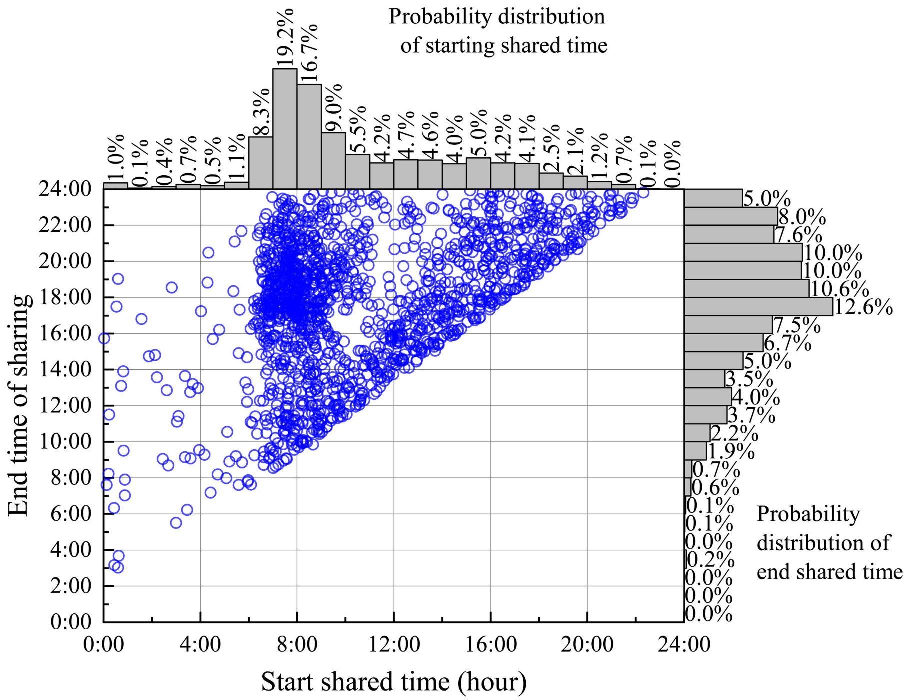


FIGURE 5. Characteristics of idle parking spaces from O-users.

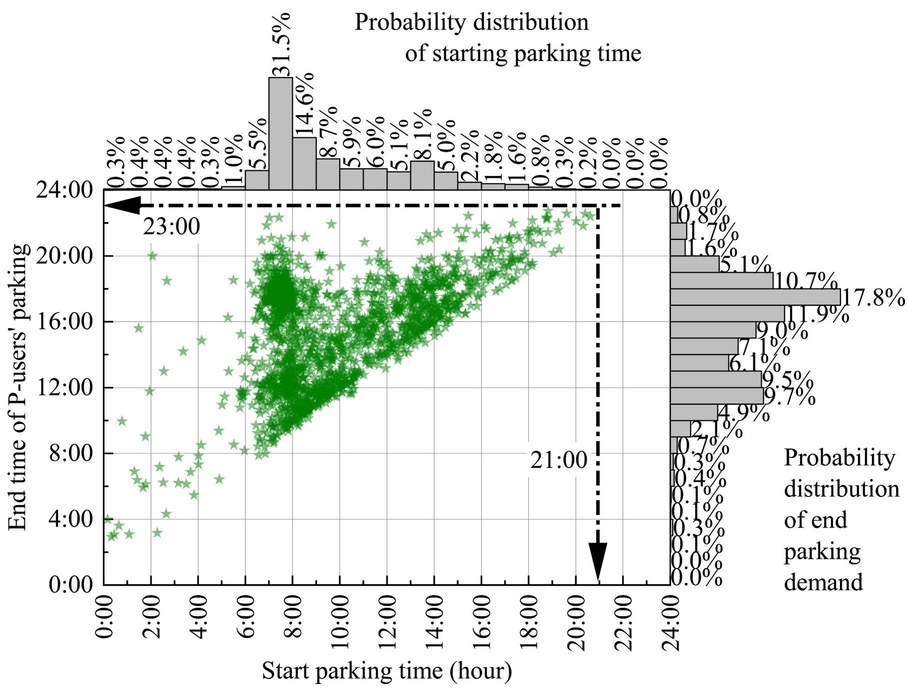
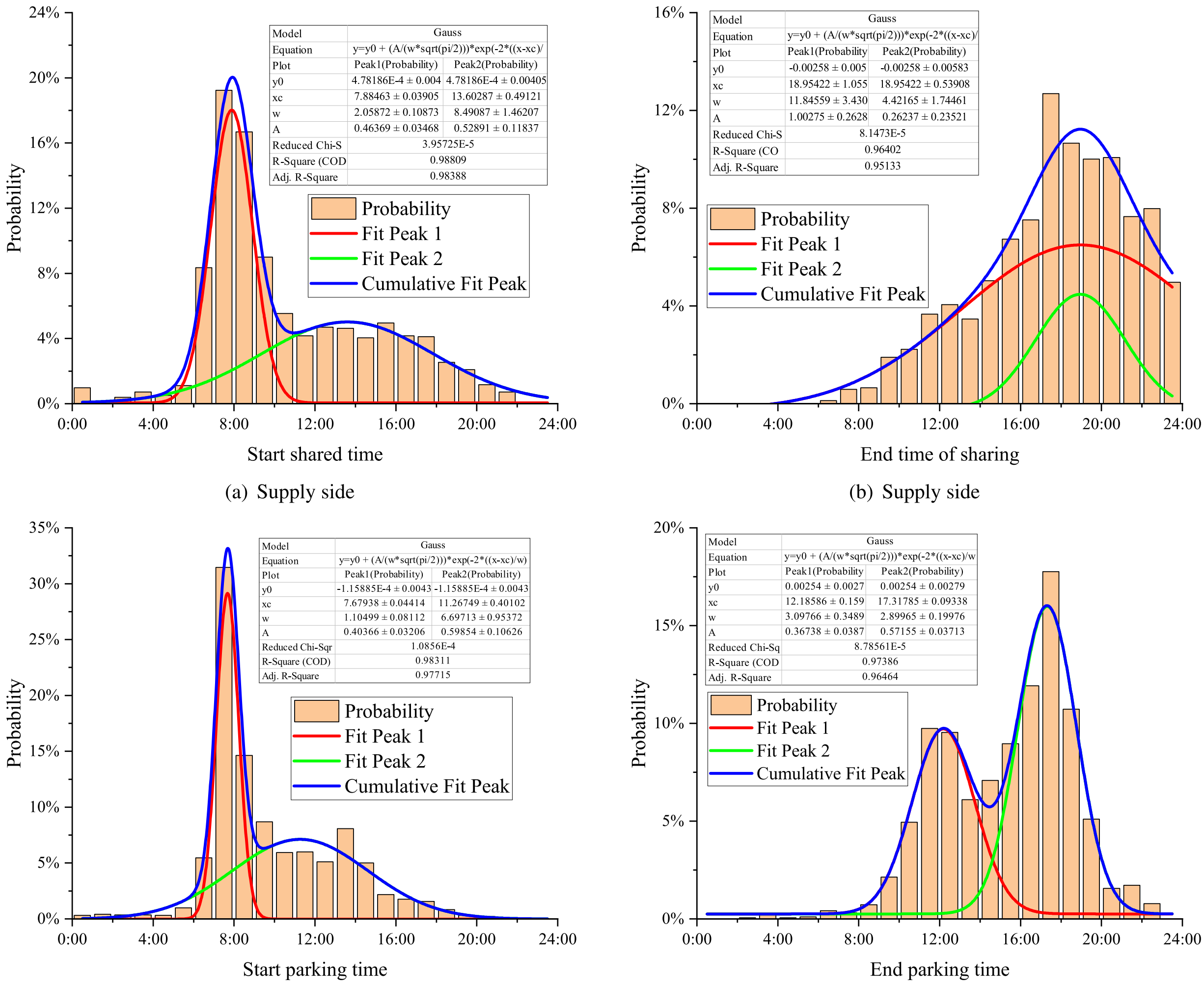


FIGURE 6. Characteristics of parking from P-users.

yards, the maximum of travelers without parking spaces is appropriately 650. Hence, the highest value of $N_{P-users}$ with 1,000 is enough to represent the parking demand from this hospital.

C. RESULT ANALYSIS

Here, we explain the results of simulations in details in terms of the optimal control strategy, total parking time, and turnover of the parking spaces.



(a) Supply side (b) Supply side

(c) Demand side (d) Demand side

FIGURE 7. Fitted distributions of probability of P-users and O-users at different times.

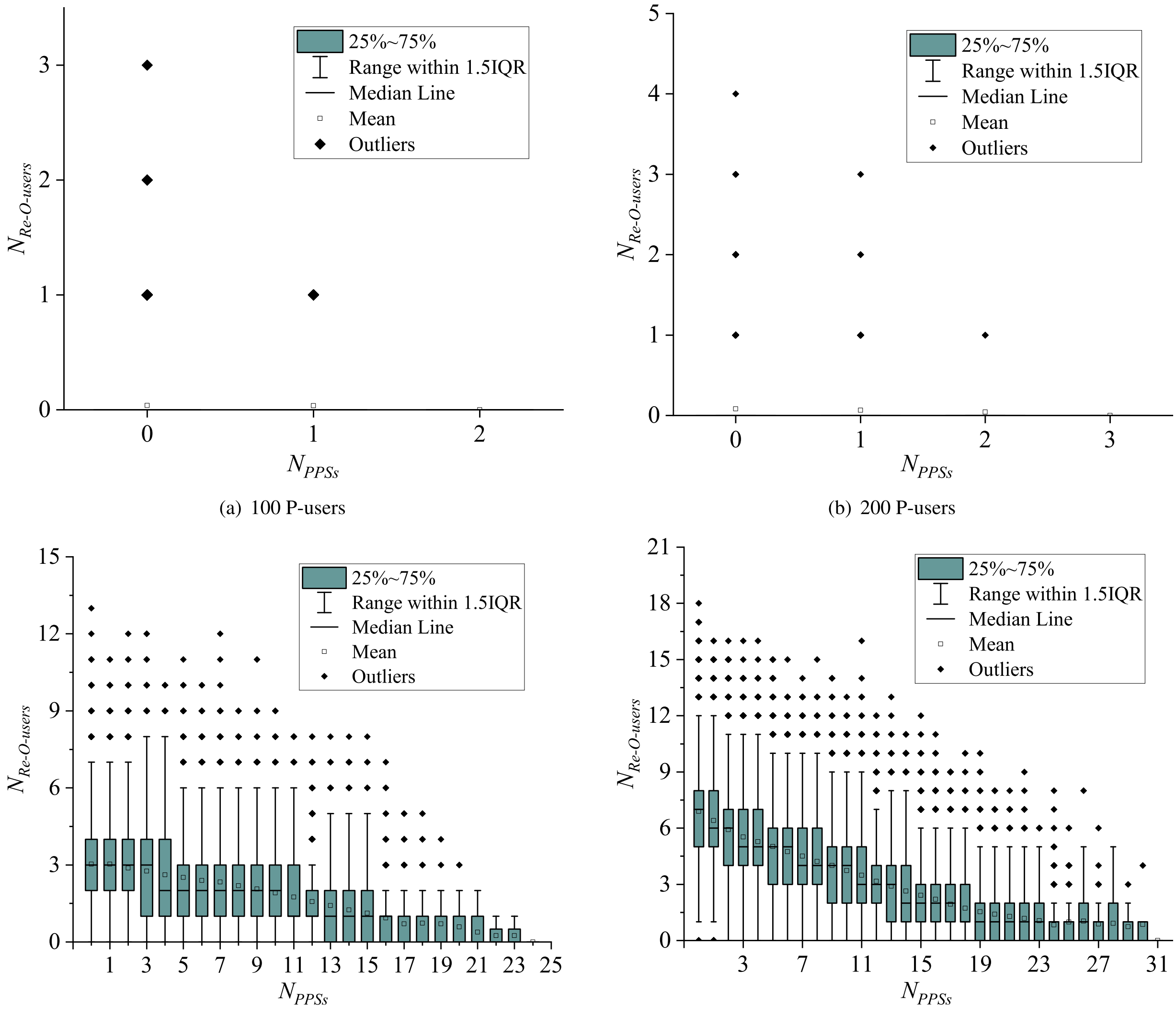
1) THE OPTIMAL CONTROL STRATEGY

Figure 8 shows the relationship between $N_{Re-P-users}$ and N_{PPSs}. On the whole, with certain $N_{P-users}$, $N_{Re-O-users}$ decreases with the increased N_{PPSs}. In addition, with given $N_{P-users}$ and given N_{PPSs}, $N_{Re-P-users}$ presents a distribution character, not a constant, which indicates that $N_{Re-P-users}$ was affected by stochastic parking characteristics in the simulation.

More specifically, the following conclusions can be drawn from Figure 8:

1) At a low public parking demand level, only a small N_{PPSs} will guarantee that there are enough parking spaces for O-users when they returned. As shown in Figure 8-(a) and Figure 8-(b), given with 100, and 200 P-users, the $N^*_{PPS_S}$s were merely 2 and 3, respectively. More than 96% of O-users' parking spaces can be shared with P-users.
2) At a medium public parking demand level, like $N_{P-users} = 600$, that six times N_{SPSs}, the maximum $N_{Re-O-users}$ went up to 13 with no PPSs equipped, as shown in Figure 8-(c). Moreover, the means of $N_{Re-O-users}$ went down gradually with increased N_{PPSs}. 24 PPSs were required to ensure all O-users have space to park, which was the optimal control strategy.
3) At a high public parking demand, like $N_{P-users} = 1000$, the maximum $N_{Re-O-users}$ increased to 18 if there is no PPSs for O-users. To guarantee all O-users have parking spaces, N^*_{PPSs} equals to 31, as shown in Figure 8-(d).

When the $N_{P-users}$ has doubled from 100 to 200, the N^*_{PPSs} increased by $(3-2)/2 \times 100\% = 50\%$ from 2 to 3. When the $N_{P-users}$ has tripled from 200 to 600, the N^*_{PPSs} has increased by $(24-3)/3 = 7$ times. When the $N_{P-users}$ increased to $(1000-600)/600 \times 100\% \approx 66.7\%$ from 600 to 1,000, the N^*_{PPSs} increased by $(31-24)/24 \times 100\% \approx 29.2\%$ from 24 to 31. Above analysis implies that there might be a little complex relationship between N^*_{PPSs} and $N_{P-users}$.



(a) 100 P-users (b) 200 P-users (c) 600 P-users (d) 1000 P-users

FIGURE 8. The relationship between $N_{Re-O-users}$ and N_{PPSs} under different demand levels.

For a given community, generally, the number of O-users' parking spaces remains stable in the short term. Under this condition, the relationship between $N_{P-users}$ and N^*_{PPSs} is represented in Figure 9. The horizontal and vertical axis represent the number of P-users and the optimal number of PPSs, respectively. The squares filled with green is the results of simulations. On the basis of these results, it can be concluded:

1) On the whole, the optimal number of PPSs increases with the increased number of P-users, therefore, there is a positive correlation between the number of P-users and the number of PPSs.
2) When the number of P-users is relatively small (less than 300), the number of PPSs increases slowly as the number of P-users increases. The reason for this is there are sufficient idle parking spaces that can be used by O-users when they come back.
3) As the number of P-users increasing, the number of PPSs increases faster than before. This is because the number of parking spaces occupied by P-users gets biger and biger. Hence, more parking spaces are assigned as PPSs to meet O-users' parking demand.
4) When the number of P-users is extremely large (more than 600), the growth rate of the number of PPSs slows down again. This means the use of parking resources achieve maximum efficiency, and excessive P-users cannot obtain parking spaces. Thus, there is no need to take much more SPSs as PPSs.

Given the positive relationship between the optimal number of PPSs and the number of P-users, two types of fitting models, namely a linear function and a non-linear model (Boltzmann model), are adopted to fit the curve of simulation results. The two fitting models can be represented as follows:

$$N^*_{PPSs} = A + B \times N_{P-users}, \tag{1}$$

$$N^*_{PPSs} = C + \frac{D - C}{1 + \exp\left(\frac{N_{P-users} - E}{F}\right)}, \tag{2}$$

TABLE 1. Parameters stetting in simulations.

Parameters	Values	Interpretation
T_b	6:00 a.m.	The starting open phase;
T_s	21:00 p.m.	The end time of open phase;
T_e	23:00 p.m.	The end time of sharing phase;
N	120	Total number of parking spaces, including PPSs and SPSs;
$N_{O-users}$	120	Number of O-users, and each of O-users provides one parking s-pace;
$N_{P-users}$	[100; 200; 300; 400; 500; 600; 700; 800; 900; 1000]	The number of P-users in each tested scenario;
N_{PPSs}	0	The initial number of PPSs, whihc increases until there are no O-users without parking spaces;
N_{SPSs}	120	The initial number of SPSs, which decreases as the number of PPSs increases;
N_{max_ite}	10, 000	The maximum number of simulations. Refer to [24], the total number of 10,000 rounds can be enough to guarantee the optimality of the results;

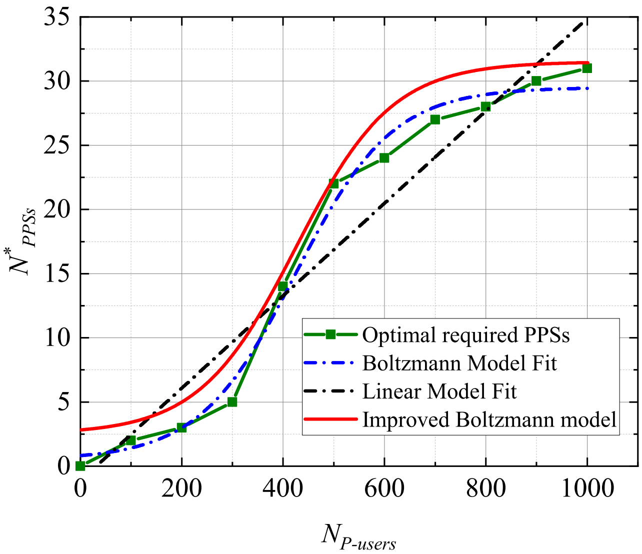


FIGURE 9. The relationship between N^*_{PPSs} and $N_{P-users}$.

where exp(·) is an exponential function; A, B, C, D, E, and F are constants, respectively.

The results of fitting are presented in Figure 9 and Table 2. The R-squares of linear fitting model and nonlinear fitting model are 0.93624 and 0.99101, respectively, which indicates that the Boltzmann fitting model is much better. Hence, to be precise, the inverse "S" relationship lies between the number of PPSs and the number of SPSs.

To further distinguish the difference between the fitting results of linear model and non-linear model, we firstly define linear residuals between fitted results and simulations, it can be formulated as following:

$$R^{linear}_{N_{P-users}} = N^{linear}_{PPSs,N_{P-users}} - N^*_{PPSs,N_{P-users}}, \quad (3)$$

$$R^{nonlinear}_{N_{P-users}} = N^{nonlinear}_{PPSs,N_{P-users}} - N^*_{PPSs,N_{P-users}}, \quad (4)$$

TABLE 2. Parameters of fitted models.

Models	Parameters	Values
Linear fit model	A	-1.12162 ± 1.85503
	B	0.036±0.00313
	Residual Sum of Squares	110.5233
	R-Square	**0.93624**
Boltzmann fit model	C	29.50976±0.85305
	D	0.49223±1.1854
	E	424.7231±16.76982
	F	95.16266±15.49138
	Reduced Sum of Squares	13.39382
	R-Square	**0.99101**
Improved Boltzmann fit model	C'	31.50976
	D'	0.49223
	E'	424.7231
	F'	95.16266

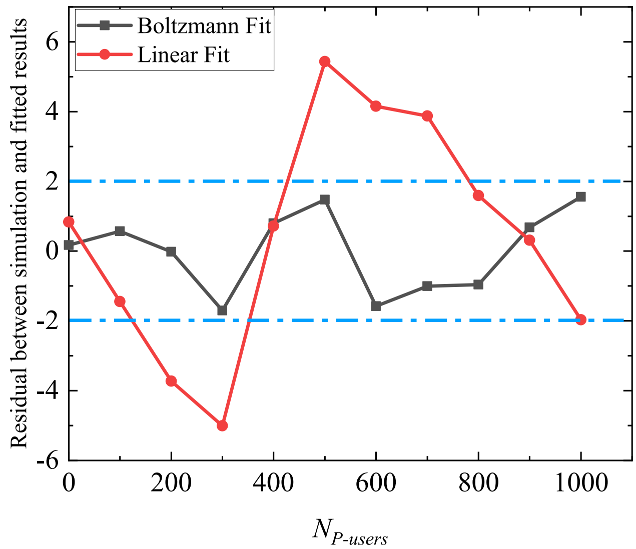


FIGURE 10. The residuals between fitted and simulated results.

where $R^{linear}_{N_{P-users}}$ is the residual between the result of linear fitted model and that of simulation with $N_{P-users}$ P-users; $R^{nonlinear}_{N_{P-users}}$ is the residual between the result of nonlinear fitted model and that of simulation with $N_{P-users}$ P-users.

Figure 10 represents the residuals between fitted and simulated results, which further proves that Boltzmann model outperforms linear model because of its smaller residual ranges than that of linear model.

In addition, it also can be seen that the maximum residuals between the results of Boltzmann fitting model and that of simulation are less than 2. As a result, an improved Boltzmann model can be obtained to improve the robustness of fitted model in practice by adjusting the parameter C, in Eq. 2 and keeping other parameters unchanged. The parameters of improved Boltzmann model are summarised in Table 2. The improved Boltzmann model is shown by the red solid line in Figure 9, it can be seen that all the simulation results of $N^*_{PPSs,N_{P-users}}$ below the curve of the improved Boltzmann model, which implies that the improved model has a strong robustness.

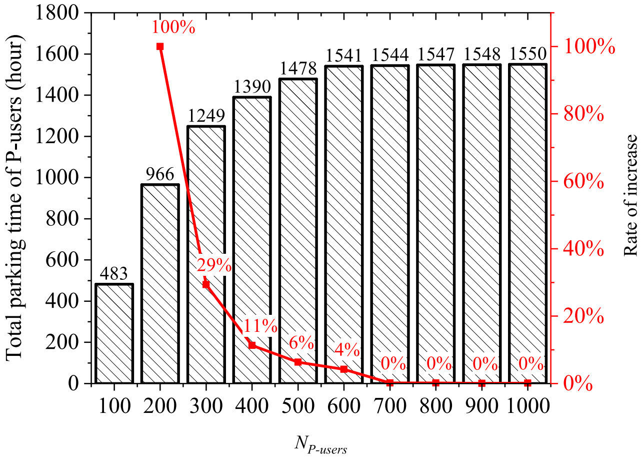


FIGURE 11. Total parking time under different demand levels.

2) TOTAL PARKING TIME OF P-USERS

The total parking time (TPT) of P-users was calculated as following:

$$TPT = \sum_{i=1}^{N_{P-users}} \Gamma_i x_i, \tag{5}$$

where Γ_i represents the i^{th} P-user's parking durartion; $x_i = \{0, 1\}$, if the i^{th} P-user obtained the parking space, $x_i = 1$; otherwise, $x_i = 0$.

Figure 11 shows the total parking time and rates of increase varies with the different configurations of parking demand. With the increase of parking demand, total parking time of P-users increased firstly with less than 700 P-users, and then nearly approached its peak of 1553 hours with 700 P-users, and remained stable with even much higher parking demand. Conversely,the rates of increase of total parking time decreased with the increase of parking demand when $N_{P-users}$ is less than 700, and it was is close to 0 with $N_{P-users}$ greater than 700, indicating that the shared parking spaces have been effectively utilized.

3) TURNOVER RATE OF PARKING SPACES

The turnover rate (TR) of parking resource is an important indicator to measure the utilization rate of parking resources. It was calculated as following:

$$TR = \frac{\sum_{i=1}^{N_{P-users}} x_i + \sum_{j=1}^{N_{O-users}} (N_{O-users}^j + 1)}{N}, \tag{6}$$

where $N_{O-users}^j$ represents the number of the j^{th} O-user's returns; $x_i = \{0, 1\}$, if the i^{th} P-user obtained the parking space, $x_i = 1$; otherwise, $x_i = 0$.

Figure 12 presents the change of turnover rate of all O-users' parking spaces before and after the implementation of shared parking. Before the implementation of parking space sharing, the average turnover rate of each parking space is approximately 2.5 vehicles/day, namely, the parking spaces merely can be used by the residents. With the increased

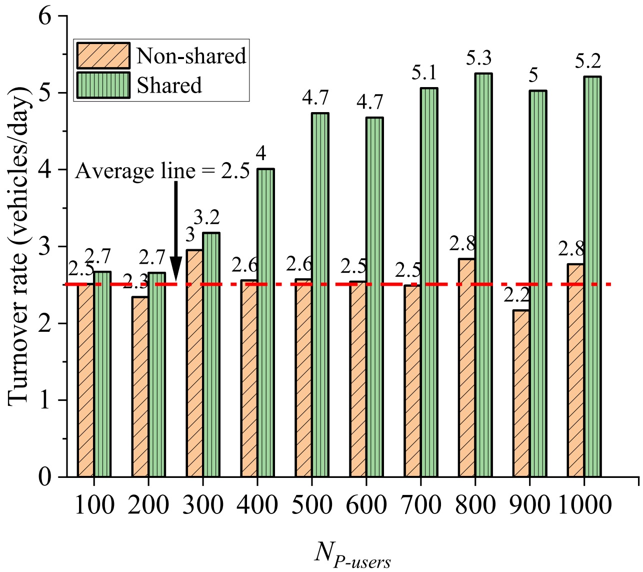


FIGURE 12. The comparison of turnover rates with and without shared parking.

$N_{P-users}$, the average turnover rate of these parking spaces has increased significantly through the implementation of shared parking. The average maximum turnover rate of each parking space was 5.2, with an average of 2.7 additional vehicles served by per parking space than without the implementation of shared parking. This shows that the efficiency of parking resources can be improved by the proposed shared parking mechanism.

V. CONCLUSION AND DISCUSSION

In order to alleviate the parking problem in cities, this paper proposed a novel shared parking resource management method. How to ensure that the parking demand of the owners of parking spaces is met during the sharing period critical to the successful implementation of shared parking. Aiming both to satisfy the parking demand of all owners of parking spaces and improving the utilization of shared parking resources while considering the uncertainty arrival and departure, we proposed a novel shared parking resource management framework. Under this framework, four phases with different accesses for different users were designed to achieve reasonable management of parking resources.

Due to the complexity of the problem, an intelligent parking management system (IPMS) was developed, regarding the reservation and allocation as a queue system, to simulate the operation of shared parking and obtain the optimal management strategy.

Further, detailed sensitivity analysis, based on real-world data and simulations, evaluates the proposed framework and IPMS of shared parking in a case study concerning parking lots in Beijing, China. The results show that the IPMS can not only obtain the optimal management strategy but also bring about vast improvements in the efficiency of parking spaces.

There are also several improvements that can be made to enhance the sharing strategy in future work. For example, we just tested a single scale of parking spaces with 120 and adopted the one week data to characterize the stochastic nature of public users' and owners' arriving/departure time for model simplicity. Other scales of parking spaces and different distribution models based on the empirical data for more accurate mathematical models and solutions can be considered and conducted. Further, the optimal design of sharing parking spaces in the mixed land-use area can be developed, due to their distinct parking demands, including work, office, commercial leisure and so on. Last but not least, since shared parking is a typical economic activity [1], [24], [39]–[41], in future research, to build a model that takes some essential economic factors and other uncertainty factors into account in shared parking seems a fruitful topic for future research work.

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